Introduction to

Parton Distribution Functions

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Outline:

- Introduction to Parton Distribution Functions (PDFs)
- PDF determination
  - theory and experimental input
  - methodology
- Overview of available PDFs
- Tools for the PDF determination

Introduction

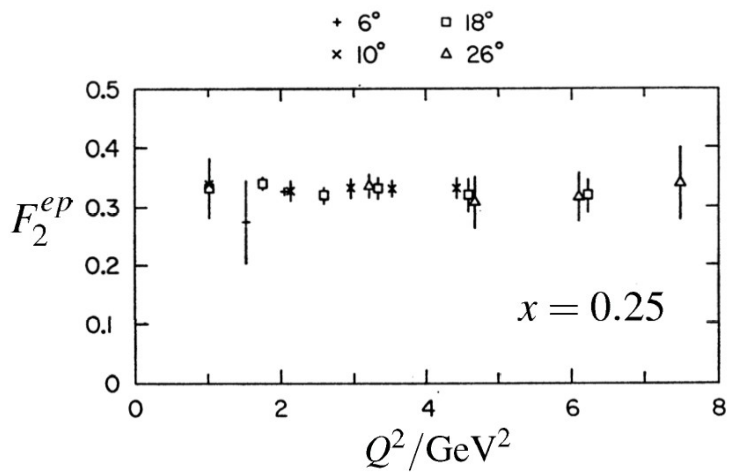
The first evidence of the structure of the proton (nucleon) has been provided with the results of the inelastic collisions

(inelastic e.g. electron-proton scattering at sufficient high scales brakes-up the proton)

The simple parton model introduced by Feynman (1969) to explain Bjorken scaling says that proton is composed of a number of point-like consituents (partons)

→ if proton is made up from point-like particles, then the cross section becomes approximately independent on the scale (**Bjorken scaling**)

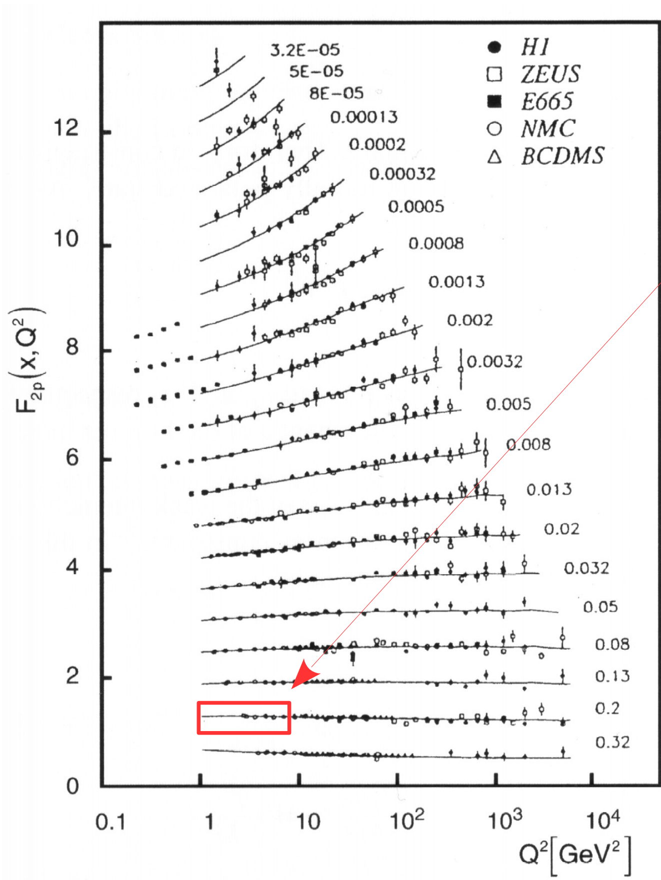
Experimentally observed at SLAC in late 60s :



J.T.Friedman + H.W.Kendall,
Ann. Rev. Nucl. Sci. 22 (1972) 203

Introduction

Soon experiments (fixed target, HERA) showed that Bjorken scaling is violated:



approximate region of
SLAC measurement

According the parton model, proton is made up from up and down quarks → the average total momentum of the proton carried by quarks is:

$$\int_0^1 xu(x)dx + \int_0^1 xd(x)dx \approx 0.36 + 0.18 = 0.54$$

... nowadays we know that gluon is responsible for the rest of the proton momentum

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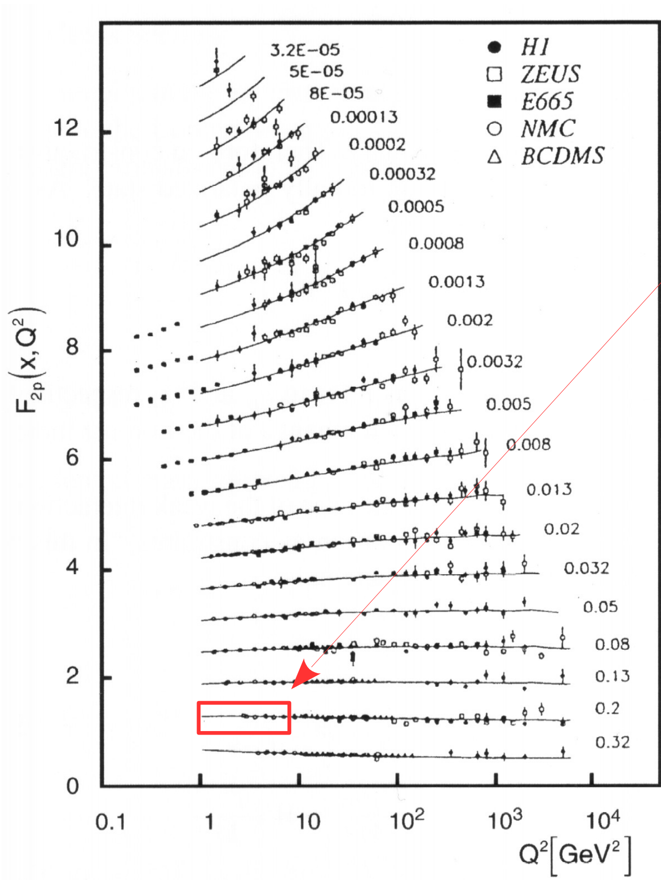

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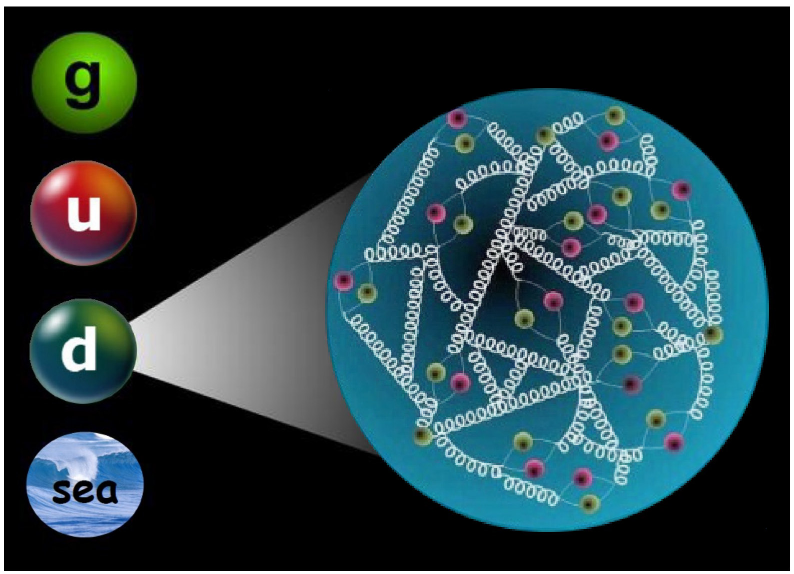
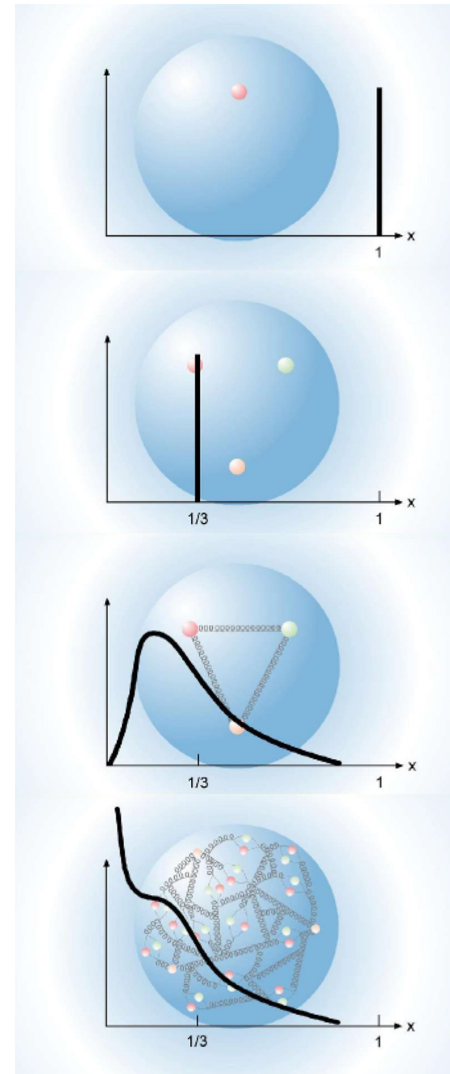
Parton Distribution Functions

What are Parton Distribution Functions (PDFs) of the proton and how are they related to partons?

PDFs → probability for a parton to carry the fraction *x* of proton momentum





PDFs are intrinsic property of nucleon,
i.e assumed to be process independent

Parton Distribution Functions

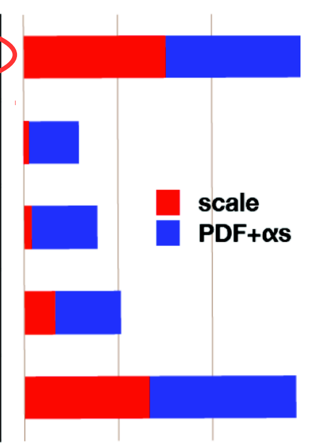
Parton Distribution Functions (PDFs) are of crucial for precision physics at hadron colliders because:

→ PDFs limit **the accuracy of the SM predictions** (including Higgs, W mass)

PDF uncertainties in Higgs productions at LHC are significant

→ similarly to top quarks, Higgs cross section is strongly gluon and α_s dependent

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
NNLO QCD +NLO EW	VBF	1.56 pb	2.9%
	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%



→ agreement with Standard Model depends on how well we know PDFs and α_s

Parton Distribution Functions

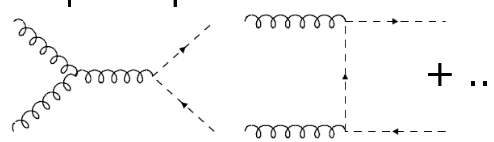
Parton Distribution Functions (PDFs) are of crucial for precision physics at hadron colliders because:

→ PDFs limit **the accuracy of the SM predictions** (including Higgs, W mass)

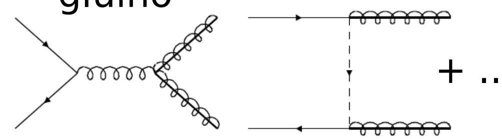
→ **reach of new physics** searches depends on PDF knowledge at high Bjorken-x

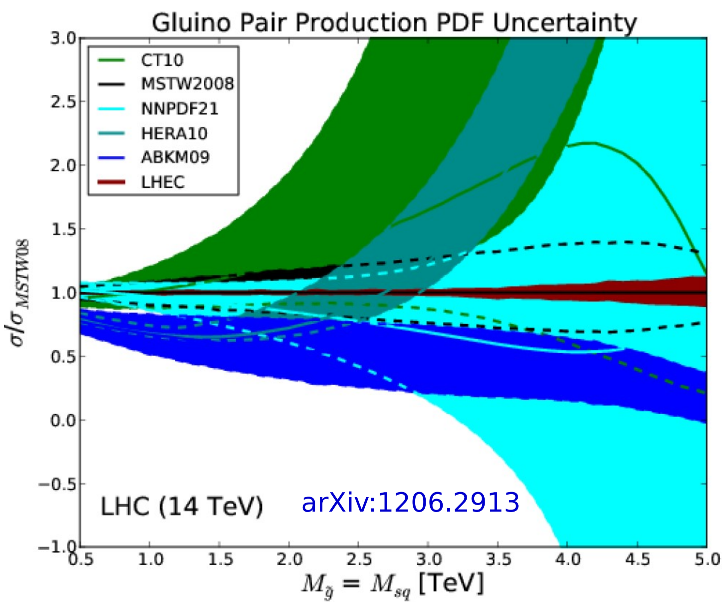
For example, the production of SUSY colored particles (squarks and gluinos) are sensitive to gluon at high $x=2m_X/\sqrt{s} \sim 0.2$ -0.7

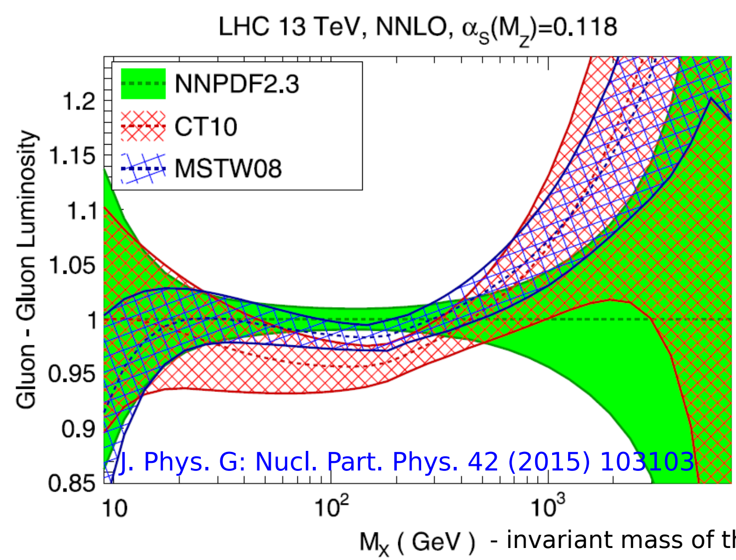
squark production



gluino







- invariant mass of the final state

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QCD factorisation:

measured cross section =

$$\sigma(\alpha_s, \mu_R^2, \mu_F^2) = \sum_{a,b} \int_0^1 \underbrace{f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2)}_{\text{PDF}} \underbrace{\hat{\sigma}(x_1, x_2; \alpha_s, \mu_R^2, \mu_F^2)}_{\text{hard-scattering ME}} + \ldots$$

precision measurements of hadron collider data

PDF determination, heavy quark treatment, QCD analysis tools

parton cross section (calculable in pQCD)

[D. Soper's lecture]

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QCD factorisation:

measured cross section = PDF hard-scattering ME

$$\sigma(\alpha_s,\mu_R^2,\mu_F^2)=\sum_{a,b}\int_0^1 \underbrace{f_a(x_1,\mu_F^2)f_b(x_2,\mu_F^2)}_{\text{PDF}}\underbrace{\hat{\sigma}(x_1,x_2;\alpha_s,\mu_R^2,\mu_F^2)}_{\text{hard-scattering ME}}+\ldots$$

precision measurements of hadron collider data

PDF determination, heavy quark treatment, QCD analysis tools

parton cross section (calculable in pQCD)

[D. Soper's lecture]

*In this lecture we focus on **proton PDFs***

→ *similar to proton case, PDFs can be extracted and for **heavy nuclei** (protons bound in nuclei)*

→ *nuclear targets is play a key role in the flavor differentiation*

→ *derived from the same basic principles but less accurate due to smaller available data sets*

Properties of PDFs (recap)

PDF determination is based on **QCD factorisation**

***x*-dependence** of PDFs is not calculable in perturbative QCD

→ parametrise PDFs at the starting scale Q^2_0

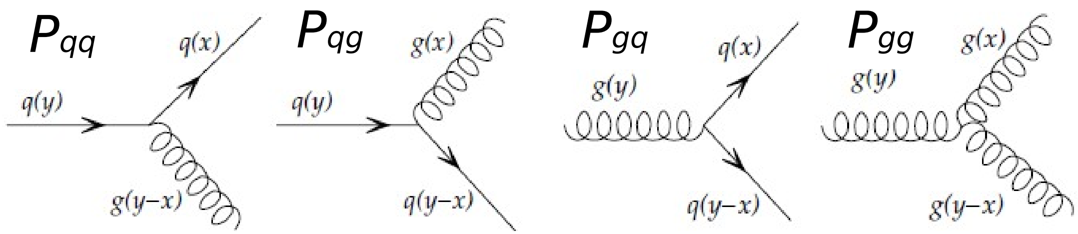
***Q*² dependence** is calculable in perturbative QCD:

evolve PDFs using DGLAP equations to $Q^2 > Q^2_0$

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) evolution equations:

$$\frac{\partial q(x,Q^2)}{\partial lnQ^2} \propto \int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{qq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{qg}\left(\frac{x}{z}\right)\right]$$
$$\frac{\partial g(x,Q^2)}{\partial lnQ^2} \propto \int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{gq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

Probability via splitting functions:



PDF Determination

A (very) general flow diagram of the PDF extraction:

- **initialisation**

choose parameterisation at starting scale and other input parameters

Initialisation

PDF Determination

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(collider, fixed target)

compared with **theory** predictions (NLO, NNLO, ...)

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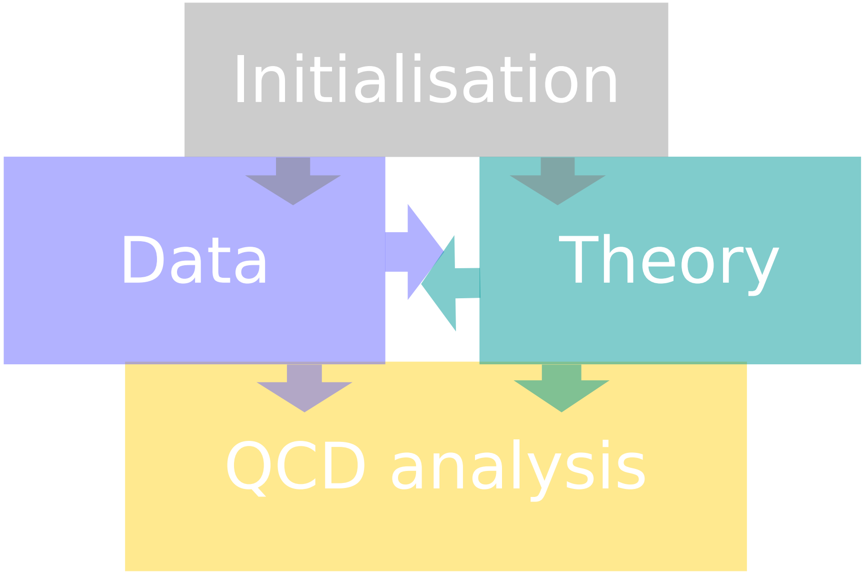
- **QCD analysis** is performed:

  experimental uncertainties

  χ^2 calculation



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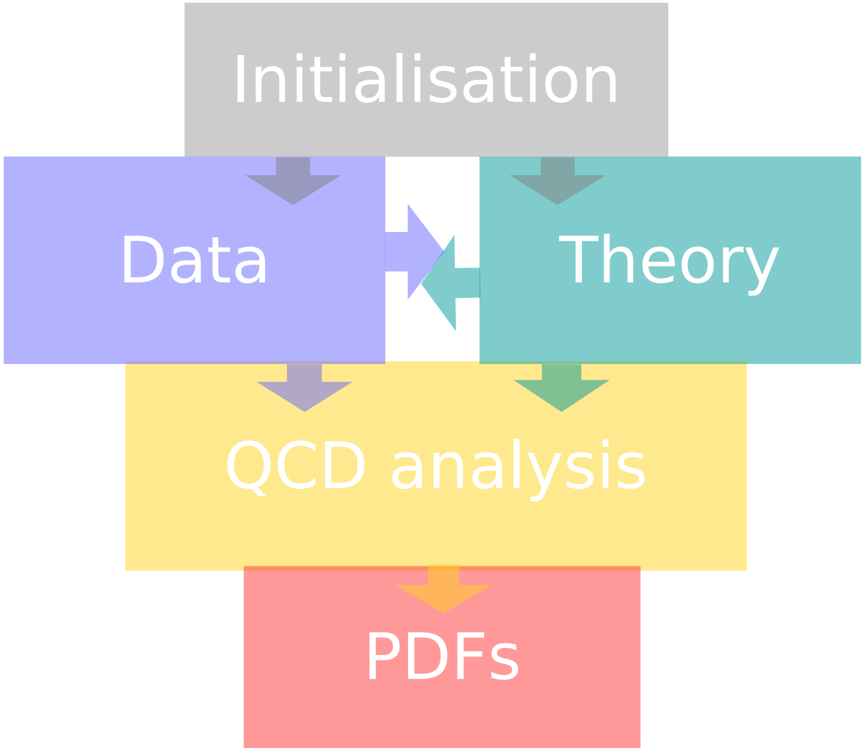
- **PDFs**

also QCD and EW parameters, comparison with data, pulls, etc...



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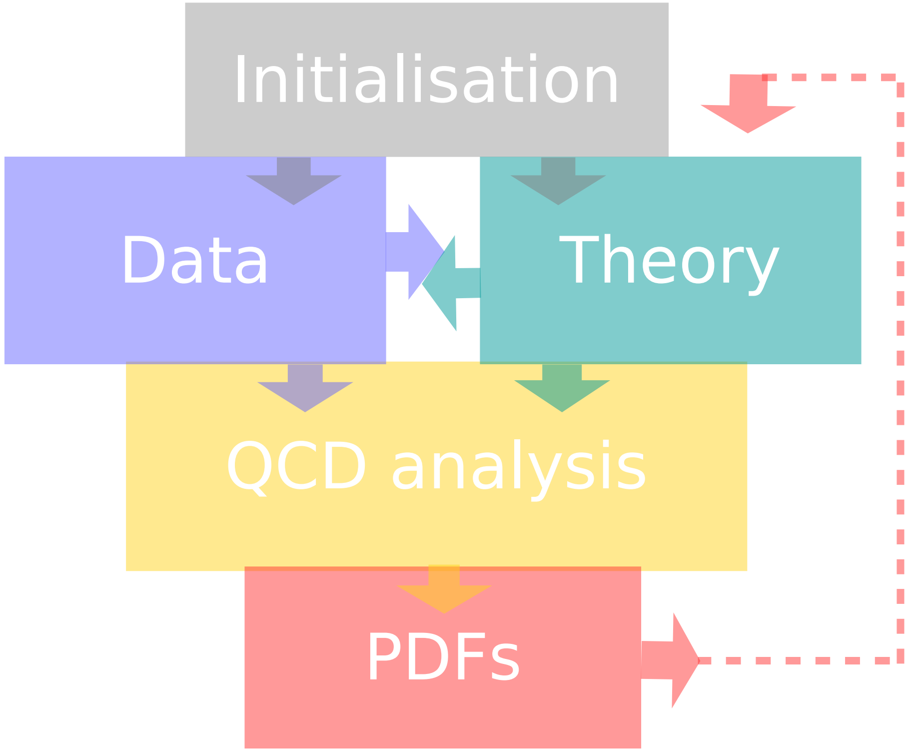

Minimisation:
adjust initial parameters and fit again

final PDFs

PDF Determination

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choose parameterisation at starting scale and other input parameters

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(collider, fixed target)

compared with **theory** predictions (NLO, NNLO, ...)

- **QCD analysis** is performed:

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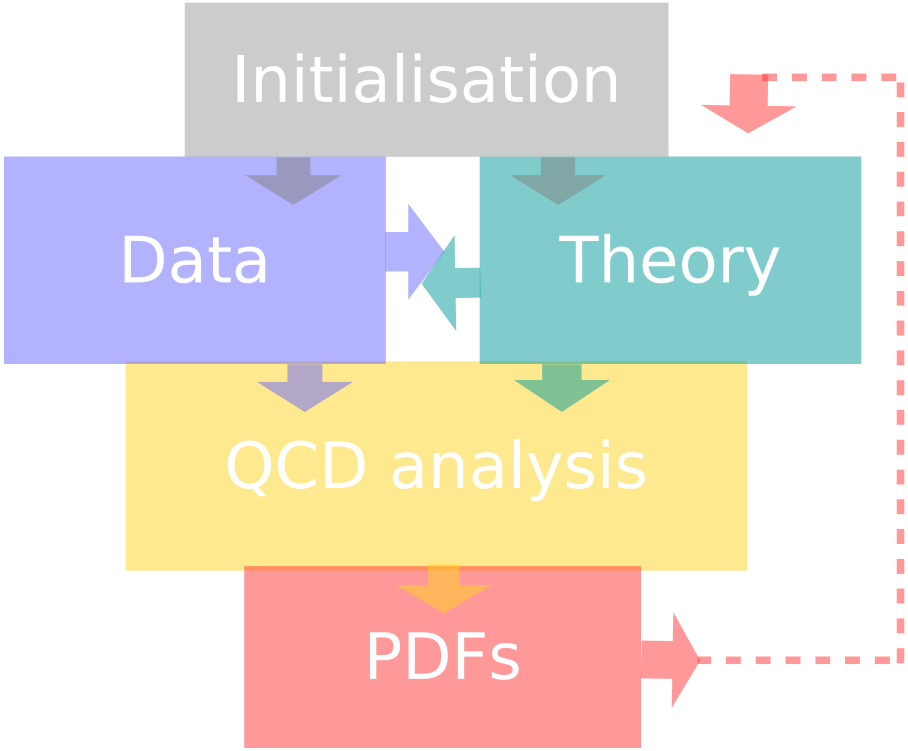
- **PDFs**

also QCD and EW parameters, comparison with data, pulls, etc...



Minimisation:
adjust initial parameters and fit again

final PDFs: CT/CJ, MMHT, NNPDF, ABM, HERAPDF, JR

PDF Determination: Main Aspects

Input data: which are sensitive to PDFs / to include in PDF fit?

→ DIS and other collider data, some examples (mainly from LHC)

Parameterisation form

Goodness of the fit - χ^2 function

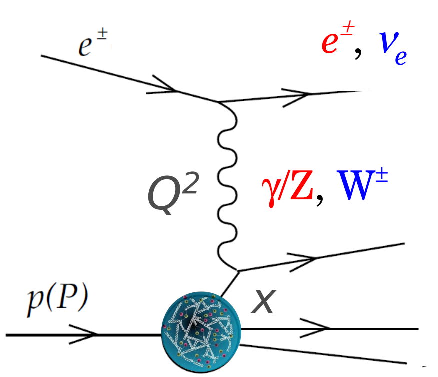
PDF uncertainties

Heavy quark treatment in PDFs

PDF Determination: Experimental Data

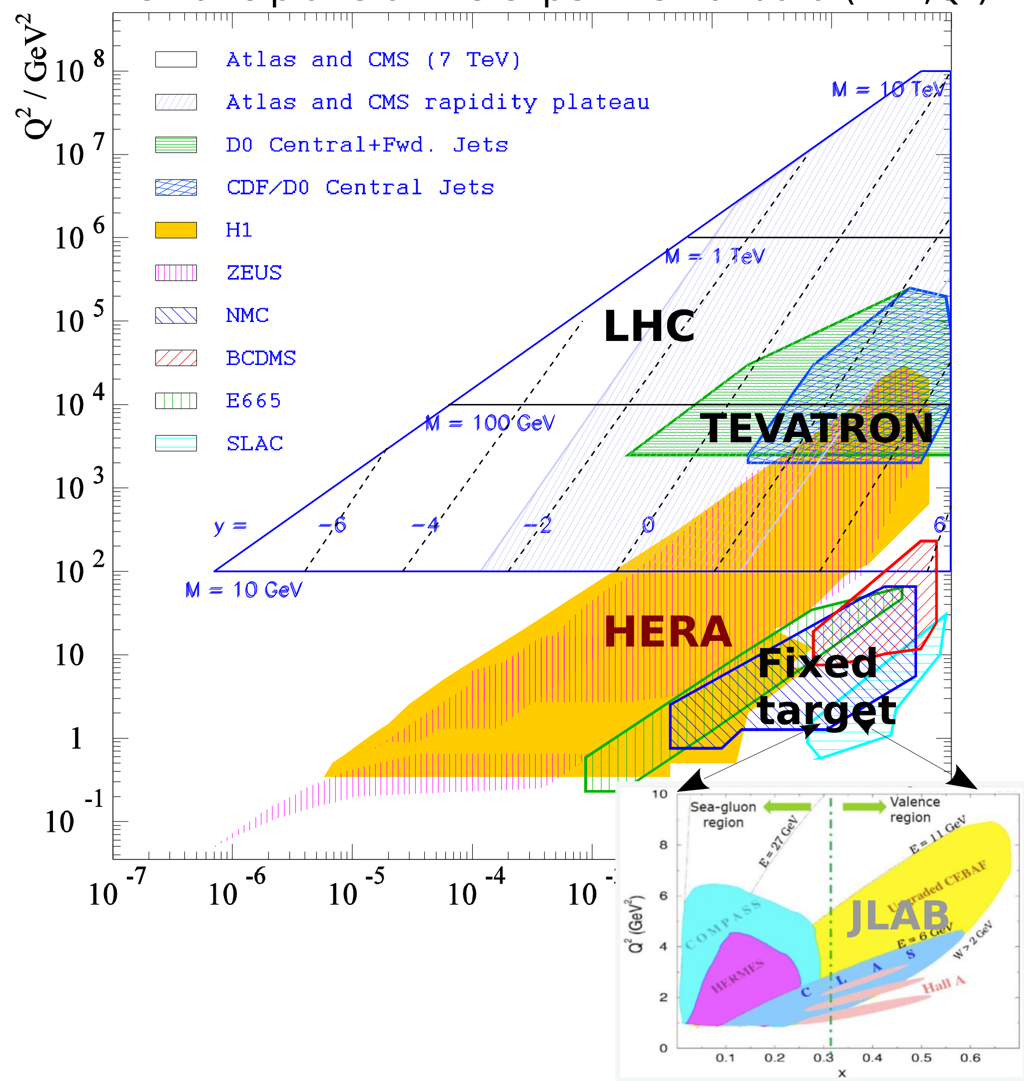
<u>Question</u>: which data include in PDF fit?

Deep Inelastic Scattering (**DIS**):
unique opportunity to study the structure of the proton (nucleon)



Neutral Current (NC): $ep \rightarrow eX$
Charged Current (CC): $ep \rightarrow \nu X$

kinematic plane of the experimental data (in x,Q^2)



PDF Determination: Experimental Data

Question: which data include in PDF fit?

Recap:
[D. Soper's lectures]

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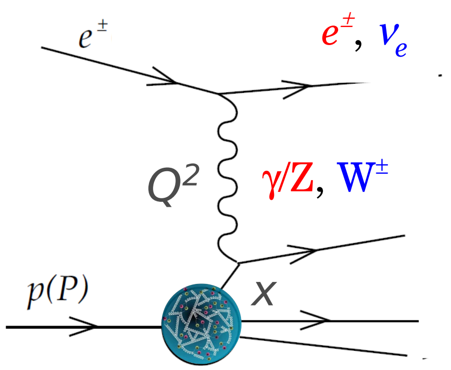


Neutral Current (NC): $ep \to eX$
Charged Current (CC): $ep \to \nu X$

Kinematics:

Q^2 - virtuality of exchanged boson

x – Bjorken scaling variable

y – inelasticity

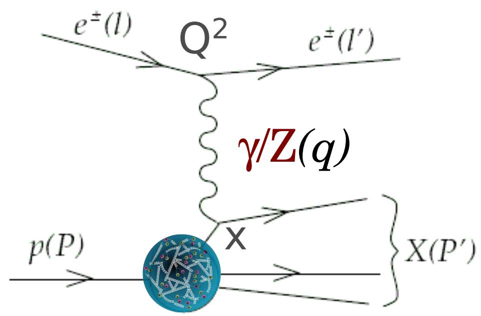
$Q^2 = sxy$ ($\sqrt{s}$ centre-of-mass energy)

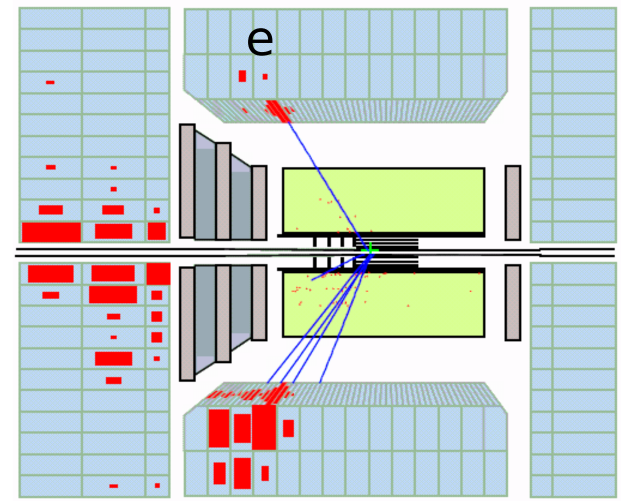
Fixed target data (SLAC, BCDMS, NMC, CCFR, NuTeV, CHORUS,...)

HERA - worlds only $e^{\pm}p$ collider (1994-2007)
collider experiments: H1 and ZEUS,
fixed-target: HERMES, HERA-B

ep scattering at HERA: Neutral Currents

Neutral current DIS cross section:





NC event in ZEUS detector

Cross section is decomposed in terms of structure functions:

$$\frac{d^2\sigma_{NC}^{e^{\pm}p}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[Y_+\tilde{F}_2^{\pm} \mp Y_- x\tilde{F}_3^{\pm} - y^2\tilde{F}_L^{\pm}\right]$$

dominant contribution

important at high Q^2

sizable at high y

$$Y_{\pm} = 1 \pm (1-y)^2$$

$$k = \frac{1}{4\sin^2\theta_w \cos^2\theta_w}\frac{Q^2}{Q^2+M_Z^2}$$

at LO sensitive to sum and difference of quarks and anti-quark densities:

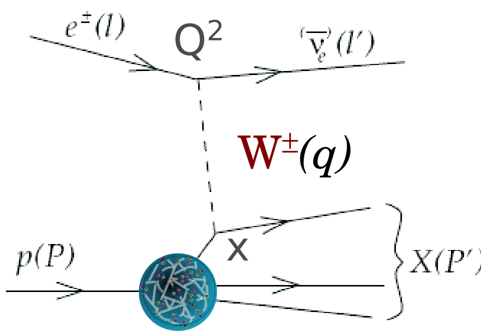
LO: $F_2 \approx x\sum e_q^2(q+\bar{q})$ (in NLO $(\alpha_s\ g)$ appear)

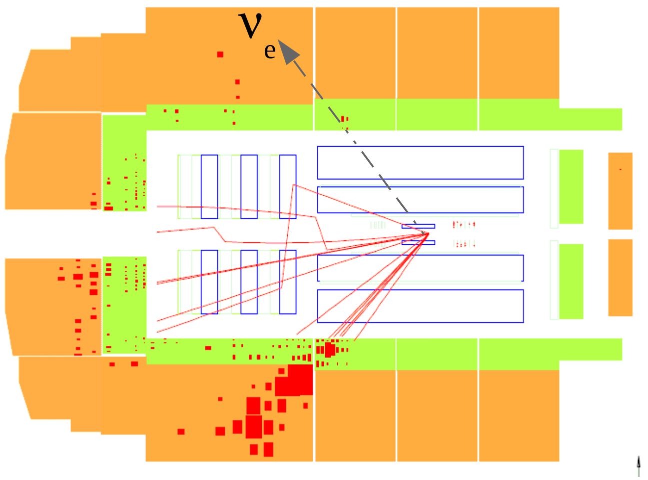
$xF_3 \approx x\sum 2e_q a_q(q-\bar{q})$

PDFs

ep scattering at HERA: Charged Currents

Charged current DIS cross section:





CC event in H1 detector

$$\frac{d^2\sigma_{CC}^{e^\pm p}}{dx dQ^2} = (1 \pm P_e)\frac{G_F^2}{2\pi x}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \tilde{\sigma}_{CC}^{e^\pm p}$$

P - electron(positron) polarisation

at LO e^+/e^- sensitive to different quark densities:

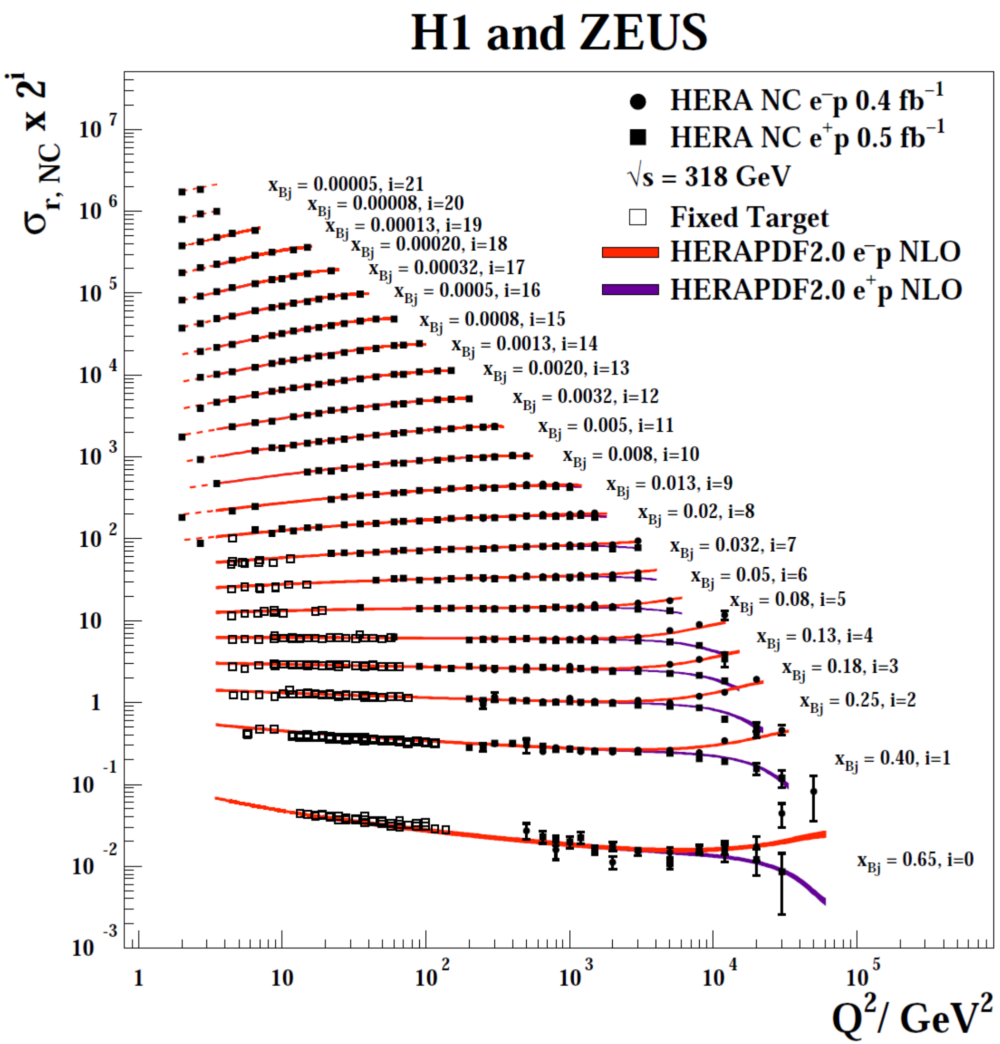
$$\sigma^{e+}_{CC} \approx x[\bar{u}+\bar{c}] + (1-y)^2 x[\mathbf{d}+s]$$

$$\sigma^{e-}_{CC} \approx x[\mathbf{u}+c] + (1-y)^2 x[\bar{d}+\bar{s}]$$

PDFs

ep scattering at HERA

Scaling violations: final word from HERA



EPJC 75 (2015) 12, 580

Scaling violations

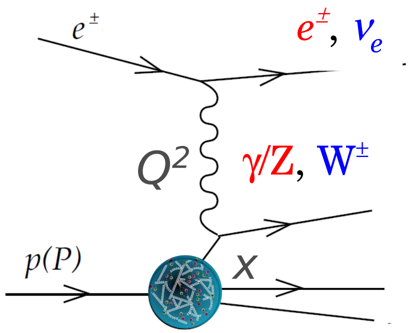
(the dependence of the structure functions on Q^2 at fixed x)

are a consequence of the strong interactions between the partons in the nucleon

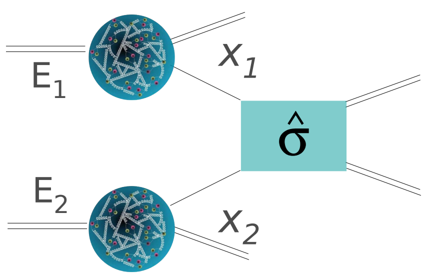
→ gluon can be extracted from these data

Data to constrain PDFs

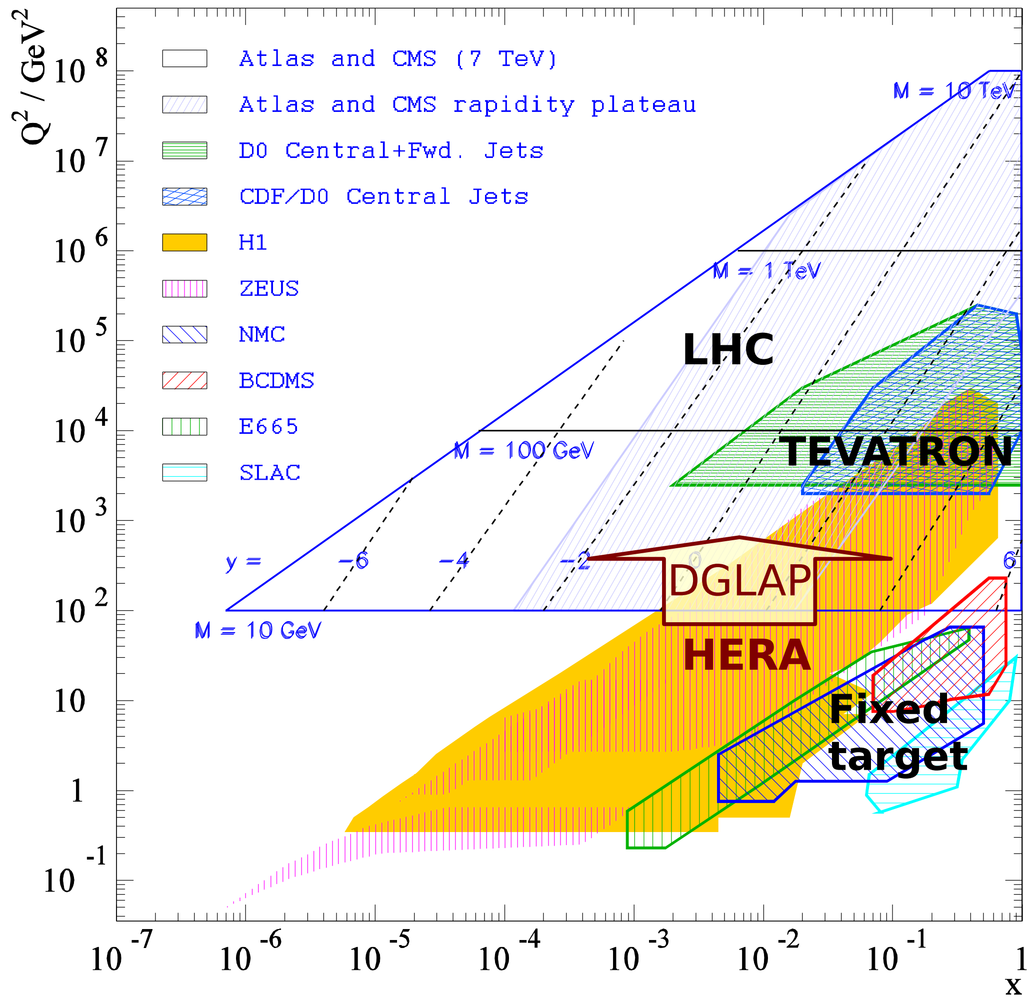
Deep Inelastic Scattering (**DIS**):



same PDFs can be used to predict *pp* collisions



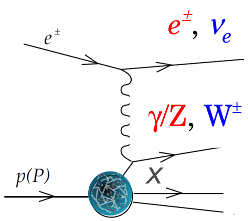
$\hat{\sigma}$ – perturbative QCD cross section



Other hadron collider data can constrain and improve PDFs further

Data in PDF fits

Deep Inelastic Scattering:



ep data: quarks and gluon at small x (F_L), flavour separation (CC)

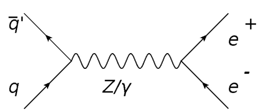
jets → gluons (moderate x) and α_s

heavy quarks → gluons, tests of heavy quark schemes, mass determination

fixed target data: higher x

neutrino DIS: flavour decomposition, x>0.01

Drell-Yan production:

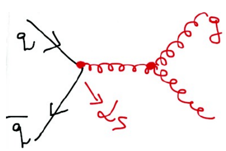


different PDF combinations (low/mid/high x), deuterium target – $\bar{u}/\bar{d}$ asymmetry

W/Z ratio, asymmetries → quark flavour separation

V+ heavy flavour → sensitivity to s quark

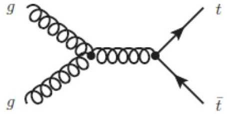
Inclusive jets, dijets and ratios:



high x gluon, α_s

Isolated photon → gluon at medium and high x

ttbar, single top:



gluon at high x, u and d quarks, α_s

some examples follow...

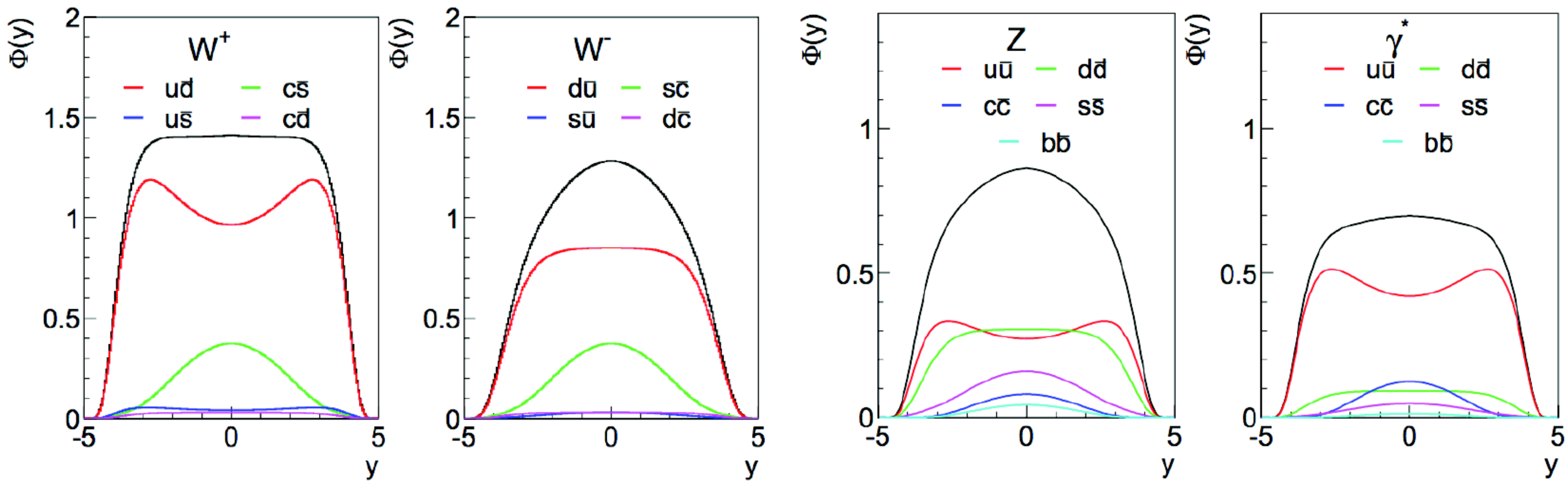
W and Z production at LHC

Z and W production at LHC

→ probe different flavour combinations
→ potential to improve quark PDFs

$$W^+ \approx 0.95(u\bar{d} + c\bar{s}) + 0.05(u\bar{s} + c\bar{d})$$
$$W^- \approx 0.95(d\bar{u} + s\bar{c}) + 0.05(d\bar{c} + s\bar{u})$$

$$Z \approx 0.29(u\bar{u} + c\bar{c}) + 0.37(d\bar{d} + s\bar{s} + b\bar{b})$$
$$\gamma^* \approx 0.44(u\bar{u} + c\bar{c}) + 0.11(d\bar{d} + s\bar{s} + b\bar{b})$$



(A.Glazov/V.Radescu)

→ u and d quarks dominate for W, all flavours contribute to Z

W Asymmetry

W lepton charge asymmetry at LHC

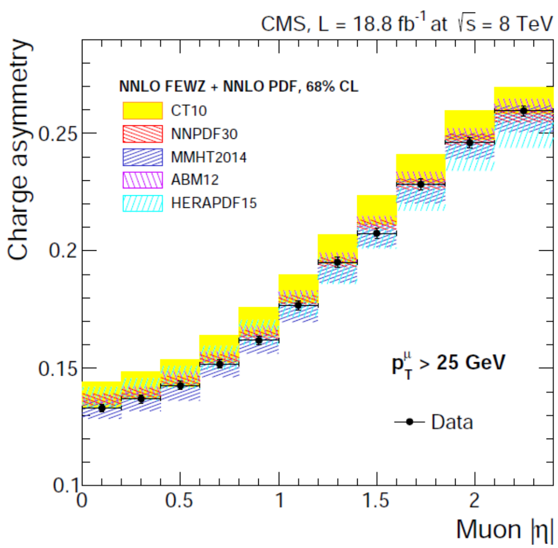
→ overall excess of W^+ over W^- due to presents of two valence *u* quarks in the proton

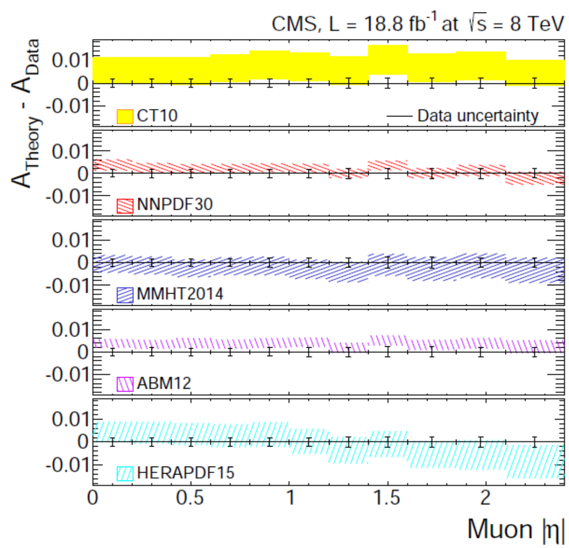
→ probe valence quarks and PDFs rations (u_v, d_v, d/u, d_v/u_v, dbar/ubar):

$$A_W = \frac{W^+ - W^-}{W^+ + W^-} \approx \frac{u_v - d_v}{u_v + d_v + 2u_{sea}}$$

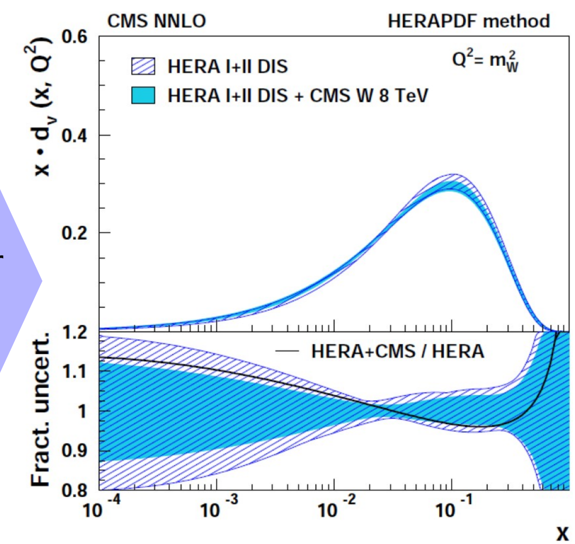
CMS W muon asymmetry data (8 TeV)

arXiv:1603.01803





after fit

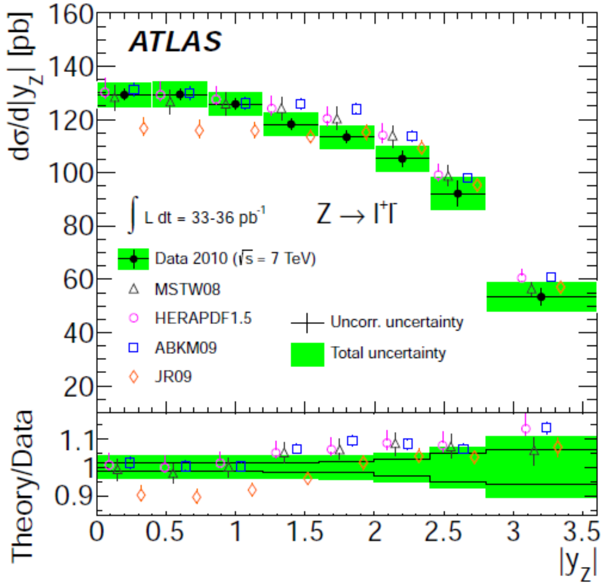


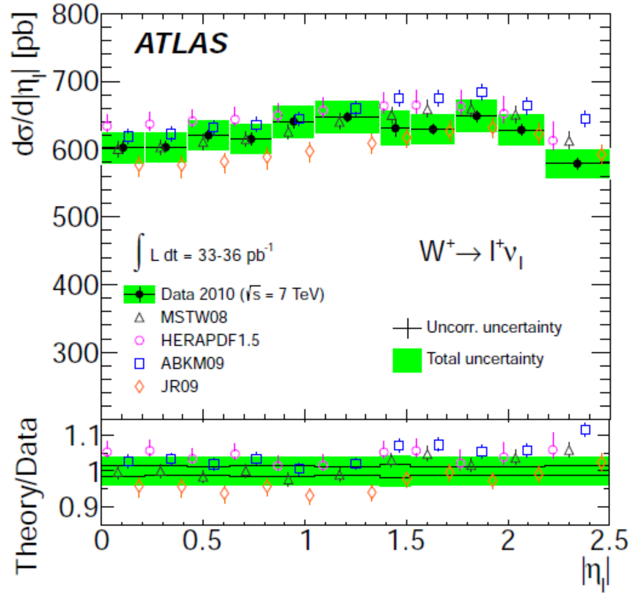
note: comparison is done with HERA data alone

W and Z production (differential cross sections)

ATLAS $W^{\pm}$ and Z inclusive differential cross sections (35 pb^{-1})

Phys Rev D 85 (2012) 072004

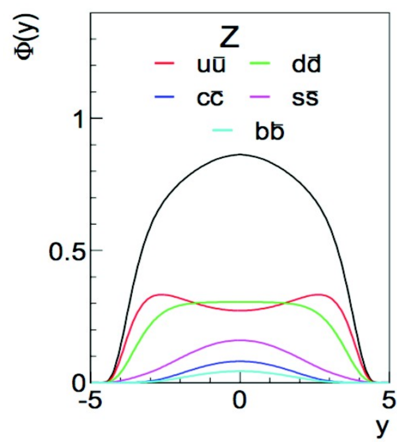




→ discrimination between PDFs

→ sensitivity to s-quark?

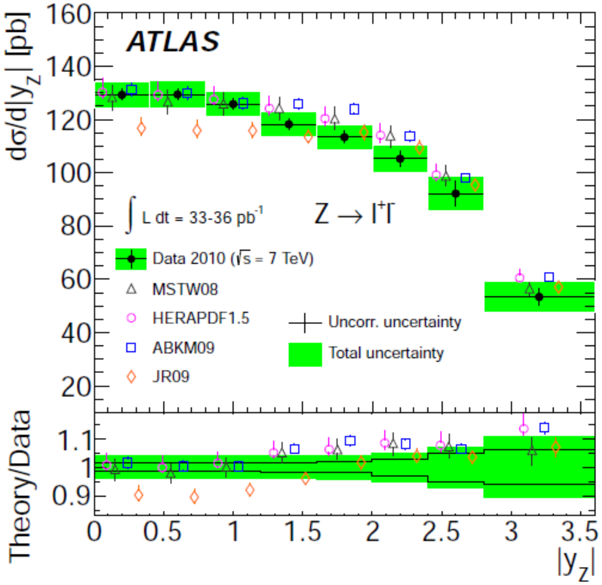
s-quark is poorly known, main constraints come from fixed-target experiments (e.g. NOMAD, Nucl.Phys. B876 (2013) 339) which suggest suppressed s-quark, i.e. less s than d at low x

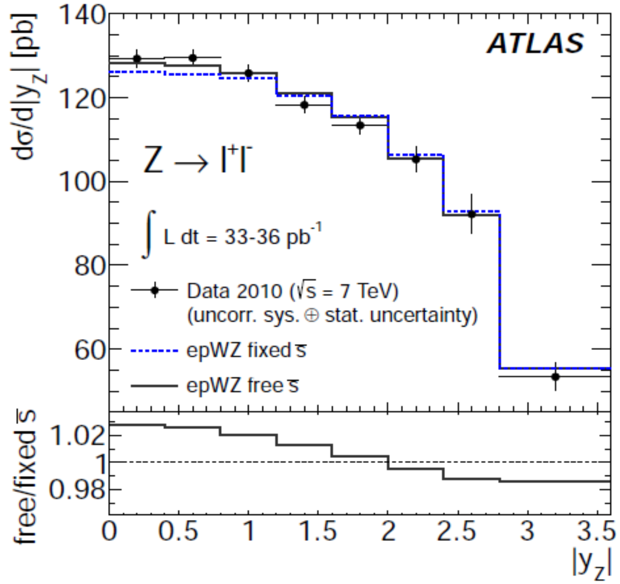


s-quark contribution largest in Z production at central rapidity

W and Z production: sensitivity to s-quark

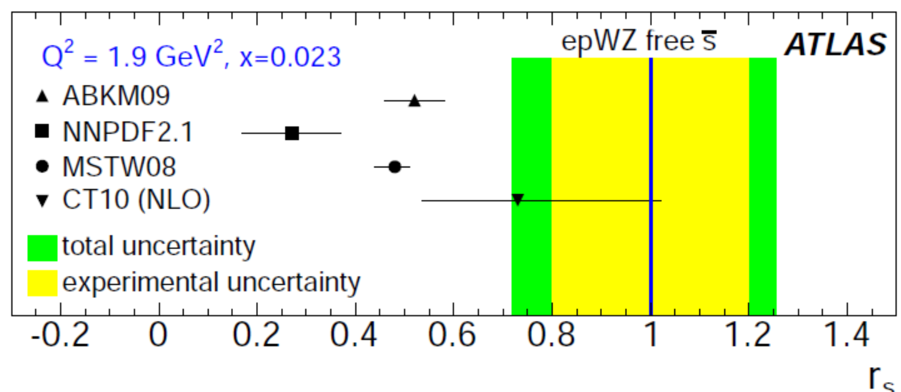
ATLAS $W^{\pm}$ and Z inclusive differential cross sections (35 pb^{-1})





Phys Rev D 85 (2012) 072004

$$r_s = 0.5(s+\bar{s})/\bar{d}$$ Phys Rev Lett 109 (2012) 012001

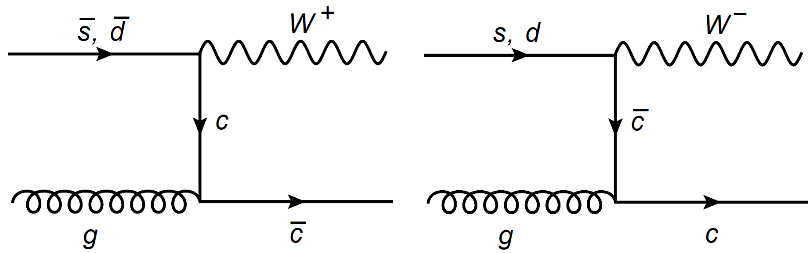


QCD analysis at NNLO of ATLAS W,Z data

→ results support symmetric light-sea

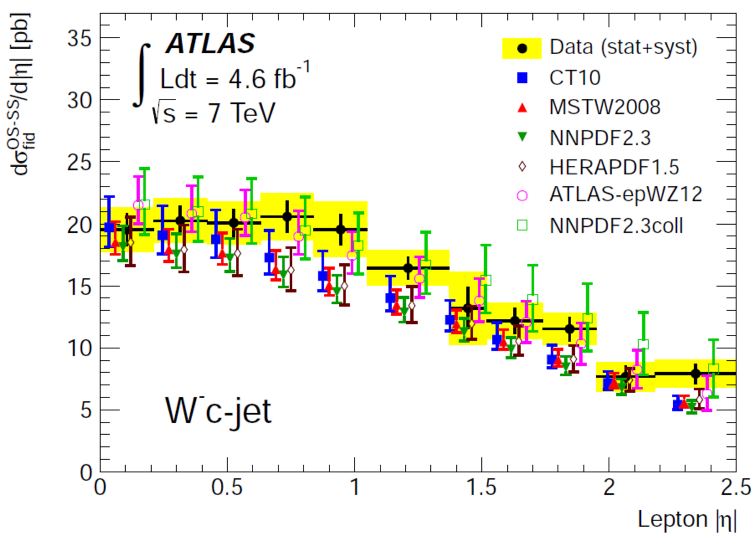
W + charm Production at LHC

Measurement of W+c at LHC provide additional constrains to the s quark

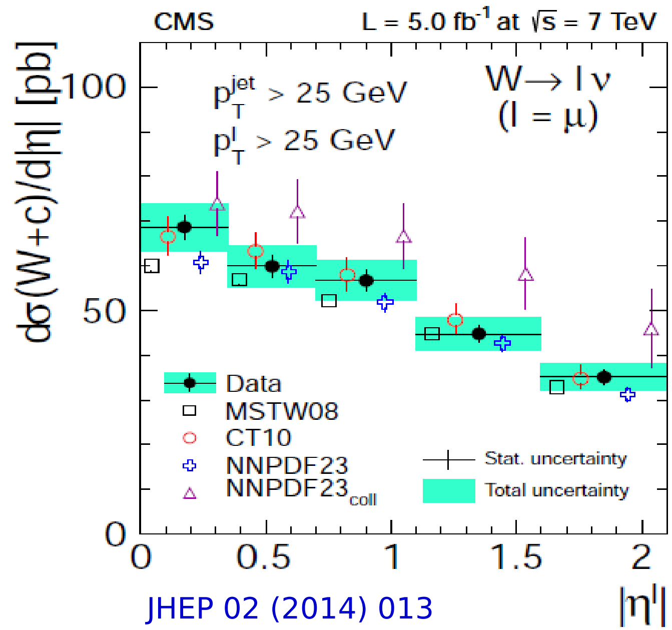


ATLAS W+ charm data (4.6 fb^{-1})

JHEP 05 (2014) 068



CMS W+ charm data (5 fb^{-1})



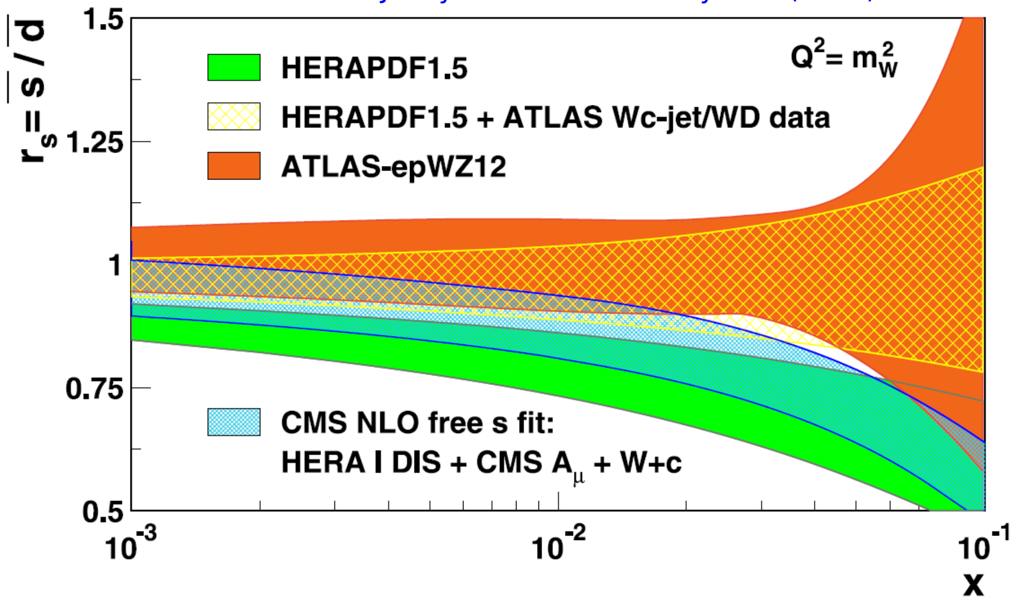
JHEP 02 (2014) 013

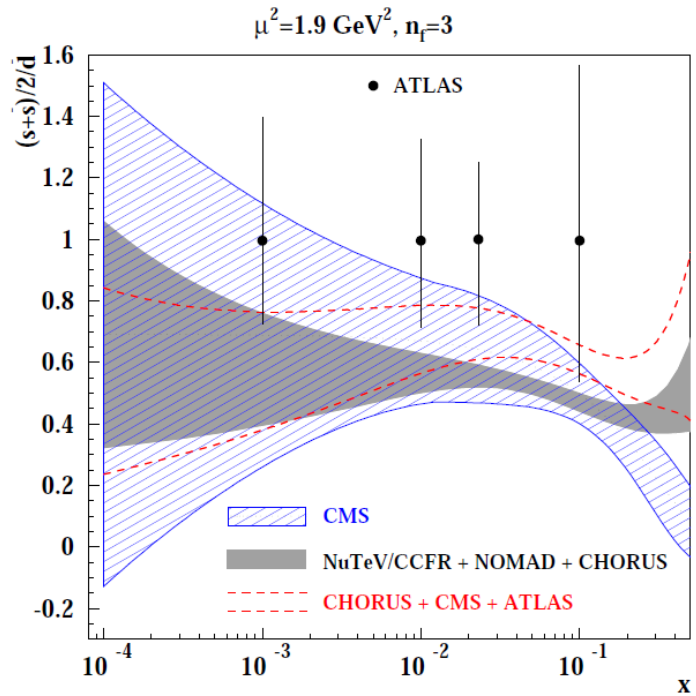
CMS in good agreement with CT10 while ATLAS data is above - indication of enhanced s fraction

Strange quark PDF

Comparison of the s-quark fraction determined by ATLAS and CMS (no fixed-target data, no additional assumptions) and with determination using fixed-target data

J. Phys. G: Nucl. Part. Phys. 42 (2015) 103103





Phys. Rev. D 91, 094002 (2015)

New data from LHC will bring more information about the s-quark

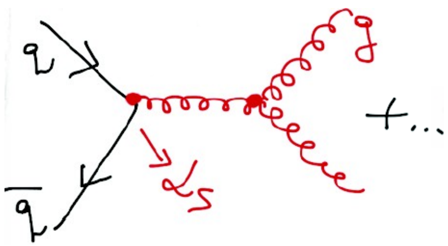
Jet Production at LHC

Jet production at LHC

→ provides information about hard QCD, PDFs, strong coupling constant α_s

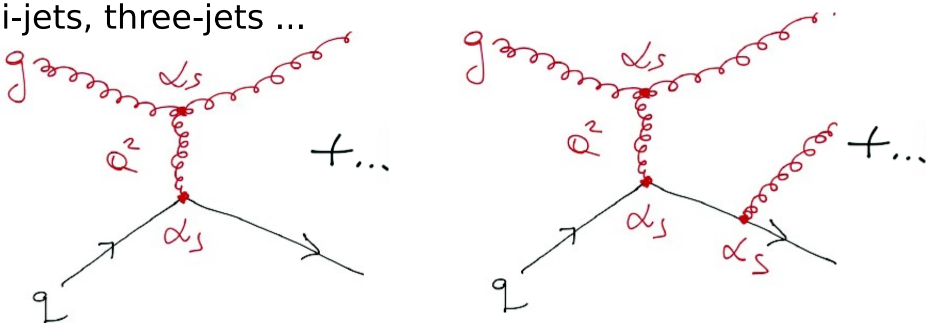
→ PDFs and α_s depend on scale of the process → P_T of the jet

Inclusive jet production at LO

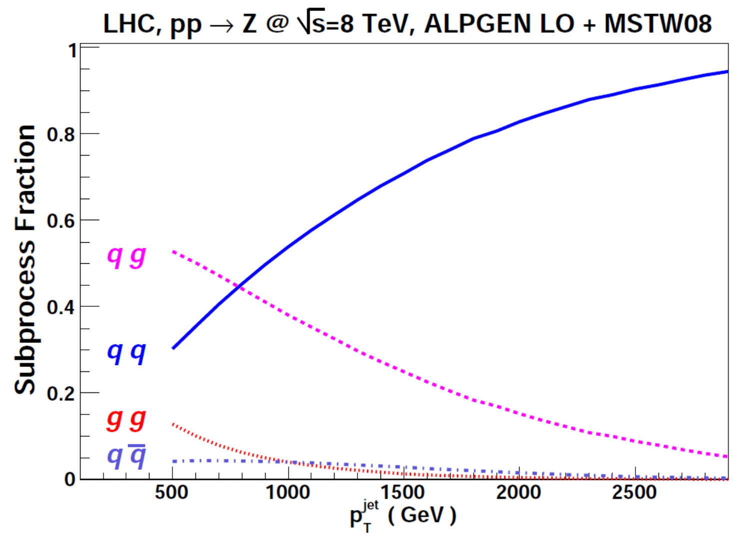


(G. Dissertori)

di-jets, three-jets ...



JHEP 08 (2012) 101



... and ratios (smart way of canceling large part of e.g. jet scale uncertainty)

→ LHC jet data provide constrains in high-x region

→ at high scales may reveal new physics (depend how well gluon at high x is known)

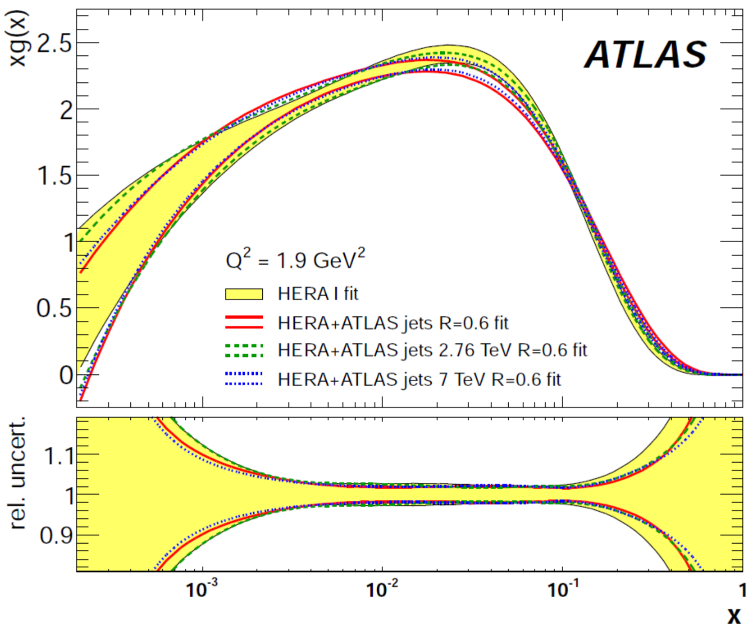
Inclusive Jet Production

Inclusive jet measurements and QCD analysis of LHC data

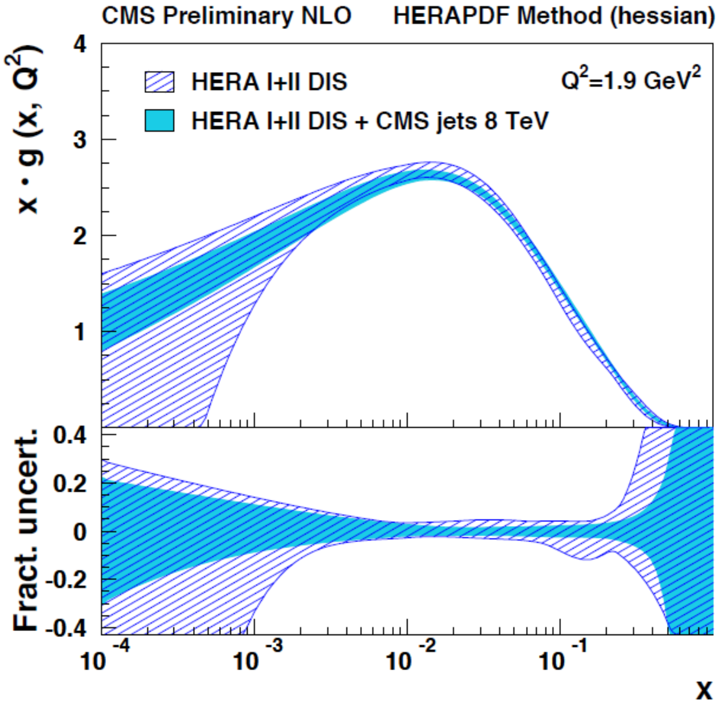
QCD analyses at NLO with (HERA and) inclusive jet data performed by

ATLAS (2.7 TeV and 7 TeV data) and CMS (8 TeV data, 19.7 fb^{-1})

EPJC (2013) 73 2509



CMS PAS SMP-14-001



→ jet data can help to improve gluon distribution function in high-x region and provides possibility to extract strong coupling constant α_s

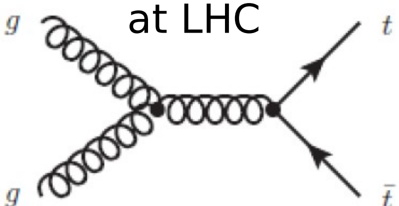
Top Quark Pairs (LHC and Tevatron)

Top quark pairs ($t\bar{t}$) provides possibility to

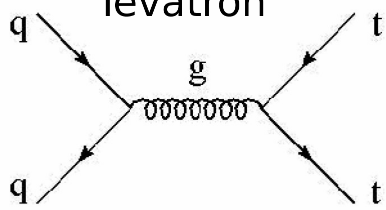
→ tests of pertubative QCD, sensitive to new physics effects

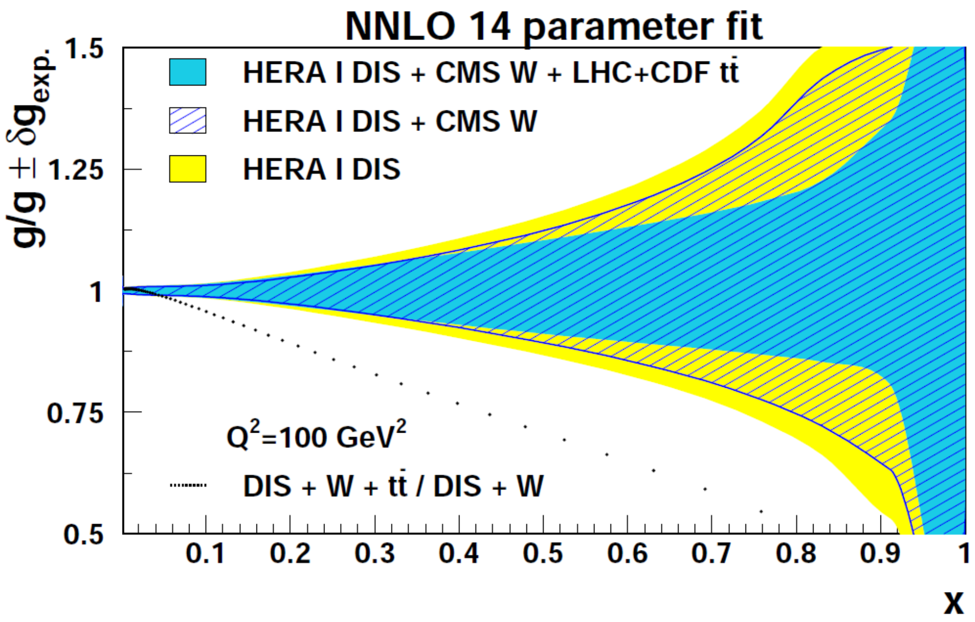
→ probe of high-x gluon (high correlation between gluon, α_s and top quark mass)

dominant processes at LHC



Tevatron





Guzzi et al., DiffTop JHEP 1501 (2015) 082

→ QCD analysis with ATLAS and CMS $t\bar{t}$ data (together with HERA, Tevatron and W production data at LHC)

→ additional top data will provide more information on PDFs

Impact of LHCb Heavy Flavour Data to PDFs

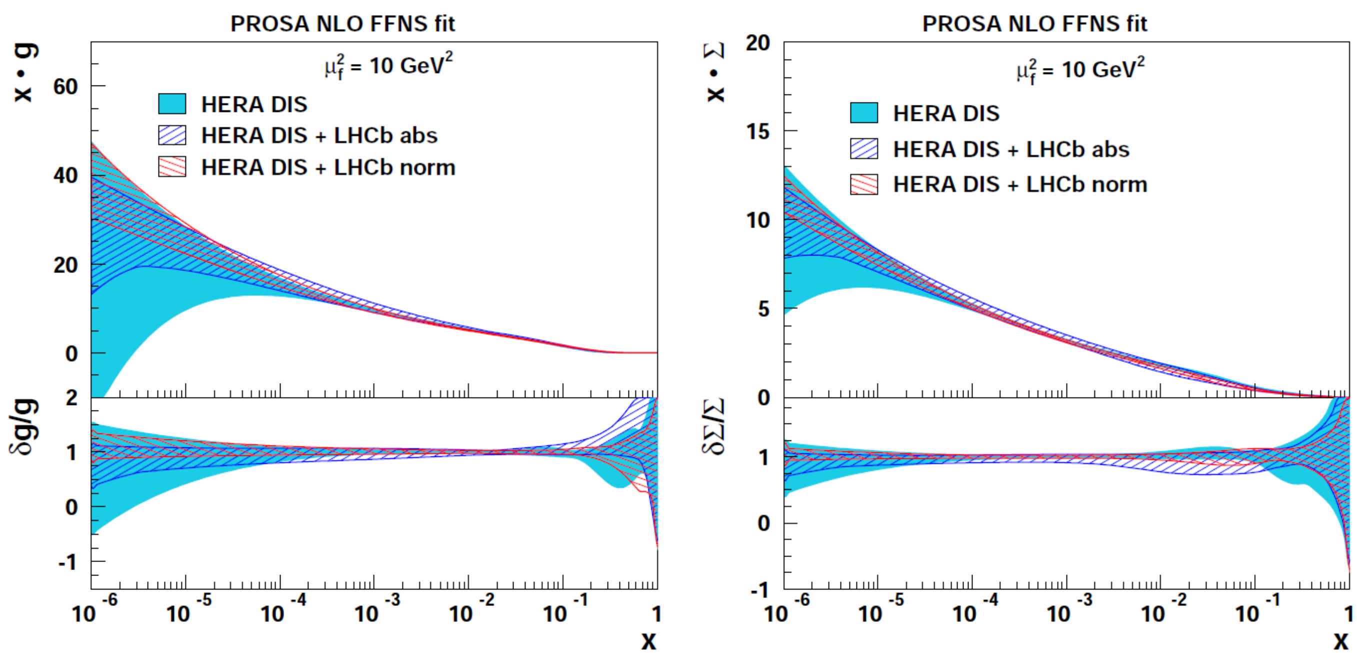
LHCb heavy-flavour data impose additional constraints on the gluon and the sea-quark distributions at low x

→ first time used to constrain PDFs   Nucl. Phys. B871 (2013)   JHEP08 (2013) 117

→ NLO QCD analysis (together with HERA data) with the fixed-flavour number scheme

→ absolute and normalised cross sections

Eur.Phys.J. C75 (2015) 8, 396



→ significant reduction of the gluon uncertainty at very low x ($x \sim 5\times10^{-6}$)

PDF Determination: Main Aspects

Input data: which are sensitive to PDFs / to include in PDF fit?

→ DIS and other collider data, some examples (mainly from LHC)

Parameterisation form

Goodness of the fit - χ^2 function

PDF uncertainties

Heavy quark treatment in PDFs

PDF Determination: Parametrisation

PDFs are parametrised (at the starting scale Q^2_0) using some flexible form

(starting scale choice is arbitrary, often $Q_0 = m_c$)

generic parametrisation form:

$$xf_j(x) = A_j x^{B_j}(1-x)^{C_j} P_j(x)$$

A: overall normalisation
B: small x behavior
C: $x \rightarrow 1$ shape

with $P_j(x) = (1 + \varepsilon_j \sqrt{x} + D_j x + E_j x^2)$ HERAPDF, MSTW/MMHT (Chebyshev polynomials), ABM, JR

or $e^{a_3 x}(1 + e^{a_4} x + e^{a_5} x^2)$ CTEQ, CT (Bernstein polynomials)

→ *parametrisation has to be flexible enough (many free parameters) to avoid bias, however too many parameters may also lead to certain bias (several minima, problems to converge, ...)*

→ *different parametrisations, if carefully chosen, will lead to similar results*

PDF Determination: Parametrisation

PDFs are parametrised (at the starting scale Q^2_0) using some flexible form

parametrised in x-space with the flexible neural network (NN) method:

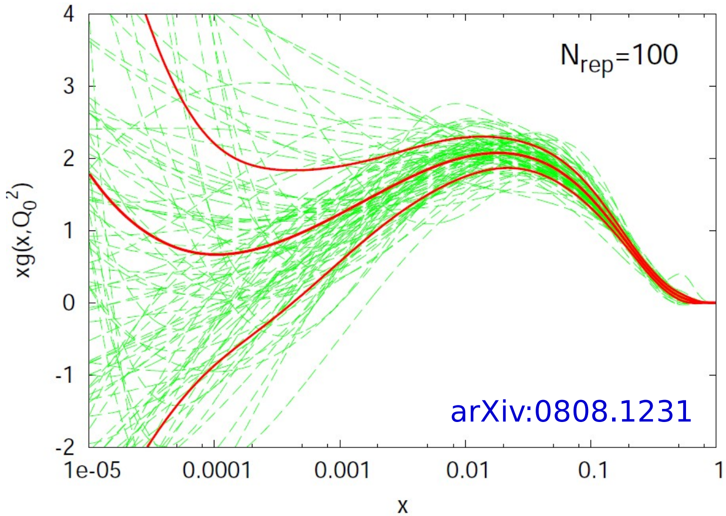
used by NNPDF collaboration

<u>Basic principle:</u>

→ Monte Carlo (MC) sampling of data
(generation of replicas of experimental data)

→ training: set of PDFs parametrised by
neural networks on each of the replicas

→ validation: fit stops when quality of fit stops
improving (determined by random selected
validation data)



advantage: unbiased parametrisation
disadvantage: requires sufficient data

Uncertainties: Monte Carlo approach (explained later)

Measure the goodness of the fit: χ^2 function

The goodness of the fit is (typically) measured by χ^2 in PDFs

Standard χ^2 function

→ with nuisance parameters:

$$\chi^2 = \sum_i \frac{\overset{\text{measurement}}{(\mu_i - \hat{m}_i)^2}}{\underset{\text{uncorrelated error}}{\Delta_i^2}} + \underset{\text{sum over correlated systematic sources}}{\sum_\alpha} b_\alpha^2 \qquad \overset{\text{theory}}{\hat{m}_i} = m_i + \sum_\alpha \underset{\text{correlated error}}{\Gamma_{i\alpha}}\, \overset{\text{nuisance parameter}}{b_\alpha}$$

→ correlations in the covariance matrix:

$$\chi^2 = (\boldsymbol{\mu} - \boldsymbol{m})^{\mathrm{T}} C^{-1} (\boldsymbol{\mu} - \boldsymbol{m}) = \sum_{ij} (\mu_i - m_i)\, C^{-1}_{ij} (\mu_j - m_j)$$

$$C = \overset{\text{statistical}}{C^{\text{stat}}} + \overset{\text{uncorrelated}}{C^{\text{unc}}} + \overset{\text{correlated}}{C^{\text{syst}}}$$

→ mixed form:

$$\chi^2 = \sum_{ij}^{N} \left(D_i - T_i - \sum_k^K r_k \beta_{ik} \right) \mathrm{C}^{-1}_{ij} \left(D_j - T_j - \sum_k^K r_k \beta_{jk} \right)$$

→ *it is important to account statistical and systematic uncertainties of data*

→ *theory (PDF) uncertainties can be accounted for*

Measure the goodness of the fit: χ^2 function

The goodness of the fit is (typically) measured by χ^2 in PDFs

Standard χ^2 function

→ same definition rewritten:

$$\chi^2_{\text{exp}}(\boldsymbol{m},\boldsymbol{b}) = \sum_i \frac{\left[m^i - \sum_\alpha \gamma^i_\alpha \mu^i b_\alpha - \mu^i\right]^2}{\left(\delta_{i,\text{stat}}\,\mu^i\right)^2 + \left(\delta_{i,\text{uncor}}\,\mu^i\right)^2} + \sum_\alpha b_\alpha^2$$

$$\Gamma^i_\alpha = \gamma^i_\alpha \mu_i$$

$$\chi^2(\boldsymbol{b}_{\text{exp}}, \boldsymbol{b}_{\text{th}}) = \sum_{i=1}^{N_{\text{data}}} \frac{\left(\sigma^{\text{exp}}_i + \sum_\alpha \Gamma^{\text{exp}}_{i\alpha} b_{\alpha,\text{exp}} - \sigma^{\text{th}}_i - \sum_\beta \Gamma^{\text{th}}_{i\beta} b_{\beta,\text{th}}\right)^2}{\Delta_i^2} + \sum_\alpha b^2_{\alpha,\text{exp}} + \sum_\beta b^2_{\beta,\text{th}}$$

Impact of experimental data on PDFs can be studied by minimizing data to theory χ^2 vs nuisance parameters corresponding to PDF eigenvectors ("profiling")

[S. Camarda, xFitter tutorial]

PDF Uncertainties: Hessian Method

PDF uncertainties come from
experimental measurement errors and from theoretical approximations

main two methods used:

PDF parameters: $\{a_i\}$ $i=1...d$

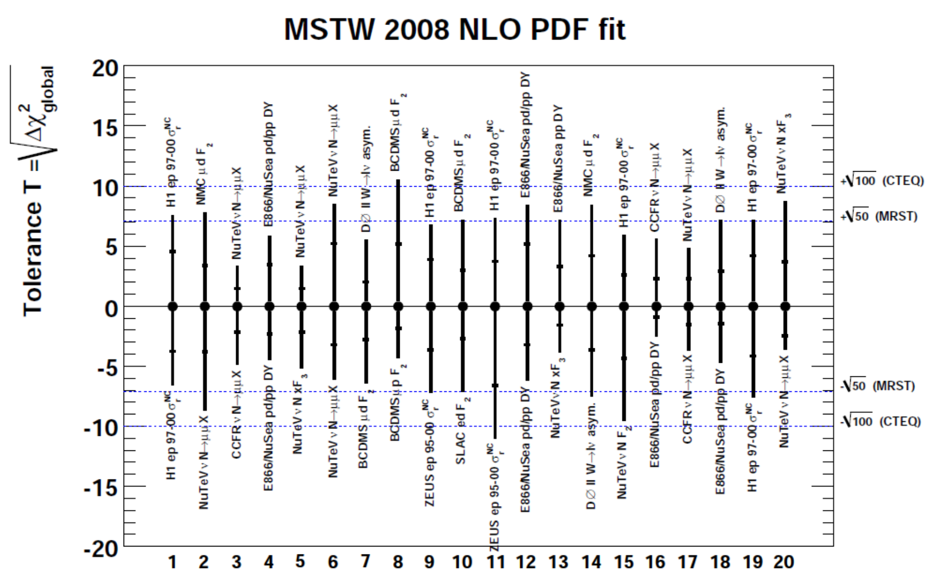
→ **Hessian**

expansion of χ^2: $$\Delta\chi^2 = \chi^2 - \chi_0^2 = \sum_{i=1}^{d}\sum_{j=1}^{d} H_{ij}\left(a_i - a_i^0\right)\left(a_j - a_j^0\right)$$

at minimum

Hessian matrix of second derivatives: $$H_{ij} = \frac{1}{2}\frac{\partial^2\chi^2_{\rm global}}{\partial a_i \partial a_j}\bigg|_0$$

Ideally $\Delta\chi^2 = 1$ for one sigma (68% CL)
→ not always the case due to not fully compatible data sets (tolerance)



arXiv:0901.0002

→ method used by
CT, MMHT, HERAPDF, ABM, JR groups

PDF Uncertainties: Monte Carlo Method

PDF uncertainties come from
experimental measurement errors and from theoretical approximations

main two methods used:

→ **Monte Carlo** (used by NNPDF)

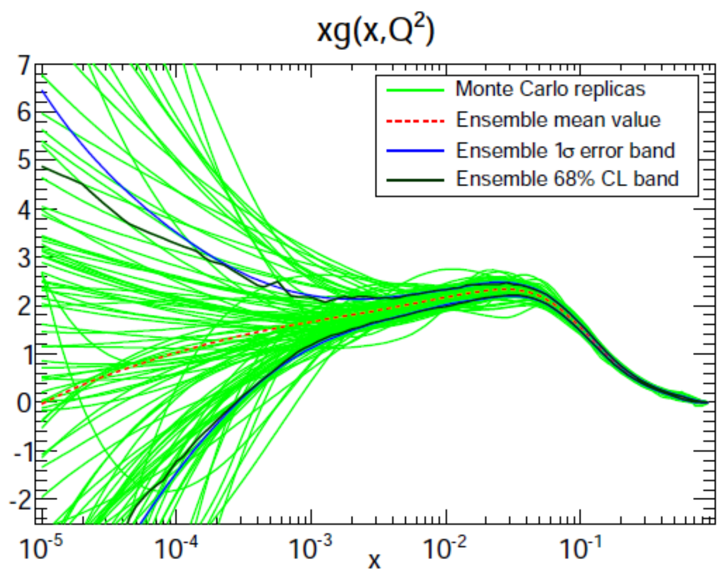
generate a large number of MC replicas of the experimental data according to multi-gaussian probability distribution

→ perform a separate fit using each data replica

Expectation values for any variable $F^{(k)}$ (of k replica) and its uncertainty can be determined directly using

$$\langle F \rangle = \frac{1}{N_{\mathrm{rep}}} \sum_{i}^{N_{\mathrm{rep}}} F^{(k)}$$

$$\sigma^2[F] = \frac{1}{N_{\mathrm{rep}} - 1} \sum_{i=1}^{N_{\mathrm{rep}}} (F^{(k)} - \langle F \rangle)^2$$



Heavy Quark Treatment in PDFs

QCD factorisation:
measured cross section =

PDF | hard-scattering ME

$$\sigma(\alpha_s, \mu_R^2, \mu_F^2) = \sum_{a,b} \int_0^1 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \hat{\sigma}(x_1, x_2; \alpha_s, \mu_R^2, \mu_F^2) + \ldots$$

a, b - partons in the proton (g, q ,qbar) of different flavours

- if #flavours fixed: **Fixed Flavour Number Scheme (FFNS)**

  only light flavours in the proton: i = 3 (4)
  c- (b-) quarks massive, produced in boson-gluon fusion,
  $Q^2 \gg m_{HQ}^2$: can be less precise, NLO coefficients contain terms ~ $ln(Q/m_{HQ})$

- if #flavours variable: **Variable Flavour Number Scheme (VFNS)**

  - Zero Mass VFNS: all flavours massless. Breaks down at $Q^2 \sim m_{HQ}^2$
  - Generalized Mass VFNS: different implementations provided by PDF groups, smooth matching with FFNS for $Q^2 \to m_{HQ}^2$ must be assured

→ m_c is a parameter (M_c)

treatment of heavy quarks is important in PDFs

PDFs on the Market

MMHT/MSTW

→ global PDF set (includes all type of data) with 25 eigenvector pairs (VFNS-TR').
Latest MMHT14 sets in LO, NLO and NNLO EPJC75 (2015) 204, arXiv:1412.3989

CT/CTEQ/CJ

→ global PDF set with 28 eigenvector pairs (VFNS-ACOT).
Latest CT14 sets in LO, NLO and NNLO (90% CL) PRD93, 033006 (2016), arXiv:1506.07443

NNPDFs

→ global PDF set with 100 and 1000 MC replicas (VFNS FONLL-B,C and FFNS) .
Latest NNPDF3.0 sets in LO, NLO and NNLO JHEP 04 (2015) 040, arXiv:1410.8849

HERAPDF

→ HERA (combined) data (VFNS-TR').
Latest HERAPDF2.0 sets in LO, NLO and NNLO EPJC 75 (2015) 12, 580

ABM/ABKM

→ global PDF set (FFNS).
Latest ABM12 sets in NLO and NNLO PRD89 (2014) 054028, arXiv:1310.3059

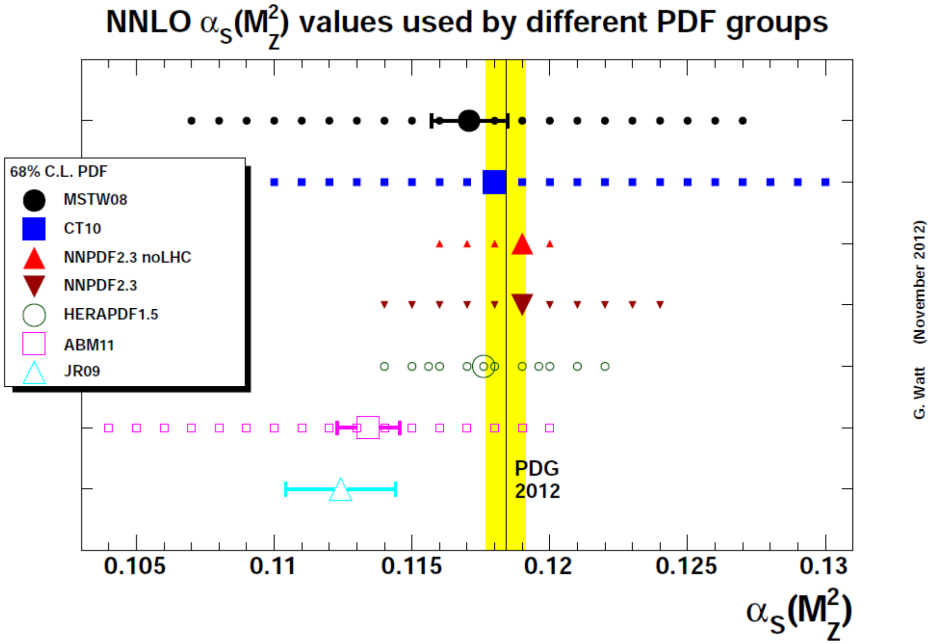
JR/GJR

→ global PDF set (no LHC data) with dynamical approach (FFNS+VFNS).
Latest JR14 sets in NLO and NNLO PRD89 (2014), no. 7 074049, arXiv:1403.1852

PDF Fitting Groups

Main sources of difference between different PDFs:

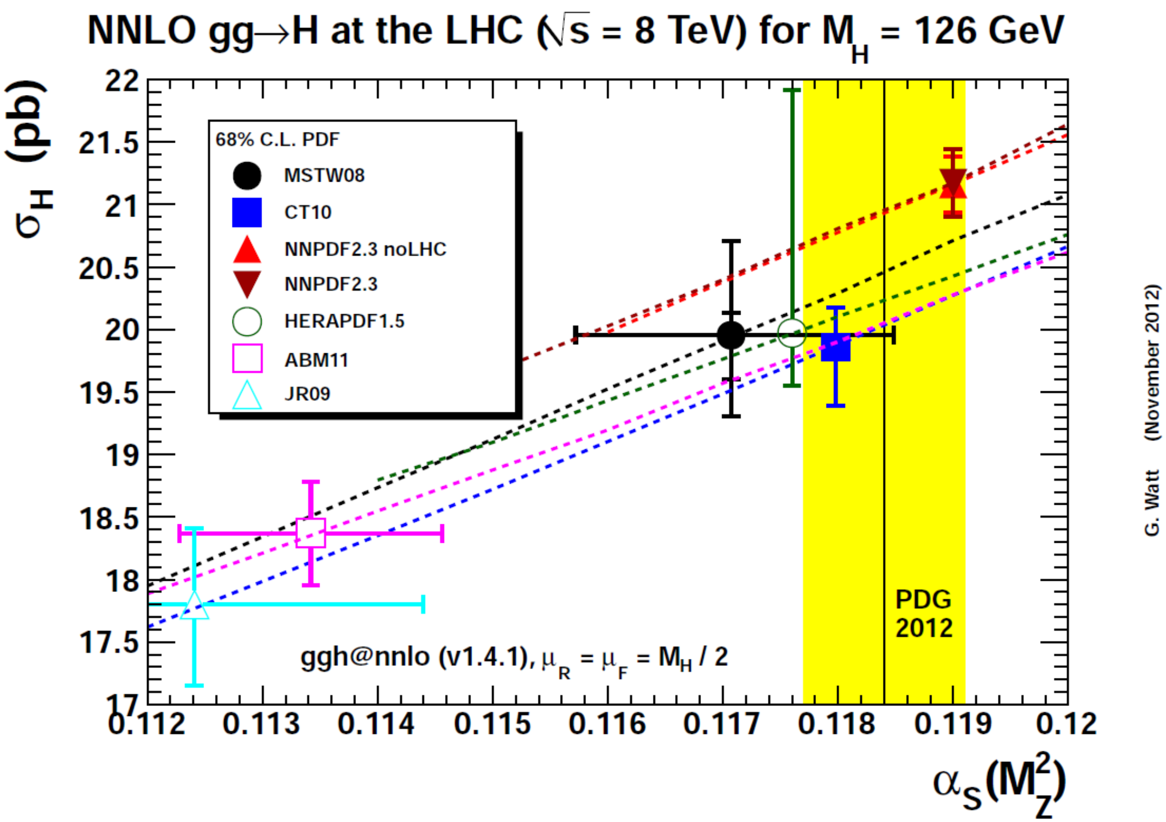
- inclusion of different data
- methods of determining 'best fit'
- uncertainty treatment/sources
- assumptions in procedure (parametrisation)
- heavy flavour treatment
- PDF and α_s correlation



... lead to differences in the cross section predictions

PDF Fit Groups: Benchmarking

Different PDF lead to differences in cross section predictions

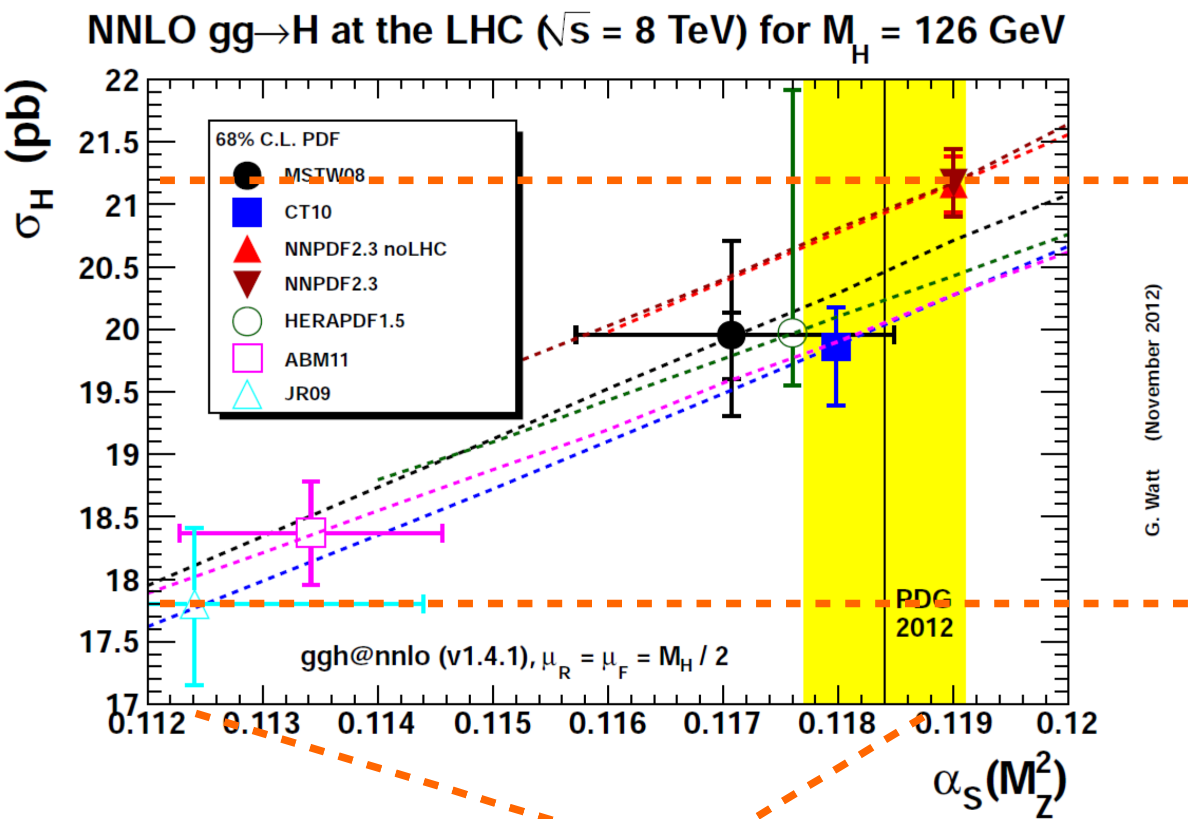


G.Watt

arXiv:1301.6754

PDF Fit Groups: Benchmarking

Different PDF lead to differences in cross section predictions



large spread in cross section predictions

different uncertainty sizes

difference in α_s

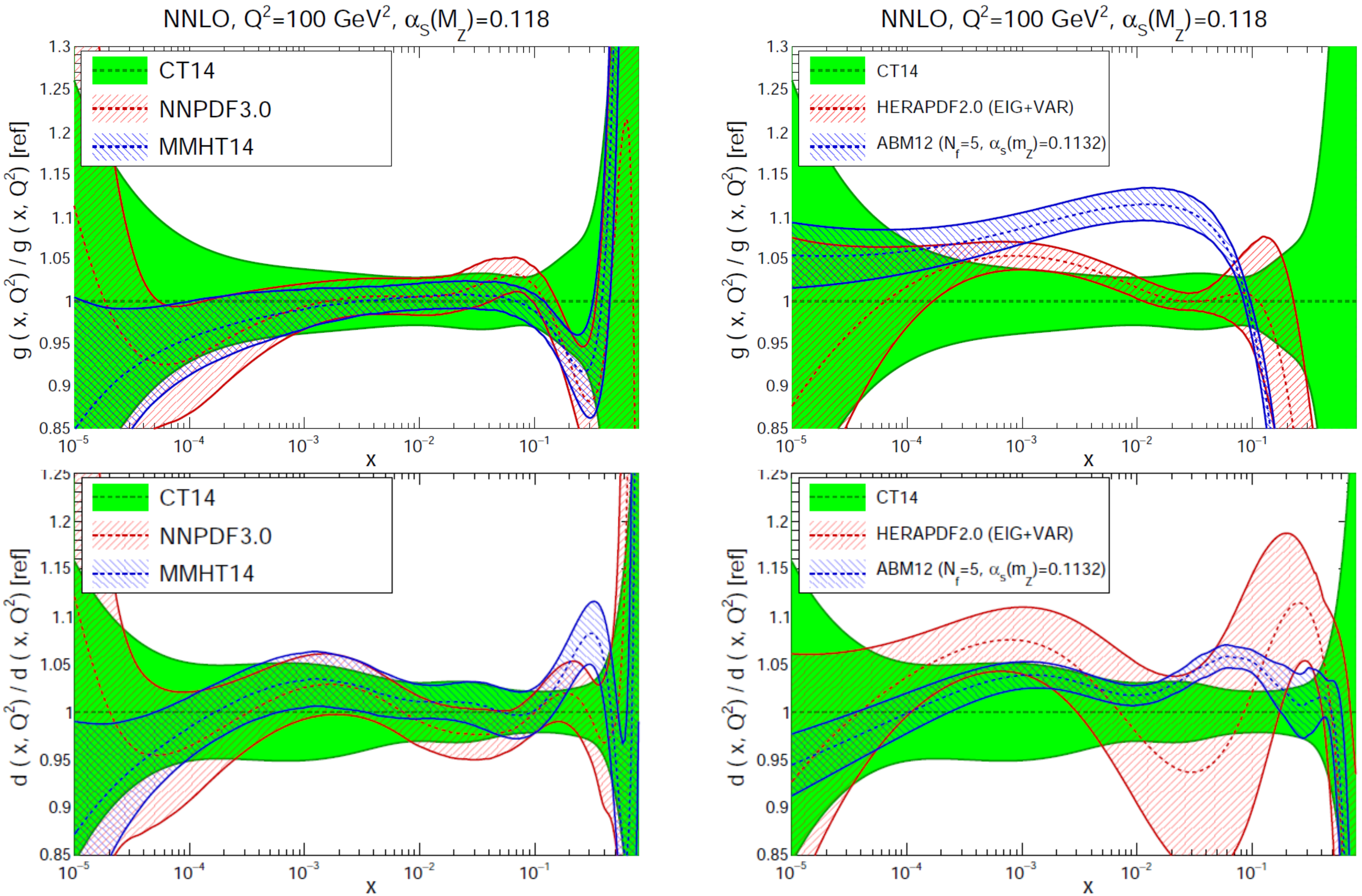
G.Watt
arXiv:1301.6754

→ *similar situation with ttbar predictions, better for W,Z*

PDF Fit Groups: Benchmarking

Different PDF lead to differences in cross section predictions

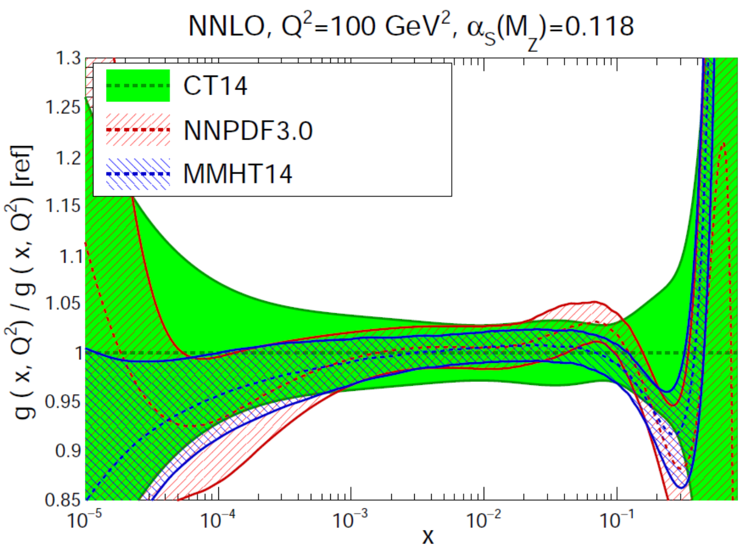
arXiv:1510.03865

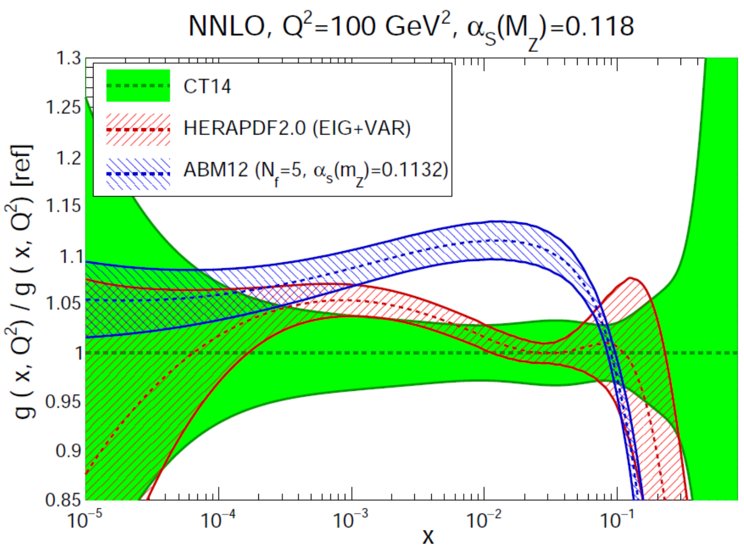


PDF Fit Groups: Benchmarking

Different PDF lead to differences in cross section predictions

arXiv:1510.03865





→ many more measurements to come
→ many theory calculations for higher order predictions are ongoing

→ collaborative theory and experiment efforts!

... which require flexible tools

PDF Tools

Often perturbative higher-order calculations are extremely time consuming
→ not possible to include into PDF fits

solution: fast grid techniques

→ based on assumption that PDF can be approximated by a set of the interpolation functions

→ after first time (full) calculation, technique with interpolation functions can be used for the fast theory prediction calculations (for any PDF)

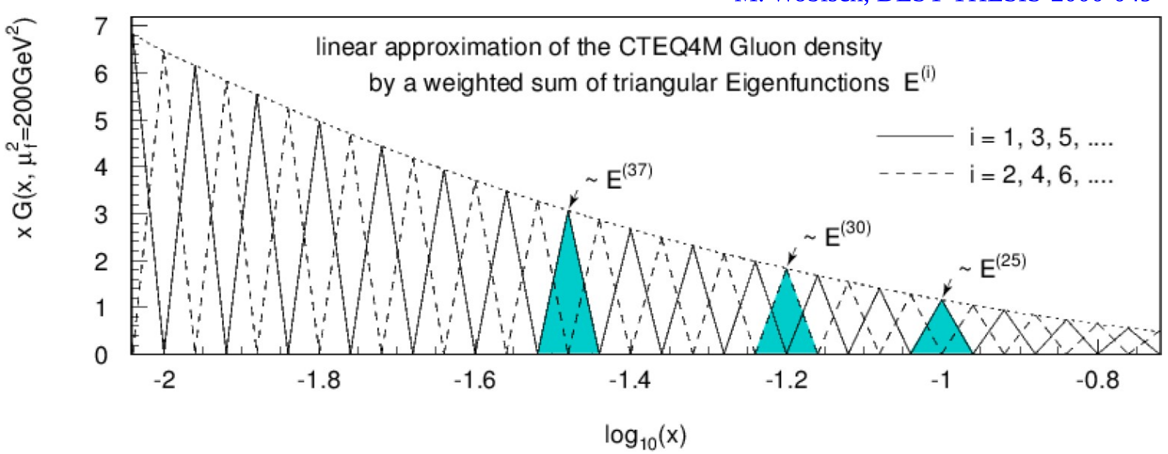
Currently available tools: FastNLO Eur.Phys.J. C19 , 289 (2001), hepph/0609285

and APPLGRID hepph/0510324, arXiv:0911.2985

PDFs ($f_{a/h}$) approximated by linear combination of the eigenfunctions $E^{(i)}$:

$$f_{a/h}(x) \simeq \sum_i f_{a/h}(x_i)\ E^{(i)}(x)$$

M. Wobisch, DESY-THESIS-2000-049



PDF Tools

Available (open-source) tools for the PDF determination:

OPENQCDRAD (ABM collaboration: numerical computation of all hard scattering cross sections (DIS structure function calculation including heavy quark contributions, W and Z production) PRD86 (2012) 054009, *www-zeuthen.desy.de/˜alekhin/OPENQCDRAD*

APFEL (NNPDF collaboration): a PDF evolution library, is a computer library specialized in the solution of DGLAP evolution equations up to NNLO in QCD and to LO in QED
arXiv.1310.1394, *apfel.hepforge.org*

xFitter (former HERAFitter): an open-source package that provides a framework for the determination of the PDFs of the proton and for many different kinds of analyses in QCD EPJC (2015), 75: 304, *xfitter.org*

ALPOS: an object-oriented data to theory comparison and fitting tool (profit from and exchange with xFitter experience) http://desy.de/~britzger/alpos/
→ access from a public svn repository (via request)

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→ access from a public svn repository (via request)

xFitter Project

www.xfitter.org

xFitter project is based on a multi-functional open source QCD software package that provides a framework for scrupulous interpretations of the QCD analyses

Schematic overview of xFitter program

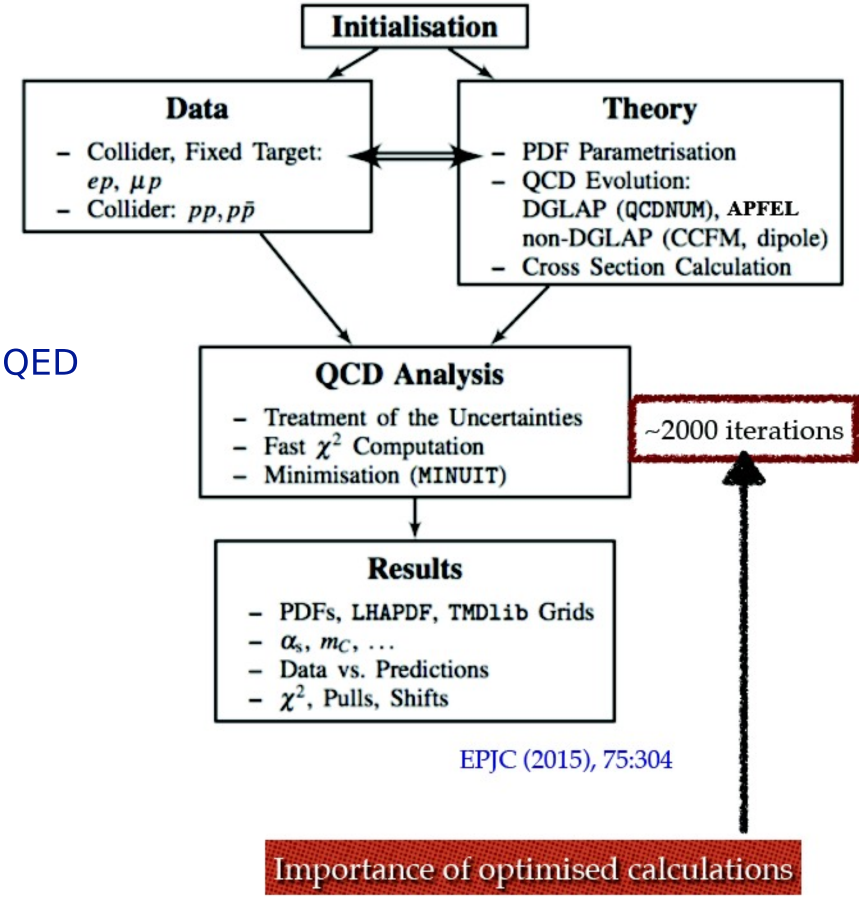
Parametrise PDFs at the starting scale
→ multiple options for functional forms
→ Standard Polynomial, Chebyshev, etc

Evolve to the scale corresponding to data point
→ QCD(DGLAP) evolution codes [QCDNUM, APFEL]
→ kt ordered evolution, dipole models, QCD(DGLAP)+QED

Calculate the cross section
→ various heavy flavour schemes:
→ RT, ACOT, FONLL, FFNS(ABM)
→ fast grid techniques interfaced to DY:
→ APPLGRID, FASTNLO

Compare with data via χ^2:
→ multiple forms to account for correlations

Minimize χ^2 with respect to PDF parameters
→ profiling, reweighting
→ fit: MINUIT, data driven regularisation

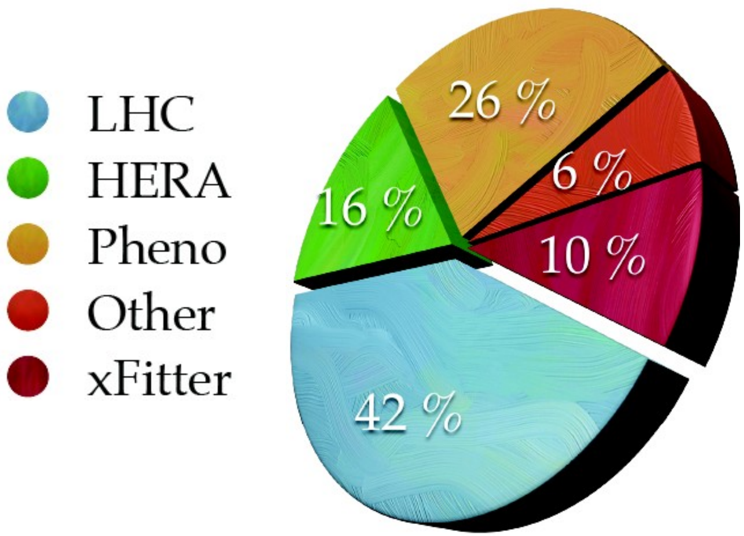


[S. Camarda, xFitter tutorial]

Results Obtained with xFitter

More than **30 public results** obtained using xFitter from the beginning of the project

https://www.xfitter.org/xFitter/xFitter/results



LHC experiments provide the main developments and usage of the xFitter platform

xFitter publications:

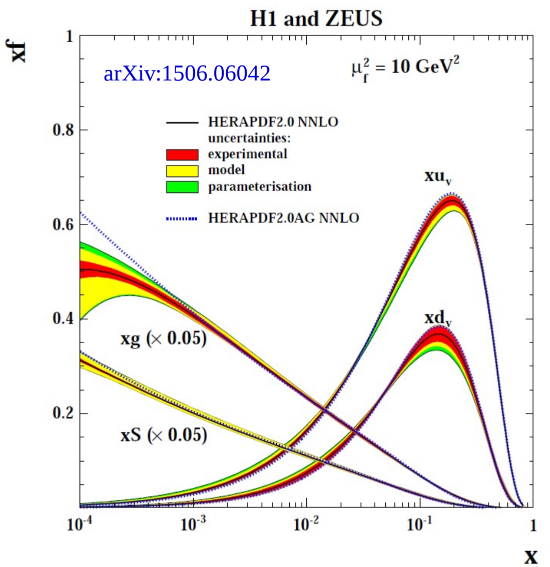
03.2016	xFitter and APFEL teams and A. Geiser	arXiv:1605.01946	A determination of mc(mc) from HERA data using a matched heavy flavor scheme

List of analyses using HERAFitter

NEW 03.2015	**HERAFitter team**	EPJC 75 (2015) 9, 458, arXiv:1503.05221	QCD analysis of W- and Z-boson production at Tevatron
10.2014	**HERAFitter team**	EPJC (2015), 75: 304, arXiv:1410.4412	HERAFitter Open Source QCD Fit Project
04.2014	**HERAFitter team**	EPJC (2014) 74: 3039, arXiv:1404.4234	Parton distribution functions at LO, NLO and NNLO with correlated uncertainties between orders

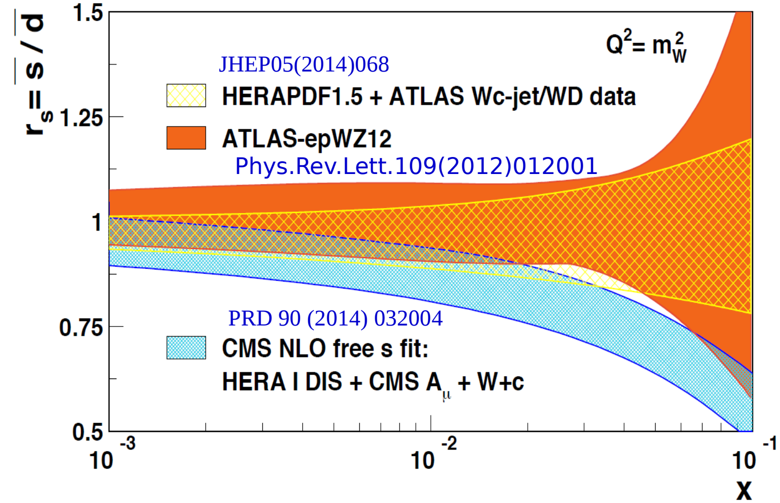
Results Obtained with xFitter: Examples

DIS inclusive processes in *ep* (fixed target)

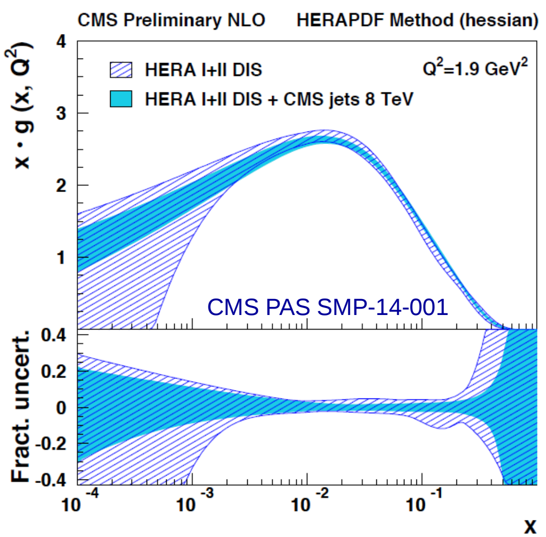


Drell-Yan processes (*pp, ppbar*)

→ strange quark density determination

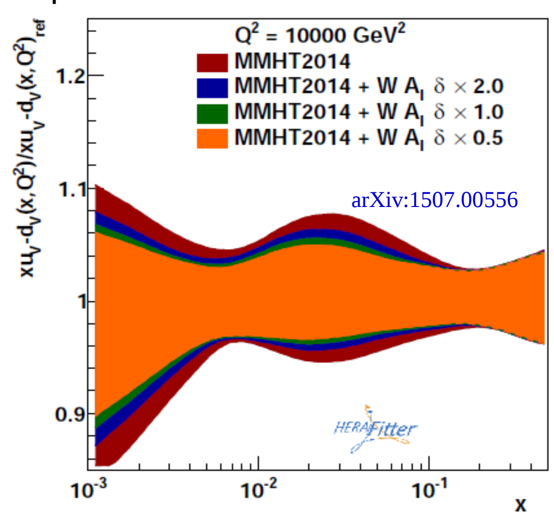


Jet production (*ep, pp, ppbar*)



PDF4LHC report (benchmarking)

→ impact of 13 TeV data



THANK YOU

xFitter tutorial (S. Camarda) after lunch today

Back-Up Slides

Data used in PDFs

PDF groups use different experimental data in the fits

→ changes in time for CT, MMHT, ABM and NNPDF PDFs

	2008		2009		2010		2011
SET MONTH	CT6.6 (02)	NN1.0 (08)	MSTW (01)	ABKM09 (08)	NN2.0 (02)	CT10(N) (07)	NN2.1(NN) (07)
F. T. DIS	✔	✔	✔	✔	✔	✔	✔
ZEUS+H1-HI	✔	✔	✔	✔	✔	✔	✔
COMB. HI	✗	✗	✗	✗	✔	✗	✔
ZEUS+H1-HII	✗	✗	✗	✗	✗	✗	some
HERA JETS	✗	✗	✔	✗	✗	✗	✗
F. T. DY	✔	✗	✔	✔	✔	✔	✔
TEV. W+Z	✔	✗	✔	✗	✔	✔	✔
TEV. JETS	✔	✗	✔	✗	✔	✔	✗
LHC W+Z	✗	✗	✗	✗	✗	✗	✗
LHC JETS	✗	✗	✗	✗	✗	✗	✗
TOP	✗	✗	✗	✗	✗	✗	✗
W+C	✗	✗	✗	✗	✗	✗	✗
W p_T	✗	✗	✗	✗	✗	✗	✗

	2012		2013		2014		2015
SET MONTH	ABM11 (02)	NN2.3 (07)	CT10(NN) (02)	ABM12 (10)	NN3.0 (10)	MMHT (12)	CT14 (06)
F. T. DIS	✔	✔	✔	✔	✔	✔	✔
ZEUS+H1-HI	✔	✔	✔	✔	✔	✔	✔
COMB. HI	✗	✔	✗	✔	✔	✗	✗
ZEUS+H1-HII	✗	✗	some	✗	✔	✗	✗
HERA JETS	✗	✗	✗	✗	✗	✔	✗
F. T. DY	✔	✔	✔	✔	✔	✔	✔
TEV. W+Z	✗	✔	✔		✔	✔	✔
TEV. JETS	✔	✔	✔	✗	✔	✔	✔
LHC W+Z	✗	✔	✗	some	✔	✔	✔
LHC JETS	✗	✔	✗	✗	✔	✔	✔
TOP	✗	✗	✗	✔	✔	✗	✗
W+C	✗	✗	✗	✗	✔	✗	✗
W p_T	✗	✗	✗	✗	✔	✗	✗

S. Forte

LHC Processes Sensitive to PDFs

LHC data
→ can help to discriminate between PDF sets
→ can help to improve PDFs

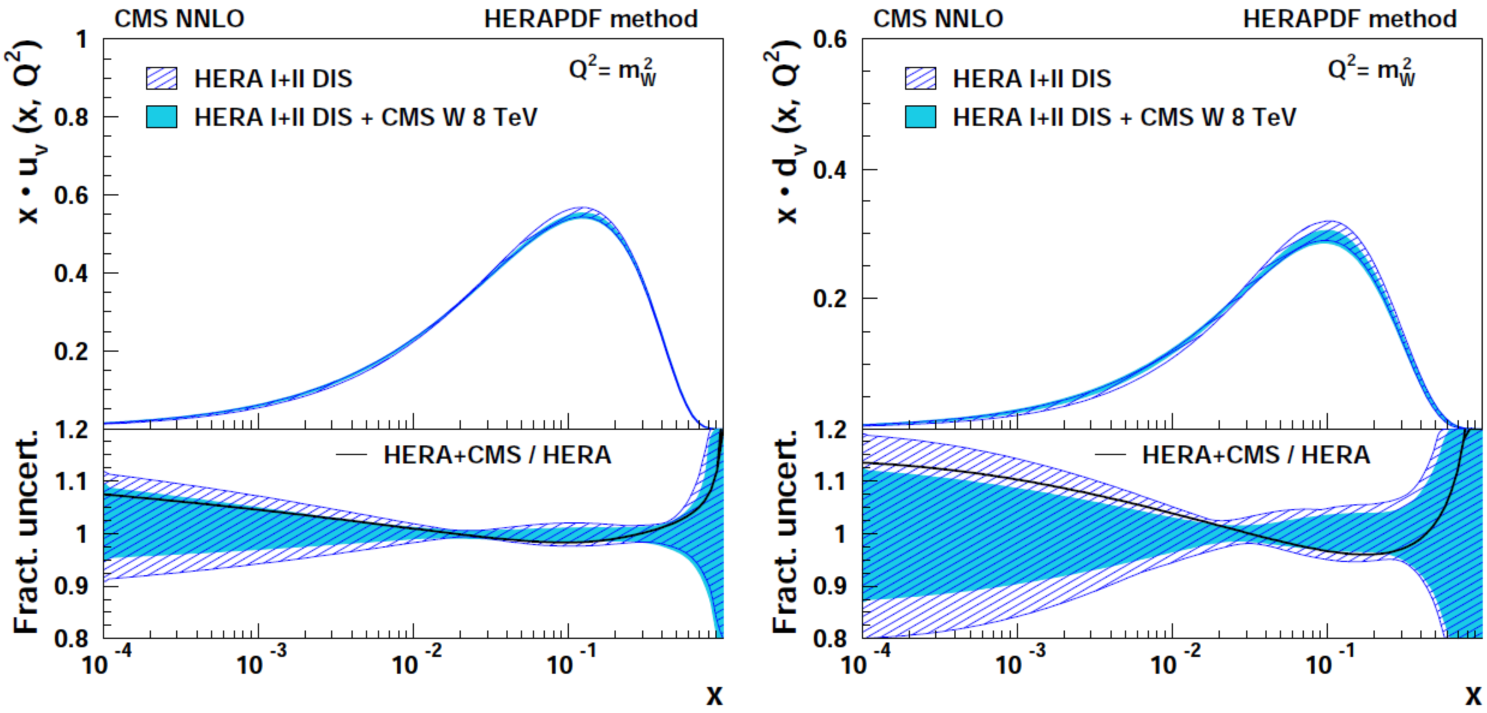
process	**sensitivity to PDFs**
W asymmetry	→ quark flavour separation
W and Z production (differential)	→ valence quarks
W+c production	→ strange quark
Drell-Yan (DY): high invariant mass	→ sea quarks, high-x
Drell-Yan (DY): low invariant mass	→ low-x
W,Z +jets	→ gluon medium-x
Inclusive jet and di-jet production	→ gluon and $\alpha_s(M_Z)$
Direct photon	→ gluon medium, high-x
ttbar, single top	→ gluon and $\alpha_s(M_Z)$

W Asymmetry

CMS W muon charge asymmetry measurement: QCD analysis at NLO

with HERA I+2 combined DIS data EPJC 75 (2015) 12, 580

arXiv:1603.01803



error bands represent total uncertainties, (experimental, model and parametrisation uncertainties)

Change of PDF shape, improved constraints on the valence distributions

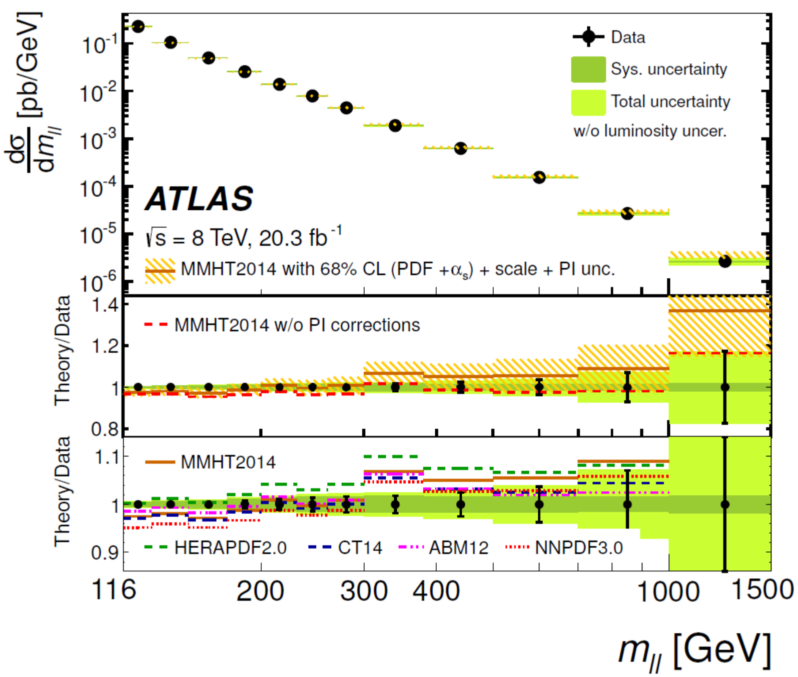
W and Z production (low-mass, high-mass)

Drell Yan data mass spectra:

→ high-mass: sensitive to sea quarks at high-x (and thus to new physics at high-scale)
→ low-mass: similar at the low-x region
(EW corrections are important)

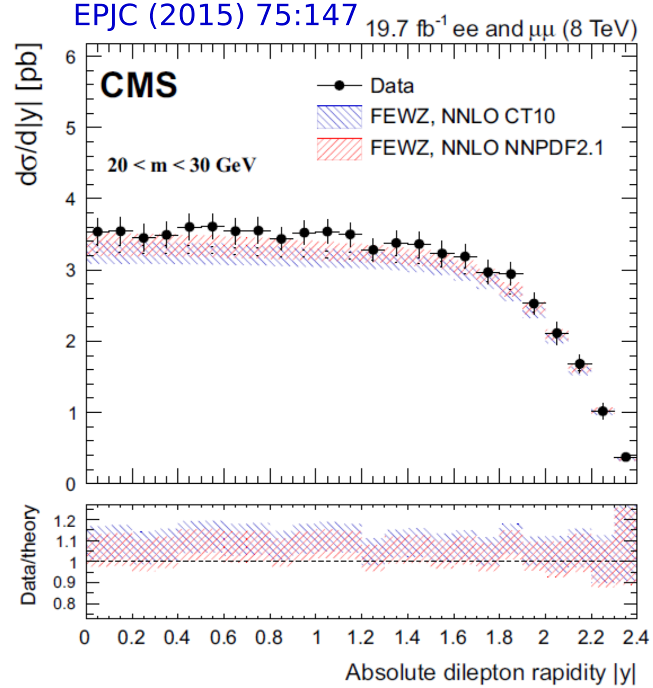
ATLAS DY high-mass measurement

arXiv:1606.01736



CMS DY from low- to high-mass measurement in P_T and Y bins

EPJC (2015) 75:147



These LHC measurements are already being included into global PDF fits

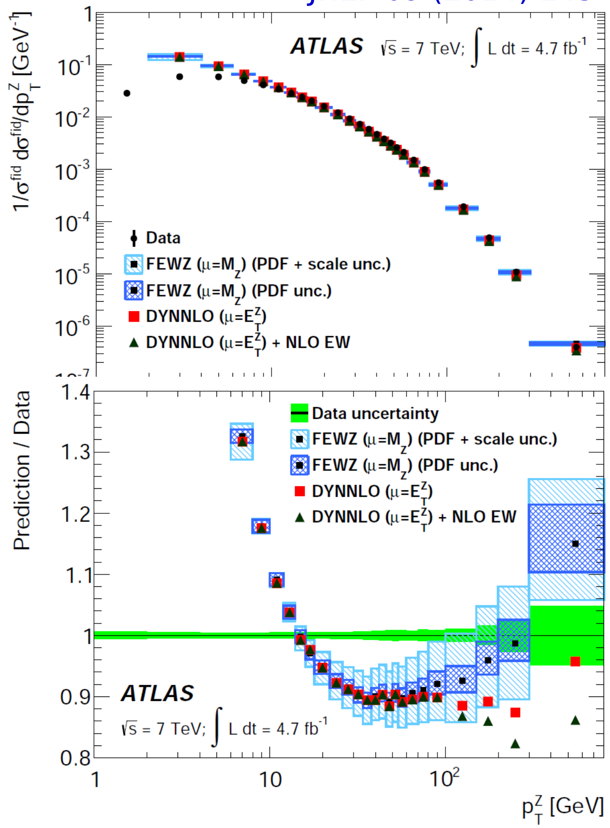
W and Z production (P_T)

ATLAS and CMS have also studied the P_T spectrum in Z rapidity bins

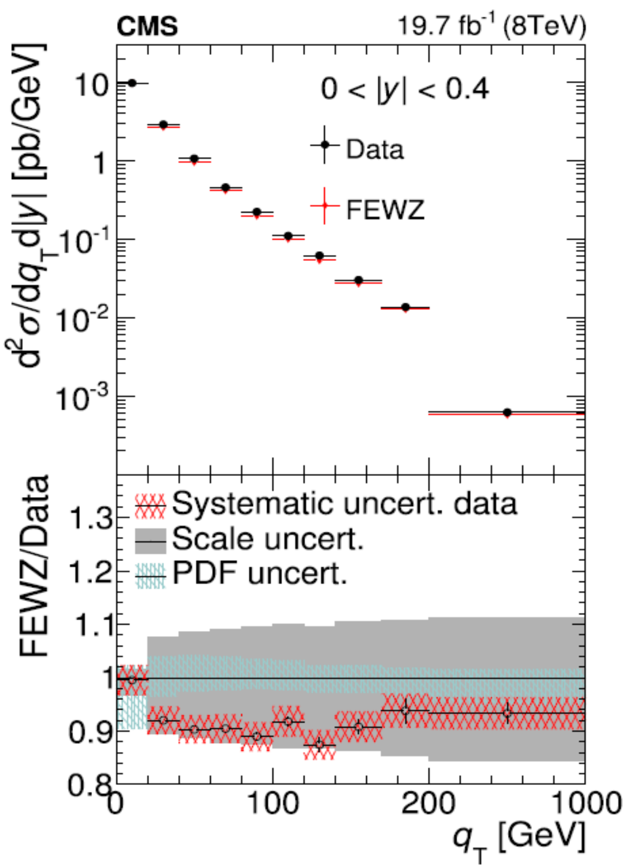
→ low P_T region dominated by the emission of soft partons (resummation and shower models)

→ high P_T region: quark-gluon scattering (PDFs)

JHEP 09 (2014) 145



PLB 749 (2015) 187



Valuable data for various purposes (e.g. W mass, PDFs), currently limited by precision in theory

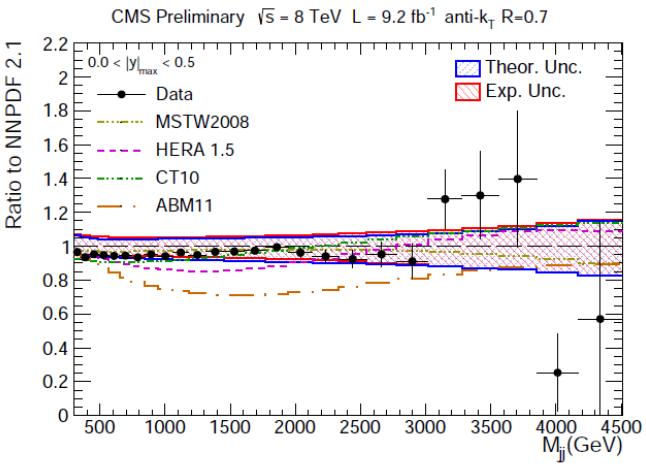
Di- and three-jet Measurements

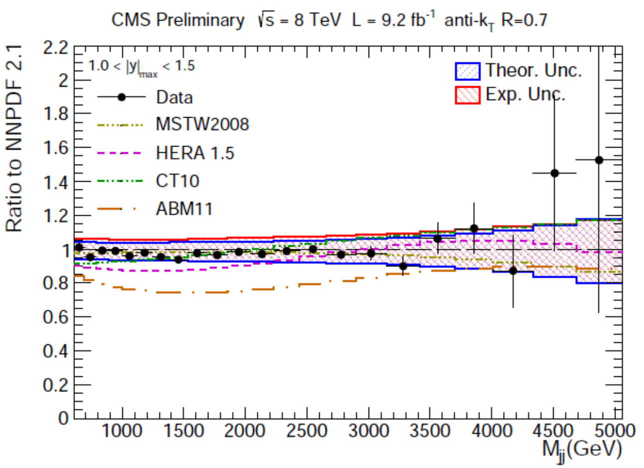
Recent di-jet and three-jet measurements from ATLAS and CMS

→ comparison with different PDFs: some tension with e.g. ABM11 PDF observed

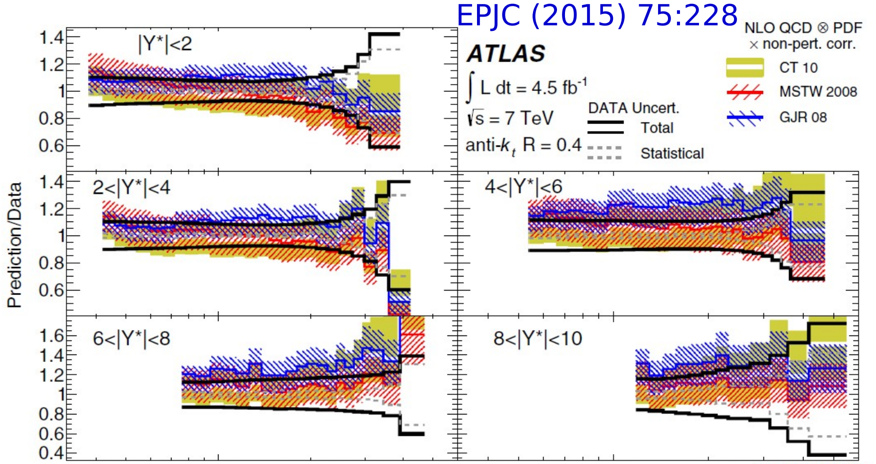
Three-jet (probe different phase space due to different combination of the initial-state partons)

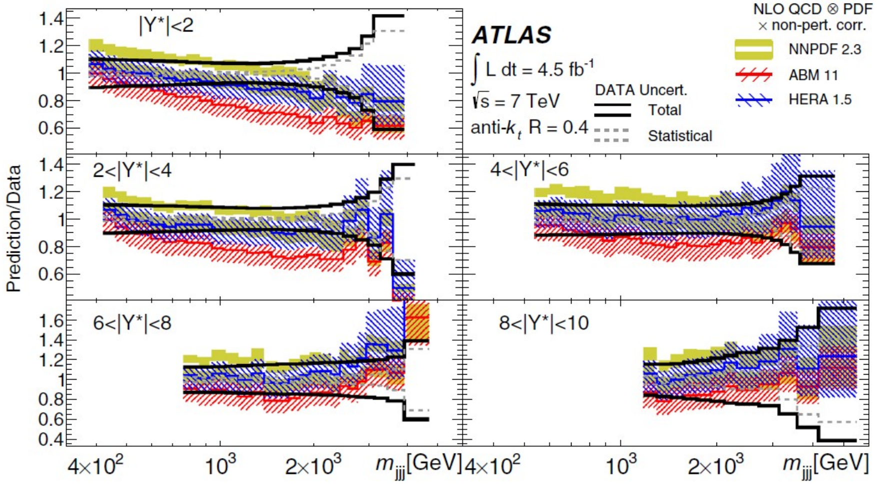
Di-jet CMS PAS SMP 14-002





EPJC (2015) 75:228

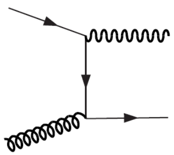




Prompt Photon Production at LHC

Prompt photon data at LHC is sensitive to gluon content at high x

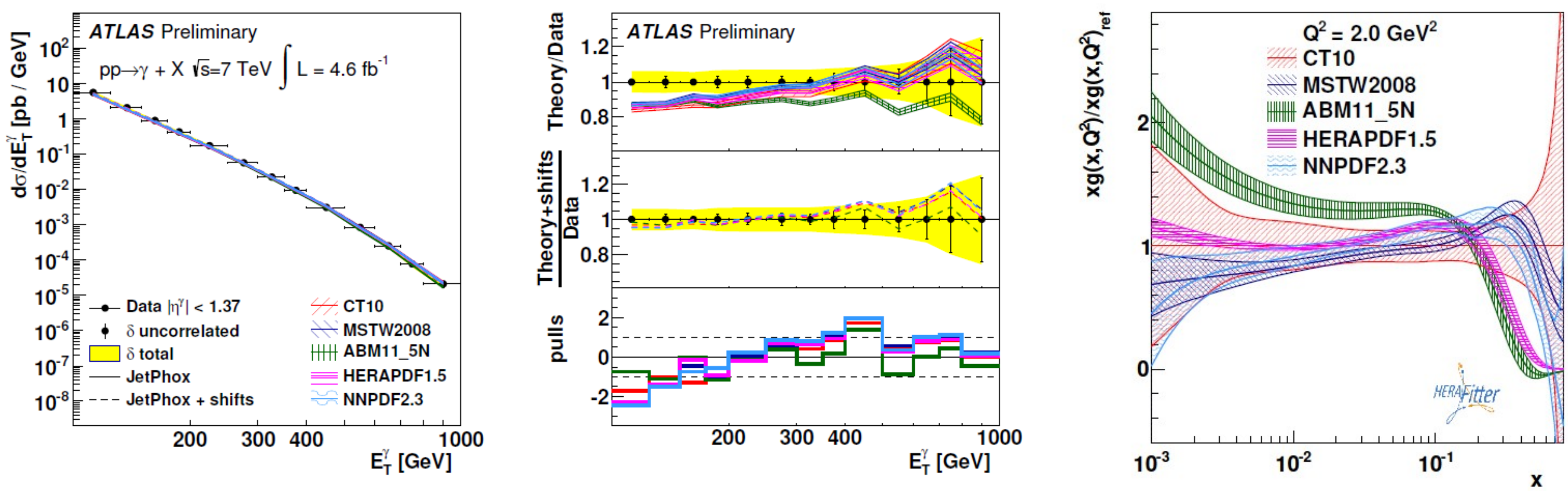
→ dominantly via Compton-like process qg->qγ



ATLAS study of the inclusive photon data sensitivity to parton distributions

→ quantitative data to theory assessment (χ^2)

ATL-PHYS-PUB-2013-018



→ large differences observed with theory (NLO) using different PDFs

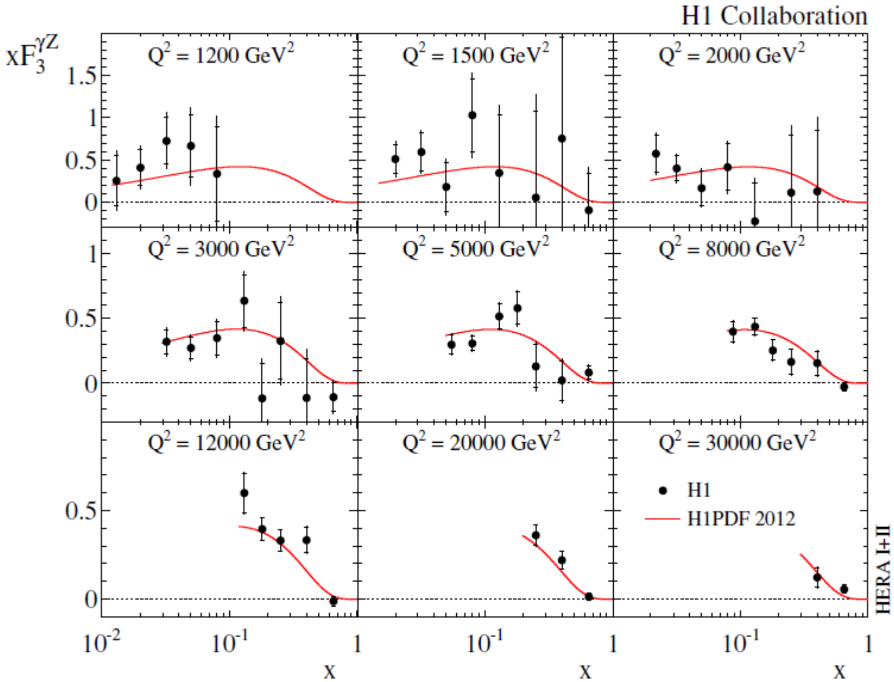
→ data show potential to improve gluon distribution (currently limited by scale uncertainty)

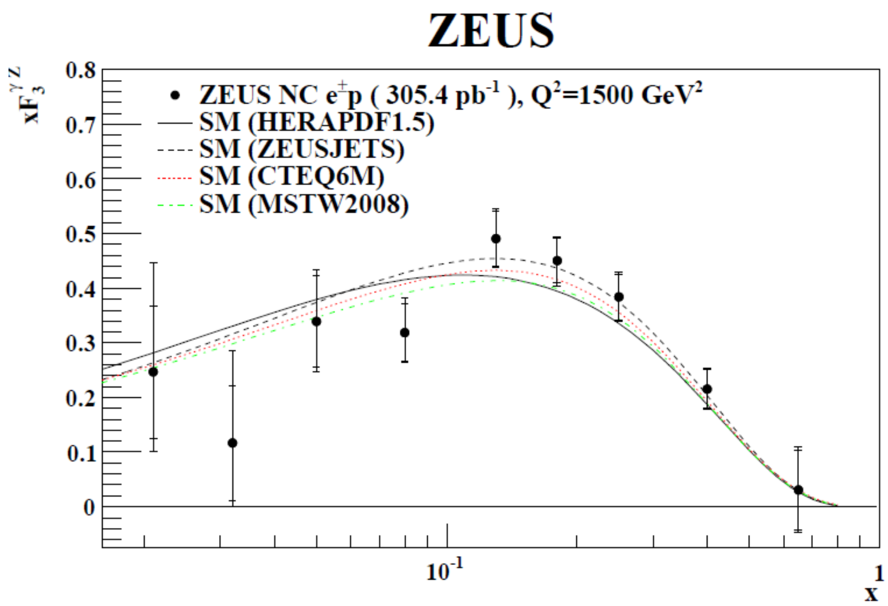
High Q^2 Neutral Current Cross Sections

Structure function xF_3

$$\boxed{x\tilde{F}_3 = \frac{Y_+}{2Y_-}[\tilde{\sigma}^- - \tilde{\sigma}^+]}$$

dominant contribution from $xF_3^{\gamma Z}$ $\quad x\tilde{F}_3^{\gamma Z} \simeq x\tilde{F}_3 \frac{(Q^2 + M_Z^2)}{\alpha^2 \kappa Q^2}$





$xF_3^{\gamma Z} \sim xq_{val}$

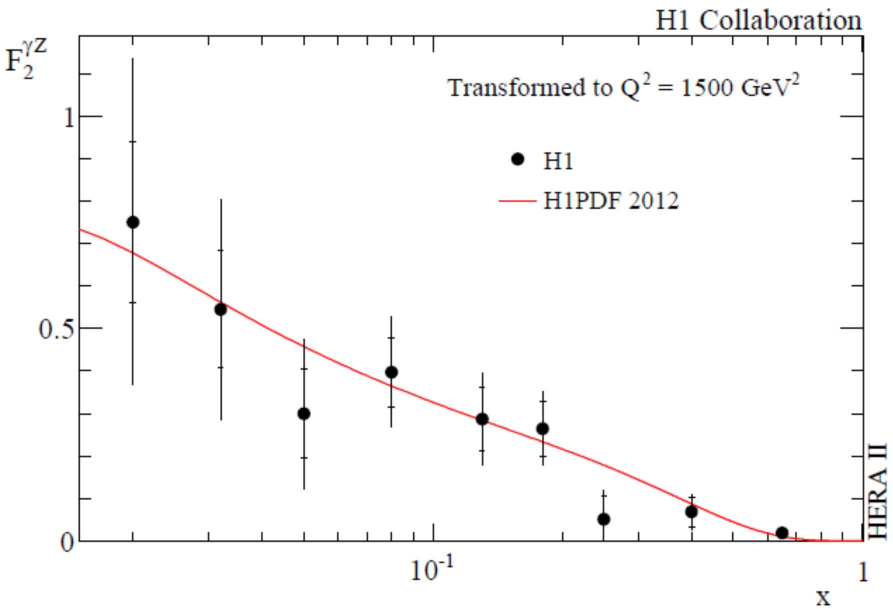
High Q^2 Neutral Current Cross Sections

Measuring the difference in NC polarised cross sections $F_2^{\gamma Z}$ can be accessed:

$$\frac{\sigma^{\pm}(P_L^{\pm}) - \sigma^{\pm}(P_R^{\pm})}{P_L^{\pm} - P_R^{\pm}} = \frac{\kappa Q^2}{Q^2 + M_Z^2}\left[\mp a_e F_2^{\gamma Z} + \frac{Y_-}{Y_+} v_e x F_3^{\gamma Z} - \frac{Y_-}{Y_+}\frac{\kappa Q^2}{Q^2 + M_Z^2}(v_e^2 + a_e^2) x F_3^Z\right]$$

$F_2^{\gamma Z}$ measurement is only possible because of e^+ and e^-, L and R data

First measurement of $F_2^{\gamma Z}$



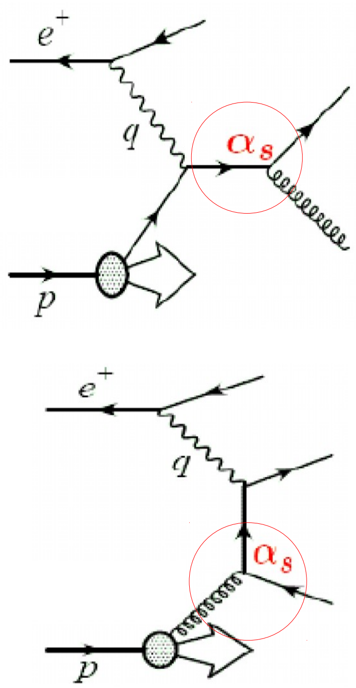
$F_2^{\gamma Z} \sim q+\bar{q}$

PDFs from HERA

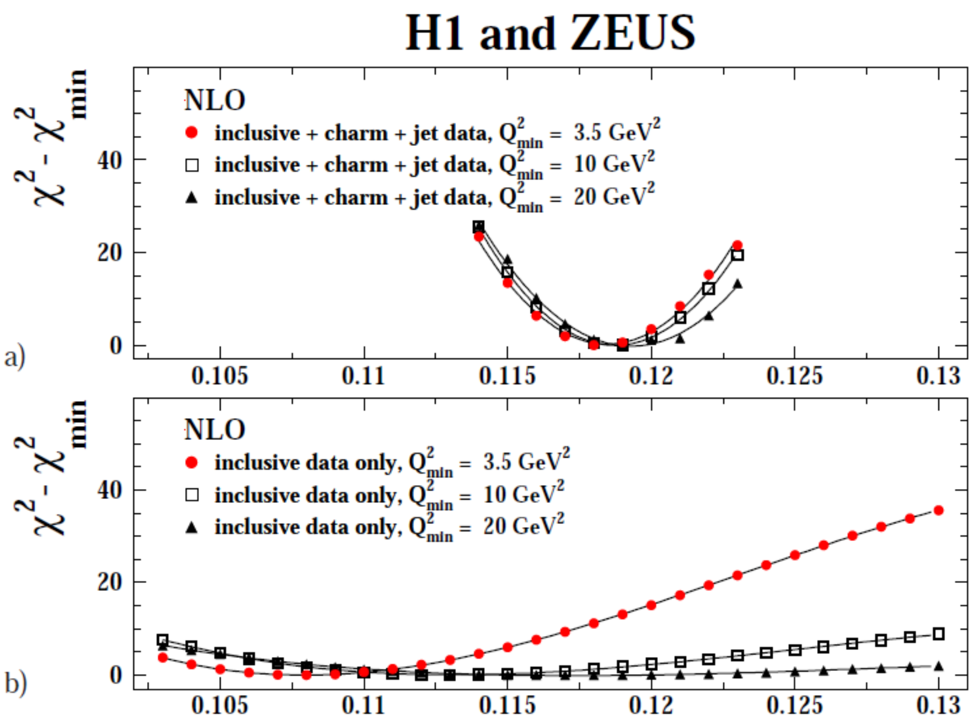
arXiv:1506.06042

HERA heavy quark and jet data bring additional sensitivity to PDFs

LO jet production in DIS:



→ direct sensitivity to gluon and strong coupling constant



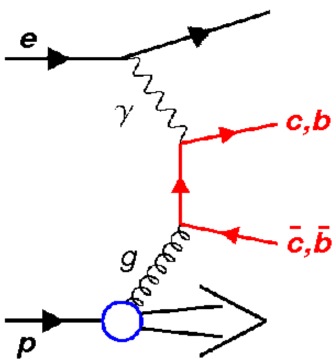
$\alpha_s(M_Z)=0.1183 \pm 0.0009(\text{exp})$
$\pm\, 0.0005(\text{mod}) \pm 0.0012(\text{had})\, ^{+0.0037}_{-0.0030}(\text{th})$

PDFs from HERA

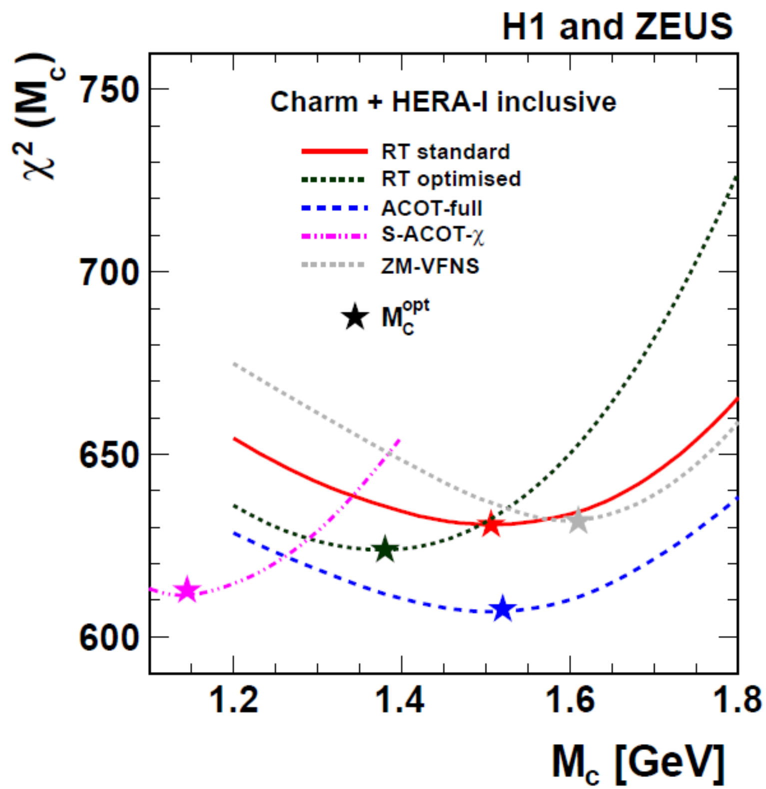
Eur.Phys. J. C73 (2012), 2311

HERA heavy quark and jet data bring additional sensitivity to PDFs

LO charm (boson-gluon-fusion) production in DIS:



→ direct sensitivity to gluon



HERA charm and beauty data provide

- → stringent tests of heavy quark treatment in PDFs
- → significant constrain on heavy quark mass

Different schemes prefer different M_C

QCD Analysis of HERA Charm Data

Eur.Phys. J. C73 (2012), 2311

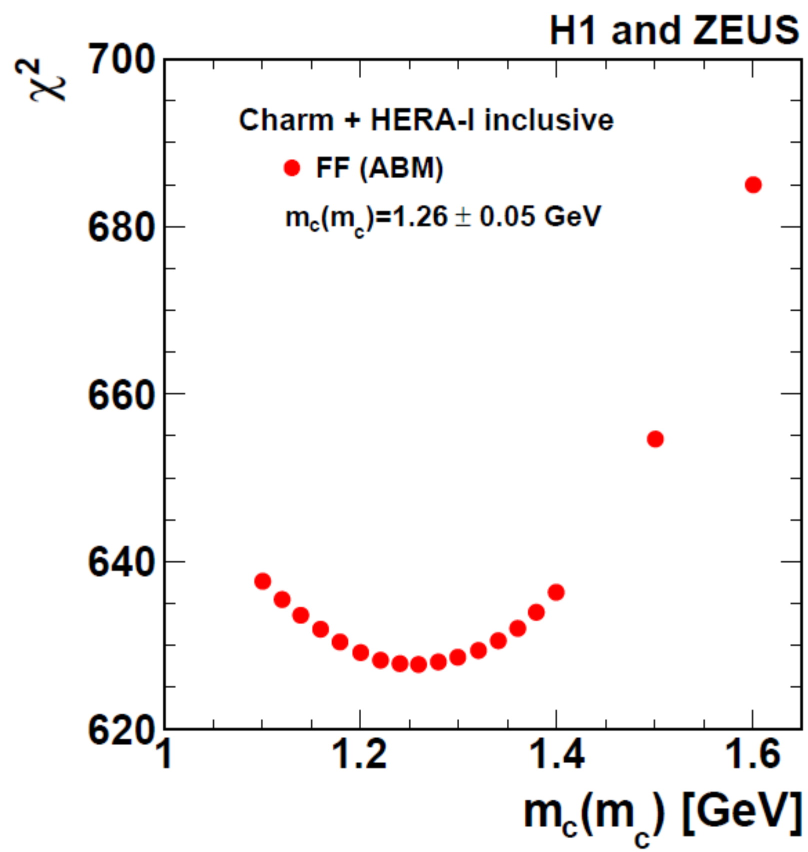
In VFN schemes the charm quark mass parameter M_c does not correspond directly to a physical mass

→ not the case for Fixed-Flavour Number Scheme (FFNS)

An NLO QCD analysis in the FFNS (FFNS of ABM, arXiv:1011.5790) performed to determine the MSbar running charm quark mass $m_c(m_c)$

$$m_c(m_c) = 1.26 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.02_{\text{param}} \pm 0.02_{\alpha_s}\ \text{GeV}$$

consistent with the world average of $m_c(m_c) = 1.275 \pm 0.025$ GeV

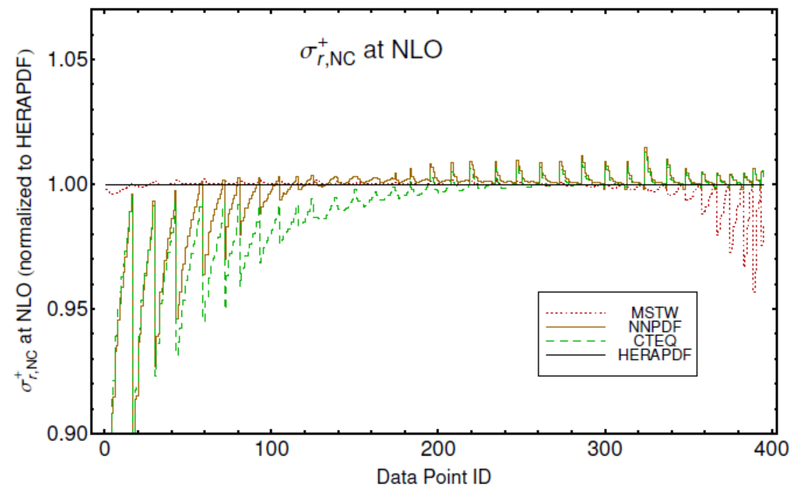


Bencharking and Future Colliders

Various benchmarking studies

→ xFitter provided unique possibility to preform PDF related studies under the same conditions

arXiv:1405.1067



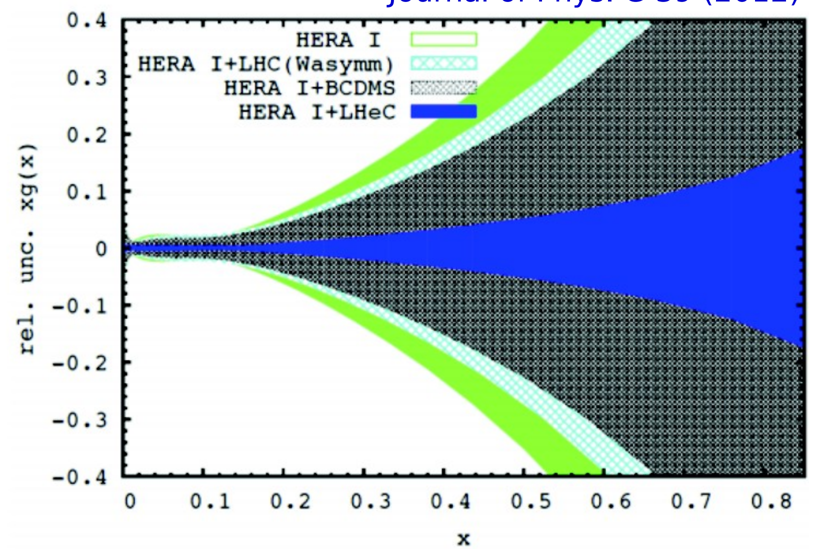
"Les Houches 2013: Physics at TeV Colliders Standard Model Working Group Report":

→ benchmark studies provide comparison of cross sections with LHC data from Run 1 and projections for future measurements in Run 2

Impact on PDF studies at LHeC

→ possibility to perform impact studies using simulated data

Journal of Phys. G 39 (2012)



per-mille accuracy on alphas:

case	cut [Q^2 in GeV	relative precision in %
HERA only (14p)	$Q^2 > 3.5$	1.94
HERA+jets (14p)	$Q^2 > 3.5$	0.82
LHeC only (14p)	$Q^2 > 3.5$	0.15
LHeC only (10p)	$Q^2 > 3.5$	0.17
LHeC only (14p)	$Q^2 > 20.$	0.25
LHeC+HERA (10p)	$Q^2 > 3.5$	0.11
LHeC+HERA (10p)	$Q^2 > 7.0$	0.20
LHeC+HERA (10p)	$Q^2 > 10.$	0.26