Matching & Merging
In Parton Shower Event Generators

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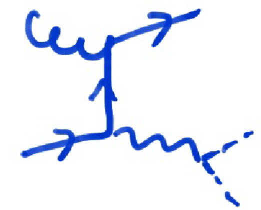
Part I
Basics & NLO Matching

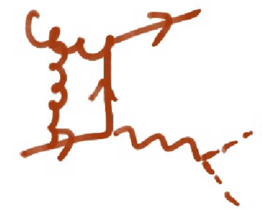
Part II
(N)LO multijet merging & combining with NNLO

Cross Sections at NLO QCD

$$\left.\frac{d\sigma}{dx}\right|_{LO} = \int_m d\sigma_{Born}(\phi_m)\,\delta\left(x - \hat{x}(\phi_m)\right)$$



$$\left.\frac{d\sigma}{dx}\right|_{NLO} = \int_m d\sigma_{Virtual}(\phi_m)\,\delta\left(x - \hat{x}(\phi_m)\right)$$



$$+ \int_{m+1} d\sigma_{Real}(\phi_{m+1})\,\delta\left(x - \hat{x}(\phi_{m+1})\right)$$

Infrared divergences cancel between virtual and real contributions.
Ultraviolet divergences in loop graphs removed by renormalization.

Fighting the Infrared Mess: The Subtraction Formalism

(Renormalized) virtual contributions in dimensional regularization:

→ poles in ε from soft and/or collinear to external loop momenta.

Real contributions divergent for soft/collinear emission:

→ poles in ε after phase space integration.

$$\left.\frac{d\sigma}{dx}\right|_{NLO} = \int_m \left[d\sigma_{Virtual}(\phi_m) + \int_1 d\sigma_{Sub}(\phi_{m+1}) \right]_{\varepsilon=0} \delta(x - \hat{x}(\phi_m))$$

$$+ \int_{m+1} \left[d\sigma_{Real}(\phi_{m+1})_{\varepsilon=0}\, \delta(x - \hat{x}(\phi_{m+1})) - d\sigma_{Sub}(\phi_{m+1})_{\varepsilon=0}\, \delta(x - \hat{x}(\phi_m)) \right]$$

Use subtraction terms to handle divergences.
Cannot generate 'events' from NLO cross section; real and subtraction
Term kinematics highly correlated.

Infrared Safety

$$\left.\frac{d\sigma}{dx}\right|_{NLO} = \int_m \left[d\sigma_{\text{Virtual}}(\Phi_m) + \int_1 d\sigma_{\text{Sub}}(\Phi_{m+1}) \right]_{\varepsilon=0} \delta\left(x - \hat{x}(\Phi_m)\right)$$

$$+ \int_{m+1} \Big[d\sigma_{\text{Real}}(\Phi_{m+1})_{\varepsilon=0}\, \delta\left(x - \hat{x}(\Phi_{m+1})\right) - d\sigma_{\text{Sub}}(\Phi_{m+1})_{\varepsilon=0}\, \delta\left(x - \hat{x}(\Phi_m)\right) \Big]$$

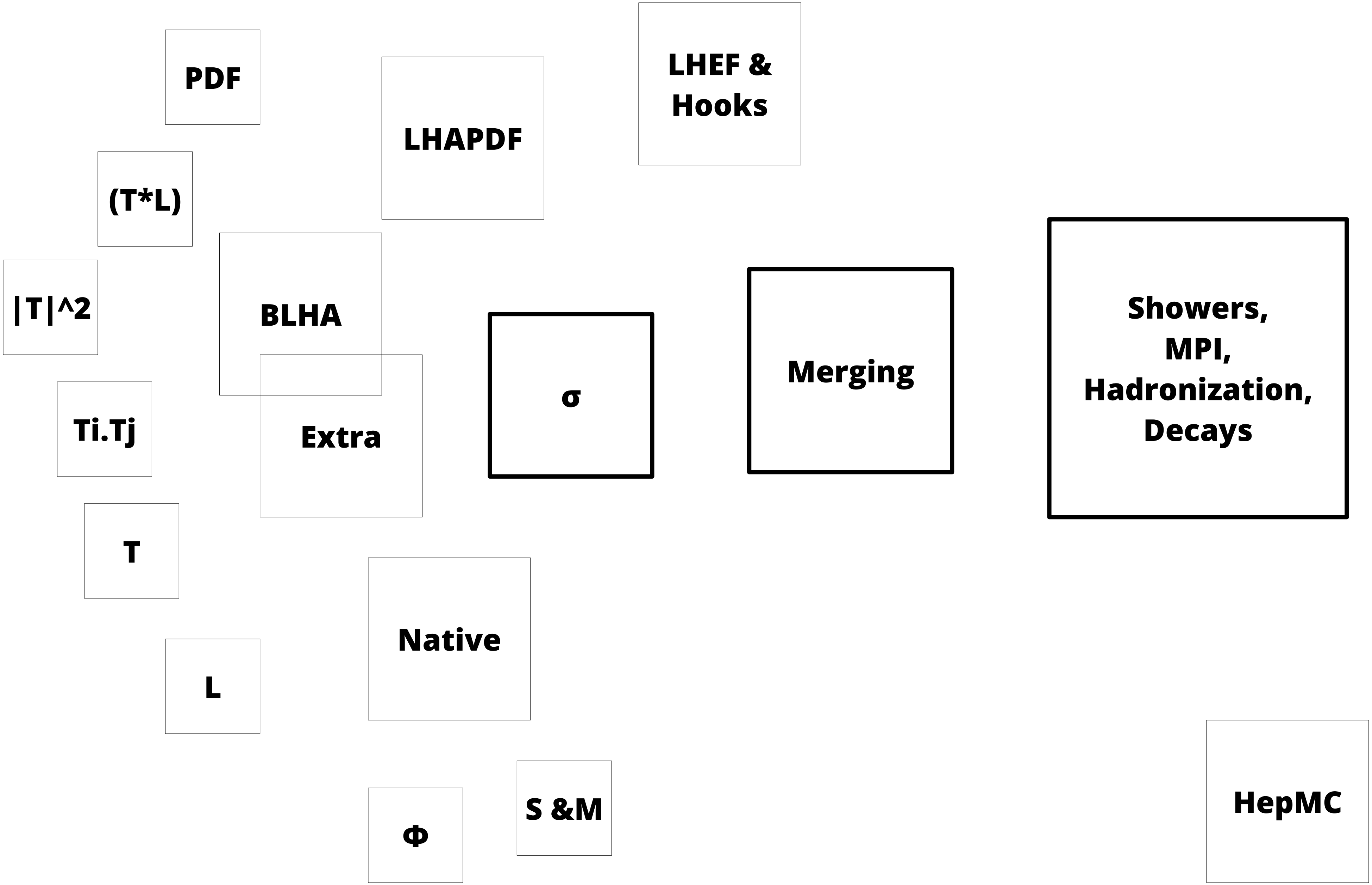
Only finite for infrared safe observables:

$$\hat{x}(\Phi_{m+1}) \to \hat{x}(\Phi_m) + \mathcal{O}\left(\frac{E_g}{Q}\right) \quad \text{for } E_g \to 0$$

$$\hat{x}(\Phi_{m+1}) \to \hat{x}(\Phi_m) + \mathcal{O}(\theta_{ij}) \quad \text{as } 1 - \beta \cos\theta_{ij} \to 0$$

No collinear divergences for massive partons.
Infrared unsafe: highly sensitive to non-perturbative contributions.

MC Frameworks and (Amplitude) Providers



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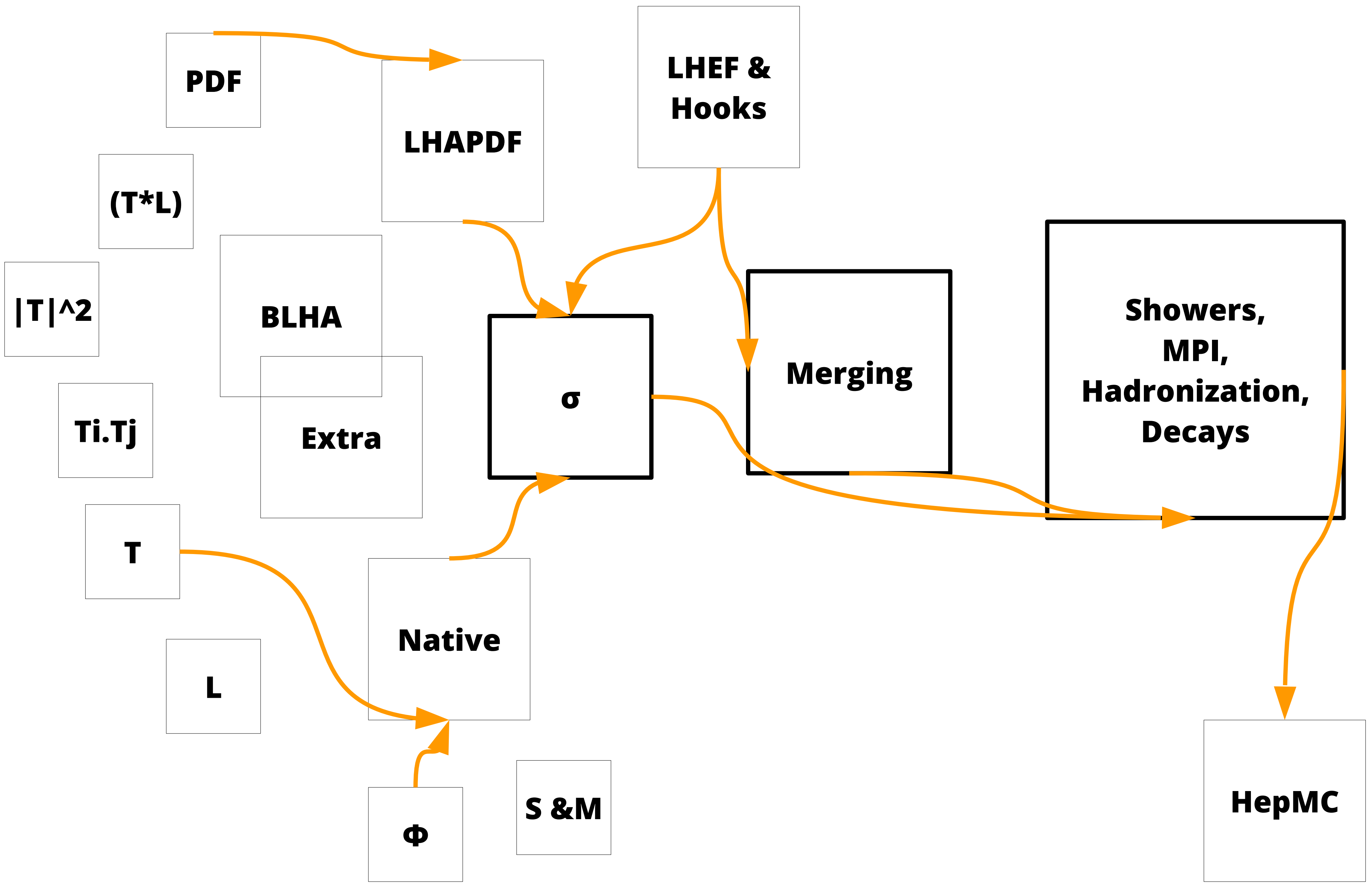

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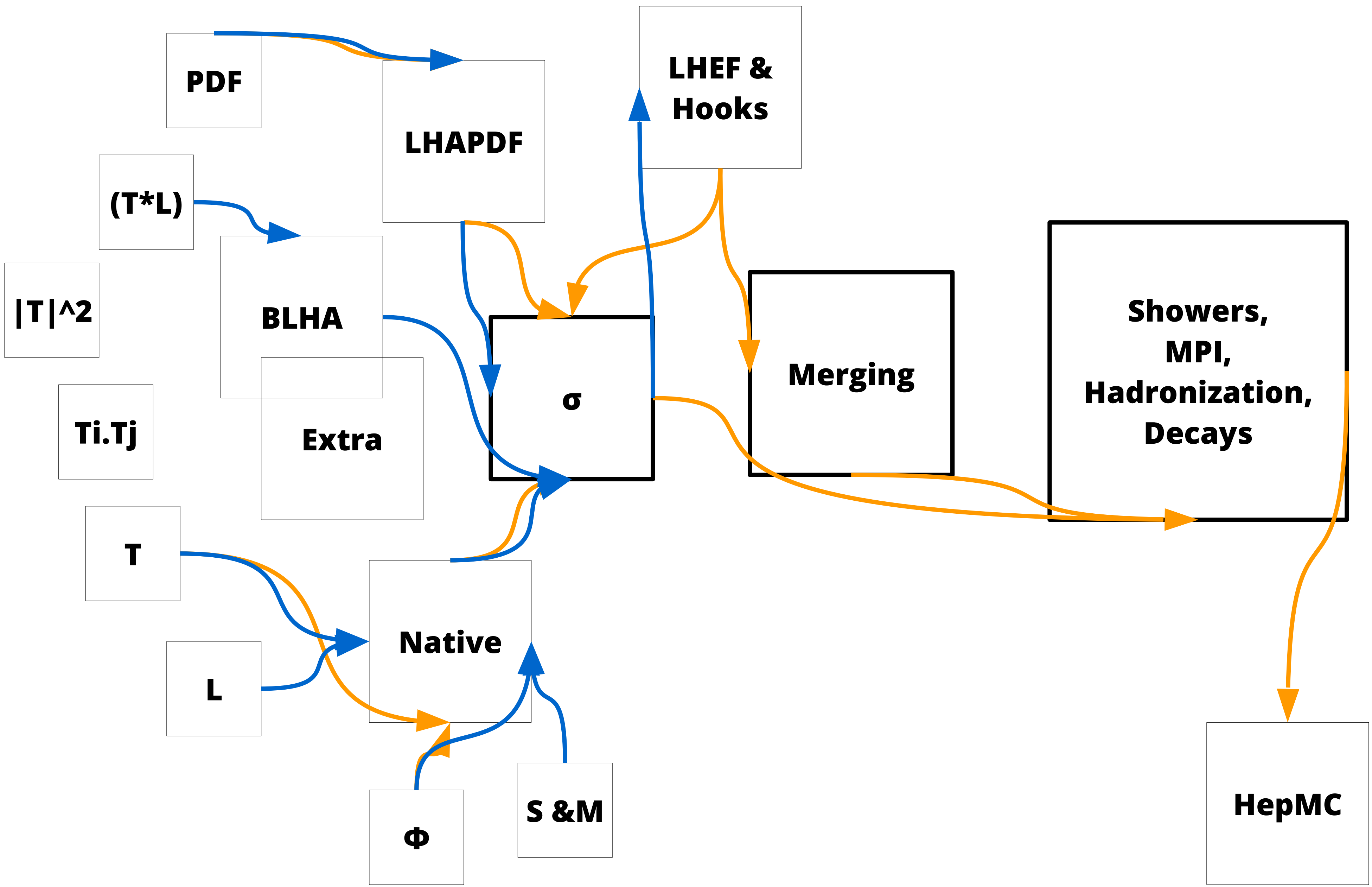

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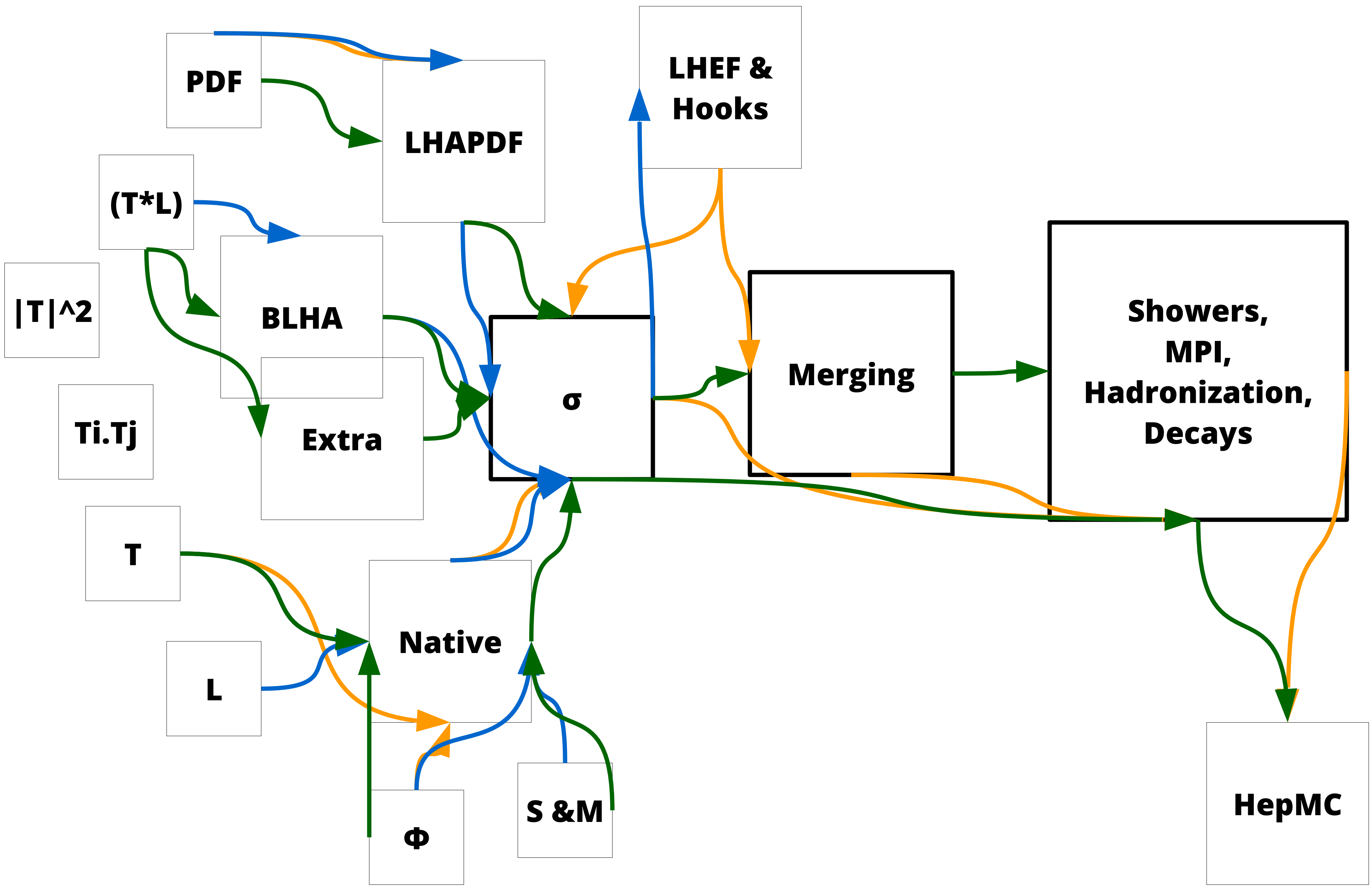

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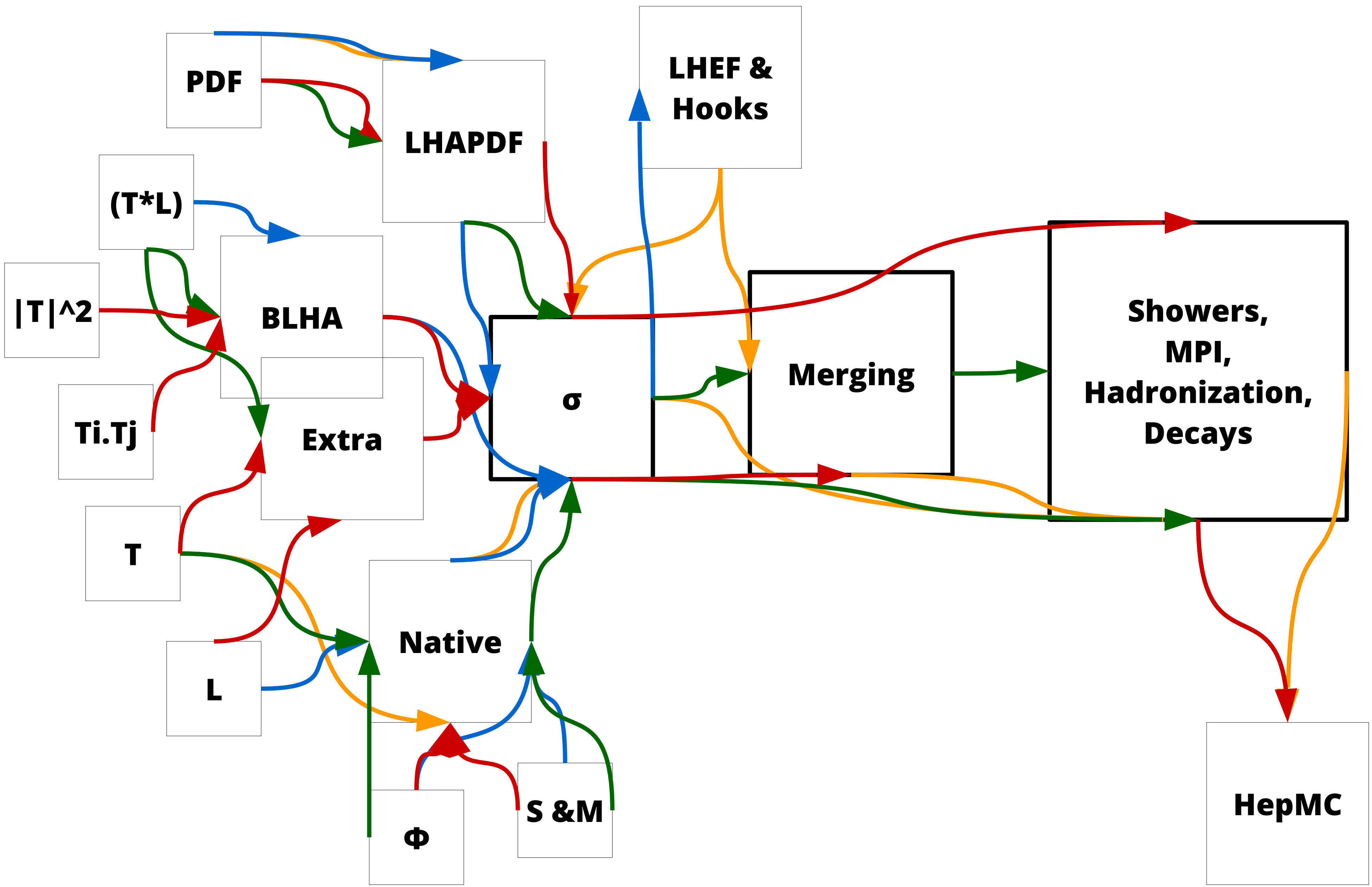


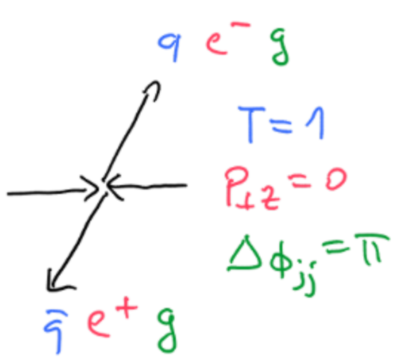
Infrared Sensitive Observables

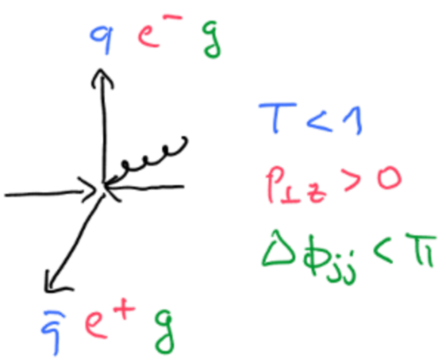
Event generators aim at predicting **highly exclusive observables:**

→ Perturbative, high-multiplicity partonic final states,
→ convoluted with phenomenological models.

Perturbative part constrained by infrared safety. Crucial is the description of infrared sensitive observables, which are

→ infrared safe, and so calculable in perturbation theory, but
→ require a minimum amount of radiation for non-trivial values, and
→ are divergent at any fixed order once zero radiation is allowed.





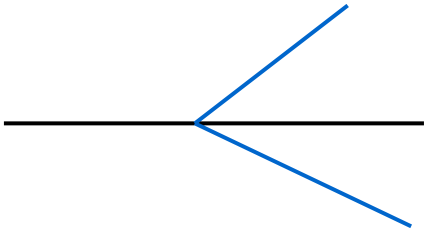
Infrared Sensitive Observables

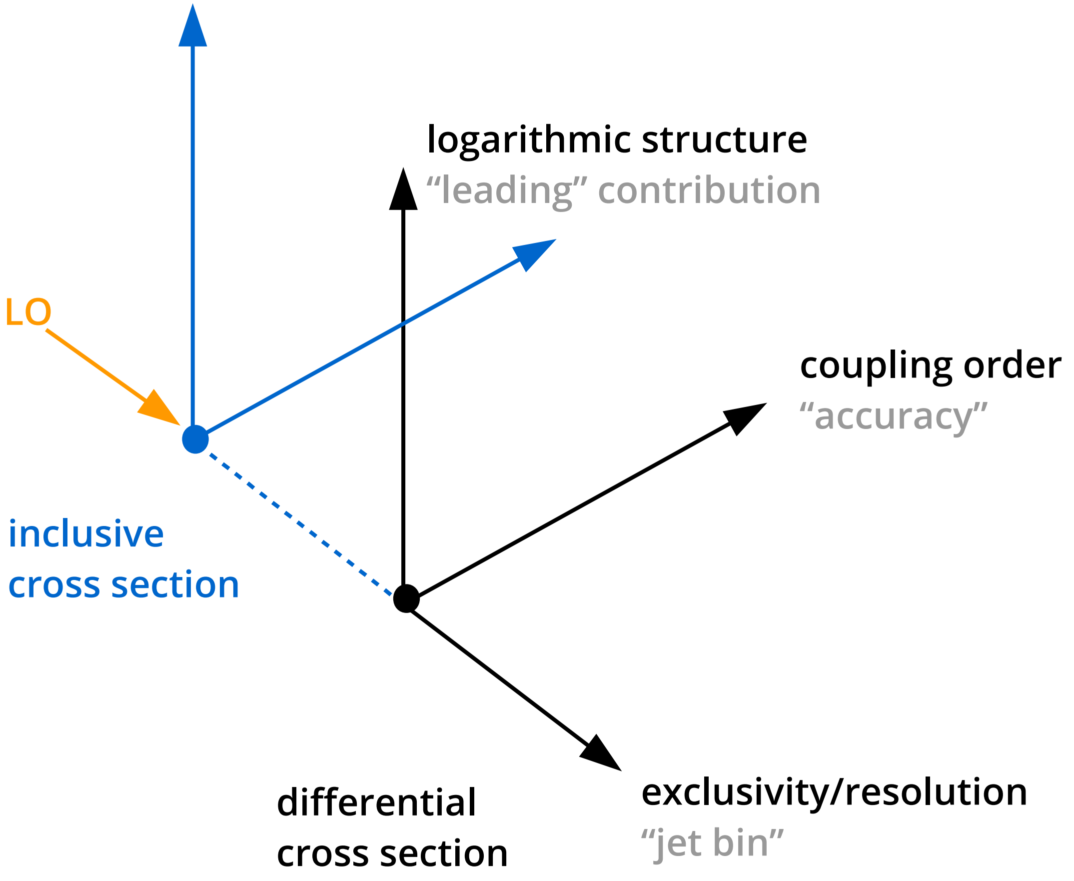
Infrared sensitive observables are divergent at any fixed order of perturbation theory, once the requirement on additional radiation is removed.

Roughly speaking: If an infrared sensitive observable requires n jets to be Present for a non-trivial value, it will diverge at the boundary to resolve n-1 jets – Divergence is entirely due to soft and/or collinear emissions.

$$\frac{d\sigma}{d\tau} \sim \sigma_0 \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} c_{n,m} \alpha_s^n \frac{\ln^{2n-m-1} \tau}{\tau} \qquad \tau \to 0$$

The Landscape of Infrared Sensitive Observables





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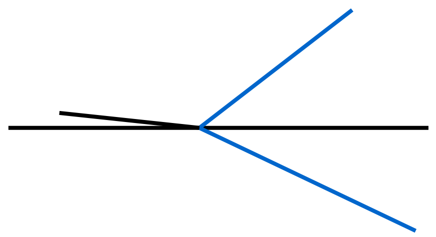

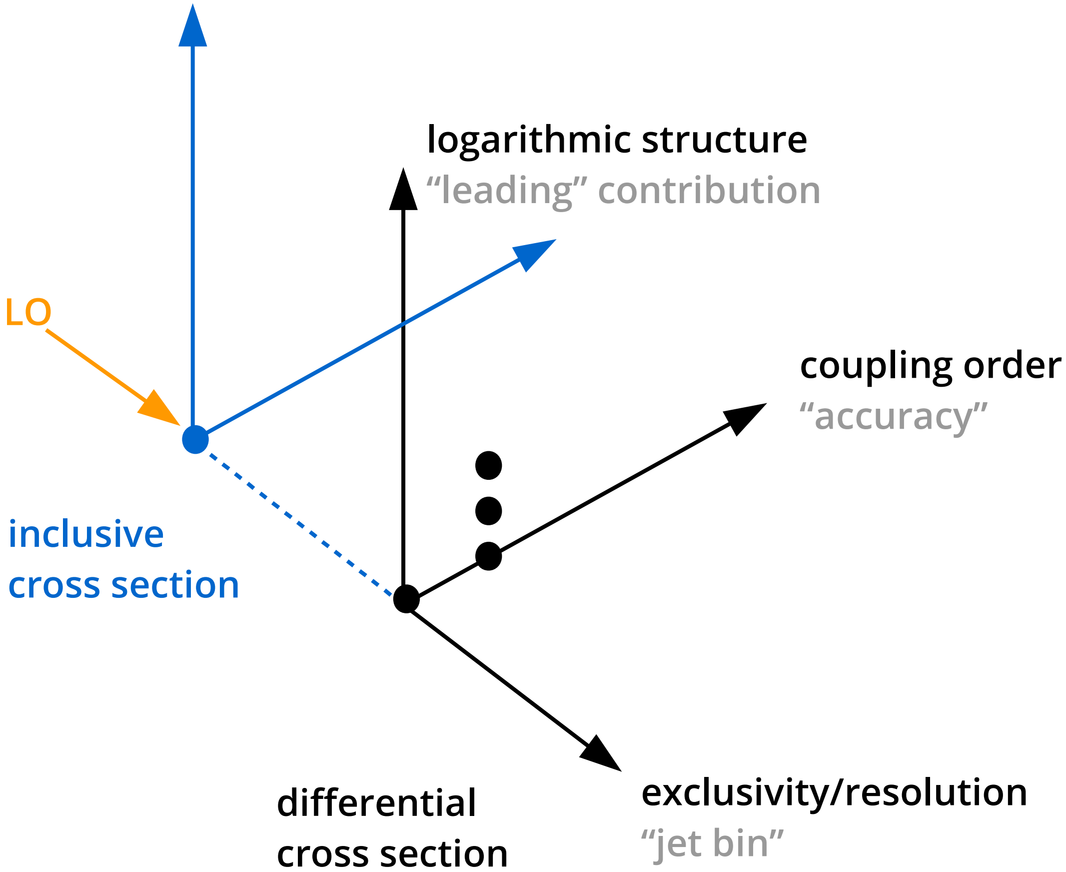

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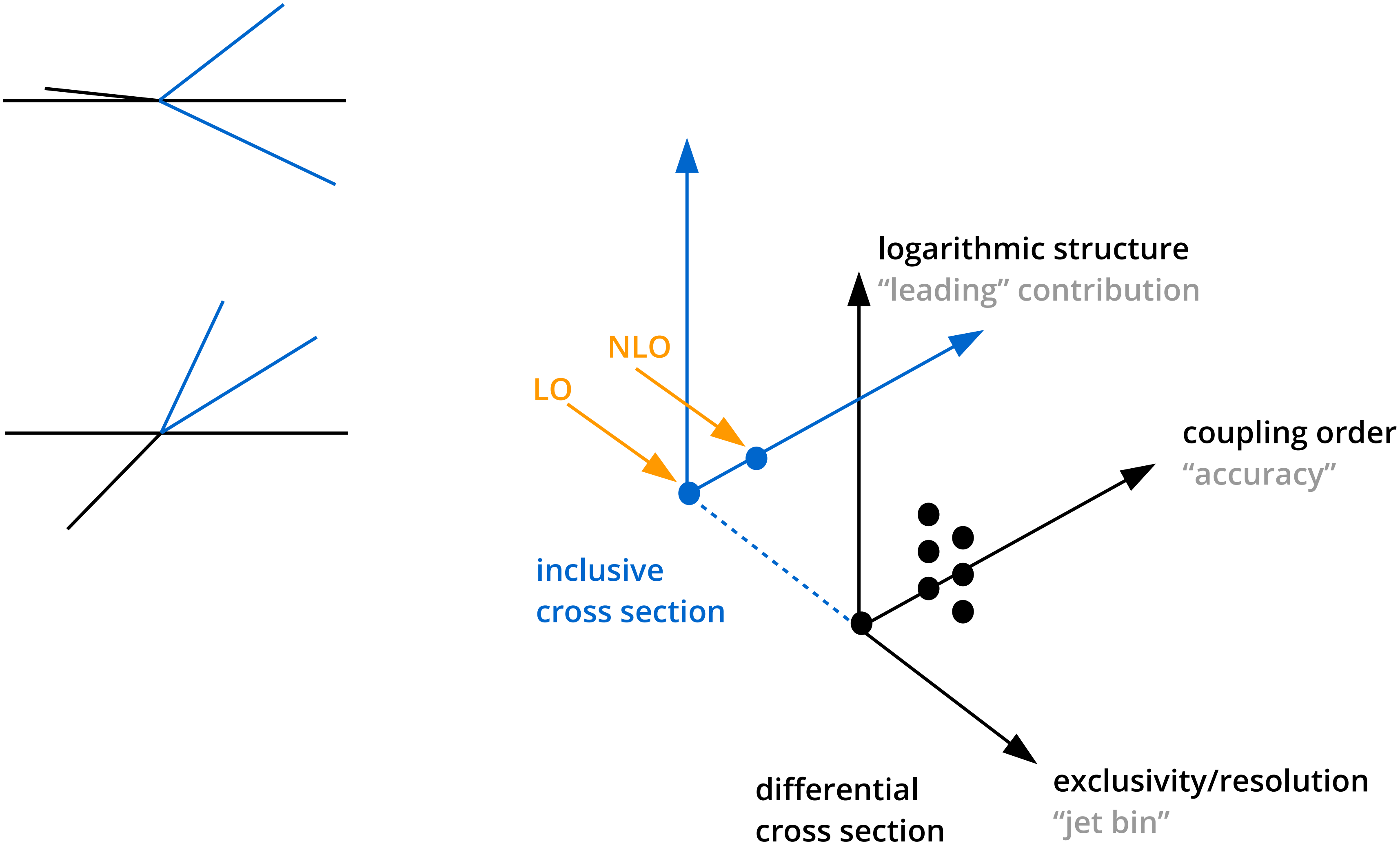

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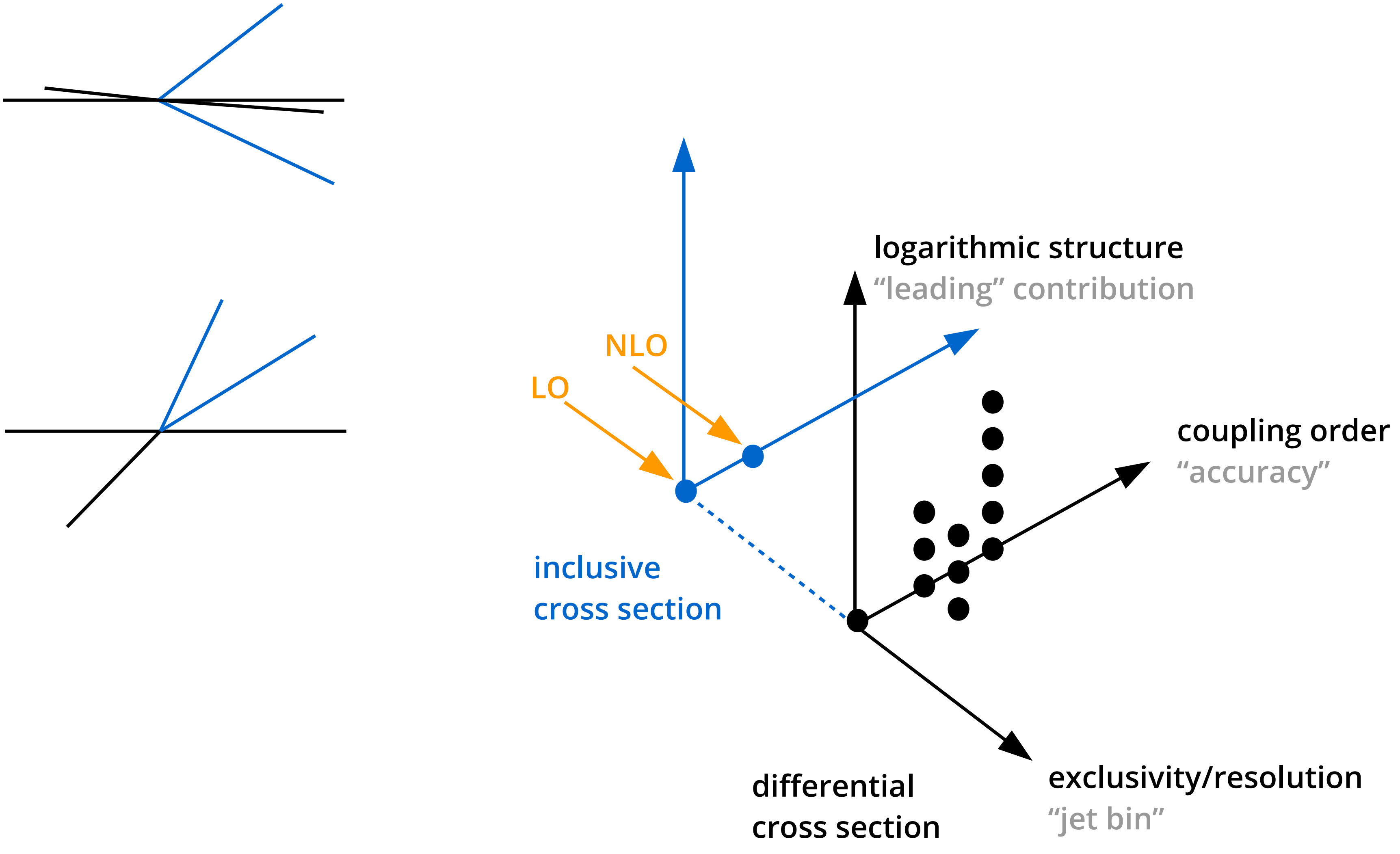

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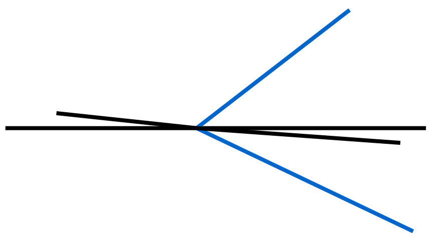

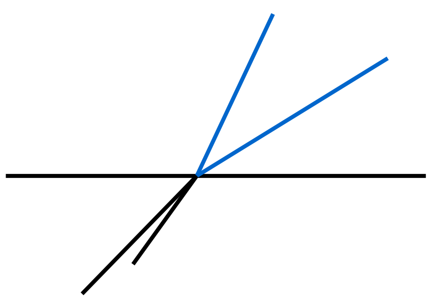

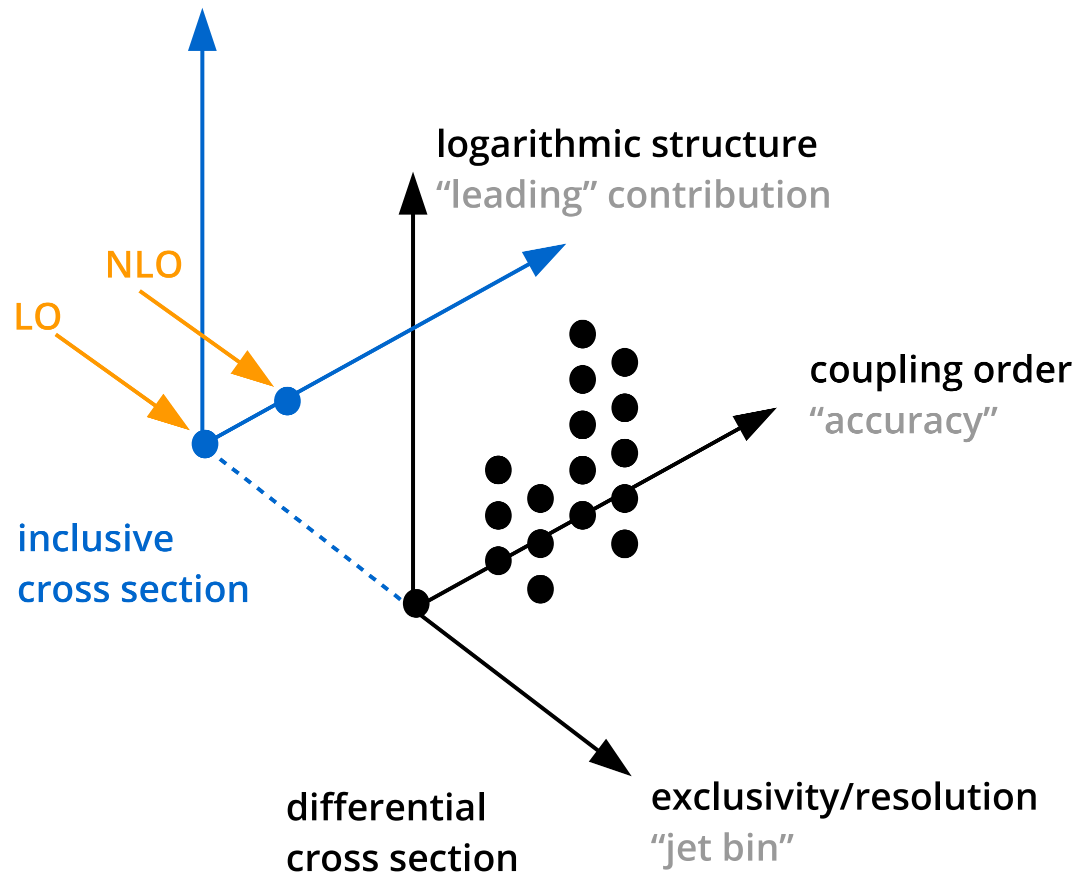

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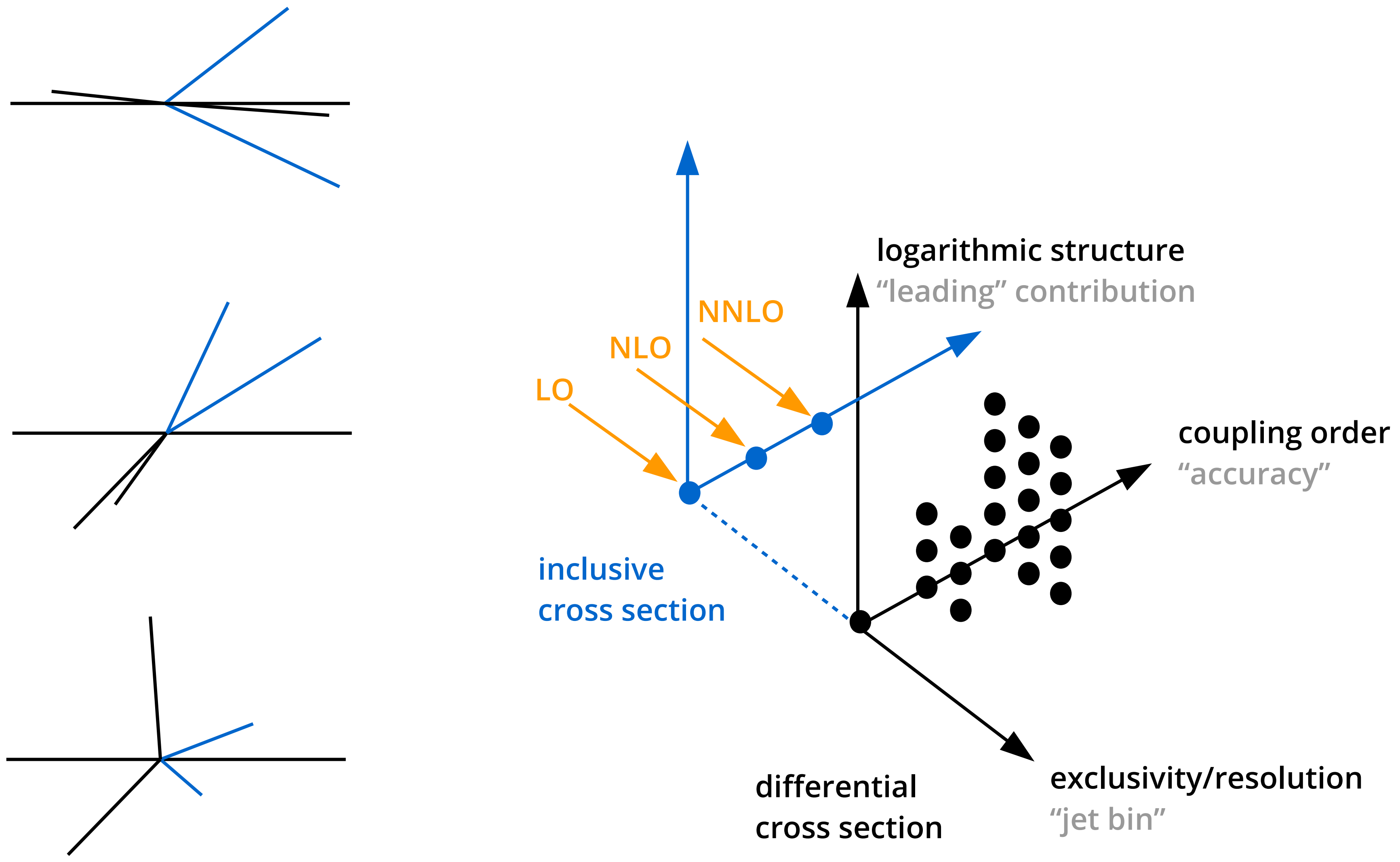
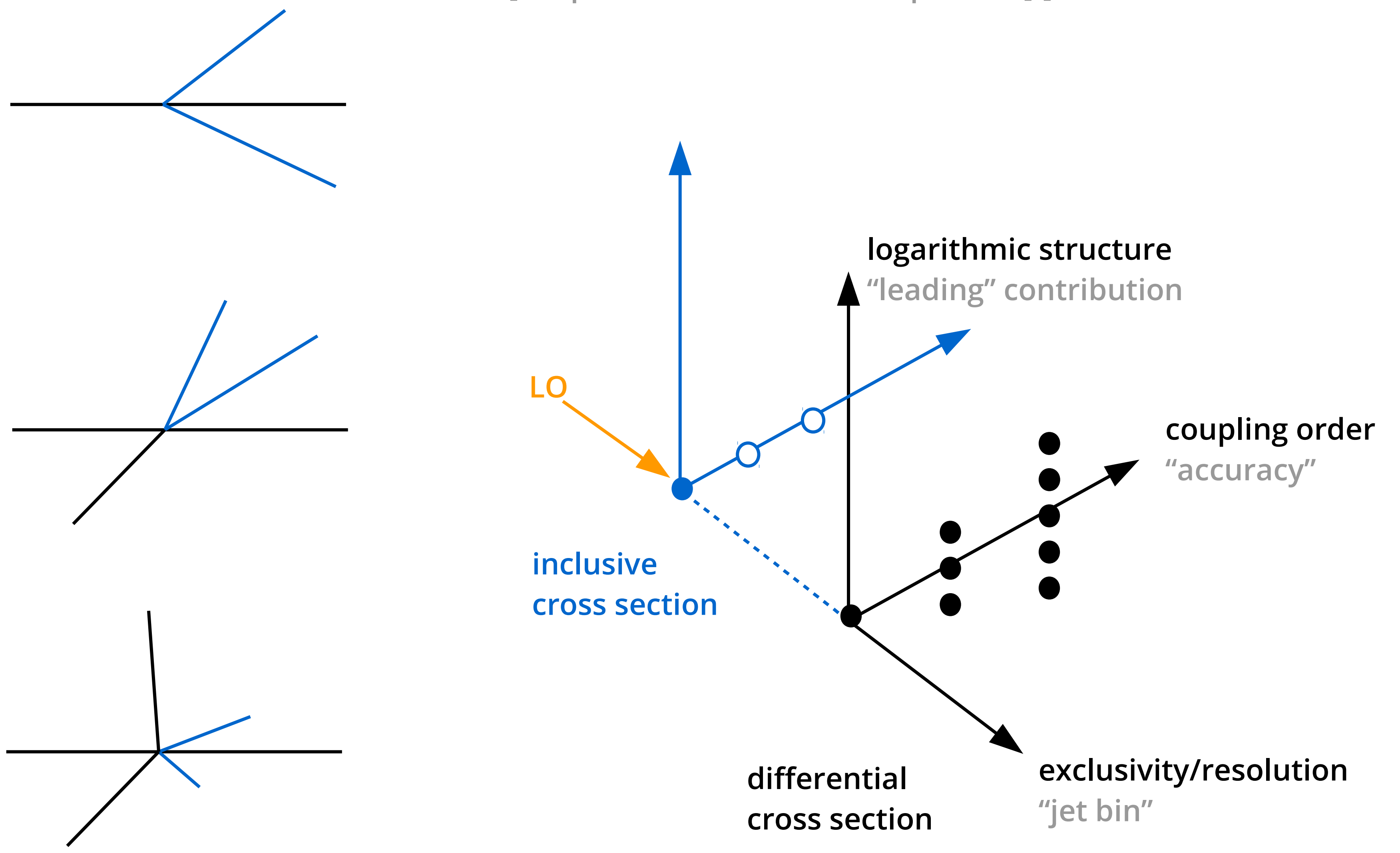


The Landscape of Infrared Sensitive Observables

[LoopSim – Rubin, Salam, Sapeta '10] [Exclusive sums – Maitre '?]



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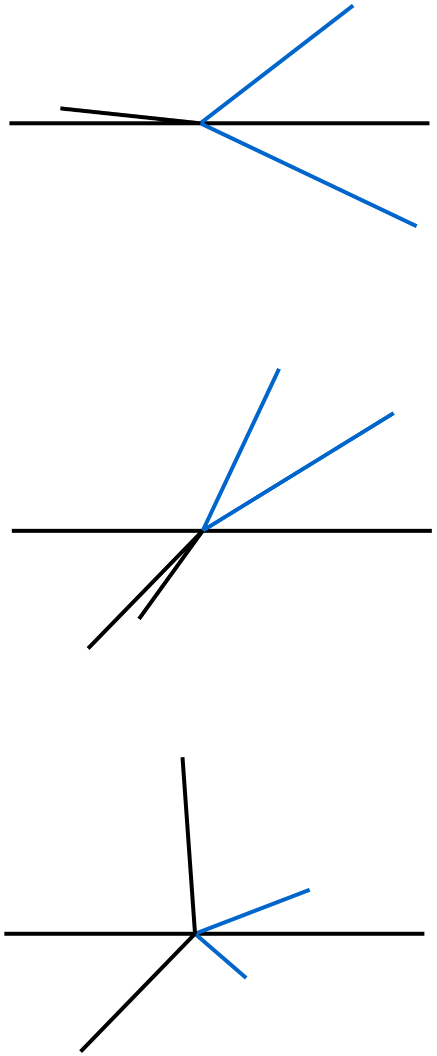

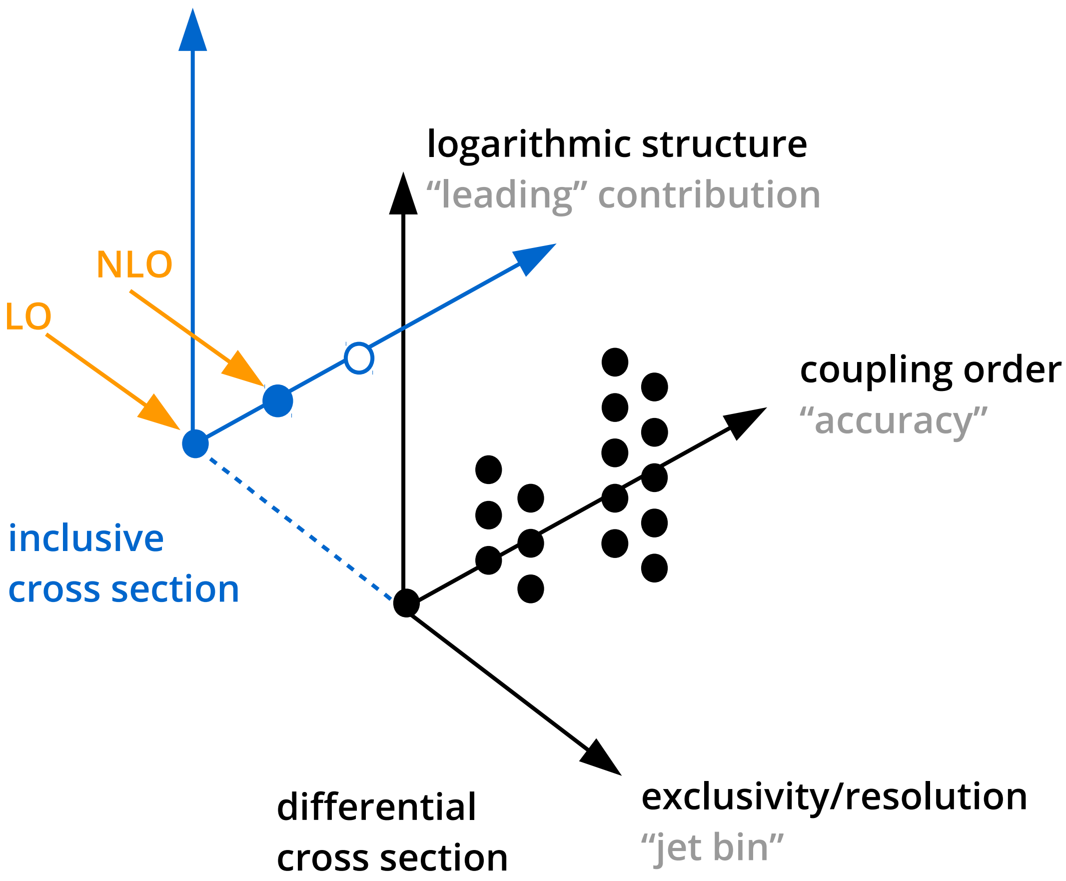

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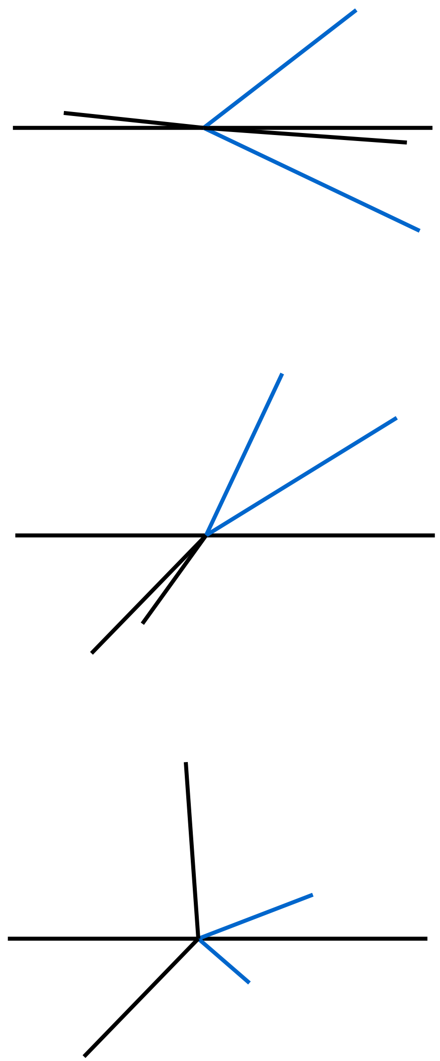

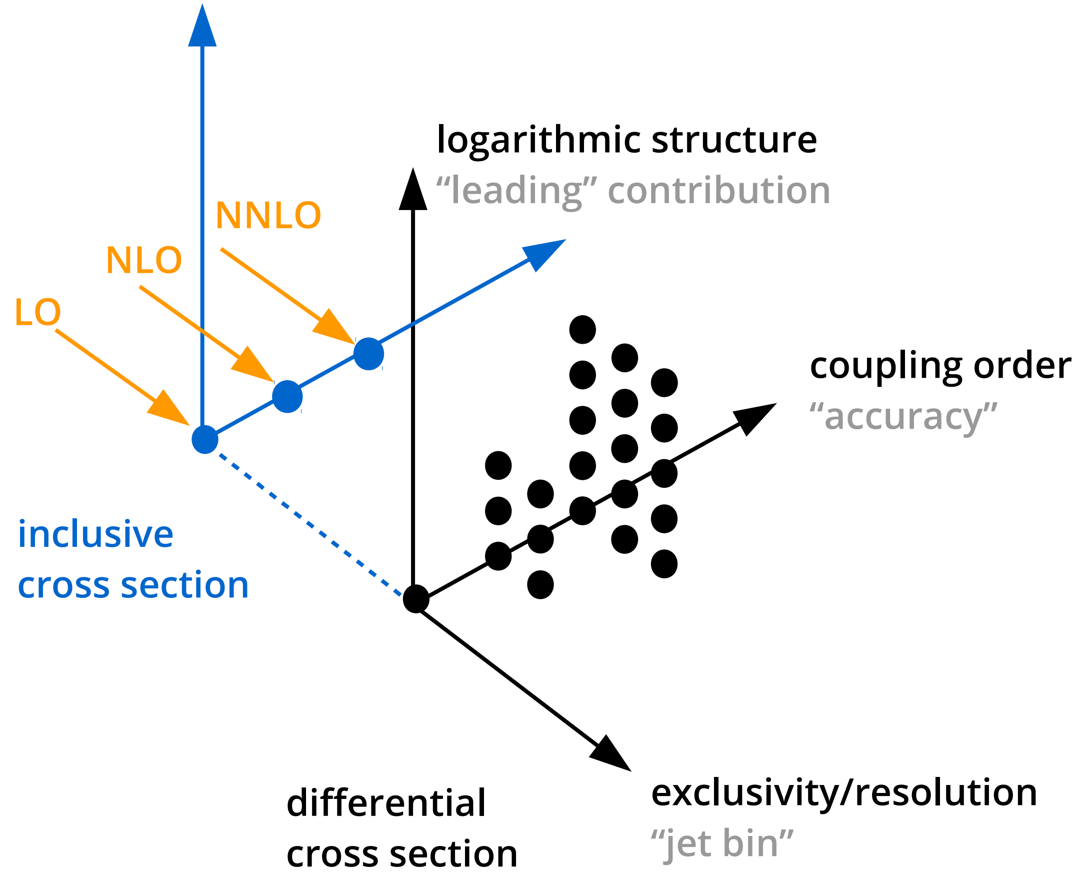

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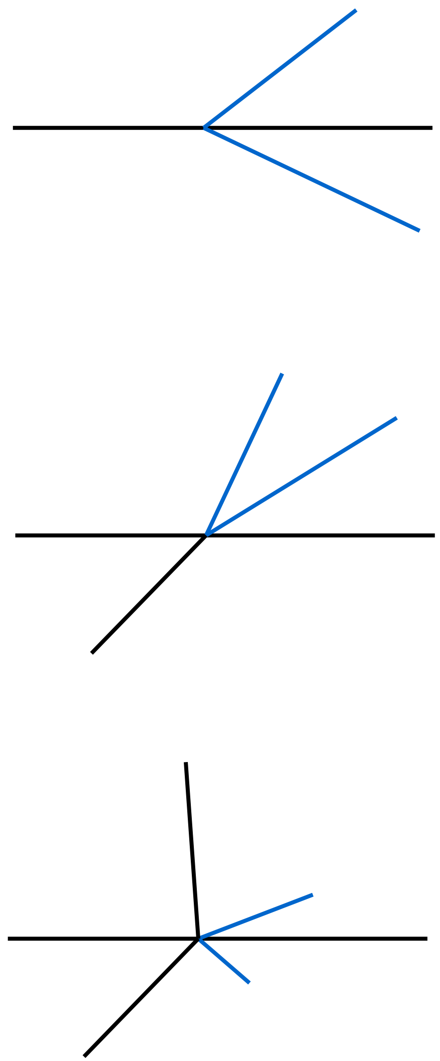

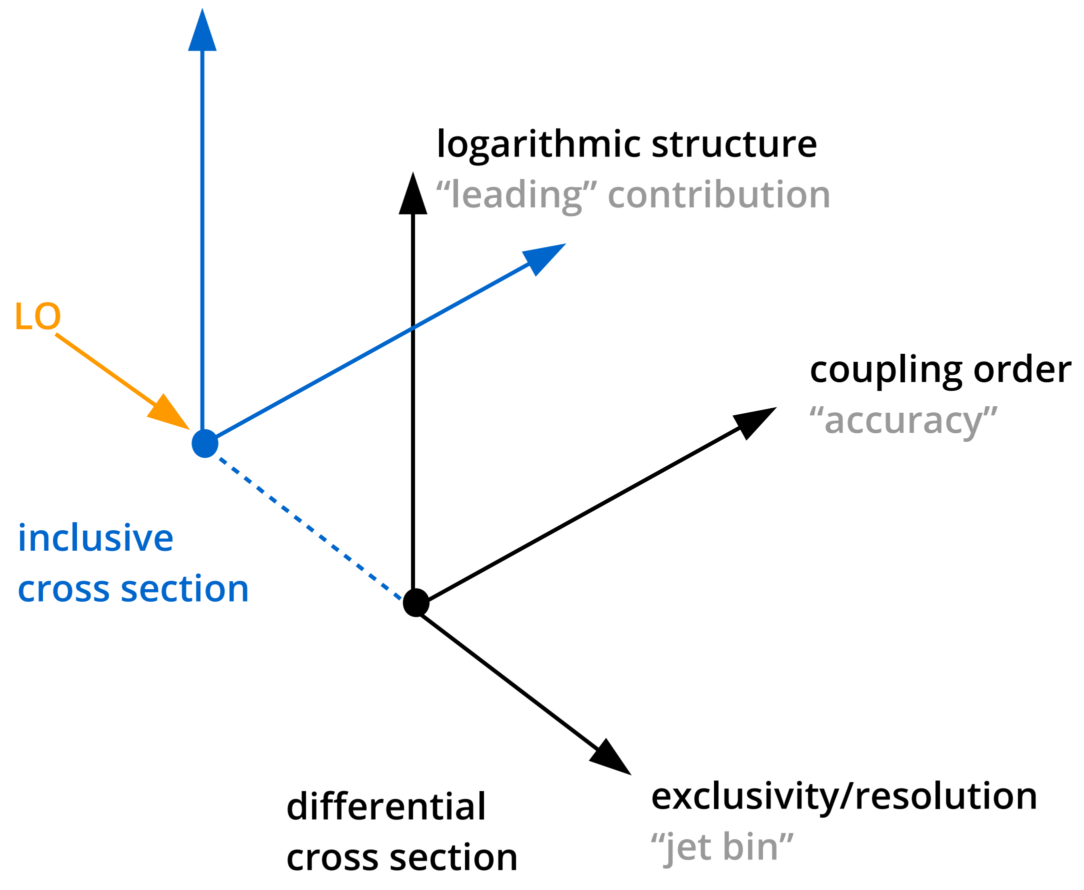

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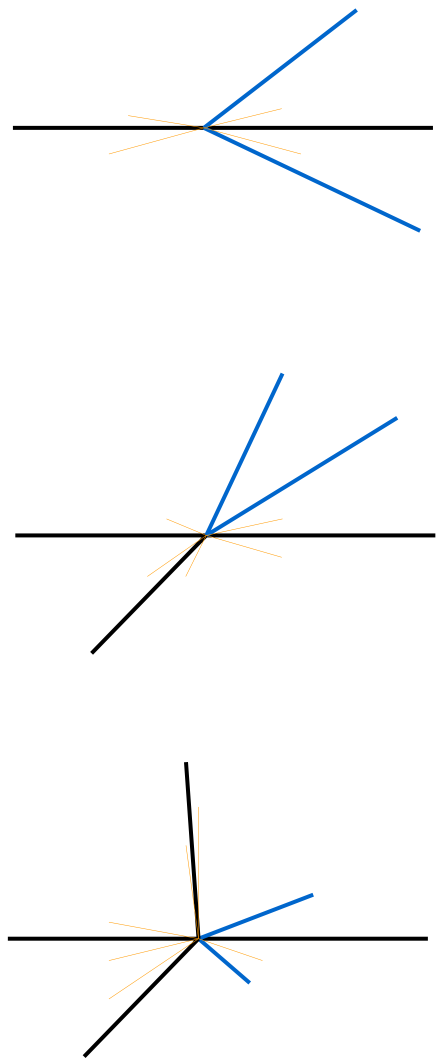

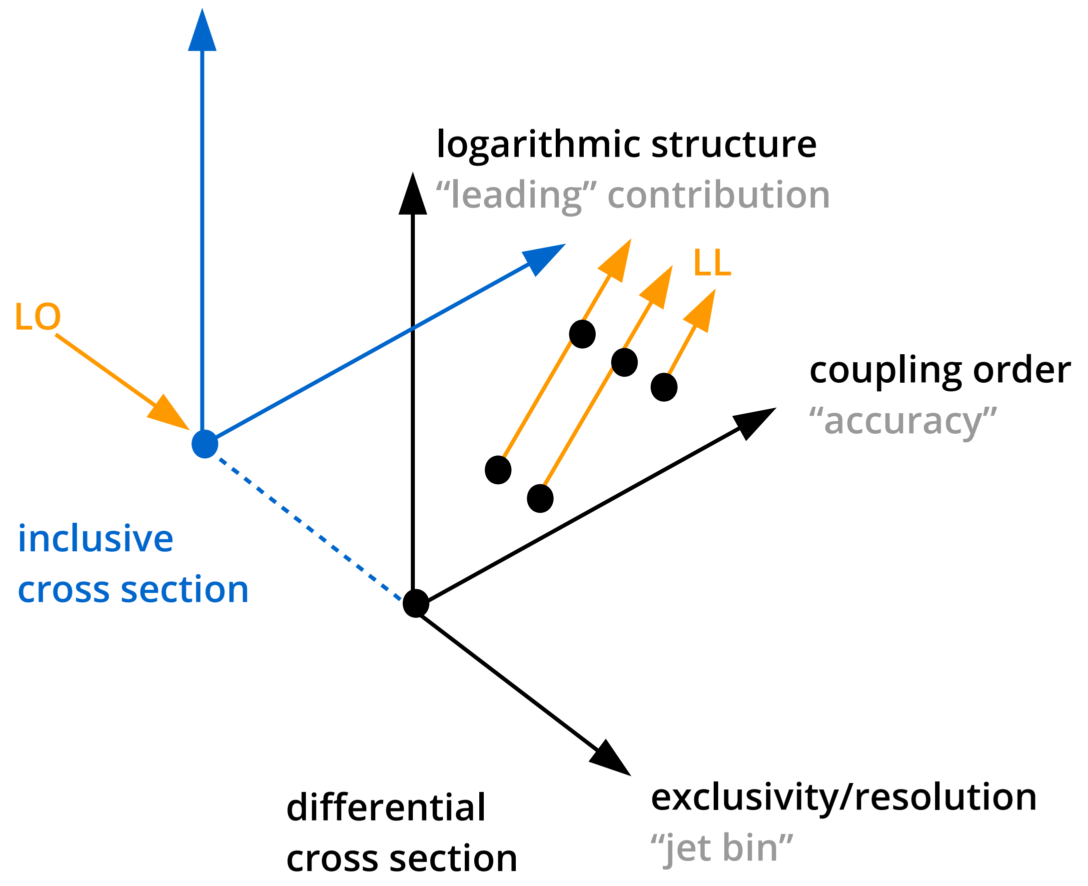

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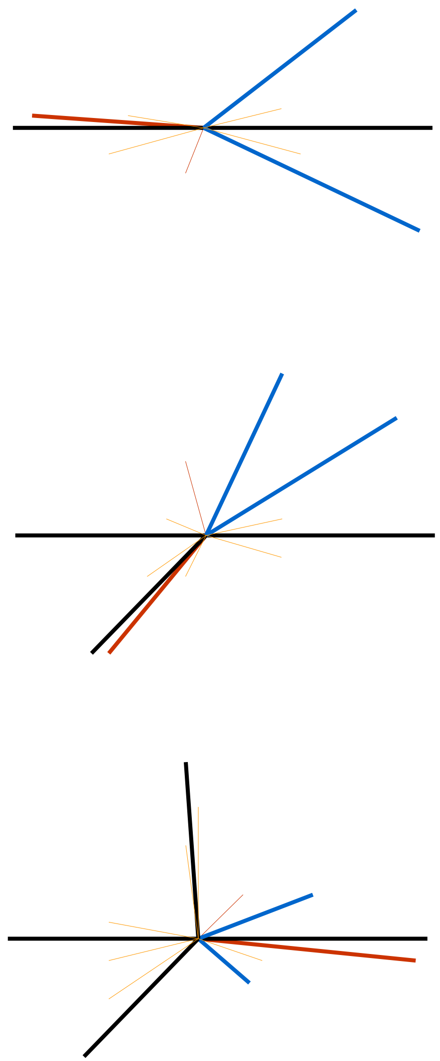

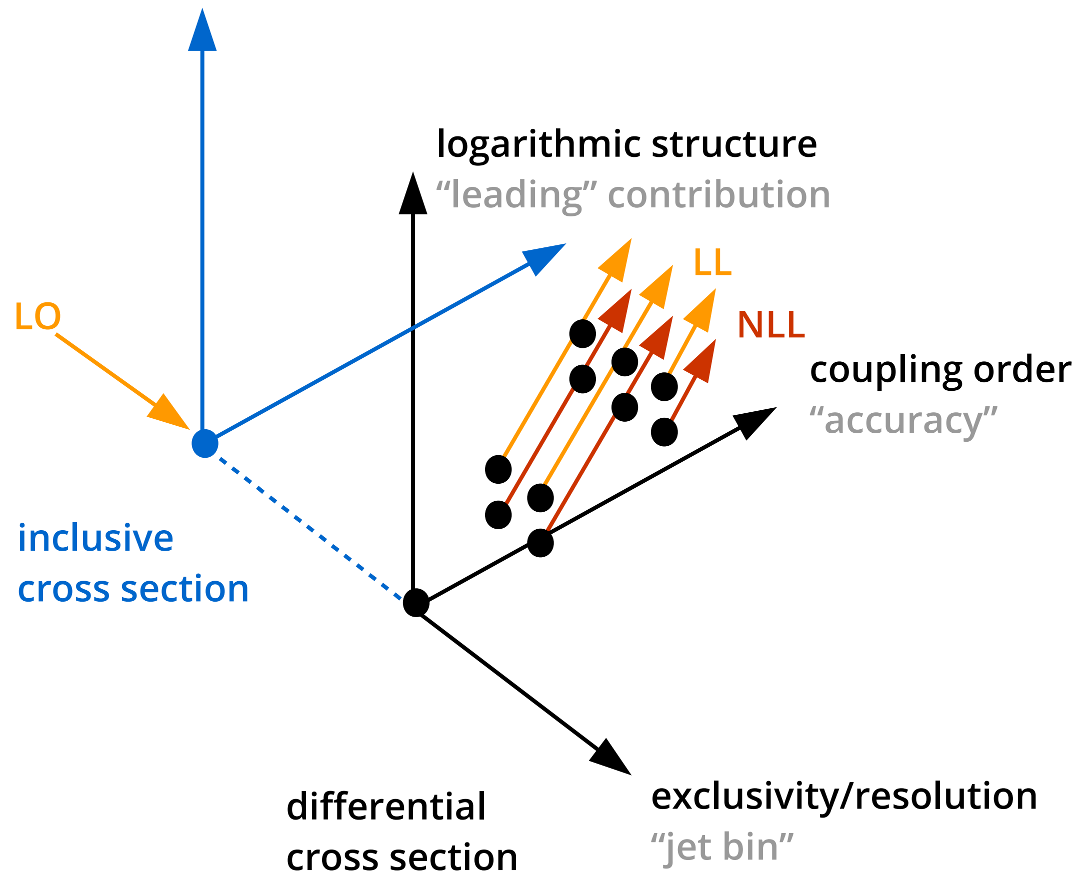

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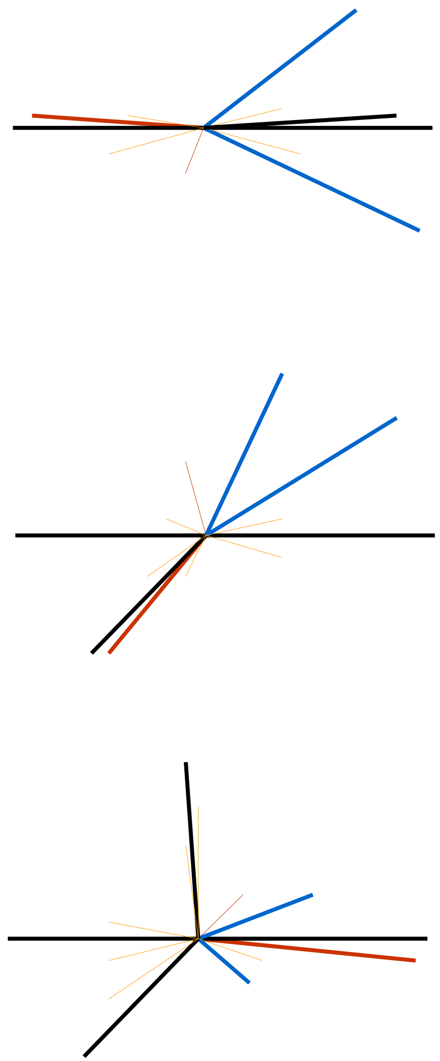
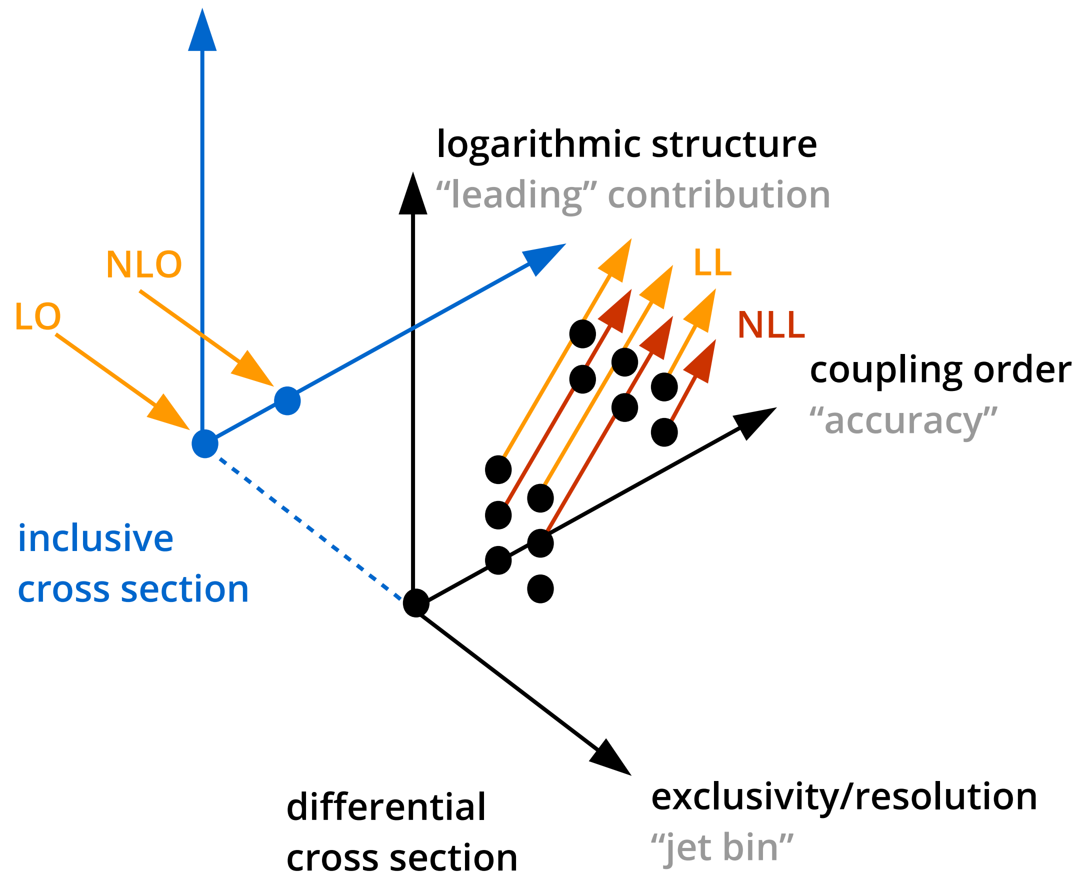
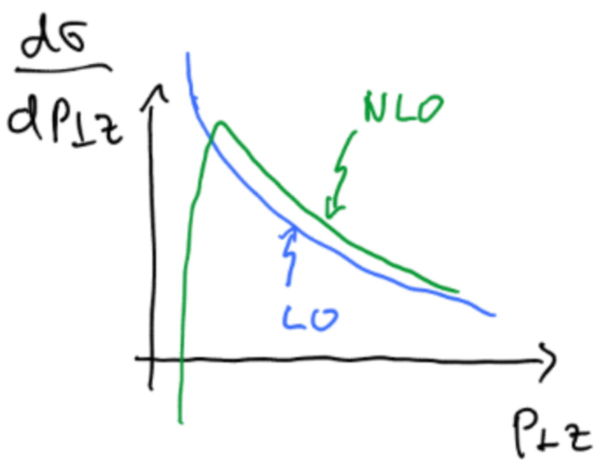
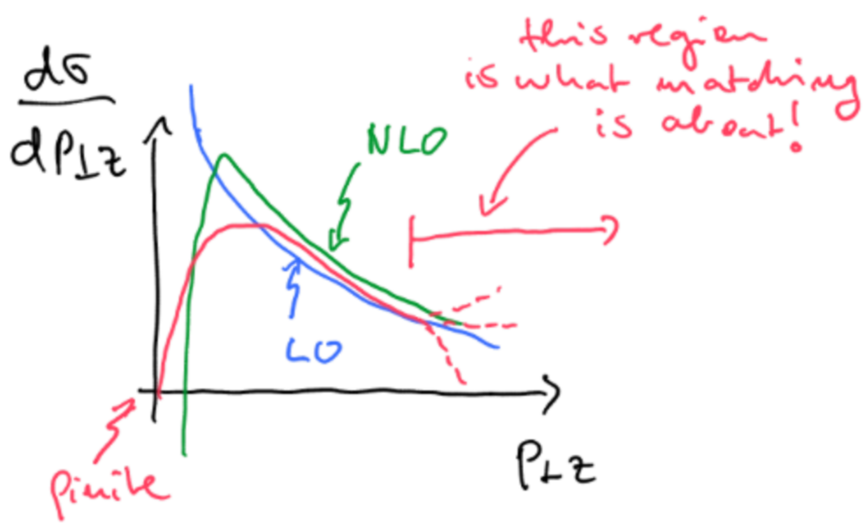




Why infrared sensitivity calls for showers

Divergences are all due to the fact that fixed order only accounts for a limited number of emissions (both real and virtual).





Once we resum any number of emissions, there will be a finite answer. The probability to emit nothing on top of a certain number of emissions is always less than one → Sudakov suppression.

What showers do

Consider a shower with generic splitting kernel P to generate emissions off a partonic state at a scale q.

Shower action on events with n partons:

$$d\sigma_n(\phi_n)\, PS_Q[u(\phi_n)]$$

$$= \Delta_n(\mu|Q)\, d\sigma_n(\phi_n)\, u(\phi_n) + d\sigma_n(\phi_n)\, P(\phi_n, q)\, \frac{d\phi_{n+1}}{d\phi_n}\, \Delta_n(q|Q)\, PS_q[u(\phi_{n+1})]$$

Sudakov form factor: Probability for no emission between two scales.
Recursive algorithm: Generate next emission off the n+1 parton state.
Evolve down to infrared cutoff μ.

What showers do

Showers have virtual and real emission contributions:

$$d\sigma_n(\phi_n)\,PS_Q[u(\phi_n)]$$

$$= \Delta_n(\mu|Q)\,d\sigma_n(\phi_n)\,u(\phi_n) + d\sigma_n(\phi_n)\,P(\phi_n,q)\,\frac{d\phi_{n+1}}{d\phi_n}\,\Delta_n(q|Q)\,PS_q[u(\phi_{n+1})]$$

Showers preserve the total inclusive cross section: Unitarity.

$$P(\phi_n,q)\,\Delta_n(q|Q) = \frac{\partial}{\partial q}\Delta_n(q|Q) \qquad \Delta_n(Q,Q) = 1$$

Showers approximate tree level matrix elements:

$$d\sigma(\phi_{n+1}) \longrightarrow d\sigma(\phi_n)\,P(\phi_n,q)\,\frac{d\phi_{n+1}}{d\phi_n}$$

In the collinear limits, and in the soft limit for large number of colours N.

What showers do

Expand showers in the strong coupling:

$$d\sigma_n(\phi_n)\, PS_Q[u(\phi_n)] =$$
$$\left[d\sigma(\phi_n) - d\sigma(\phi_n) \int_{\mu}^{Q} dq\, P(\phi_n, q) \frac{d\phi_{n+1}}{d\phi_n\, dq} \right] u(\phi_n)$$
$$+ d\sigma(\phi_n)\, P(\phi_n, q) \frac{d\phi_{n+1}}{d\phi_n}\, u(\phi_{n+1}) + \mathcal{O}(\alpha_s^2)$$

Scale choices in the running coupling are beyond this order.

Virtual shower contributions are minus the integral of its real contributions.

→ Cross sections after showering are preserved order by order.

Matching, merging – egal: Hauptsache higher orders!

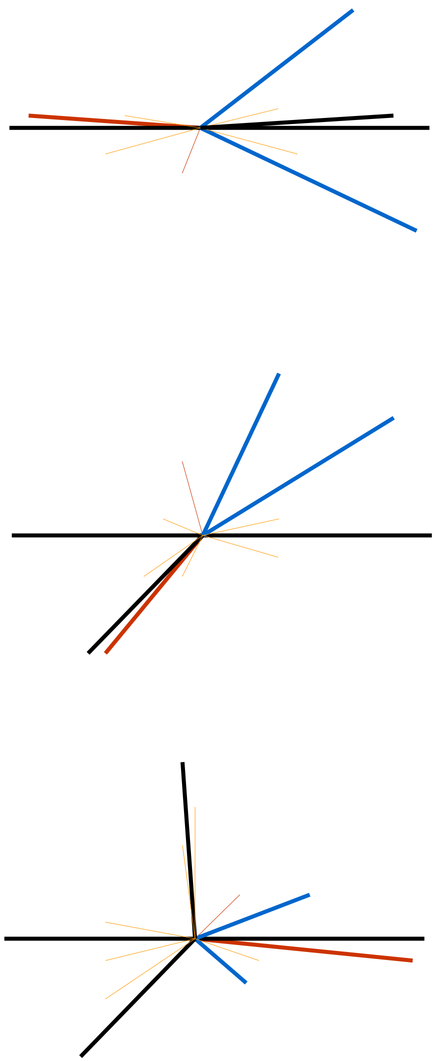
Matching:

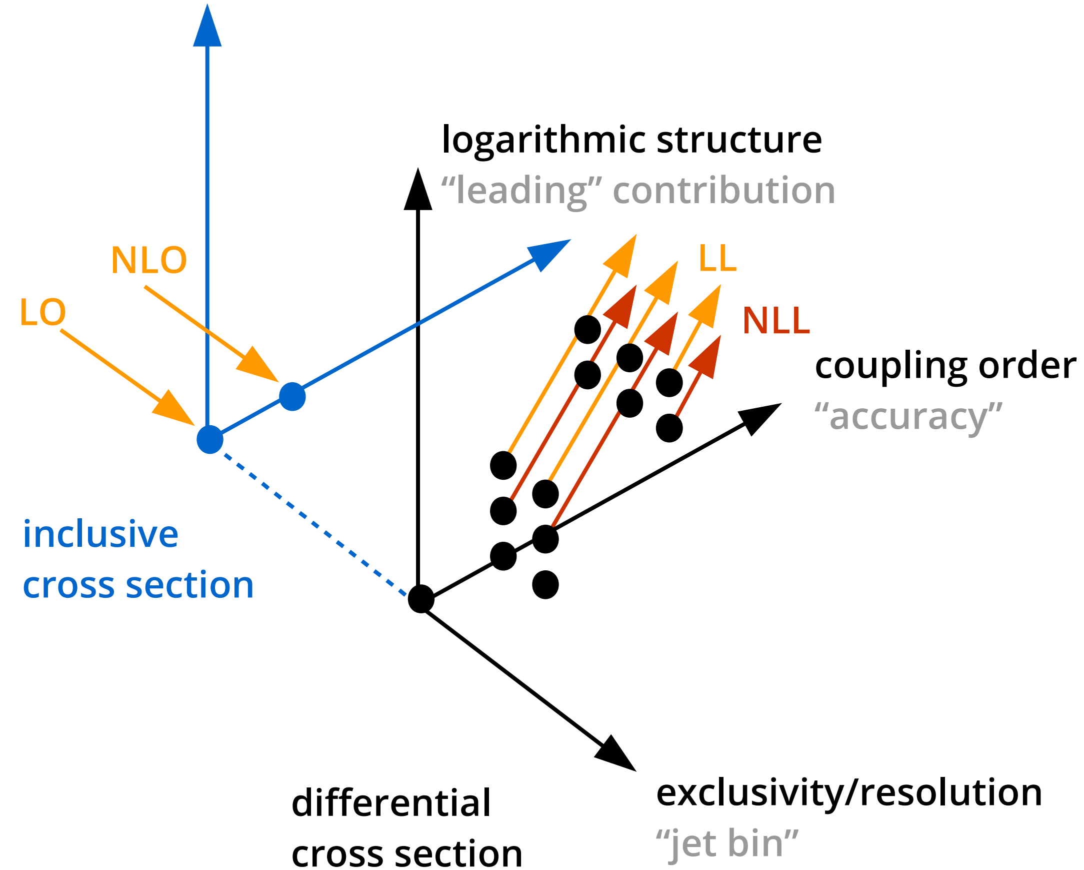
→ Combine resummation with a fixed order.
→ Here: Combine a parton shower and a NLO calculation.
→ Applicable only where the fixed order calculation is reliable.

Merging:

→ Combine calculations for different hard jet multiplicities.
→ Add parton shower on top.
→ Applicable when crossing 'jet bins'.
→ At low scales typically only parton shower predictions.

NLO Matching



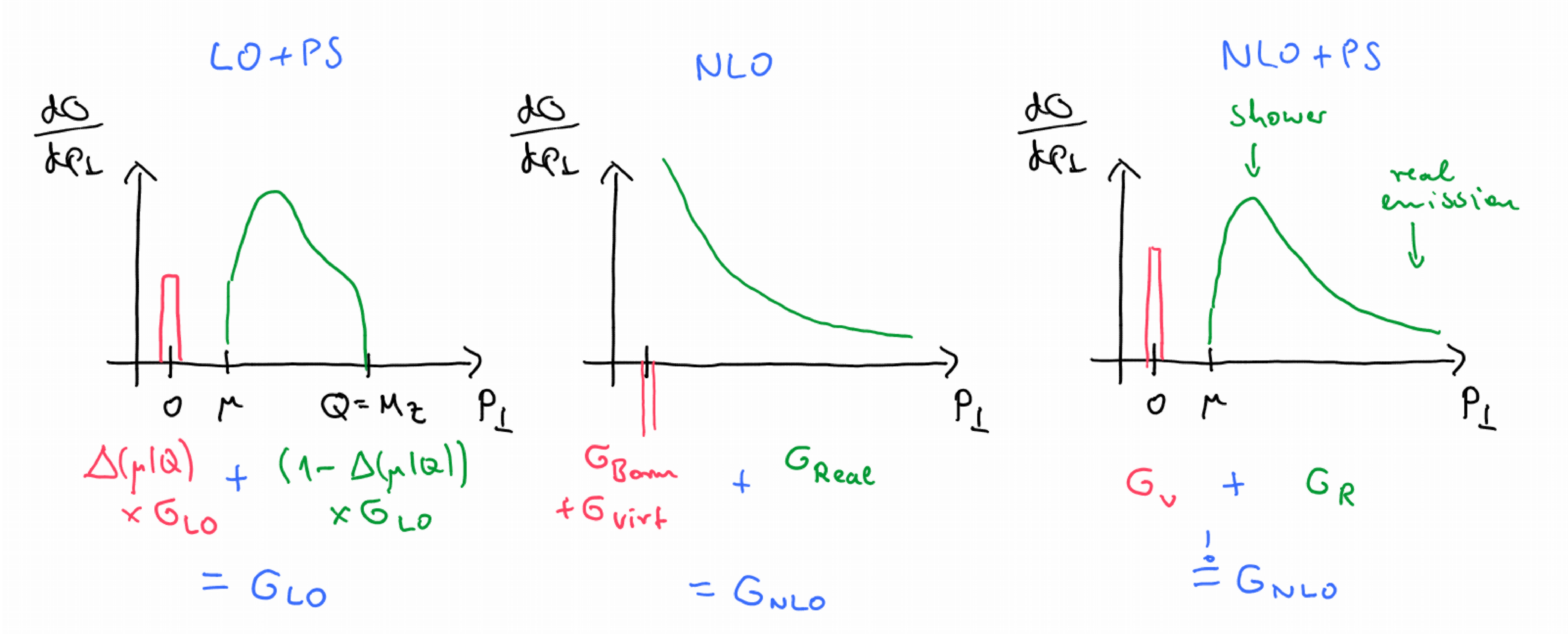


Matching

[Frixione, Webber, ...]

NLO matching in a nutshell:

- → Inclusive cross section given by NLO result
- → First additional jet described at LO (i.e. NLO real emission)
- → 1 → 0 jet limit exhibits proper Sudakov supression



The Matching Condition

$$PS[d\sigma_{NLO}^{matched}] = d\sigma_{NLO} + \mathcal{O}(\alpha_s^{n+2})$$

$$\int PS[d\sigma_{NLO}^{matched}] = \sigma_{NLO}$$

Solving the Matching Condition

$$PS[d\sigma_{NLO}] =$$

$$d\sigma_{LO}(\phi_n)\, u(\phi_n)$$

$$+ \left(d\sigma_{Virtual}(\phi_n) + \int_1 d\sigma_{Sub}(\phi_{n+1}) \right) u(\phi_n)$$

$$- d\sigma_{Sub}(\phi_{n+1})\, u(\phi_n) \qquad + d\sigma_{Real}(\phi_{n+1})\, u(\phi_{n+1})$$

$$- \int_\mu^Q dk\, P(\phi_n, k) \frac{d\phi_{n+1}}{d\phi_n\, dk} d\sigma_{LO}(\phi_n)\, u(\phi_n) \qquad + P(\phi_n, k) \frac{d\phi_{n+1}}{d\phi_n\, dk} d\sigma_{LO}(\phi_n)\, u(\phi_{n+1})$$

$$+ \mathcal{O}(\alpha_s^2)$$

Solving the Matching Condition

$d\sigma_{matched}$ =

$$d\sigma_{LO}(\phi_n)\, u(\phi_n)$$

$$+ \left(d\sigma_{Virtual}(\phi_n) + \int_1 d\sigma_{Sub}(\phi_{n+1}) \right) u(\phi_n)$$

$$- d\sigma_{Sub}(\phi_{n+1})\, u(\phi_n) \qquad + d\sigma_{Real}(\phi_{n+1})\, u(\phi_{n+1})$$

$$+ \int_{\mu}^{Q} dk\, P(\phi_n, k) \frac{d\phi_{n+1}}{d\phi_n\, dk} d\sigma_{LO}(\phi_n)\, u(\phi_n) \quad - \quad P(\phi_n, k) \frac{d\phi_{n+1}}{d\phi_n\, dk} d\sigma_{LO}(\phi_n)\, u(\phi_{n+1})$$

$$+ \mathcal{O}(\alpha_s^2)$$

Solving the Matching Condition

$d\sigma_{\text{matched}} =$

$$d\sigma_{LO}(\phi_n)\, u(\phi_n)$$

$$+\left(d\sigma_{\text{Virtual}}(\phi_n) + \int_1 d\sigma_{\text{sub}}(\phi_{n+1}) \right) u(\phi_n)$$

$$- d\sigma_{\text{sub}}(\phi_{n+1})\, u(\phi_n) \qquad + d\sigma_{\text{Real}}(\phi_{n+1})\, u(\phi_{n+1})$$

$$+ \int_\mu^Q dk\, P(\phi_n, k) \frac{d\phi_{n+1}}{d\phi_n\, dk} d\sigma_{LO}(\phi_n)\, u(\phi_n) \times \theta(k-\mu) \quad - P(\phi_n, k) \frac{d\phi_{n+1}}{d\phi_n\, dk} d\sigma_{LO}(\phi_n)\, u(\phi_{n+1}) \times \theta(k-\mu) + \mathcal{O}(\alpha_s^2)$$

Infrared cutoff prevents finite weights.

Solving the Matching Condition

$$
\begin{aligned}
d\sigma_{\mathrm{matched}} = \; & d\sigma_{LO}(\Phi_n)\, u(\Phi_n) \\
& + \left(d\sigma_{\mathrm{Virtual}}(\Phi_n) + \int_1 d\sigma_{\mathrm{Sub}}(\Phi_{n+1}) \right) u(\Phi_n) \\
& - d\sigma_{\mathrm{Sub}}(\Phi_{n+1})\, u(\Phi_n) \\
& + d\sigma_{\mathrm{Bridge}}(\Phi_{n+1})\, \Theta(\mu - t)\, u(\Phi_n) \\
& + \int_{\mu}^{Q} dt\, P(\Phi_n, t)\, \frac{d\Phi_{n+1}}{d\Phi_n\, dt}\, d\sigma_{LO}(\Phi_n)\, u(\Phi_n) \times \Theta(t - \mu) \\
& + d\sigma_{\mathrm{Real}}(\Phi_{n+1})\, u(\Phi_{n+1}) \\
& - d\sigma_{\mathrm{Bridge}}(\Phi_{n+1})\, \Theta(\mu - t)\, u(\Phi_{n+1}) \\
& - P(\Phi_n, t)\, \frac{d\Phi_{n+1}}{d\Phi_n\, dt}\, d\sigma_{LO}(\Phi_n)\, u(\Phi_{n+1}) \times \Theta(t - \mu) + \mathcal{O}(\alpha_s^2)
\end{aligned}
$$

Infrared cutoff prevents finite weights.

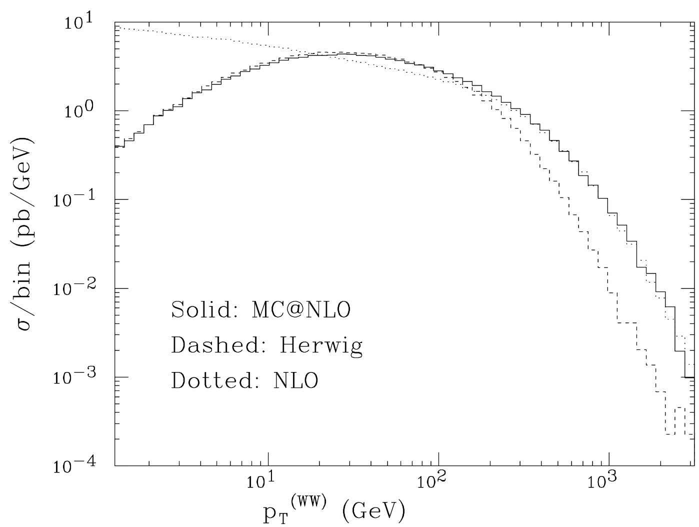
Add power correction (IR safe observables!) to fix divergences.

Matching Variants – Multi-purpose frameworks

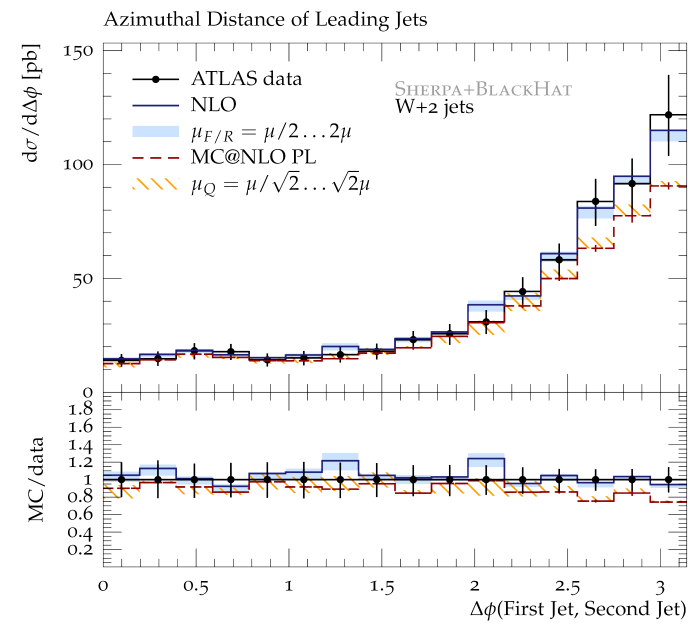
Depending on the choice of terms below the cutoff, the shower, and the subtraction terms chosen, there is a host of different matching Implementations.

Framework	Subtraction	Special hard emission	Subsequent shower
MC@NLO	FKS	no	Herwig 6/++
Powheg	FKS	ME correction	any
aMC@NLO	FKS	no	Herwig 6/++, Pythia 6/8
Sherpa-MC@NLO	CS	Colour-corrected dipole	CSS dipoles
Herwig7 NLO+PS	CS	no	Herwig7 Qtilde/Dipoles
Herwig7 NLOxPS	CS	ME correction	Herwig7 Qtilde/Dipoles

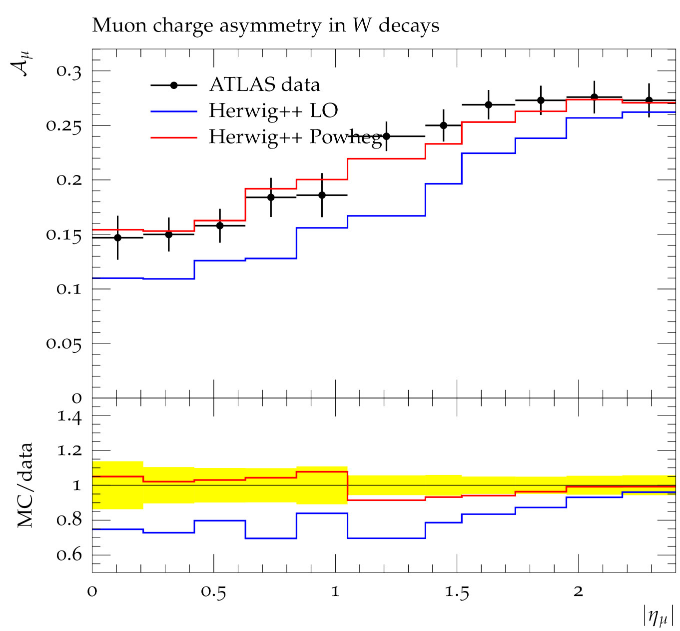
(Random) Examples



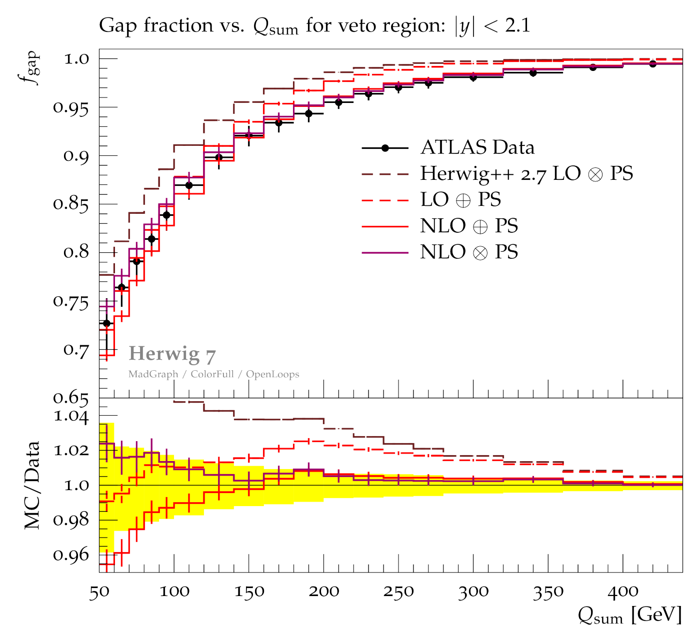
[Frixione, Webber – 2002]



[Höche, Krauss, Schönherr, Siegert – 2012]



[Hamilton, Richardson, Tully – 2009]



[Herwig7 collaboration – 2015]

Summary & Outlook – Part I

NLO calculations are automated.

→ Enabled by dedicated libraries and flexible interfaces.

NLO+PS matching is settled, different variants all fit into the same framework.

→ New standard for multipurpose event generators.

Watch out where you get NLO!

→ Observables driven by real emission are LO, further shower only!

Uncertainties in matching/showers are subject to current research.

→ Event generator uncertainty involves more than these variations.