Electroweak Theory and Higgs Mechanism

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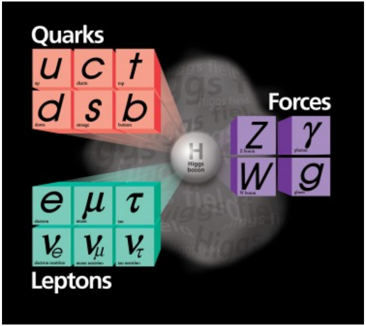
Standard Model of Particle Physics



Gauge interactions (3 parameters)

- SU(3)×SU(2)×U(1) symmetry, universality

Symmetry breaking sector (2 parameters)

- W, Z masses via gauge interactions
- minimal and weakly coupled but many alternatives (2HDM, little Higgs, extra dim)

Yukawa sector (20 parameters)

- fermion masses via Yukawa interactions
- rich flavour structure, mixing and CP violation

Simple and highly predictive

- almost all natural phenomena down to 10^{-9} times the atomic scale
- renormalizable ⇒ very accurate predictions and tests

Huge progress in perturbative calculations

$$\mathrm{d}\sigma \;=\; \mathrm{d}\sigma_{\mathrm{LO}} + \alpha_S\,\mathrm{d}\sigma_{\mathrm{NLO}} + \alpha_{\mathrm{EW}}\,\mathrm{d}\sigma^{\mathrm{EW}}_{\mathrm{NLO}} + \alpha_S^2\,\mathrm{d}\sigma_{\mathrm{NNLO}} + \alpha_S^3\,\mathrm{d}\sigma_{\mathrm{N^3LO}}$$

Perturbative calculations for hard scattering processes

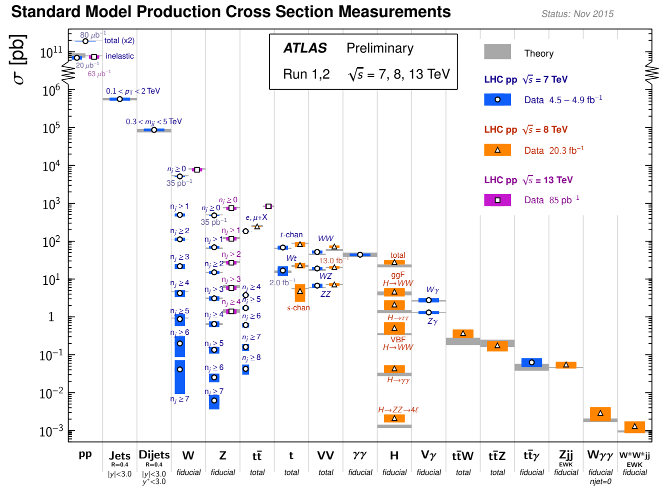
- more general, automated and widely applicable methods

⇒ drastic improvements for vast range of LHC processes

when	order	automation	availability	precision
2009–now	NLO QCD+EW	full	$2 \to 2, 3, 4(5)$ processes	$\mathcal{O}\,(10\%)$
2011–now	NNLO QCD	partial	$15\;\; 2 \to 2$ processes	few %
2015–now	$\mathrm{N^3LO}$ QCD	–	$gg \to H$	$\sim 1\%$

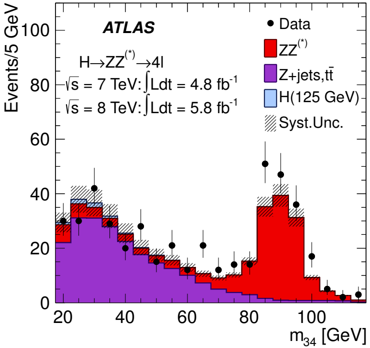
⇒ crucial ingredient of any SM precision test and many new-physics searches

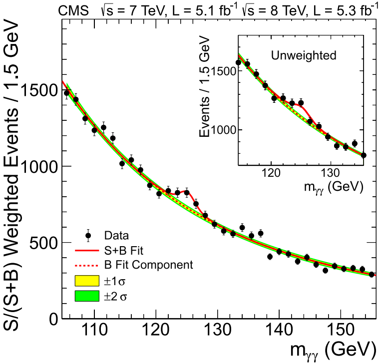
Success of LHC and Standard Model at Run 1 (+2)



(N)NLO calculations already crucial to explain several measurements

2012 Higgs Boson Discovery (48 years after hypothesis)





at

$$m_H = \begin{cases} 125.5 \pm 0.2\,(\text{stat})^{+0.5}_{-0.6}\,(\text{syst}) & \text{ATLAS} \\ 125.7 \pm 0.3\,(\text{stat}) \pm 0.3\,(\text{syst}) & \text{CMS} \end{cases}$$

Higgs mass measurement has turned the SM into a fully predictive theory at the quantum level!

First Higgs measurements [Run1 combination ATLAS-CONF-2015-044]

Theory predictions mostly NNLO QCD+NLO EW

- few to 10% uncertainty (already outdated)
- backgrounds typically less precise

Production process	Cross section [pb] $\sqrt{s} = 7$ TeV	Cross section [pb] $\sqrt{s} = 8$ TeV	Order of calculation
ggF	15.0 ± 1.6	19.2 ± 2.0	NNLO(QCD)+NLO(EW)
VBF	1.22 ± 0.03	1.58 ± 0.04	NLO(QCD+EW)+~NNLO(QCD)
WH	0.577 ± 0.016	0.703 ± 0.018	NNLO(QCD)+NLO(EW)
ZH	0.334 ± 0.013	0.414 ± 0.016	NNLO(QCD)+NLO(EW)
$[ggZH]$	0.023 ± 0.007	0.032 ± 0.010	NLO(QCD)
bbH	0.156 ± 0.021	0.203 ± 0.028	5FS NNLO(QCD) + 4FS NLO(QCD)
ttH	0.086 ± 0.009	0.129 ± 0.014	NLO(QCD)
tH	0.012 ± 0.001	0.018 ± 0.001	NLO(QCD)
Total	17.4 ± 1.6	22.3 ± 2.0	

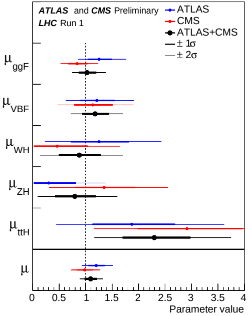
Channels established over 5σ (15%–25% unc.)

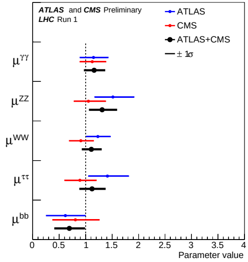
- ggH+VBF production and $H \to \gamma\gamma, ZZ, WW, \tau\tau$

Some tensions driven by $t\bar{t}H(WW)$ and $ZH(WW)$

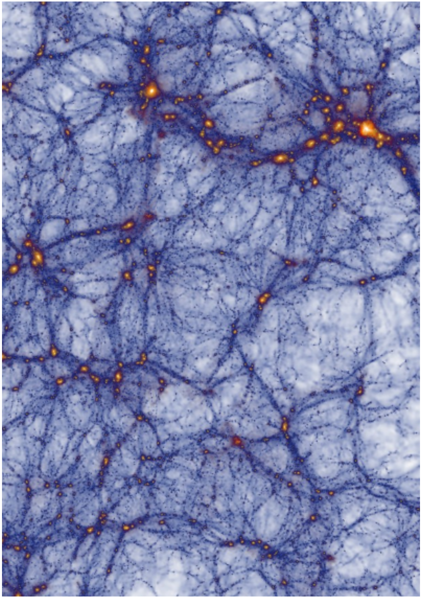
- $\mu_{ttH} = 2.3^{+0.7}_{-0.6}$ and $\mu^{bb} = 0.69^{+0.29}_{-0.27}$

$\Rightarrow$ still limited precision and room for surprises





Searches for Physics Beyond the Standard Model (BSM)



Millennium XXL simulation of cosmic web: Cluster formation at the intersection of filaments

Motivations

- **dark matter** in the universe
- quantum instability of Higgs mass
- ...

High discovery potential at LHC

- vast campaign of SM tests and BSM searches at higher energies
- precise SM predictions crucial for direct/indirect BSM sensitivity and interpretation of possible discoveries

This talk

1. Gauge symmetry and symmetry breaking
2. Electroweak Standard Model
3. Theoretical implications of a light Higgs Boson
4. Higgs production and decays at the LHC

Reference and Acknowledgement

This lecture is largely based on slides kindly made available by Laura Reina

- see lectures by L. Reina at 2011 Maria Laach & 2013 CTEQ schools

And additional material taken from

- lectures by Sally Dawson at 2012 CERN–FERMILAB & 2015 CTEQ schools

Particles and forces are a realization of fundamental symmetries of nature

Very old story: Noether's theorem in *classical mechanics*

$$L(q_i, \dot{q}_i) \text{ such that } \frac{\partial L}{\partial q_i} = 0 \longrightarrow p_i = \frac{\partial L}{\partial \dot{q}_i} \text{ conserved}$$

to any symmetry of the Lagrangian is associated a conserved physical quantity:

- $q_i = x_i \quad \longrightarrow \quad p_i$ linear momentum;
- $q_i = \theta_i \quad \longrightarrow \quad p_i$ angular momentum.

Generalized to the case of a *relativistic quantum theory* at multiple levels:

- $q_i \to \phi_j(x)$ coordinates become "fields" $\leftrightarrow$ "particles"
- $\mathcal{L}(\phi_j(x), \partial_\mu \phi_j(x))$ can be symmetric under many transformations.
- To any continuous symmetry of the Lagrangian we can associate a conserved current

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} \delta\phi_j \quad \text{such that} \quad \partial_\mu J^\mu = 0$$

The symmetries that make the world as we know it ...

- ▷ *translations*:
  conservation of energy and momentum;
- ▷ *Lorentz transformations* (rotations and boosts):
  conservation of angular momentum (orbital and spin);
- ▷ *discrete transformations* (P,T,C,CP,...):
  conservation of corresponding quantum numbers;
- ▷ *global transformations of internal degrees of freedom* (ϕ_j "rotations")
  conservation of "isospin"-like quantum numbers;
- ▷ *local transformations of internal degrees of freedom* ($\phi_j(x)$ "rotations"):
  define the interaction of fermion (s=1/2) and scalar (s=0) particles in terms of exchanged vector (s=1) massless particles $\longrightarrow$ "forces"

Requiring different global and local symmetries defines a theory

AND

Keep in mind that they can be broken

From Global to Local: gauging a symmetry

Abelian case

A theory of free Fermi fields described by the Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$$

is invariant under a global $U(1)$ transformation (α=constant phase)

$$\psi(x) \to e^{i\alpha}\psi(x) \quad \text{such that} \quad \partial_\mu\psi(x) \to e^{i\alpha}\partial_\mu\psi(x)$$

and the corresponding Noether's current is conserved,

$$J^\mu = \bar{\psi}(x)\gamma^\mu\psi(x) \quad \to \quad \partial_\mu J^\mu = 0$$

The same is not true for a local $U(1)$ transformation ($\alpha = \alpha(x)$) since

$$\psi(x) \to e^{i\alpha(x)}\psi(x) \quad \underline{\text{but}} \quad \partial_\mu\psi(x) \to e^{i\alpha(x)}\partial_\mu\psi(x) + ige^{i\alpha(x)}\partial_\mu\alpha(x)\psi(x)$$

Need to introduce a covariant derivative D_μ such that

$$D_\mu\psi(x) \to e^{i\alpha(x)}D_\mu\psi(x)$$

Only possibility: introduce a vector field $A_\mu(x)$ trasforming as

$$A_\mu(x) \to A_\mu(x) - \frac{1}{g}\partial_\mu \alpha(x)$$

and define a covariant derivative D_μ according to

$$D_\mu = \partial_\mu + igA_\mu(x)$$

modifying $\mathcal{L}$ to accomodate D_μ and the gauge field $A_\mu(x)$ as

$$\mathcal{L} = \bar{\psi}(x)(i\not{D} - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)$$

where the last term is the Maxwell Lagrangian for a vector field A^μ, i.e.

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \ .$$

Requiring invariance under a local $U(1)$ symmetry has:

$\longrightarrow$ promoted a free theory of fermions to an interacting one;

$\longrightarrow$ defined univoquely the form of the interaction in terms of a new vector field $A^\mu(x)$:

$$\mathcal{L}_{int} = -g\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)$$

$\longrightarrow$ no mass term $A^\mu A_\mu$ allowed by the symmetry $\to$ this is QED.

Non-abelian case: Yang-Mills theories

Consider the same Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$$

where $\psi(x) \to \psi_i(x)$ $(i = 1, \dots, n)$ is a n-dimensional representation of a non-abelian compact Lie group (e.g. $SU(N)$).
$\mathcal{L}$ is invariant under the global transformation $U(\alpha)$

$$\psi(x) \to \psi'(x) = U(\alpha)\psi(x) \ , \qquad U(\alpha) = e^{i\alpha^a T^a} = 1 + i\alpha^a T^a + O(\alpha^2)$$

where T^a $((a = 1, \dots, d_{adj}))$ are the generators of the group infinitesimal transformations with algebra,

$$[T^a, T^b] = if^{abc}T^c$$

and the corresponding Noether's current are conserved. <u>However</u>, requiring $\mathcal{L}$ to be invariant under the corresponding local transformation $U(x)$

$$U(x) = 1 + i\alpha^a(x)T^a + O(\alpha^2)$$

brings us to replace ∂_μ by a covariant derivative

$$D_\mu = \partial_\mu - igA^a_\mu(x)T^a$$

in terms of vector fields $A_\mu^a(x)$ that transform as

$$A_\mu^a(x) \to A_\mu^a(x) + \frac{1}{g}\partial_\mu \alpha^a(x) + f^{abc} A_\mu^b(x)\alpha^c(x)$$

such that

$$\begin{aligned} D_\mu &\to U(x) D_\mu U^{-1}(x) \\ D_\mu \psi(x) &\to U(x) D_\mu U^{-1}(x) U(x)\psi = U(x) D_\mu \psi(x) \\ F_{\mu\nu} \equiv \frac{i}{g}[D_\mu, D_\nu] &\to U(x) F_{\mu\nu} U^{-1}(x) \end{aligned}$$

The invariant form of $\mathcal{L}$ or Yang Mills Lagrangian will then be

$$\mathcal{L}_{YM} = \mathcal{L}(\psi, D_\mu\psi) - \frac{1}{4}\mathrm{Tr} F_{\mu\nu}F^{\mu\nu} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu}$$

where $F_{\mu\nu} = F_{\mu\nu}^a T^a$ and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

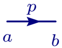
We notice that:

- as in the abelian case:
  - ⟶ mass terms of the form $A^{a,\mu}A^a_\mu$ are forbidden by symmetry: gauge bosons are massless
  - ⟶ the form of the interaction between fermions and gauge bosons is fixed by symmetry to be

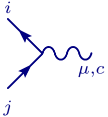
$$\mathcal{L}_{int} = -g\bar{\psi}(x)\gamma_\mu T^a \psi(x) A^{a,\mu}(x)$$

- at difference from the abelian case:
  - ⟶ gauge bosons carry a group charge and therefore ...
  - ⟶ gauge bosons have self-interaction.

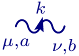
Feynman rules, Yang-Mills theory:



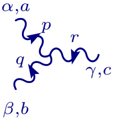
$$= \quad \frac{i\delta^{ab}}{\not{p} - m}$$



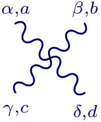
$$= \quad ig\gamma^{\mu}(T^{c})_{ij}$$



$$= \quad \frac{-i}{k^2}\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]\delta^{ab}$$



$$= \quad gf^{abc}\left(g^{\beta\gamma}(q-r)^{\alpha} + g^{\gamma\alpha}(r-p)^{\beta} + g^{\alpha\beta}(p-q)^{\gamma}\right)$$



$$= \quad -ig^2\left[f^{abe}f^{cde}(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}) + f^{ace}f^{bde}(\cdots) + f^{ade}f^{bce}(\cdots)\right]$$

Spontaneous Breaking of a Gauge Symmetry

Abelian Higgs mechanism: one vector field $A^\mu(x)$ and one complex scalar field $\phi(x)$:

$$\boxed{\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi}$$

where

$$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

and ($D^\mu = \partial^\mu + igA^\mu$)

$$\mathcal{L}_\phi = (D^\mu\phi)^* D_\mu\phi - V(\phi) = (D^\mu\phi)^* D_\mu\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

$\mathcal{L}$ invariant under local phase transformation, or local $U(1)$ symmetry:

$$\begin{aligned}\phi(x) &\rightarrow e^{i\alpha(x)}\phi(x)\\ A^\mu(x) &\rightarrow A^\mu(x) + \frac{1}{g}\partial^\mu\alpha(x)\end{aligned}$$

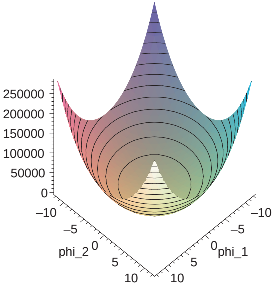
Mass term for A^μ breaks the $U(1)$ gauge invariance.

Can we build a gauge invariant massive theory? Yes.

Consider the potential of the scalar field:

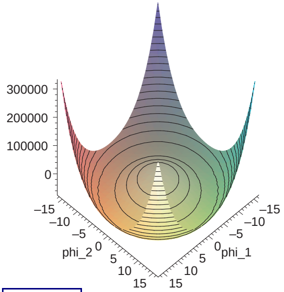
$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

where $\lambda > 0$ (to be bounded from below), and observe that:



$\mu^2 > 0$ $\rightarrow$ unique minimum:

$$\phi^* \phi = 0$$



$\mu^2 < 0$ $\rightarrow$ degeneracy of minima:

$$\phi^* \phi = \frac{-\mu^2}{2\lambda}$$

- $\mu^2 > 0 \longrightarrow$ electrodynamics of a massless photon and a massive scalar field of mass μ ($g = -e$).

- $\mu^2 < 0 \longrightarrow$ when we choose a minimum, the original $U(1)$ symmetry is spontaneously broken or hidden.

$$\phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} = \frac{v}{\sqrt{2}} \;\longrightarrow\; \phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\left(\phi_1(x) + i\phi_2(x)\right)$$

$$\Downarrow$$

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2A^\mu A_\mu}_{\text{massive vector field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_1)^2 + \mu^2\phi_1^2}_{\text{massive scalar field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_2)^2 + gvA_\mu\partial^\mu\phi_2}_{\text{Goldstone boson}} + \ldots$$

Side remark: The ϕ_2 field actually generates the correct transverse structure for the mass term of the (now massive) A^μ field propagator:

$$\langle A^\mu(k)A^\nu(-k)\rangle = \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \cdots$$

More convenient parameterization (unitary gauge):

$$\phi(x) = \frac{e^{i\frac{\chi(x)}{v}}}{\sqrt{2}}(v + H(x)) \quad \stackrel{U(1)}{\longrightarrow} \quad \frac{1}{\sqrt{2}}(v + H(x))$$

The $\chi(x)$ degree of freedom (Goldstone boson) is rotated away using gauge invariance, while the original Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_A + \frac{g^2v^2}{2}A^\mu A_\mu + \frac{1}{2}\left(\partial^\mu H \partial_\mu H + 2\mu^2 H^2\right) + \ldots$$

which describes now the dynamics of a system made of:

- a massive vector field A^μ with $m_A^2 = g^2v^2$;
- a real scalar field H of mass $m_H^2 = -2\mu^2 = 2\lambda v^2$: the Higgs field.

$$\Downarrow$$

Total number of degrees of freedom is balanced

Non-Abelian Higgs mechanism: several vector fields $A^a_\mu(x)$ and several (real) scalar field $\phi_i(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi \quad , \quad \mathcal{L}_\phi = \frac{1}{2}(D^\mu \phi)^2 - V(\phi) \quad , \quad V(\phi) = \mu^2 \phi^2 + \frac{\lambda}{2}\phi^4$$

($\mu^2 < 0$, $\lambda > 0$) invariant under a non-Abelian symmetry group G:

$$\phi_i \longrightarrow (1 + i\alpha^a t^a)_{ij}\phi_j \overset{t^a = iT^a}{\longrightarrow} (1 - \alpha^a T^a)_{ij}\phi_j$$

(s.t. $D_\mu = \partial_\mu + g A^a_\mu T^a$). In analogy to the Abelian case:

$$\frac{1}{2}(D_\mu \phi)^2 \longrightarrow \ldots + \frac{1}{2} g^2 (T^a\phi)_i (T^b\phi)_i A^a_\mu A^{b\mu} + \ldots$$
$$\overset{\phi_{min} = \phi_0}{\longrightarrow} \ldots + \frac{1}{2} \underbrace{g^2 (T^a\phi_0)_i (T^b\phi_0)_i}_{m^2_{ab}} A^a_\mu A^{b\mu} + \ldots =$$

$\boxed{T^a\phi_0 \neq 0}$ $\longrightarrow$ massive vector boson + (Goldstone boson)

$\boxed{T^a\phi_0 = 0}$ $\longrightarrow$ massless vector boson + massive scalar field

Classical $\longrightarrow$ Quantum : $\boxed{V(\phi) \longrightarrow V_{eff}(\varphi_{cl})}$

The stable vacuum configurations of the theory are now determined by the extrema of the Effective Potential:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{VT}\Gamma_{eff}[\phi_{cl}] \ , \ \phi_{cl} = \text{constant} = \varphi_{cl}$$

where

$$\Gamma_{eff}[\phi_{cl}] = W[J] - \int d^4y J(y)\phi_{cl}(y) \quad , \quad \phi_{cl}(x) = \frac{\delta W[J]}{\delta J(x)} = \langle 0|\phi(x)|0\rangle_J$$

$W[J] \longrightarrow$ generating functional of connected correlation functions
$\Gamma_{eff}[\phi_{cl}] \longrightarrow$ generating functional of 1PI connected correlation functions

$V_{eff}(\varphi_{cl})$ can be organized as a loop expansion (expansion in $\hbar$), s.t.:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}$$

SSB $\longrightarrow$ non trivial vacuum configurations

Outline

Towards the Standard Model of particle physics

Translating experimental evidence into the right gauge symmetry group.

- Electromagnetic interactions → QED
  - ▷ well established example of abelian gauge theory
  - ▷ extremely succesful quantum implementation of field theories
  - ▷ useful but very simple template
- Strong interactions → QCD
  - ▷ evidence for strong force in hadronic interactions
  - ▷ Gell-Mann-Nishijima quark model interprets hadron spectroscopy
  - ▷ need for extra three-fold quantum number (color)
    (ex.: hadronic spectroscopy, $e^+e^- \rightarrow$hadrons, ...)
  - ▷ natural to introduce the gauge group → $SU(3)_C$
  - ▷ DIS experiments: confirm parton model based on $SU(3)_C$
  - ▷ ... and much more!
- Weak interactions → most puzzling ...
  - ▷ discovered in neutron β-decay: $n \rightarrow p + e^- + \bar{\nu}_e$
  - ▷ new force: small rates/long lifetimes
  - ▷ universal: same strength for both hadronic and leptonic processes
    ($n \rightarrow pe^-\nu_e$, $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$, $\mu^- \rightarrow e^-\bar{\nu}_e + \nu_\mu$, ...)

- violates parity (P)
- charged currents only affect left-handed particles (right-handed antiparticles)
- neutral currents not of electromagnetic nature
- First description: Fermi Theory (1934)

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}}\,(\bar{p}\gamma_\mu(1-\gamma_5)n)(\bar{e}\gamma^\mu(1-\gamma_5)\nu_e)$$

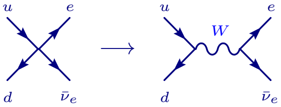
$G_F \rightarrow$ Fermi constant, $[G_F] = m^{-2}$ (in units of $c = \hbar = 1$).

- Easely accomodates a massive intermediate vector boson

$$\mathcal{L}_{IVB} = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^- + \text{h.c.}$$

with (in a proper quark-based notation)

$$J_\mu^- = \bar{u}\gamma_\mu \frac{1-\gamma_5}{2} d + \bar{\nu_e}\gamma^\mu \frac{1-\gamma_5}{2} e$$



provided that,

$$q^2 \ll M_W^2 \quad \longrightarrow \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

- Promote it to a gauge theory: natural candidate $SU(2)_L$, but if $T^{1,2}$ can generate the charged currents ($T^{\pm} = (T^1 \pm iT^2)$), T^3 cannot be the electromagnetic charge (Q) ($T^3 = \sigma^3/2$'s eigenvalues do not match charges in $SU(2)$ doublets)
- Need extra $U(1)_Y$, such that $Y = T^3 - Q$!
- Need massive gauge bosons → SSB

$$\Downarrow$$

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$$

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW}$$

where

$$\mathcal{L}_{EW} = \mathcal{L}_{EW}^{\text{ferm}} + \mathcal{L}_{EW}^{\text{gauge}} + \mathcal{L}_{EW}^{SSB} + \mathcal{L}_{EW}^{Yukawa}$$

Strong interactions: Quantum Chromodynamics

Exact Yang-Mills theory based on $SU(3)_C$ (quark fields only):

$$\mathcal{L}_{QCD} = \sum_i \bar{Q}_i (i\not{D} - m_i)\, Q_i - \frac{1}{4} F^{a,\mu\nu} F^a_{\mu\nu}$$

with

$$D_\mu = \partial_\mu - igA^a_\mu T^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- $Q_i \to (i = 1, \ldots, 6 \to u, d, s, c, b, t)$ fundamental representation of $SU(3) \to$ triplets:

$$Q_i = \begin{pmatrix} Q_i \\ Q_i \\ Q_i \end{pmatrix}$$

- $A^a_\mu \to$ adjoint representation of $SU(3) \to N^2 - 1 = 8$ massless gluons
  $T^a \to SU(3)$ generators (Gell-Mann's matrices)

Electromagnetic and weak interactions: unified into Glashow-Weinberg-Salam theory

Spontaneously broken Yang-Mills theory based on $SU(2)_L \times U(1)_Y$.

- $SU(2)_L \rightarrow$ weak isospin group, gauge coupling g:
  - ▷ three generators: $T^i = \sigma^i/2$ (σ^i = Pauli matrices, $i = 1, 2, 3$)
  - ▷ three gauge bosons: W_1^μ, W_2^μ, and W_3^μ
  - ▷ $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ fields are doublets of $SU(2)$
  - ▷ $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$ fields are singlets of $SU(2)$
  - ▷ mass terms not allowed by gauge symmetry
- $U(1)_Y \rightarrow$ weak hypercharge group ($Q = T_3 + Y$), gauge coupling g':
  - ▷ one generator $\rightarrow$ each field has a Y charge
  - ▷ one gauge boson: B^μ

Example: first generation

$$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}_{Y=-1/2} \qquad (\nu_{eR})_{Y=0} \qquad (e_R)_{Y=-1}$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_{Y=1/6} \qquad (u_R)_{Y=2/3} \qquad (d_R)_{Y=-1/3}$$

Three fermionic generations, summary of gauge quantum numbers:

				$\underline{SU(3)_C}$	$\underline{SU(2)_L}$	$\underline{U(1)_Y}$	$\underline{U(1)_Q}$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{matrix} \frac{2}{3} \\ -\frac{1}{3} \end{matrix}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{matrix} 0 \\ -1 \end{matrix}$
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

where a minimal extension to include ν_R^i has been allowed (notice however that it has zero charge under the entire SM gauge group!)

Lagrangian of fermion fields

For each generation (here specialized to the first generation):

$$\mathcal{L}_{EW}^{\text{ferm}} = \bar{L}_L(i\not{D})L_L+\bar{e}_R(i\not{D})e_R+\bar{\nu}_{eR}(i\not{D})\nu_{eR}+\bar{Q}_L(i\not{D})Q_L+\bar{u}_R(i\not{D})u_R+\bar{d}_R(i\not{D})d_R$$

where in each term the covariant derivative is given by

$$D_\mu = \partial_\mu - igW_\mu^i T^i - ig'\frac{1}{2}YB_\mu$$

and $T^i = \sigma^i/2$ for L-fields, while $T^i = 0$ for R-fields ($i = 1, 2, 3$), i.e.

$$\begin{aligned} D_{\mu,L} &= \partial_\mu - \frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} gW_\mu^3 - g'YB_\mu & 0 \\ 0 & -gW_\mu^3 - g'YB_\mu \end{pmatrix} \\ D_{\mu,R} &= \partial_\mu + ig'\frac{1}{2}YB_\mu \end{aligned}$$

with

$$W^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

$\mathcal{L}_{EW}^{\text{ferm}}$ can then be written as

$$\mathcal{L}_{EW}^{\text{ferm}} = \mathcal{L}_{kin}^{\text{ferm}} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

where

$$\begin{aligned}
\mathcal{L}_{kin}^{\text{ferm}} &= \bar{L}_L(i\partial\!\!\!/)L_L + \bar{e}_R(i\partial\!\!\!/)e_R + \ldots \\
\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_{eL}\gamma^\mu e_L + W_\mu^- \bar{e}_L\gamma^\mu \nu_{eL} + \ldots \\
\mathcal{L}_{NC} &= \frac{g}{2} W_\mu^3 \left[\bar{\nu}_{eL}\gamma^\mu \nu_{eL} - \bar{e}_L\gamma^\mu e_L\right] + \frac{g'}{2} B_\mu \left[Y(L)(\bar{\nu}_{eL}\gamma^\mu \nu_{eL} + \bar{e}_L\gamma^\mu e_L)\right. \\
&+ \left. Y(e_R)\bar{\nu}_{eR}\gamma^\mu \nu_{eR} + Y(e_R)\bar{e}_R\gamma^\mu e_R\right] + \ldots
\end{aligned}$$

where

$W^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \rightarrow$ mediators of Charged Currents

W_μ^3 and $B_\mu \rightarrow$ mediators of Neutral Currents.

$$\Downarrow$$

<u>However</u> neither W_μ^3 nor B_μ can be identified with the photon field A_μ, because they couple to neutral fields.

Rotate W_μ^3 and B_μ introducing a weak mixing angle (θ_W)

$$
\begin{aligned}
W_\mu^3 &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\
B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu
\end{aligned}
$$

such that the kinetic terms are still diagonal and the neutral current lagrangian becomes

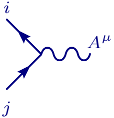
$$
\mathcal{L}_{NC} = \bar{\psi}\gamma^\mu \left(g\sin\theta_W T^3 + g'\cos\theta_W \frac{Y}{2} \right) \psi A_\mu + \bar{\psi}\gamma^\mu \left(g\cos\theta_W T^3 - g'\sin\theta_W \frac{Y}{2} \right) \psi Z_\mu
$$

for $\psi^T = (\nu_{eL}, e_L, \nu_{eR}, e_R, \ldots)$. One can then identify ($Q \to$ e.m. charge)

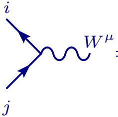
$$
eQ = g\sin\theta_W T^3 + g'\cos\theta_W \frac{Y}{2}
$$

and, e.g., from the leptonic doublet L_L derive that

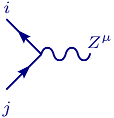
$$
\begin{cases}
\frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W = 0 \\
-\frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W = -e
\end{cases}
\longrightarrow g\sin\theta_W = g'\cos\theta_W = e
$$



$$= \quad -ieQ_f\gamma^\mu$$



$$= \quad \frac{ie}{2\sqrt{2}s_w}\gamma^\mu(1-\gamma_5)$$



$$= \quad ie\gamma^\mu(v_f - a_f\gamma_5)$$

where

$$v_f = -\frac{s_w}{c_w}Q_f + \frac{T_f^3}{2s_W c_W}$$

$$a_f = \frac{T_f^3}{2s_W c_W}$$

Lagrangian of gauge fields

$$\mathcal{L}_{EW}^{\text{gauge}} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

where

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc} W_\mu^b W_\nu^c \end{aligned}$$

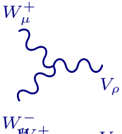
in terms of physical fields:

$$\mathcal{L}_{EW}^{\text{gauge}} = \mathcal{L}_{kin}^{\text{gauge}} + \mathcal{L}_{EW}^{3V} + \mathcal{L}_{EW}^{4V}$$

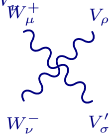
where

$$\begin{aligned} \mathcal{L}_{kin}^{\text{gauge}} &= -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\ &- \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ \mathcal{L}_{EW}^{3V} &= (\text{3-gauge-boson vertices involving } ZW^+W^- \text{ and } AW^+W^-) \\ \mathcal{L}_{EW}^{4V} &= (\text{4-gauge-boson vertices involving } ZZW^+W^-,\ AAW^+W^-, \\ & \quad AZW^+W^-, \text{ and } W^+W^-W^+W^-) \end{aligned}$$

$$= \quad \frac{-i}{k^2 - M_V^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_V^2}\right)$$



$$= \quad ieC_V \left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\right]$$



$$= \quad ie^2 C_{VV'}\left(2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$$

where

$$C_\gamma = 1 \;\; , \quad C_Z = -\frac{c_W}{s_W}$$

and

$$C_{\gamma\gamma} = -1 \;\; , \quad C_{ZZ} = -\frac{c_W^2}{s_W^2} \;\; , \quad C_{\gamma Z} = \frac{c_W}{s_W} \;\; , \quad C_{WW} = \frac{1}{s_W^2}$$

The Higgs sector of the Standard Model: $SU(2)_L \times U(1)_Y \stackrel{SSB}{\longrightarrow} U(1)_Q$

Introduce one complex scalar doublet of $SU(2)_L$ with $Y = 1/2$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \longleftrightarrow \quad \mathcal{L}_{EW}^{SSB} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

where $D_\mu \phi = (\partial_\mu - igW_\mu^a T^a - ig' Y_\phi B_\mu)$, $(T^a = \sigma^a/2,\ a = 1,2,3)$.

The SM symmetry is spontaneously broken when $\langle \phi \rangle$ is chosen to be (e.g.):

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left(\frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0,\ \lambda > 0)$$

The gauge boson mass terms arise from:

$$\begin{aligned}(D^\mu \phi)^\dagger D_\mu \phi \quad &\longrightarrow \quad \cdots + \frac{1}{8} (0 \;\; v) \left(g W_\mu^a \sigma^a + g' B_\mu \right) \left(g W^{b\mu} \sigma^b + g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} + \cdots \\ &\longrightarrow \quad \cdots + \frac{1}{2} \frac{v^2}{4} \left[g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-g W_\mu^3 + g' B_\mu)^2 \right] + \cdots \end{aligned}$$

And correspond to the weak gauge bosons:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \longrightarrow \boxed{M_W = g\frac{v}{2}}$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu) \longrightarrow \boxed{M_Z = \sqrt{g^2+g'^2}\frac{v}{2}}$$

while the linear combination orthogonal to Z_μ remains massless and corresponds to the photon field:

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu) \longrightarrow \boxed{M_A = 0}$$

Notice: using the definition of the weak mixing angle, θ_w:

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}} \quad , \quad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$$

the W and Z masses are related by: $\boxed{M_W = M_Z\cos\theta_w}$

The scalar sector becomes more transparent in the unitary gauge:

$$\phi(x) = \frac{e^{\frac{i}{v}\vec{\chi}(x)\cdot\vec{\tau}}}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \xrightarrow{SU(2)} \quad \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

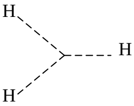
after which the Lagrangian becomes

$$\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4$$

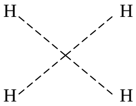
Three degrees of freedom, the $\chi^a(x)$ Goldstone bosons, have been reabsorbed into the longitudinal components of the $W_\mu^\pm$ and Z_μ weak gauge bosons. One real scalar field remains:

the Higgs boson, H, with mass $\boxed{M_H^2 = -2\mu^2 = 2\lambda v^2}$

and self-couplings:

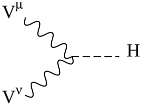


$= -3i \frac{M_H^2}{v}$

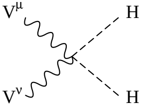


$= -3i \frac{M_H^2}{v^2}$

From $(D^\mu \phi)^\dagger D_\mu \phi \longrightarrow$ Higgs-Gauge boson couplings:



$= 2i \frac{M_V^2}{v} g^{\mu\nu}$



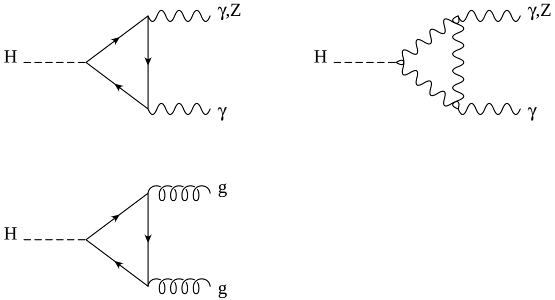
$= 2i \frac{M_V^2}{v^2} g^{\mu\nu}$

Notice: The entire Higgs sector depends on only two parameters, e.g.

$$M_H \text{ and } v$$

v measured in μ-decay:
$v = (\sqrt{2} G_F)^{-1/2} = 246$ GeV $\longrightarrow$ **SM Higgs Physics depends on M_H**

Also: remember Higgs-gauge boson loop-induced couplings:



They will be discussed in the context of Higgs boson decays.

Higgs boson couplings to quarks and leptons

The chiral nature of gauge symmetry in the SM also forbids fermion mass terms ($\mathcal{L}_m = m_f \bar{f}_L f_R$+h.c.), but all fermions are massive.

Fermion masses are generated via gauge-invariant Yukawa couplings:

$$\mathcal{L}_{Yukawa} = \sum_{i,j=1}^{3} \left\{ -Y_u^{ij} \bar{Q}_L^i \phi^c u_R^j - Y_d^{ij} \bar{Q}_L^i \phi d_R^j - Y_\ell^{ij} \bar{L}_L^i \phi l_R^j + h.c. \right\}$$

such that, upon spontaneous symmetry breaking:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_{Yukawa} &= -Y_u^{ij} \bar{u}_L^i \frac{v+H}{\sqrt{2}} u_R^j - Y_d^{ij} \bar{d}_L^i \frac{v+H}{\sqrt{2}} d_R^j - Y_\ell^{ij} \bar{l}_L^i \frac{v+H}{\sqrt{2}} l_R^j + \text{h.c.} \\ &= -\sum_{f=u,d,\ell} \sum_{i,j=1}^{3} \bar{f}_L^i M_f^{ij} f_R^j \left(1 + \frac{H}{v}\right) + \text{h.c.} \end{aligned}$$

with a complex-valued non-diagonal mass matrix

$$M_f^{ij} = Y_f^{ij} \frac{v}{\sqrt{2}}$$

Upon diagonalization (by unitary transformations U_L and U_R)

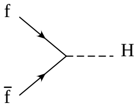
$$M_D = (U_L^f)^\dagger M_f U_R^f$$

and defining mass eigenstates:

$$f_L'^{\,i} = (U_L^f)_{ij} f_L^j \quad \text{and} \quad f_R'^{\,i} = (U_R^f)_{ij} f_R^j$$

the fermion masses are extracted as

$$\begin{aligned}\mathcal{L}_{Yukawa} &= \sum_{f,i,j} \bar{f}_L'^{\,i} [(U_L^f)^\dagger M_f U_R^f] f_R'^{\,j} \left(1 + \frac{H}{v}\right) + \text{h.c.} \\ &= \sum_{f,i,j} m_f \left(\bar{f}_L' f_R' + \bar{f}_R' f_L'\right) \left(1 + \frac{H}{v}\right)\end{aligned}$$



$$= -i\frac{m_f}{v} = -iy_f$$

In terms of the new mass eigenstates the quark part of $\mathcal{L}_{CC}$ now reads

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{u}'^{\,i}_L [(U^u_L)^\dagger U^d_L] \gamma^\mu d^j_L + \text{h.c.}$$

where

$$V_{CKM} = (U^u_L)^\dagger U^d_L$$

is the Cabibbo-Kobayashi-Maskawa matrix, origin of flavor mixing and CP violation in the SM.

No flavour changing neutral currents ⇔ minimal form of SM Higgs sector