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Theory of QCD jets

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CTEQ/MCNET/DESY School

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Outline of lecture

- Introduction to basics of jet physics
- Jets in e^+e^- annihilation : definition and jet fraction calculation
- Jets in hadron collisions : definition and properties
- Perturbative and non-perturbative effects in jets at small R
- Optimal R values for resonance reconstruction

Note: Jet substructure covered in next lecture.

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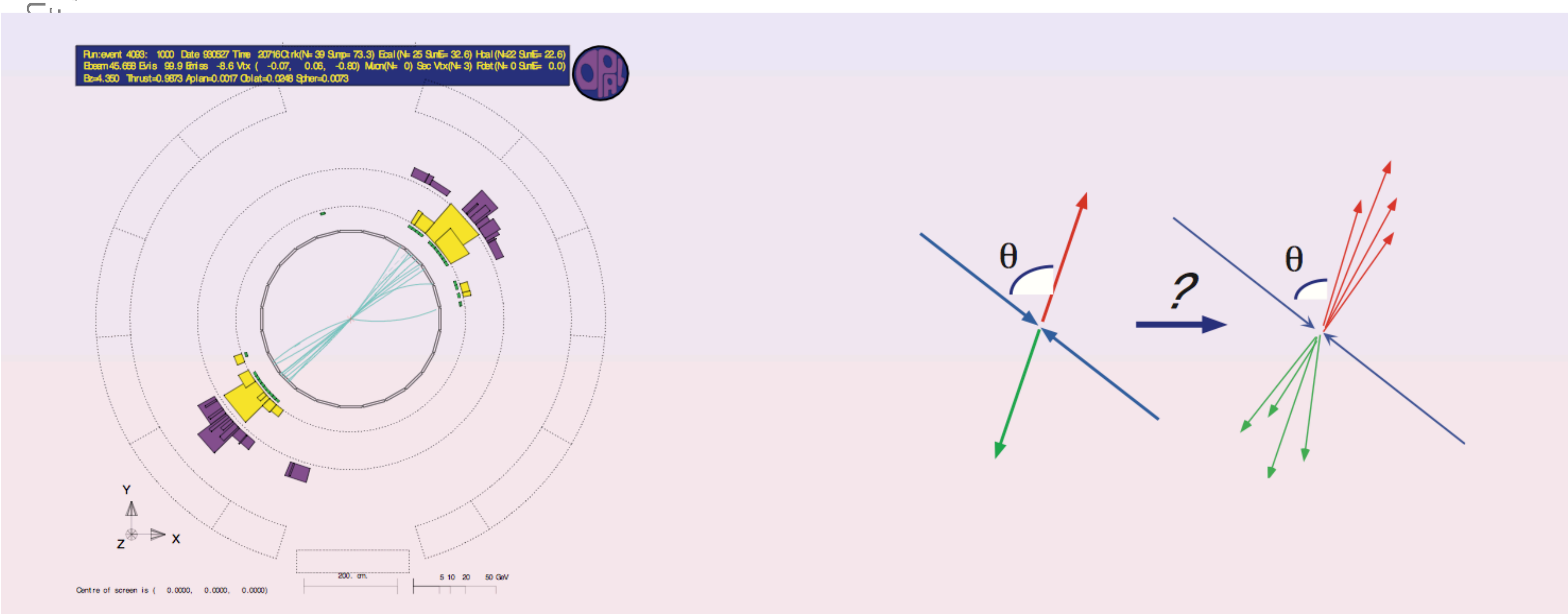
Basics of jets

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^A_{\alpha\beta} F_A^{\alpha\beta} + \sum_{\rm flavours} \bar{q}_a (i\not{D} - m)_{ab} q_b + \mathcal{L}_{\rm gauge-fixing} + \mathcal{L}_{\rm ghost}$$

- QCD poses a unique challenge: Quarks and gluons appearing in Lagrangian not seen in detectors.
- This fact can have fatal consequences for perturbative QCD predictions. It is not a minor detail.

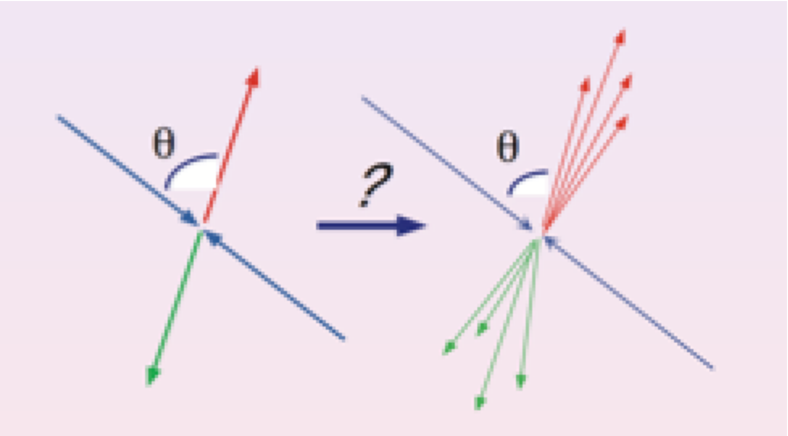
Basics : from partons to jets



- At high energy colliders we do see footprints of underlying partonic scattering.
- Collimated sprays of particles or hadronic jets. Jets are proxies for the energetic partons that emerge from hard scattering.

Sterman TASI Lectures 2004

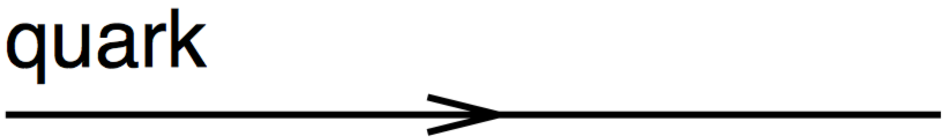
Basics : from partons to jets



- Jets are seen in place of partons in experiments.

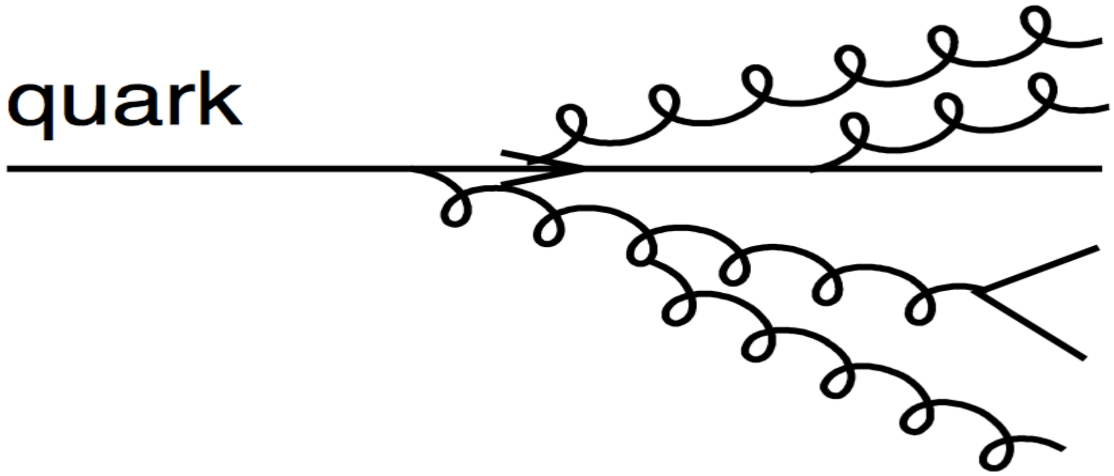
- But introduction of jets as a theoretical concept is also suggested by theory itself. One sees this when attempting to explain the transition of partons into the experimental picture.

Basics : from partons to jets



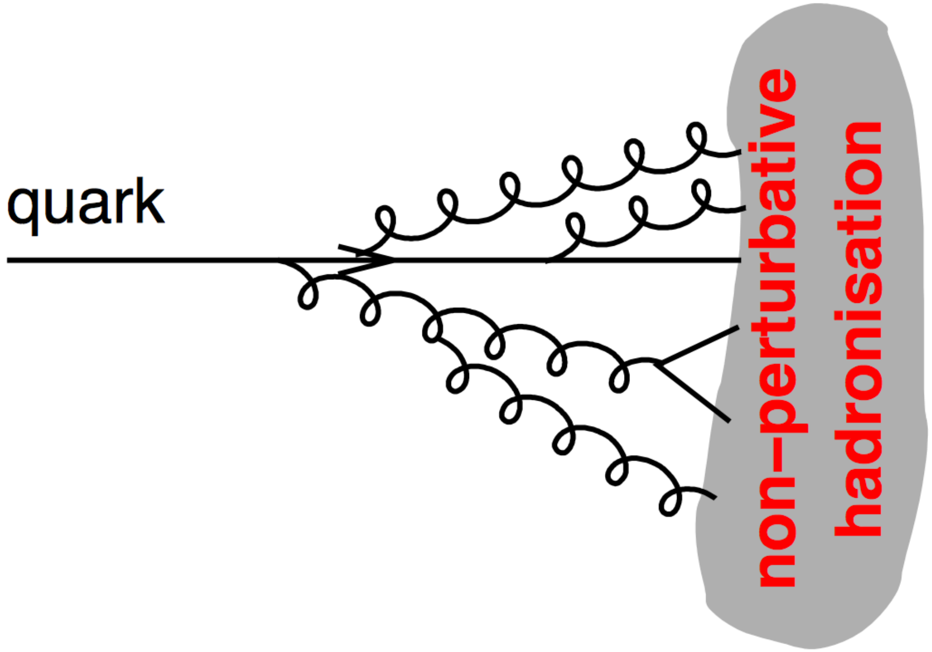
Start with an energetic parton produced in hard process.

Basics : from partons to jets



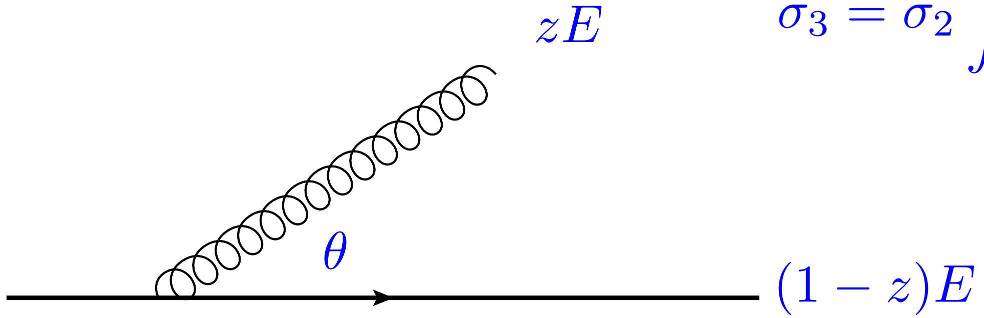
Need to account for production of many particles. Naïve perturbation theory suggests n additional parton radiation ~ α_s^n

Basics: from partons to jets



Also need to account for hadronisation of partons.

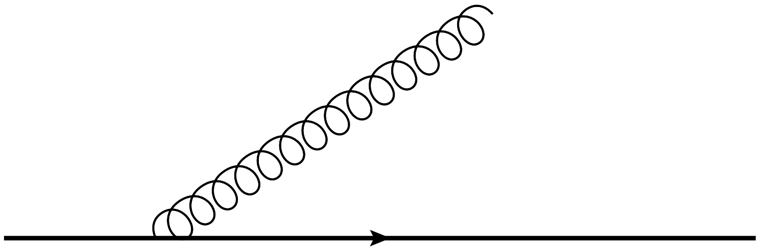
Basics : from partons to jets



$$\sigma_3 = \sigma_2 \int_0^1 \frac{dz}{z} \int_0^1 \frac{d\theta^2}{\theta^2} \frac{C_F \alpha_s}{\pi}$$

Computing just one additional particle production already fails due to IRC divergences.

Basics : from partons to jets



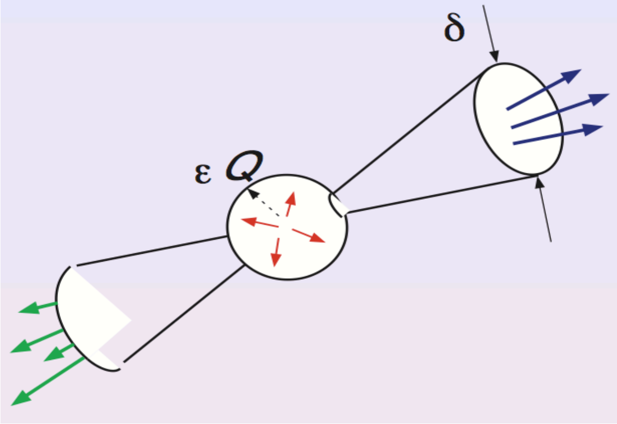
$$\sigma_3 = \sigma_2 \int_0^1 \frac{dz}{z} \int_0^1 \frac{d\theta^2}{\theta^2} \frac{C_F \alpha_s(z\theta E)}{\pi}$$

Running coupling also diverges in soft and collinear limit.

Lesson: Multiple particle production probability cannot be computed perturbatively. Probability of particle production can be $\mathcal{O}(1)$ rather than $\mathcal{O}(\alpha_s)$.

Defining jets : SW cones

Previous calculation shows that to calculate perturbatively one needs energy and angular resolution parameters.



Sterman and Weinberg: Define di-jet event by including emissions below energy fraction ϵ or those within angle δ into hard jets.

G.Sterman and S.Weinberg 1977

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Defining jets : SW cones

$$\sigma_{3\mathrm{jets}} \sim \sigma_{2\mathrm{jets}} \frac{C_F \alpha_s}{\pi} \ln \frac{1}{\epsilon} \ln \frac{1}{\delta^2}$$

plus higher order corrections and non-logarithmic terms.

- Probability of producing extra parton is $\mathcal{O}(1)$
  That of producing extra jet is $\mathcal{O}(\alpha_s)$
- But beware of large logarithms in resolution parameters.

Defining jets : Snowmass accord

- SW jet definition not very practical though provides basic idea. IRC safety is cruical from theory viewpoint.
- Experimental considerations equally important. A number of key requirements were laid out at Snowmass 1990.

Defining jets : Snowmass accord

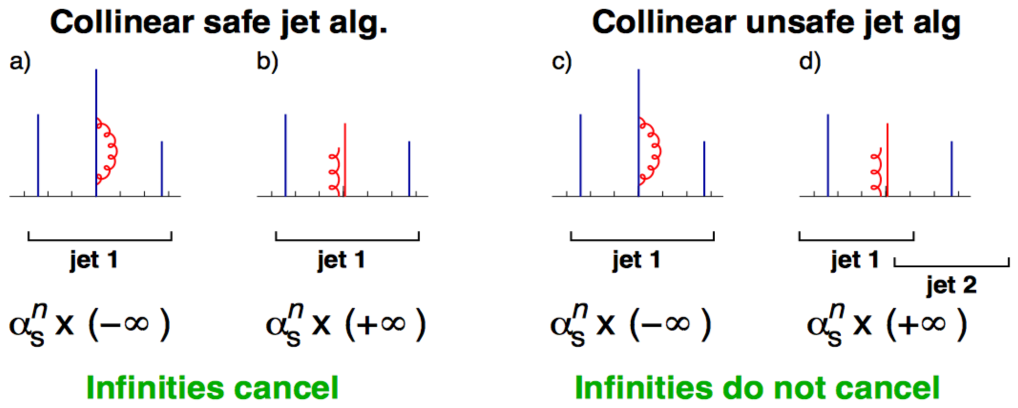
Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.

Simple to state but proved notoriously hard to satisfy. Many variants of cone algorithms exist but virtually all had problems.

QCD and Collider Physics (ESW) “Snowmass accord more honoured in the breach than in the observance.”

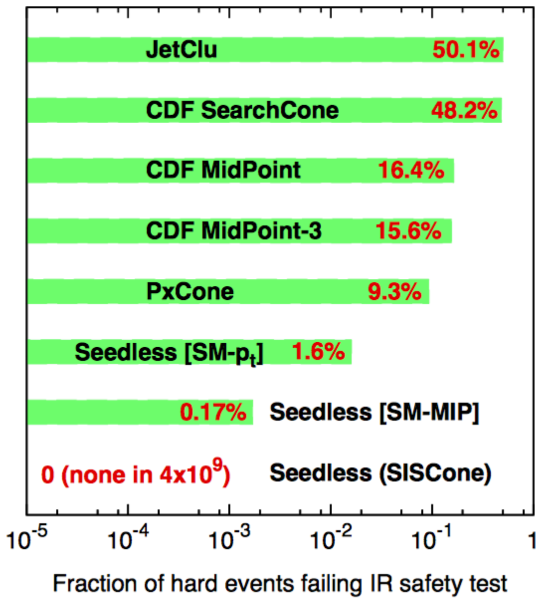
Defining jets : history of problems



Example of iterative cone algorithms that use the hardest particle as a seed for jet finding.

Collinear splitting changes jet structure leading to divergence.

Defining jets : issues with cones



- Several variants of cone jet algorithms proposed and used in theory calculations and experiment.
- Nearly all cone definitions have suffered from IRC safety issues.

Seedless infrared safe cone SISCONE is free from this issue.

Salam and Soyez 2007

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Defining jets : sequential recombination

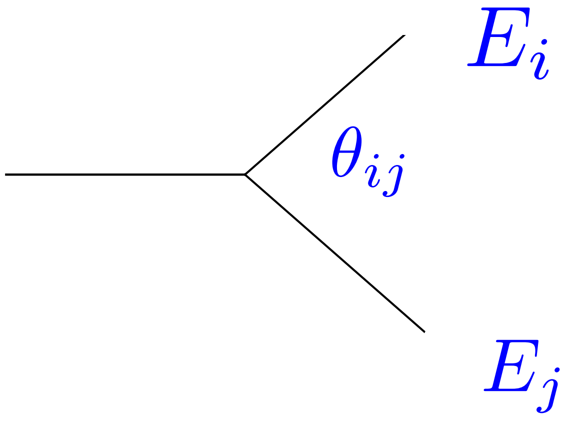
- Another class of algorithms involves building up jets using a distance measure y_{ij}

- One of the earliest examples still in use is the k_t algorithm. Originally formulated for studies of jets in e^+e^- annihilation.

$$y_{ij} = \frac{2\min\left(E_i, E_j\right)^2}{Q^2}\left(1 - \cos\theta_{ij}\right)$$

Catani, Dokshitzer, Olsson, Turnock and Webber 1991.

Defining jets: k_t algorithm



$$E_i < E_j \qquad \theta_{ij} \ll 1$$

$$y_{ij} = \frac{2\min\left(E_i, E_j\right)^2}{Q^2}\left(1 - \cos\theta_{ij}\right)$$

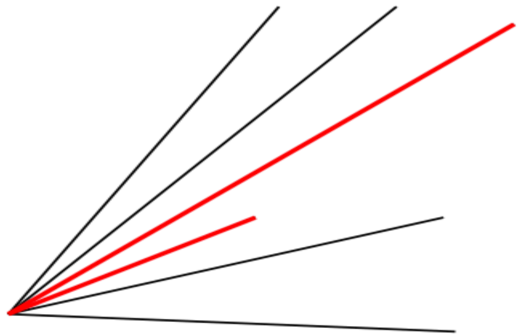
$$\approx \frac{E_i^2}{Q^2}\theta_{ij}^2$$

$$\approx k_{t,ij}^2 / Q^2$$

In small angle limit y_{ij} is the transverse momentum of softer particle wrt direction of harder one.

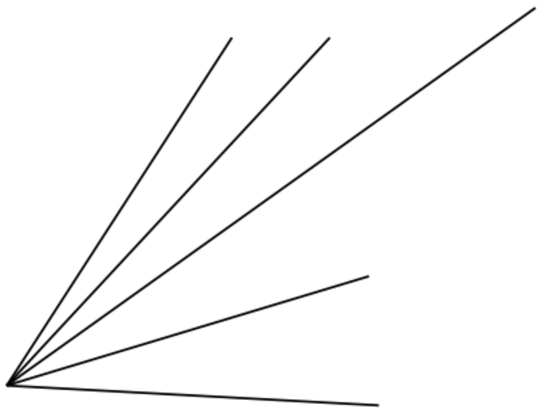
Defining jets : k_t algorithm

- For each pair of particles compute the y_{ij} distance and find the minimum value $y_{min.}$



Defining jets : k_t algorithm

- If $y_{min} < y_{cut}$ then recombine i and j into a single particle and repeat with new list of particles.



$$p_k = p_i + p_j$$

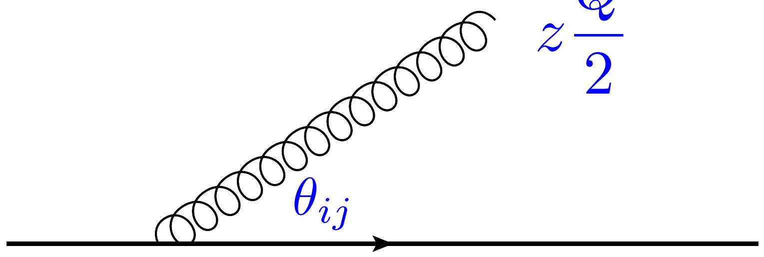
- Recombination is done according to some scheme. Most commonly used is E-scheme where one adds four-momenta and produces massive jets.
- Otherwise declare all particles to be jets and terminate.

Pedagogical example : jet fractions

Let us do a leading order calculation of the three jet fraction f_3.

We shall work in the soft-collinear approximation.

$z \ll 1 \;\; \theta_{ij} \ll 1$



$z\frac{Q}{2}$

θ_{ij}

$(1-z)\frac{Q}{2}$

Condition to have three jets : $z^2\theta_{ij}^2 > 4y_{\mathrm{cut}}$

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Three jet fraction

$$f_3 = \frac{\sigma_3}{\sigma}$$

$$\approx \frac{C_F \alpha_s}{\pi} \int \frac{dz}{z} \frac{d\theta^2}{\theta^2} \Theta\left(z^2\theta^2 - 4y_{\mathrm{cut}}\right)$$

Both infrared and collinear divergence removed by single jet resolution condition.

$$f_3 \approx \frac{C_F \alpha_s}{2\pi} \ln^2 \frac{1}{y_{\mathrm{cut}}} + \mathcal{O}\left(\alpha_s \ln \frac{1}{y_{\mathrm{cut}}}\right)$$

Double logarithmic term is leading for $y_{\mathrm{cut}} \ll 1$

Three jet fraction

Also straightforward to compute single logarithmic terms. Need to extend the soft approximation by using the full QCD splitting function

$$\frac{dz}{z} \to \frac{1+(1-z)^2}{2z} dz$$

Then one gets

$$f_3 \approx \frac{C_F \alpha_s}{2\pi} \left(\ln^2 \frac{1}{y_{\mathrm{cut}}} - \frac{3}{2} \ln \frac{1}{y_{\mathrm{cut}}} \right)$$

Jet fractions at small y_{cut}

At this order one also has

$$f_2 = 1 - f_3$$

$$\approx 1 - \frac{C_F \alpha_s}{2\pi} \left(\ln^2 \frac{1}{y_{\rm cut}} - \frac{3}{2} \ln \frac{1}{y_{\rm cut}} \right)$$

At small $y_{\rm cut}$ the logarithms are large and can compensate the smallness of the coupling.

Indicates inadequacy of fixed-order calcs. Need to resum large logarithms to all orders.

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Resummation for jet fractions

- If y_{cut} is small enough that $\alpha_s L^2 \sim 1$ one needs all order resummation of terms $\alpha_s^n L^{2n}$. This restores sensible behaviour at small y_{cut}.

- But for accurate phenomenology one also generally needs resummation of single logarithmic terms $\alpha_s^n L^n$

Resummation of f_2

Resummed result for f_2 and higher jet fractions can be expressed in terms of Sudakov form factors.

$$f_2(Q^2, y) = [\Delta_q]^2$$

$$\Delta_q = \exp\left[-\int_{y_{\mathrm{cut}}Q^2}^{Q^2} dt'\,\Gamma_q(t,t')\right]$$

$$\Gamma_q(t,t') = \frac{C_F}{2\pi}\frac{\alpha_s(t')}{t'}\left(\ln\frac{t}{t'} - \frac{3}{2}\right)$$

At fixed coupling this is just exponentiation of leading order result derived before.

Jet fractions

$$\Delta_q = \exp\left[-\int_{y_{\rm cut}Q^2}^{Q^2} dt' \, \Gamma_q\left(t,t'\right)\right]$$

- The Sudakov form factor expresses the probability that there are no emissions above some scale y_{cut} Q^2 . Resummation also possible and known for higher jet fractions.

- If y_{cut} is not too small $\alpha_s \ln^2 y_{\rm cut} \sim \alpha_s$ then resummed calculations alone are not meaningful. One should then use exact fixed-order calculations without a soft-collinear approximation.

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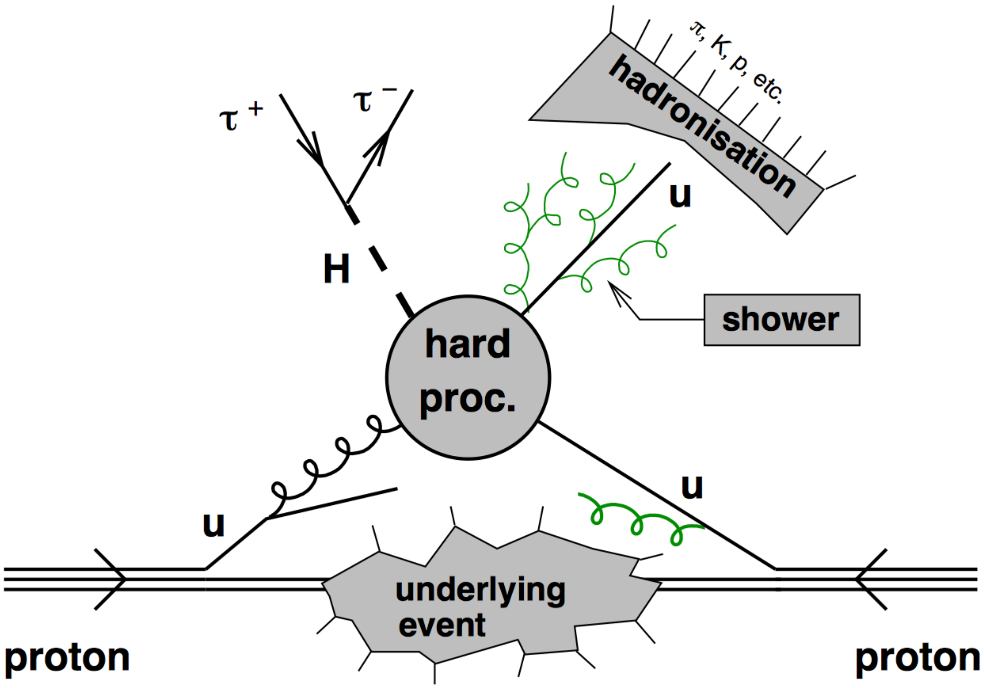
Jet fractions at fixed-order

$$f_{n+2}(Q^2, y) = \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^n \sum_{j=0}^{\infty} C_{nj}\left(y, \frac{Q^2}{\mu^2}\right)\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^j$$

- For complete calculations valid over a wide range of y_{cut} one needs to combine resummed calculations with fixed-order predictions without double counting i.e. perform matched resummed calculations.

- Comparison of theoretical predictions (with and without resummation) for jet rates against data from LEP has been one of the standard tests of QCD

Jets in hadron-hadron collisions



Much more complicated environment. Need to make significant changes to jet algorithms.

SR algorithms at hadron colliders

- The kt algorithm has to be modified for use at hadron colliders
- In hh collisions the centre-of-mass energy is not really relevant to the hard process
- One also has two incoming partons and to ensure there are no divergences associated to emission off these partons.

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k_t algorithm for hadron colliders

Uses dimensionful distance measures formulated in terms of variables invariant under longitudinal boosts along the beam axis.

Definition

$$d_{ij} = \min(p_{t,i}^2, p_{t,j}^2)\frac{\Delta_{ij}}{R^2}, \ \Delta_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} = p_{t,i}^2$$

$p_{t,i}$ is transverse momentum wrt the beam direction.

$y_i = \frac{1}{2}\ln\left(\frac{E + p_{z,i}}{E - p_{z,i}}\right)$ is rapidity wrt beam.

ϕ_i is azimuthal angle in plane transverse to beam.

Note that at small angles $\Delta_{ij} \propto \theta_{ij}^2$ and R plays the role of a cone radius parameter.

Ellis and Soper 1993. Catani, Dokshitzer, Seymour and Webber 1993.

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Inclusive k_t algorithm

In "inclusive mode" algorithm proceeds as follows:

- Find the smallest among d_{ij} and d_{iB}. If it is a d_{iB} call the object a jet and remove from list. If d_{ij} then merge *i* and *j*.
- Repeat until all particles are removed.

Parameter R sets jet opening angle. If object i has no objects within distance R then d_{iB} is always smallest and i is called jet.

Note :To prevent arbitrarily soft particles being called a jet one needs also $p_{T,i} > p_{T,\min}$

Other SR algorithms

We can generalise the k_t distance measure to define a family of SR algorithms

$$d_{ij} = \min(p_{t,i}^{2p}, p_{t,j}^{2p}) \frac{\Delta_{ij}}{R^2}$$

$$d_{iB} = p_{t,i}^{2p}$$

- p=1 is k_t algorithm
- p=0 is Cambridge-Aachen (C-A) algorithm

Wobisch and Wengler 1999

- p = -1 is anti-k_t algorithm

Cacciari, Salam and Soyez 2008

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The anti-k_t algorithm

Has become the standard choice for most LHC studies involving QCD.

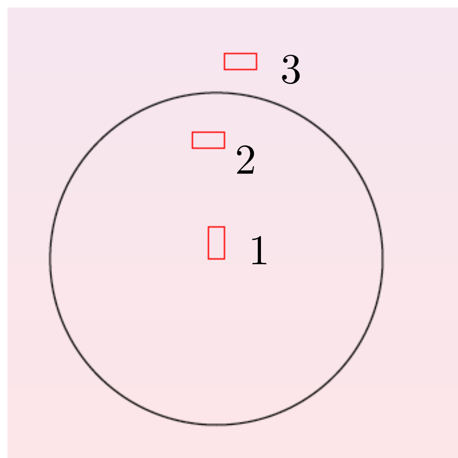
While k_t algorithm has distance measure with simple relationship to QCD dynamics in soft-collinear limit:

$$[dk_j]|M^2_{g\to g_i g_j}(k_j)| \sim \frac{\alpha_s C_A}{2\pi}\frac{dk_{tj}}{\min(k_{ti},k_{tj})}\frac{d\Delta R_{ij}}{\Delta R_{ij}}\,, \qquad (k_{tj} \ll k_{ti}\,,\ \ \Delta R_{ij} \ll 1)$$

Anti-kt does not reflect the divergence structure of QCD matrix elements. Cannot be simply related to sequential parton branchings.

Anti-k_t algorithm

Has the property that a hard particle clustering with a soft particle is preferred to the clustering of soft particles.



$$p_{t1} \gg p_{t2}, p_{t3}$$

$$d_{1j} \sim \frac{1}{p_{t1}^2}\frac{\theta_{1j}^2}{R^2}$$

$$d_{1B} \sim \frac{1}{p_{t1}^2}$$

$$d_{23} \sim \min\left(\frac{1}{p_{t2}^2}, \frac{1}{p_{t3}^2}\right)\frac{\theta_{23}^2}{R^2}$$

Only soft particles within distance R of the hard particle are clustered. Algorithm works like a perfect cone.

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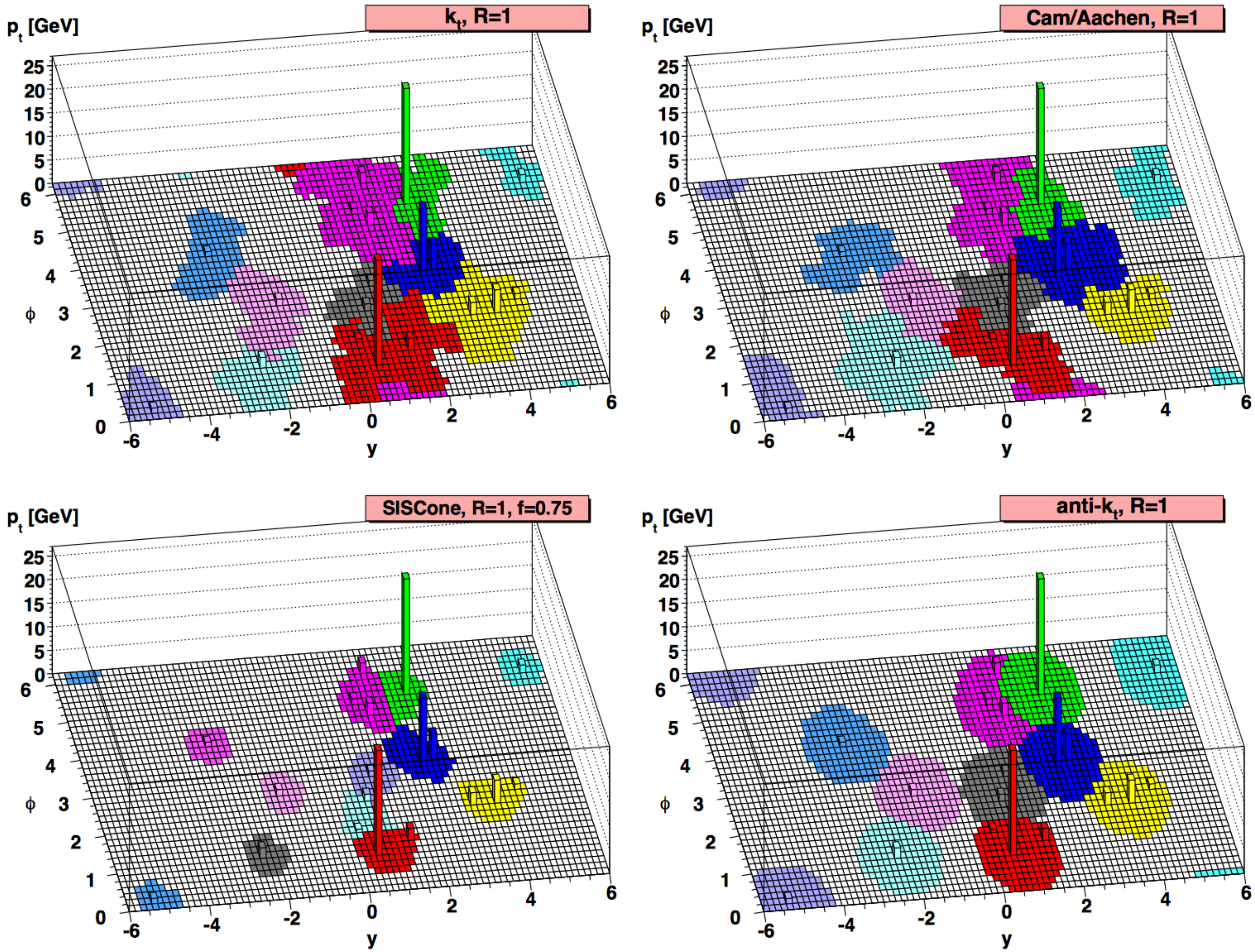
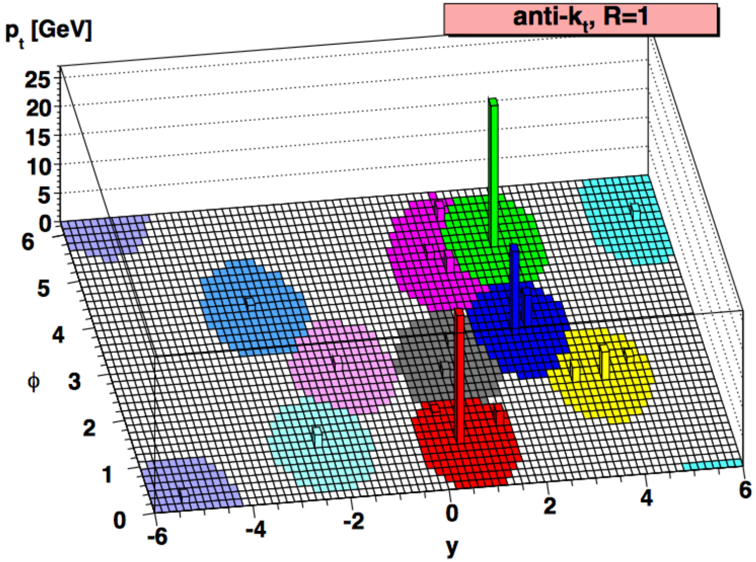
SR algorithms



G. Salam, "Towards Jetography" 2009

Anti k_t jets



- Circular geometry similar to that seen with some cone variants.
- Highly prized property from experimental viewpoint.

(Facilitates estimating detector corrections and removing some non-perturbative effects)

Application : jets with small R

- Small R limit is a useful one for calculations that give some physical intuition about jet properties
- Results derived in formal small R limit are often seen to hold over substantial range of R values.
- One may expect to get insight only about perturbative properties but analytical calculations shed some light on non-perturbative behaviour too.

MD, Magnea and Salam 2007

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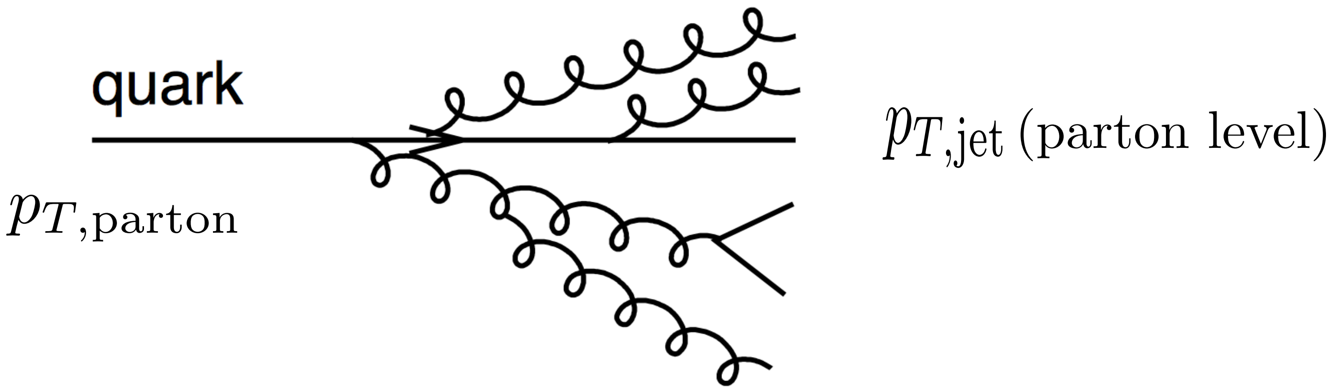
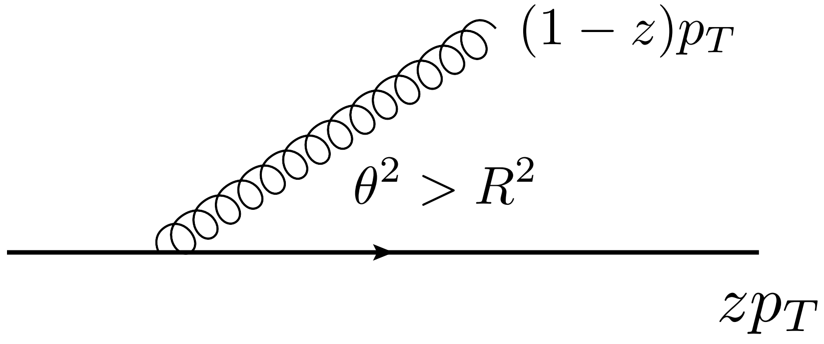
Peturbative radiation at small R



A key question is how does the energy of the parton that emerges from hard process relate to that of measured jet ?

Important for resonance reconstruction as mass of resonance is related to parton energy while measurements are made on jets.

Perturbative radiation at small R



Find change in p_T for hardest jet.

$$\delta p_t = (1-z)p_t - p_t = -zp_t, \; 1-z > z$$
$$\delta p_t = zp_t - p_t = -(1-z)p_t, \; z > 1-z$$

$$\langle \delta p_t \rangle_q = -\frac{C_F \alpha_s}{2\pi} p_t \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \frac{1+z^2}{1-z} \min\left[(1-z), z\right]$$

Perturbative radiation loss

The results are

$$\langle \delta p_t \rangle_q = -C_F \frac{\alpha_s}{\pi} p_t \ln \frac{1}{R} \left(2 \ln 2 - \frac{3}{8} \right)$$

$$\langle \delta p_t \rangle_g = -\frac{\alpha_s}{\pi} p_t \ln \frac{1}{R} \left[C_A \left(2 \ln 2 - \frac{43}{96} \right) + T_R n_f \frac{7}{48} \right]$$

for quark and gluon jets respectively. Derive gluon result yourself as an exercise. Results hold for any SR algorithm.

Perturbative radiation loss

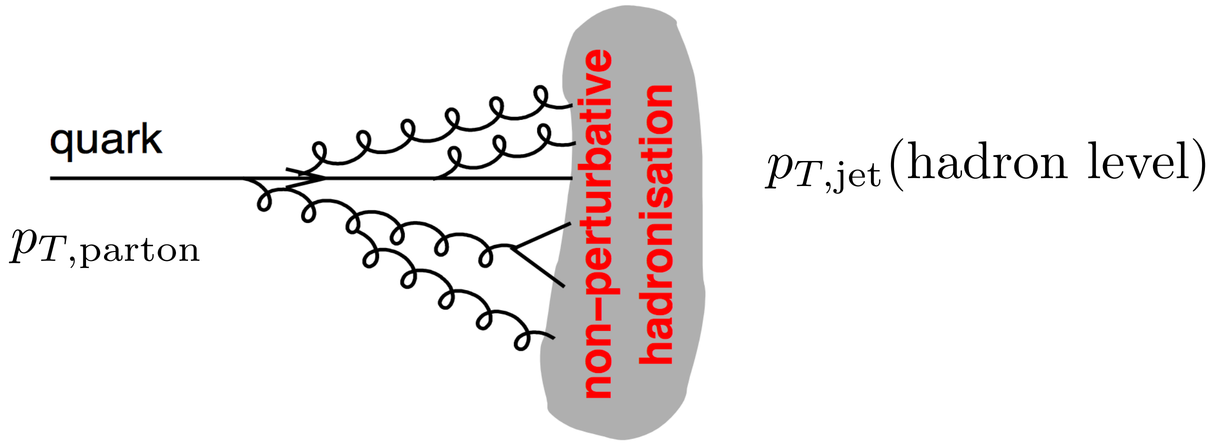
Our calculations imply

$$\frac{\langle \delta p_t \rangle_q}{p_t} = -0.43 \alpha_s \ln \frac{1}{R}$$
$$\frac{\langle \delta p_t \rangle_g}{p_t} = -1.02 \alpha_s \ln \frac{1}{R}$$

For $R = 0.4$ quark jet will have 5 percent less and gluon jet 11 percent less p_t than parent parton.

The numbers will receive corrections from higher order and finite R effects.

Hadronisation effects



We have dealt with perturbative degradation of a jet's momentum. But how does hadronisation affect the same?

Surprisingly analytical models exist that can help answer this question. They are based on universal infrared finite extensions of the strong coupling.

Hadronisation effects

The results from the analytical models (again in small R limit)

$$\langle \delta p_t \rangle_q = -\frac{2C_F}{\pi} \int_0^{\mu_I} \alpha_s(k_t) dk_t \times \frac{1}{R}$$

$$\langle \delta p_t \rangle_q \approx \frac{-0.5\mathrm{GeV}}{R}$$

quark jets

$$\langle \delta p_t \rangle_g \approx \frac{-1\mathrm{GeV}}{R}$$

gluon jets

The thing to note is striking singular 1/R behaviour at small R.

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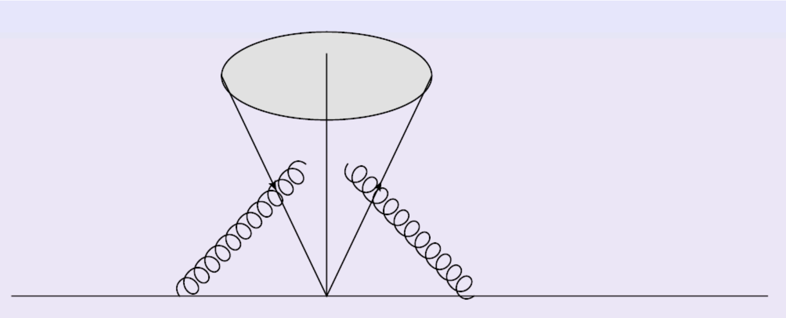
Underlying event

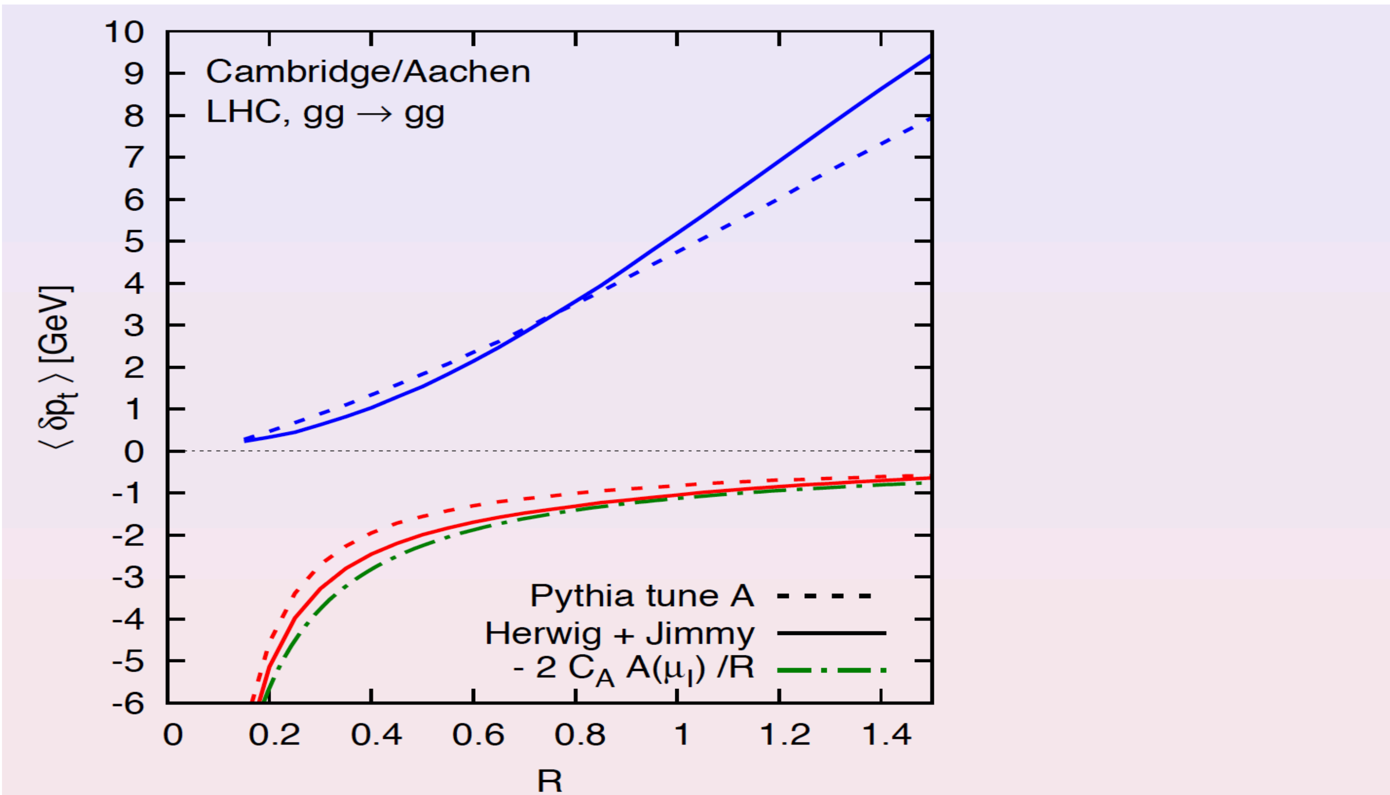


To a reasonable approximation one can think of UE corrections as spraying a constant amount of energy uniformly in rapidity and azimuth.

Then the change in p_T simply comes out proportional to the jet area.

$$\langle \delta p_t \rangle_{\mathrm{UE}} = \Lambda_{\mathrm{UE}} \int_{\eta^2+\phi^2<R^2} d\eta \frac{d\phi}{2\pi} = \Lambda_{\mathrm{UE}} \frac{R^2}{2}$$

Comparison to MC models



Good agreement between analytical hadronisation and MC.
Results are very similar for all algorithms.

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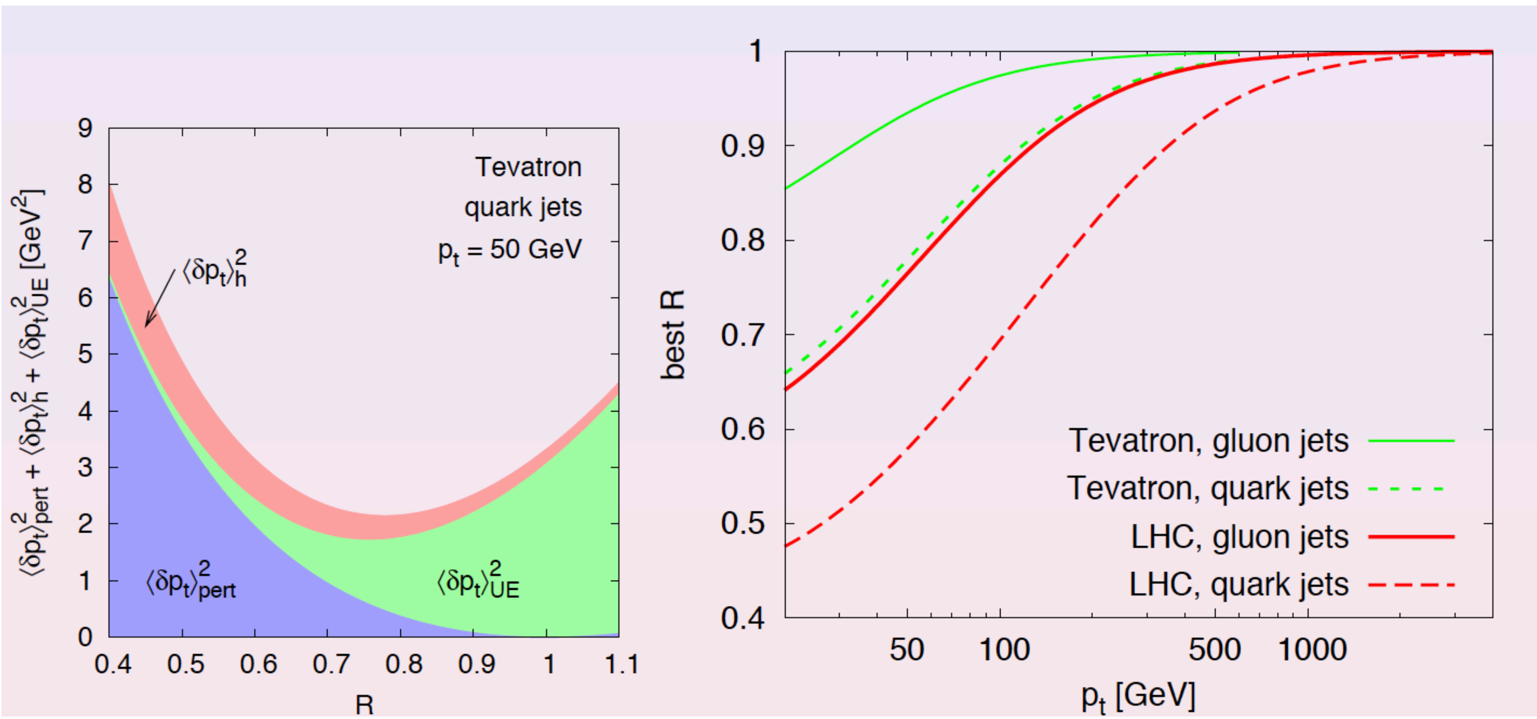
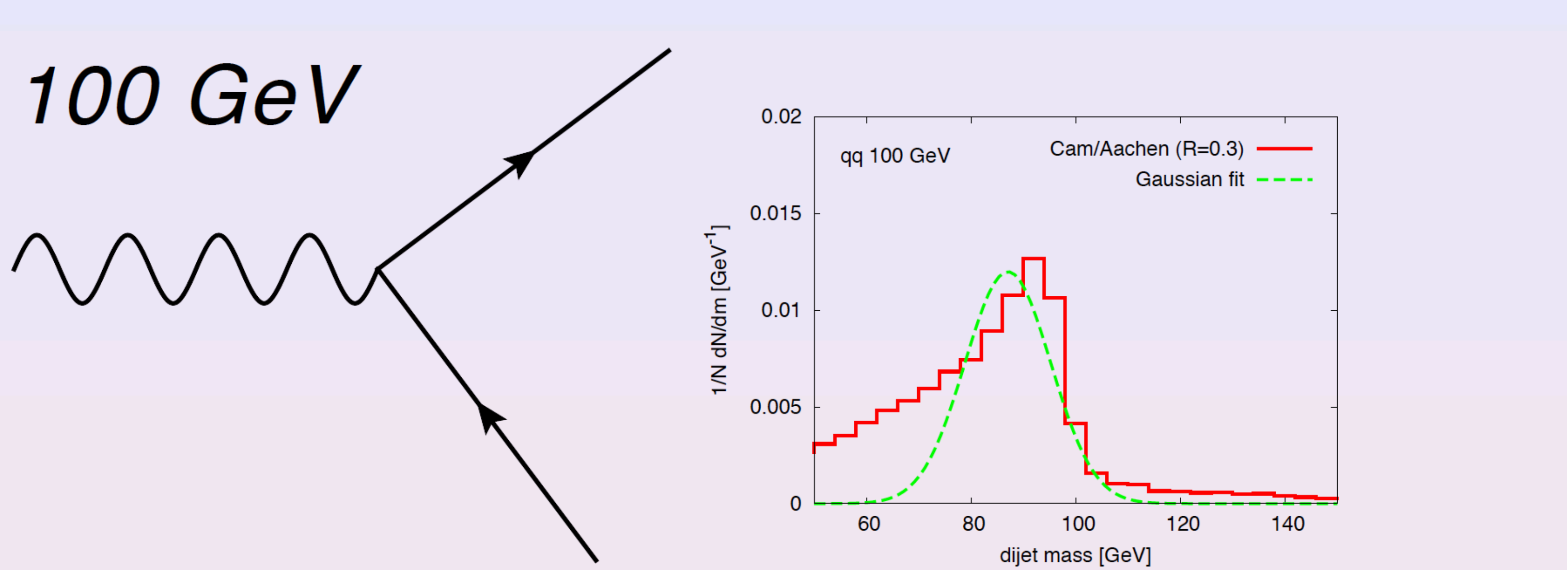
Optimal R value

For resonance reconstruction we want to minimise the dispersion on a jet. Rough estimate : $\langle \delta p_t^2 \rangle = \langle \delta p_t \rangle_{\mathrm{h}}^2 + \langle \delta p_t \rangle_{\mathrm{UE}}^2 + \langle \delta p_t \rangle_{\mathrm{PT}}^2$



At high p_T larger R is suggested. For gluon jets use a larger R than quark jets. Use a smaller R at LHC than Tevatron.

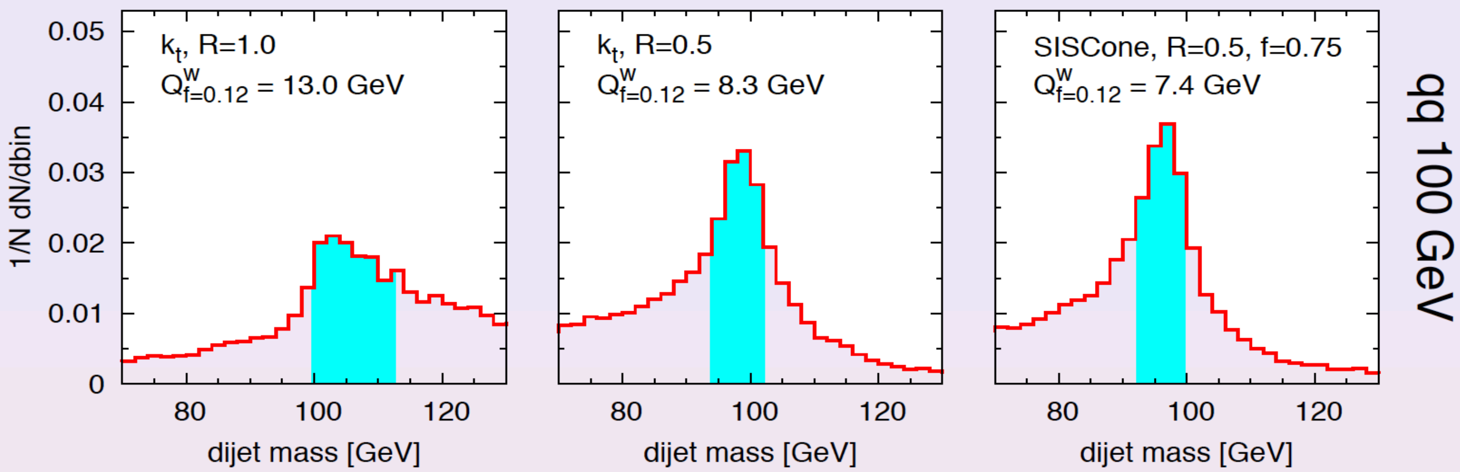
Best R for peak reconstruction



- Want to illustrate effect of finding best R on peak reconstruction.
- Use example of 100 GeV $q\bar{q}$ resonance.
- First need a good measure of peak width

Salam "Towards Jetography" 2009

Best R for peak reconstruction



Quality measure $Q^W_{f=z}$ is width of narrowest window that contains specified fraction f = z of events.
R=0.5 appears to do best here.

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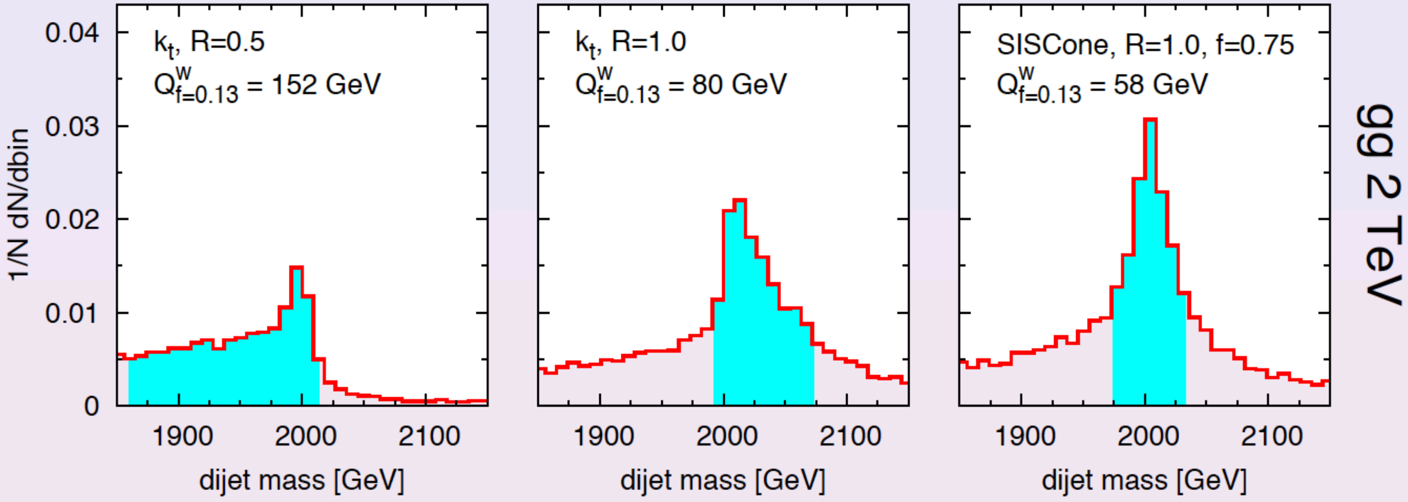
Best R for 2 TeV gg resonance



Here R=0.5 would be a terrible choice. Large perturbative radiation loss suggests a larger R. Suggests the importance of being flexible in choice of R.

Summary

- We introduced the concept of jets from a QCD theory viewpoint.
- Discussed jet definitions in $e^+ e^-$ annihilation focussing on k_t algorithm
- Briefly discussed calculation of jet fractions in soft and collinear approximation.
- Motivated and discussed hadron collider jet definitions
- Showed how one may make simple estimates of jet properties in certain limits (small R)
- Showed that these simple estimates can go quite a long way in enabling us to do better phenomenology with jets

NEXT LECTURE : JET SUBSTRUCTURE STUDIES IN THE BOOSTED REGIME.