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MCnet

Introduction to Event Generators 2

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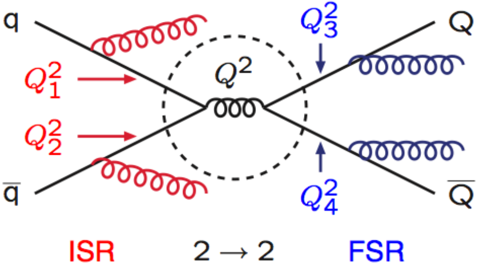
$$2 \to n \quad = \quad (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$

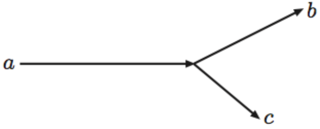


FSR = Final-State Radiation = timelike shower
$Q_i^2 \sim m^2 > 0$ decreasing
ISR = Initial-State Radiation = spacelike showers
$Q_i^2 \sim -m^2 > 0$ increasing

Why "time" like and "space" like?

Consider four-momentum conservation in a branching $a \to b\,c$

$$\mathbf{p}_{\perp a} = 0 \quad \Rightarrow \quad \mathbf{p}_{\perp c} = -\mathbf{p}_{\perp b}$$
$$p_{+} = E + p_{\mathrm{L}} \quad \Rightarrow \quad p_{+a} = p_{+b} + p_{+c}$$
$$p_{-} = E - p_{\mathrm{L}} \quad \Rightarrow \quad p_{-a} = p_{-b} + p_{-c}$$



Define $p_{+b} = z\,p_{+a}, \quad p_{+c} = (1-z)\,p_{+a}$

Use $p_{+}p_{-} = E^2 - p_{\mathrm{L}}^2 = m^2 + p_{\perp}^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z\,p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1-z)\,p_{+a}}$$

$$\Rightarrow \quad m_a^2 = \frac{m_b^2 + p_{\perp}^2}{z} + \frac{m_c^2 + p_{\perp}^2}{1-z} = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_{\perp}^2}{z(1-z)}$$

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_{\perp}^2}{z(1-z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_{\perp}^2}{1-z} < 0 \Rightarrow$ spacelike

Showers and cross sections

Shower evolution is viewed as a probabilistic process, which occurs with unit total probability:
the cross section is not directly affected

Showers and cross sections

Shower evolution is viewed as a probabilistic process, which occurs with unit total probability:
the cross section is not directly affected

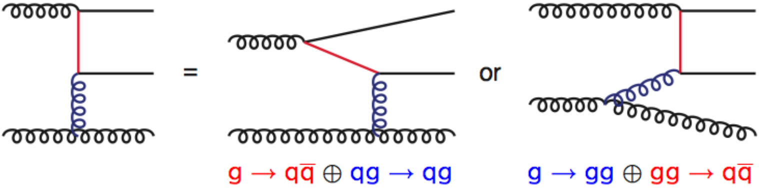
However, more complicated than that

- PDF evolution $\approx$ showers $\Rightarrow$ enters in convoluted cross section, e.g. for $2 \to 2$ processes

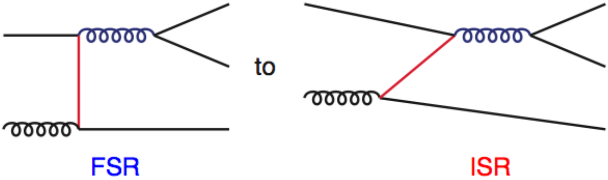
$$\sigma = \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \; f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

- Shower affects event shape
  E.g. start from 2-jet event with $\mathrm{p}_{\perp 1} = \mathrm{p}_{\perp 2} = 100$ GeV.
  ISR gives third jet, plus recoil to existing two, so
  $\mathrm{p}_{\perp 1} = 110$ GeV, $\mathrm{p}_{\perp 2} = 90$ GeV, $\mathrm{p}_{\perp 1} = 20$ GeV:
  - inclusive $\mathrm{p}_{\perp \mathrm{jet}}$ spectrum goes up
  - hardest $\mathrm{p}_{\perp \mathrm{jet}}$ spectrum goes up
  - two-jets with both jets above some $p_{\perp \mathrm{min}}$ comes down
  - three-jet rate goes up

A $2 \to n$ graph can be "simplified" to $2 \to 2$ in different ways:



or deform

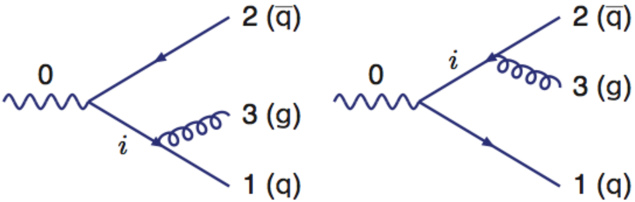


Do not doublecount: $2 \to 2$ = most virtual = shortest distance

(detailed handling of borders ⇒ **match & merge**)

Final-state radiation

Standard process $\boxed{e^+e^- \to q\bar{q}g}$ by two Feynman diagrams:



$$x_i = \frac{2E_i}{E_{\rm cm}}$$

$$x_1 + x_2 + x_3 = 2$$

$$\boxed{\frac{d\sigma_{\rm ME}}{\sigma_0} = \frac{\alpha_s}{2\pi}\,\frac{4}{3}\,\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\,dx_1\,dx_2}$$

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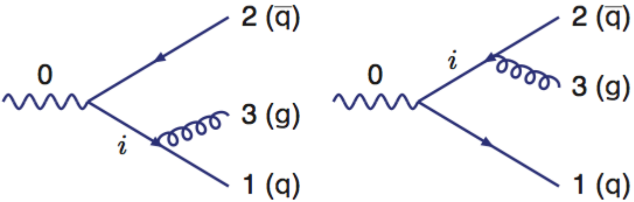

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$$\frac{d\sigma_{\mathrm{ME}}}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \, dx_1 \, dx_2$$

Convenient (but arbitrary) subdivision to "split" radiation:

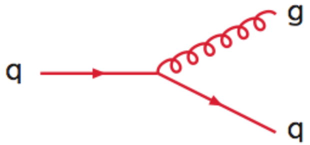
$$\frac{1}{(1-x_1)(1-x_2)} \frac{(1-x_1)+(1-x_2)}{x_3} = \frac{1}{(1-x_2)x_3} + \frac{1}{(1-x_1)x_3}$$

From matrix elements to parton showers

Rewrite for $x_2 \to 1$, i.e. q–g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\mathrm{cm}}^2} = \frac{Q^2}{E_{\mathrm{cm}}^2} \Rightarrow \mathrm{d}x_2 = \frac{\mathrm{d}Q^2}{E_{\mathrm{cm}}^2}$$

define z as fraction q retains
in branching $\mathrm{q} \to \mathrm{qg}$



$$\begin{aligned} x_1 &\approx z \Rightarrow \mathrm{d}x_1 \approx \mathrm{d}z \\ x_3 &\approx 1 - z \end{aligned}$$

$$\Rightarrow \mathrm{d}\mathcal{P} = \frac{\mathrm{d}\sigma}{\sigma_0} = \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{\mathrm{d}x_2}{(1 - x_2)} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1 - x_1)} \mathrm{d}x_1 \approx \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} \frac{4}{3} \frac{1 + z^2}{1 - z} \mathrm{d}z$$

In limit $x_1 \to 1$ same result, but for $\overline{\mathrm{q}} \to \overline{\mathrm{q}}\mathrm{g}$.

$\mathrm{d}Q^2/Q^2 = \mathrm{d}m^2/m^2$: "mass (or collinear) singularity"

$\mathrm{d}z/(1 - z) = \mathrm{d}\omega/\omega$ "soft singularity"

The DGLAP equations

Generalizes to

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$
\begin{aligned}
\mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \\
P_{\mathrm{q}\to\mathrm{qg}} &= \frac{4}{3}\,\frac{1+z^2}{1-z} \\
P_{\mathrm{g}\to\mathrm{gg}} &= 3\,\frac{(1-z(1-z))^2}{z(1-z)} \\
P_{\mathrm{g}\to\mathrm{q}\overline{\mathrm{q}}} &= \frac{n_f}{2}\,(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})
\end{aligned}
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\end{aligned}
$$

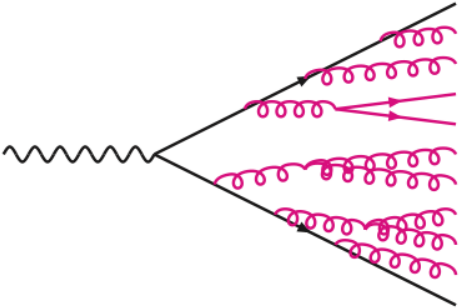
Universality: any matrix element reduces to DGLAP in collinear limit.

$$
\text{e.g.}\quad \frac{\mathrm{d}\sigma(\mathrm{H}^0\to\mathrm{q\overline{q}g})}{\mathrm{d}\sigma(\mathrm{H}^0\to\mathrm{q\overline{q}})} = \frac{\mathrm{d}\sigma(\mathrm{Z}^0\to\mathrm{q\overline{q}g})}{\mathrm{d}\sigma(\mathrm{Z}^0\to\mathrm{q\overline{q}})} \quad \text{in collinear limit}
$$

The iterative structure

Generalizes to many consecutive emissions if strongly ordered, $Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots$ ($\approx$ time-ordered).
To cover "all" of phase space use DGLAP in whole region $Q_1^2 > Q_2^2 > Q_3^2 \ldots$.

Iteration gives final-state parton showers:



Need soft/collinear cuts to stay away from nonperturbative physics.
Details model-dependent, but around 1 GeV scale.

The Sudakov form factor – 1

Time evolution, conservation of total probability:
$\mathcal{P}(\text{no emission}) = 1 - \mathcal{P}(\text{emission})$.

Multiplicativeness, with $T_i = (i/n)T$, $0 \le i \le n$:

$$\begin{aligned}
\mathcal{P}_{\text{no}}(0 \le t < T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{no}}(T_i \le t < T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{em}}(T_i \le t < T_{i+1})) \\
&= \exp\left(- \lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{em}}(T_i \le t < T_{i+1}) \right) \\
&= \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{em}}(t)}{\mathrm{d}t} \mathrm{d}t \right) \\
\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) &= \mathrm{d}\mathcal{P}_{\text{em}}(T) \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{em}}(t)}{\mathrm{d}t} \mathrm{d}t \right)
\end{aligned}$$

cf. radioactive decay in lecture 1.

The Sudakov form factor – 2

Expanded, with $Q \sim 1/t$ (Heisenberg)

$$\begin{aligned} \mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\alpha_{\mathrm{s}}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z \\ &\times \exp\left(-\sum_{b,c} \int_{Q^2}^{Q^2_{\mathrm{max}}} \frac{\mathrm{d}Q'^2}{Q'^2} \int \frac{\alpha_{\mathrm{s}}}{2\pi} P_{a\to bc}(z')\,\mathrm{d}z' \right) \end{aligned}$$

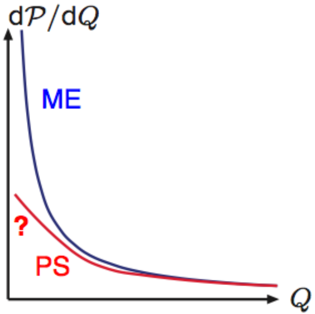
where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c} \int\int \mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo*
($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).

The Sudakov form factor – 3

Sudakov regulates singularity for *first* emission . . .



. . . but in limit of *repeated soft* emissions $\mathrm{q} \to \mathrm{qg}$ (but no $\mathrm{g} \to \mathrm{gg}$) one obtains the same inclusive Q emission spectrum as for ME,

i.e. divergent ME spectrum
$\Longleftrightarrow$ infinite number of PS emissions

More complicated in reality:

- energy-momentum conservation effects big since α_{s} big, so hard emissions frequent
- $\mathrm{g} \to \mathrm{gg}$ branchings leads to accelerated multiplication of partons

The ordering variable

In the evolution with

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z$$

Q^2 orders the emissions (memory).

If $Q^2 = m^2$ is one possible evolution variable
then $Q'^2 = f(z)Q^2$ is also allowed, since

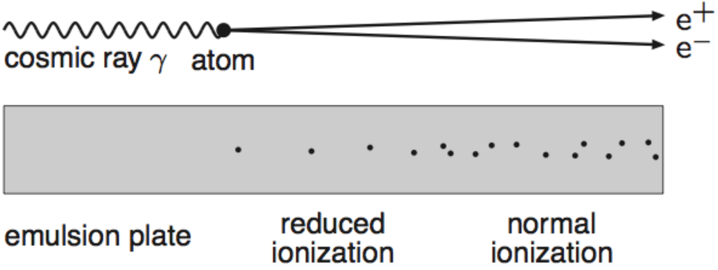
$$\left|\frac{\mathrm{d}(Q'^2, z)}{\mathrm{d}(Q^2, z)}\right| = \left|\begin{array}{cc} \frac{\partial Q'^2}{\partial Q^2} & \frac{\partial Q'^2}{\partial z} \\ \frac{\partial z}{\partial Q^2} & \frac{\partial z}{\partial z} \end{array}\right| = \left|\begin{array}{cc} f(z) & f'(z)Q^2 \\ 0 & 1 \end{array}\right| = f(z)$$

$$\Rightarrow \mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{f(z)\mathrm{d}Q^2}{f(z)Q^2} P_{a\to bc}(z)\,\mathrm{d}z = \frac{\alpha_s}{2\pi} \frac{\mathrm{d}Q'^2}{Q'^2} P_{a\to bc}(z)\,\mathrm{d}z$$

- $Q'^2 = E_a^2\theta_{a\to bc}^2 \approx m^2/(z(1-z))$; angular-ordered shower
- $Q'^2 = p_\perp^2 \approx m^2 z(1-z)$; transverse-momentum-ordered

Coherence

QED: Chudakov effect (mid-fifties)



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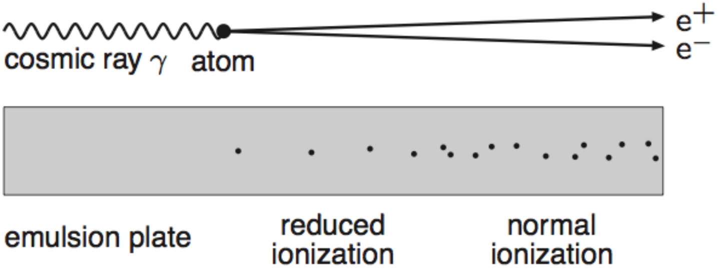
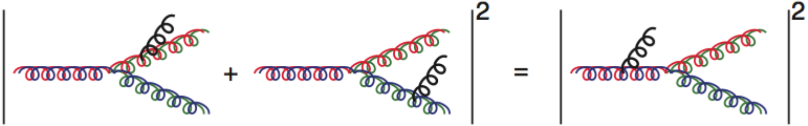


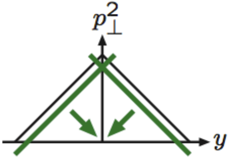
QCD: colour coherence for **soft** gluon emission



solved by • requiring emission angles to be decreasing
or • requiring transverse momenta to be decreasing

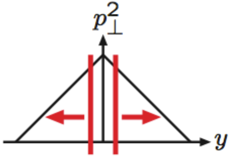
Ordering variables in the LEP/Tevatron era

PYTHIA: $Q^2 = m^2$



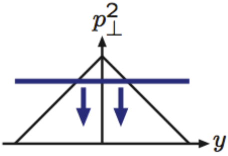
large mass first
$\Rightarrow$ "hardness" ordered
coherence brute force
covers phase space
ME merging simple
g $\rightarrow$ q$\overline{\text{q}}$ simple
not Lorentz invariant
no stop/restart
ISR: $m^2 \rightarrow -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first
$\Rightarrow$ **hardness not ordered**
coherence inherent
gaps in coverage
ME merging messy
g $\rightarrow$ q$\overline{\text{q}}$ simple
not Lorentz invariant
no stop/restart
ISR: $\theta \rightarrow \theta$

ARIADNE: $Q^2 = p_\perp^2$



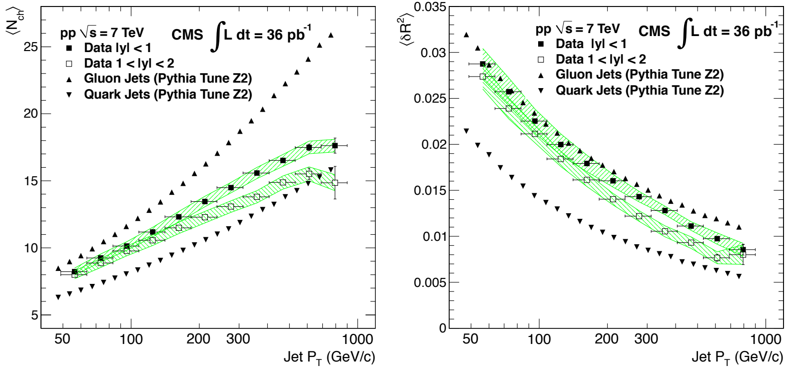
large $p_\perp$ first
$\Rightarrow$ "hardness" ordered
coherence inherent

covers phase space
ME merging simple
g $\rightarrow$ q$\overline{\text{q}}$ **messy**
Lorentz invariant
can stop/restart
ISR: more messy

Quark vs. gluon jets

$$\frac{P_{\mathrm{g}\to\mathrm{gg}}}{P_{\mathrm{q}\to\mathrm{qg}}} \approx \frac{N_c}{C_F} = \frac{3}{4/3} = \frac{9}{4} \approx 2$$

$\Rightarrow$ gluon jets are softer and broader than quark ones
(also helped by hadronization models, lecture 4).



Note transition g jets $\longrightarrow$ q jets for increasing $p_\perp$.

Heavy flavours: the dead cone

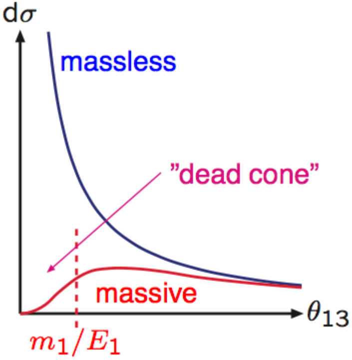
Matrix element for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\overline{\mathrm{q}}\mathrm{g}$ for small θ_{13}

$$\frac{\mathrm{d}\sigma_{\mathrm{q}\overline{\mathrm{q}}\mathrm{g}}}{\sigma_{\mathrm{q}\overline{\mathrm{q}}}} \propto \frac{x_1^2 + x_2^2}{(1 - x_1)\,(1 - x_2)} \approx \frac{\mathrm{d}\omega}{\omega}\,\frac{\mathrm{d}\theta_{13}^2}{\theta_{13}^2}$$

is modified for heavy quark Q:

$$\frac{\mathrm{d}\sigma_{\mathrm{q}\overline{\mathrm{q}}\mathrm{g}}}{\sigma_{\mathrm{q}\overline{\mathrm{q}}}} \propto \frac{\mathrm{d}\omega}{\omega}\,\frac{\mathrm{d}\theta_{13}^2}{\theta_{13}^2}\left(\frac{\theta_{13}^2}{\theta_{13}^2 + m_1^2/E_1^2}\right)^2 = \frac{\mathrm{d}\omega}{\omega}\,\frac{\theta_{13}^2\,\mathrm{d}\theta_{13}^2}{(\theta_{13}^2 + m_1^2/E_1^2)^2}$$

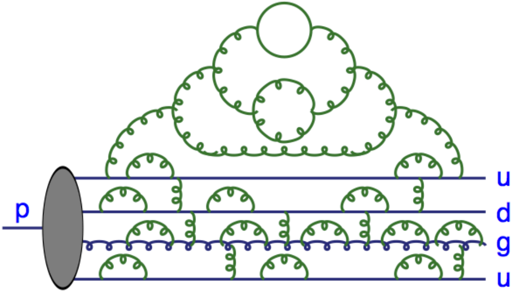
so "dead cone" for $\theta_{13} < m_1/E_1$



For charm and bottom lagely filled in by their decay products.

Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2)$ = number density of partons i
at momentum fraction x and probing scale Q^2.

Linguistics (example):

$$F_2(x, Q^2) \quad = \quad \sum_i e_i^2 \, x f_i(x, Q^2)$$

structure function — parton distributions

PDF evolution

Initial conditions at small Q_0^2 unknown: nonperturbative.

Resolution dependence perturbative, by DGLAP:

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$\frac{\mathrm{d}f_b(x, Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} \, f_a(y, Q^2) \, \frac{\alpha_\mathrm{s}}{2\pi} \, P_{a\to bc} \left(z = \frac{x}{y} \right)$$

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DGLAP already introduced for (final-state) showers:

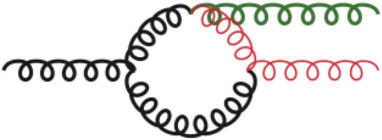
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \, \frac{\mathrm{d}Q^2}{Q^2} \, P_{a\to bc}(z) \, \mathrm{d}z$$

Same equation, but different context:

- $\mathrm{d}\mathcal{P}_{a\to bc}$ is probability for the individual parton to branch; while
- $\mathrm{d}f_b(x, Q^2)$ describes how the ensemble of partons evolve by the branchings of individual partons as above.

Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- Structure at Q is resolved at a time $t \sim 1/Q$ *before* collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



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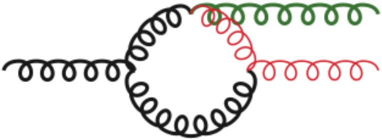
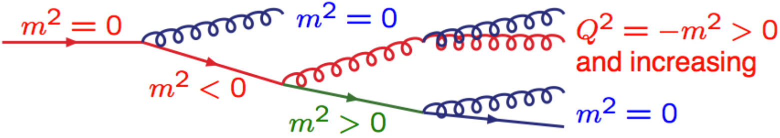


- Convenient reinterpretation:



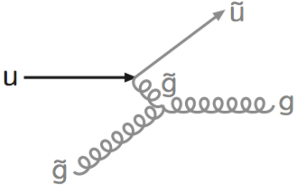
Forwards vs. backwards evolution

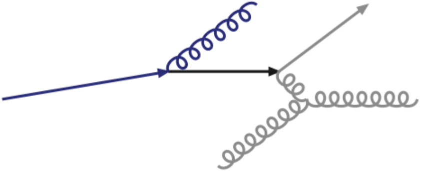
Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low Q_0 and evolve,
consider collisions at different Q^2 and pick by σ of those.

Inefficient:

1. have to evolve and check for *all* potential collisions, but 99.9. . . % inert
2. impossible (or at least very complicated) to steer the production, e.g. of a narrow resonance (Higgs)

Backwards evolution is viable and ∼equivalent alternative:
start at hard interaction and trace what happened "before"





Backwards evolution master formula

Monte Carlo approach, based on *conditional probability*: recast

$$\frac{\mathrm{d}f_b(x, Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} \, f_a(x', Q^2) \, \frac{\alpha_s}{2\pi} \, P_{a\to bc}(z)$$

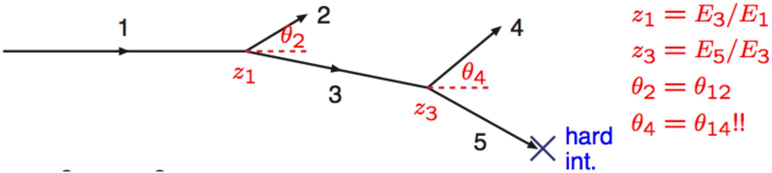
with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \, \frac{x' f_a(x', t)}{x f_b(x, t)} \, \frac{\alpha_s}{2\pi} \, P_{a\to bc}(z)$$

then solve for *de*creasing t, i.e. backwards in time,
starting at high Q^2 and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$.

Extra factor $x' f_a / x f_b$ relative to final-state equations.

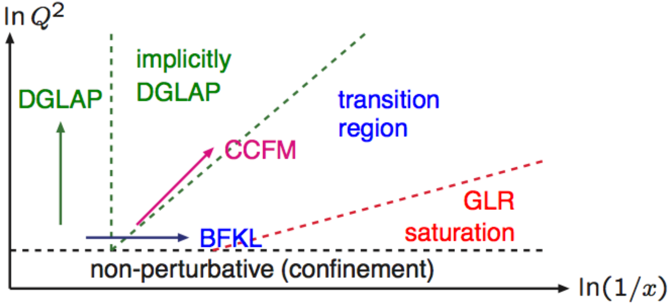
Coherence in spacelike showers



with $Q^2 = -m^2 =$ spacelike virtuality

- kinematics only:
  $Q_3^2 > z_1 Q_1^2$, $Q_5^2 > z_3 Q_3^2$, ...
  i.e. Q_i^2 need not even be ordered
- coherence of leading collinear singularities:
  $Q_5^2 > Q_3^2 > Q_1^2$, i.e. Q^2 ordered
- coherence of leading soft singularities (more messy):
  $E_3\theta_4 > E_1\theta_2$, i.e. $z_1\theta_4 > \theta_2$
  $z \ll 1$: $E_1\theta_2 \approx p_{\perp 2}^2 \approx Q_3^2$, $E_3\theta_4 \approx p_{\perp 4}^2 \approx Q_5^2$
  i.e. reduces to Q^2 ordering as above
  $z \approx 1$: $\theta_4 > \theta_2$, i.e. angular ordering of soft gluons
  $\Longrightarrow$ reduced phase space

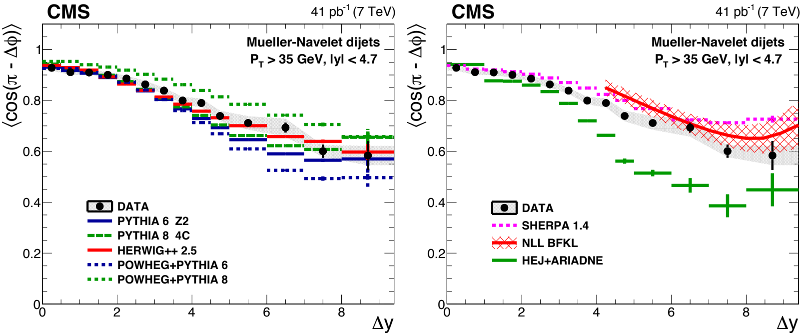
Evolution procedures



DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger Q^2 and (implicitly) towards smaller x
BFKL: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller x (with small, unordered Q^2)
CCFM: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL
GLR: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region,
where partons recombine, not only branch

Did we reach BFKL regime?

Study events with ≥ 2 jets as a function of their y separation; $\cos(\pi - \Delta\phi) = 1$ is back-to-back jets, i.e. little extra radiation.



Analytic BFKL calculations describe data for $\Delta y > 4$,
but HEJ BFKL-inspired generator overshoots effect,
and standard DGLAP Herwig++ almost spot on.
No strong indications for BFKL/CCFM behaviour onset so far!

Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \;\cdot\; (\mathrm{Sudakov})$$

Initial- vs. final-state showers

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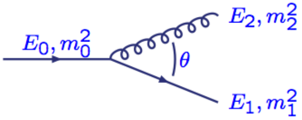
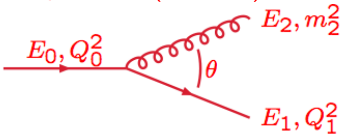
but

Final-state showers:
Q^2 timelike ($\sim m^2$)



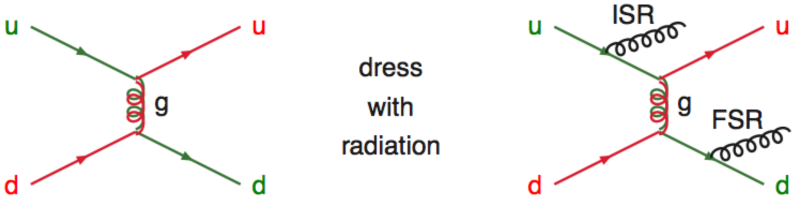
decreasing E, m^2, θ
both daughters $m^2 \geq 0$
physics relatively simple
$\Rightarrow$ "minor" variations:
Q^2, shower vs. dipole, ...

Initial-state showers:
Q^2 spacelike ($\approx -m^2$)



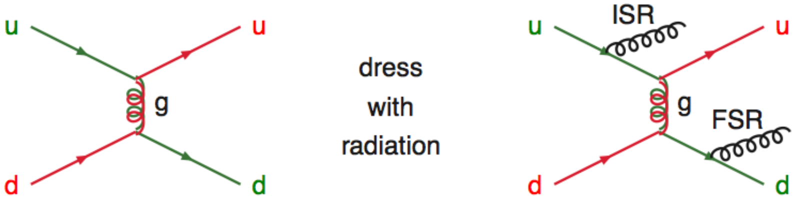
decreasing E, increasing Q^2, θ
one daughter $m^2 \geq 0$, one $m^2 < 0$
physics more complicated
$\Rightarrow$ more formalisms:
DGLAP, BFKL, CCFM, GLR, ...

Combining FSR with ISR



Separate processing of ISR and FSR misses interference
($\sim$ colour dipoles)

Combining FSR with ISR



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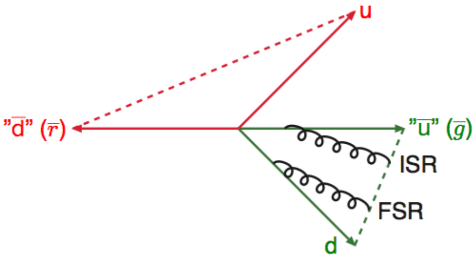


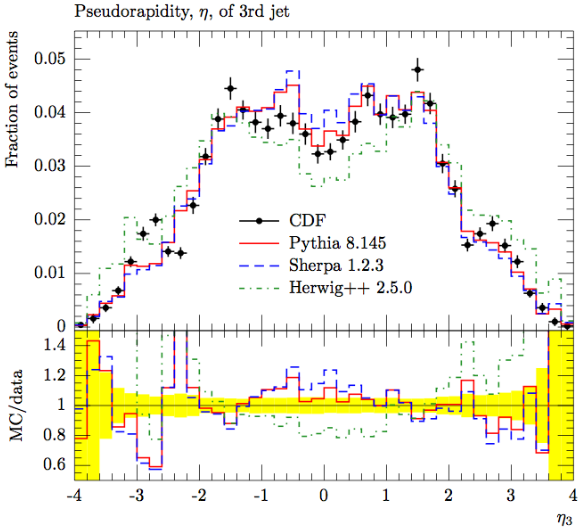
ISR+FSR add coherently in regions of colour flow and destructively else

in "normal" shower by azimuthal anisotropies

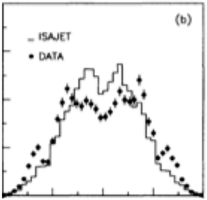
automatic in dipole (by proper boosts)

Coherence tests

Current-day generators for pseudorapidity of third jet:

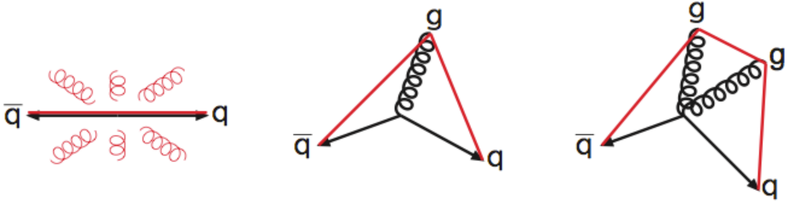


and past incoherent:



The dipole picture – 1

$1 \to 2$ branching = replace $m = 0$ parton by pair with $m > 0$.
Breaks energy–momentum conservation.
Herwig angular-ordered shower: post-facto rescaling machinery.



Alternative: dipole picture (first Ariadne, now everybody else).
$2 \to 3$ parton branching, or $1 \to 2$ colour dipole branching.
Can be viewed as radiator $a \to bc$ with recoiler r.

The dipole picture – 2

Ariadne main splitting expressions for final-state radiation:

$$\begin{aligned}
\mathrm{d}P_{\mathrm{q\bar{q}\to q\bar{q}g}} &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{4}{3}\,\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\,\mathrm{d}x_1\,\mathrm{d}x_2 \\
\mathrm{d}P_{\mathrm{qg\to qgg}} &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{3}{2}\,\frac{x_1^2 + x_2^3}{(1-x_1)(1-x_2)}\,\mathrm{d}x_1\,\mathrm{d}x_2 \\
\mathrm{d}P_{\mathrm{gg\to ggg}} &= \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{3}{2}\,\frac{x_1^3 + x_2^3}{(1-x_1)(1-x_2)}\,\mathrm{d}x_1\,\mathrm{d}x_2
\end{aligned}$$

does not define angular orientation.

The Catani–Seymour dipole is primarily a kinematics recipe how to map 2 partons $ar \leftrightarrow$ 3 partons bcr' for both initial and final state:

$$\begin{aligned}
p_a &= p_b + p_c - \frac{y}{1-y} p_{r'} \\
p_r &= \frac{1}{1-y} p_{r'}
\end{aligned}$$

$$y = \frac{p_b p_c}{p_b p_c + p_b p_{r'} + p_c p_{r'}}$$

Some shower programs

Herwig	angular-ordered shower (QTilde) $p_\perp$-ordered CS dipoles (Dipoles)
PYTHIA	$p_\perp$-ordered dipoles (TimeShower, SpaceShower) VINCIA antennae (plugin) DIRE dipoles (plugin)
Sherpa	$p_\perp$-ordered CS dipoles (CSSHOWER++) DIRE dipoles
Ariadne	first dipole parton shower program
DIPSY	evolution and collision of dipoles in transverse space
Deductor	improved handling of colour, partitioned dipoles, all final partons share recoil, q^2/E evolution variable
HEJ	(High Energy Jets) BFKL-inspired description of well-separated multijets, with approximate matrix elements and virtual corrections

...

VINCIA: an Interleaved Antennae shower

Markovian process: no memory of path to reach current state.

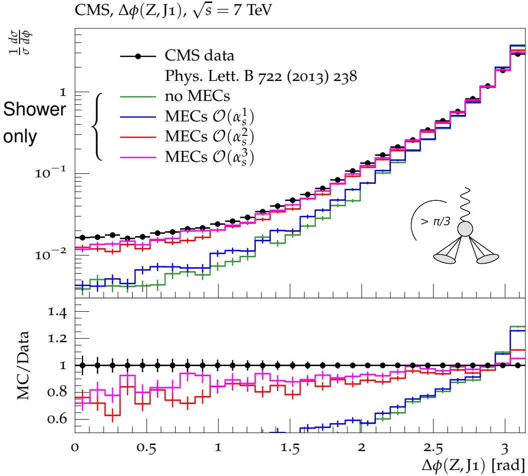
Based on antenna factorization of amplitudes and phase space.

Smooth ordering fills whole phase space.

Step-by-step reweighting to new matrix elements:
$Z \to Zj \to Zjj \to Zjjj$
(also Sudakov), e.g.

$$W = \frac{|\mathcal{M}_{\mathrm{Zj}}|^2}{\sum_i a_i |\mathcal{M}_{\mathrm{Z}}|_i^2}$$

Replaces PYTHIA normal showers; recent release.



DIRE: a Dipole Resummation shower

Joint Sherpa/PYTHIA development, but separate implementations, means technically well tested.

"Midpoint between dipole and parton shower", dipole with emitter & spectator, but not quite CS ones: unified initial–initial, initial–final, final–initial, final–final.

Soft term of kernels in all dipole types is less singular

$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2 + p_{\perp}^2/M^2}$$