



DIS Lectures at CTEQ 2017 Summer School- Lecture 1

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Overview

➤ The Universe as an infinite source of puzzles

*The world is large –
It contains multitudes.
I look with all-embracing eyes
And I tell you what I see.
Do I contradict myself?
Very well, I contradict myself.
If you are not bedazzled yet:
Look differently, and marvel.*

Walt Whitman

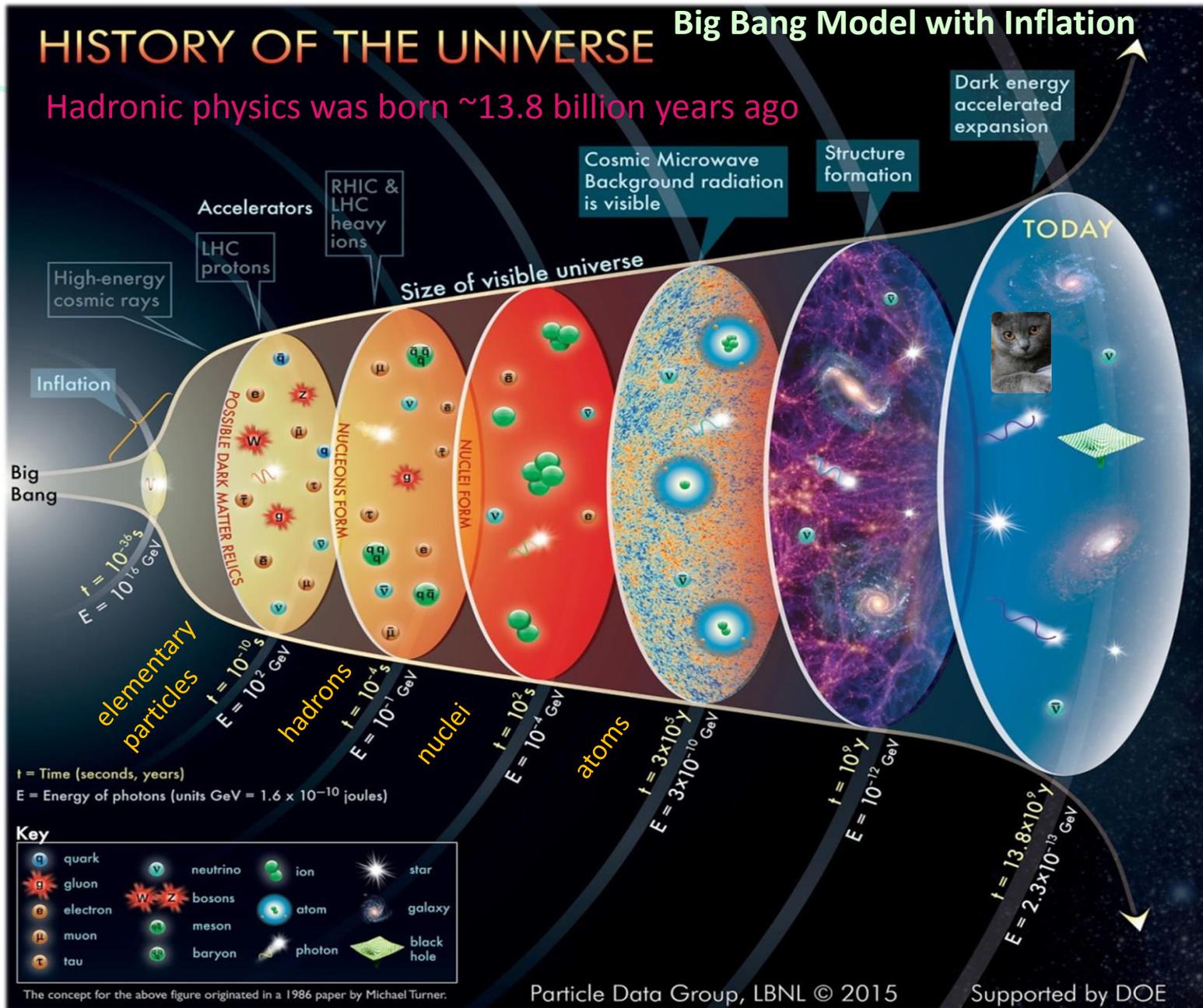
➤ The normal matter puzzle: from atoms to protons to quarks and gluons a wild ride that continues

- “The basic building blocks of Nature are few and profoundly simple, their properties fully specified by equations of high symmetry.
- The world of objects is vast, infinitely various, and inexhaustible.”

A Beautiful Question – Frank Wilczek

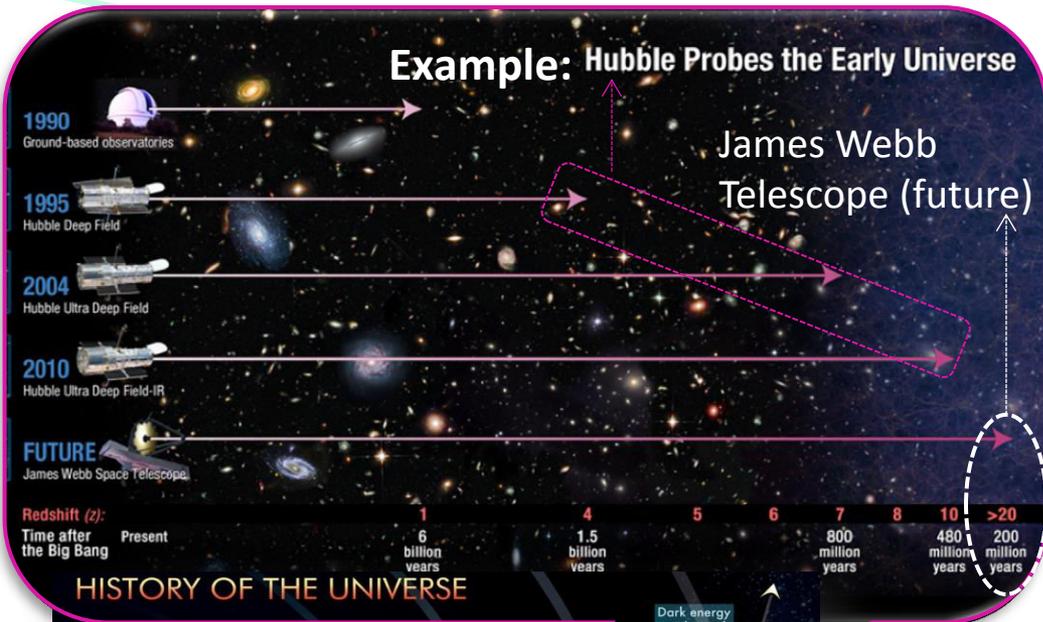
➤ The strongly interacting normal matter puzzle: probing nucleons to understand the strong interactions

The Universe: Infinite Source of Puzzle

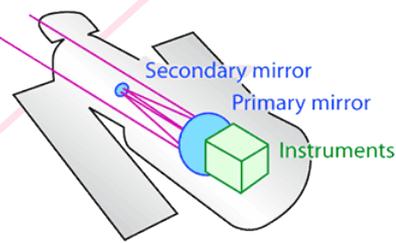


What Do We Do to Figure Out the Universe Puzzle?

➤ **Micro-cosm and Macro-cosm: with Space Telescopes** (example)

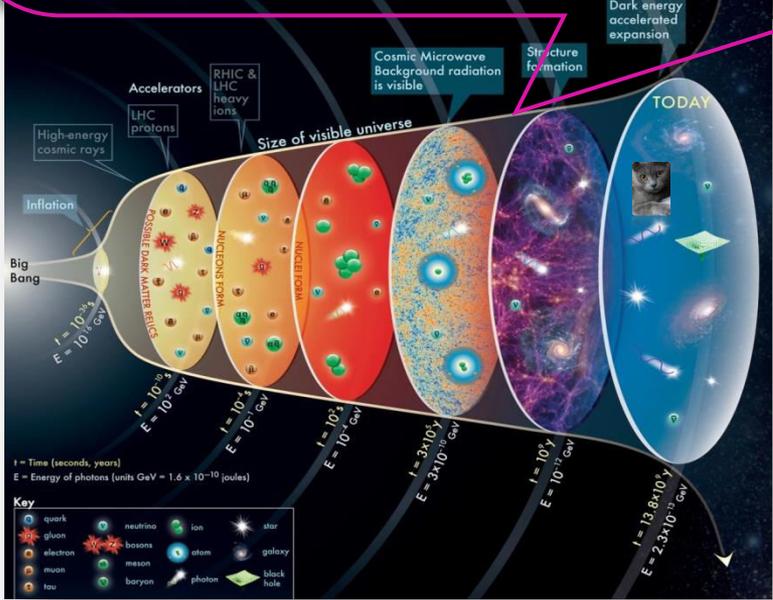


Light: UV, near UV, Visible, near-IR, IR

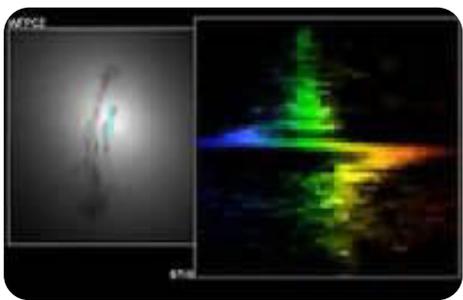


Instruments: cameras, spectrographs, spectrometers

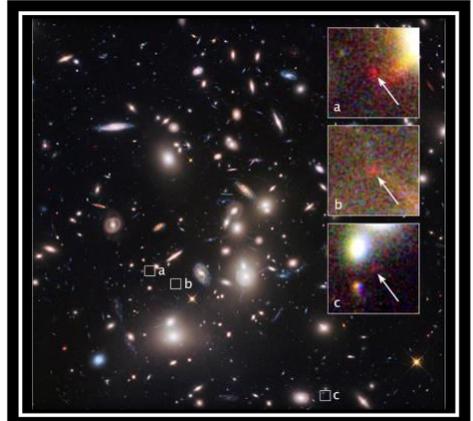
Hubble legacy



Hubble legacy



Black Hole's Signature (1997)



Galaxy as it was 500 million years after Big Bang (2014)

With Hubble we go back in time ~13.3 billion years

What Do We Do to Figure Out the Universe Puzzle?

➤ Macro-cosm and Micro-cosm: with accelerators – colliders

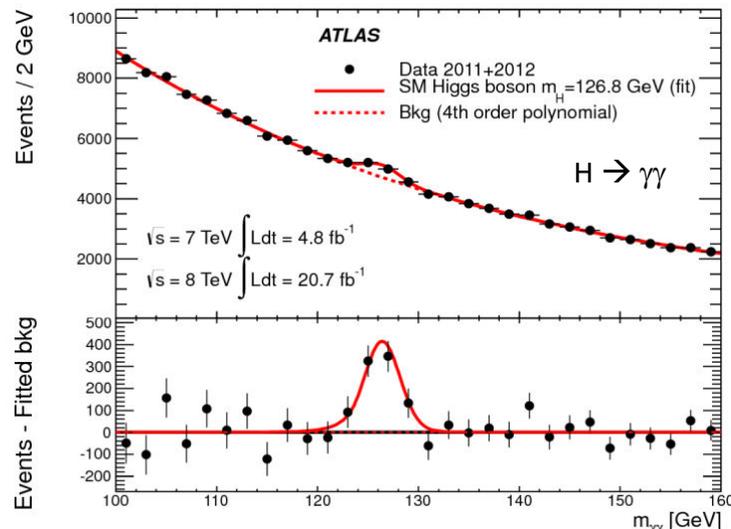
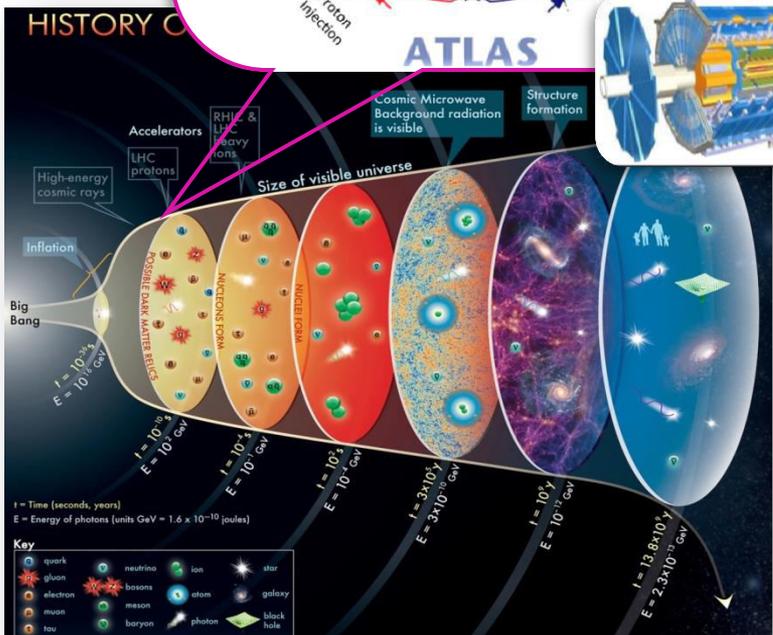
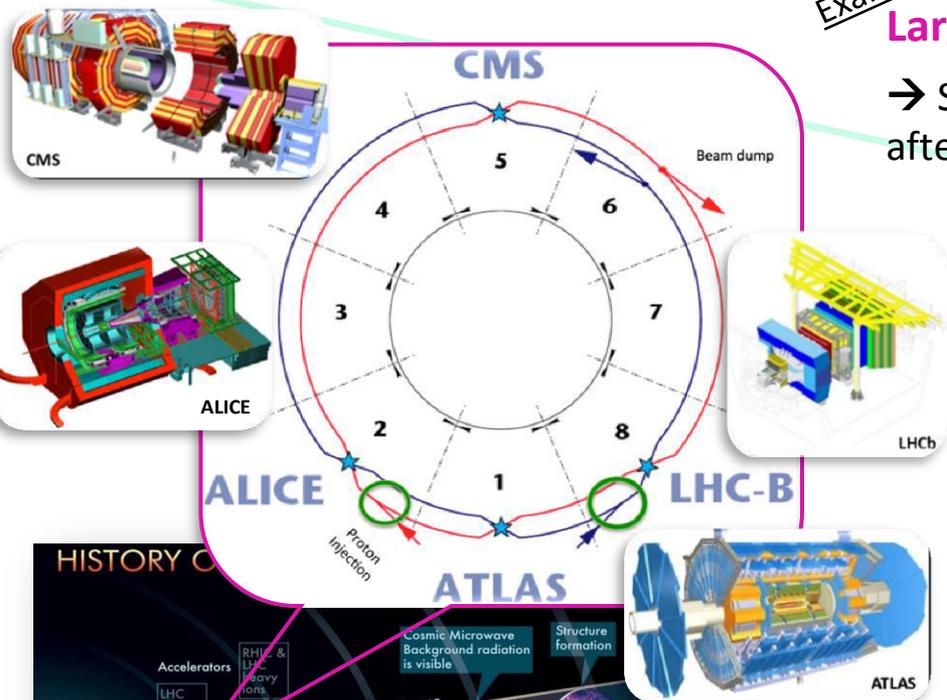
Example

Large Hadron Collider

→ Study physics laws that governed the first moments after the Big Bang, $\sim 10^{-16}$ s

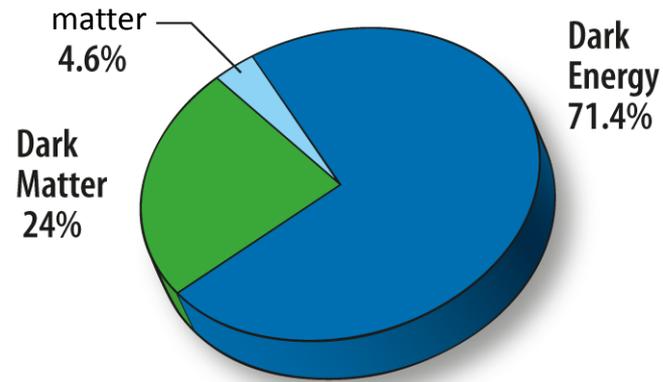
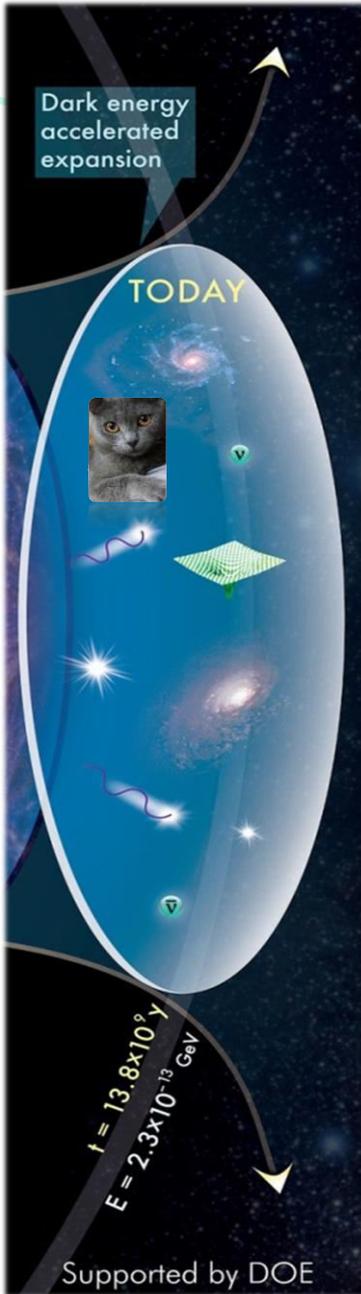
to elucidate questions pertaining to:

- Origin of mass
- Nature of dark matter
- Primordial plasma
- Matter vs antimatter
- ...



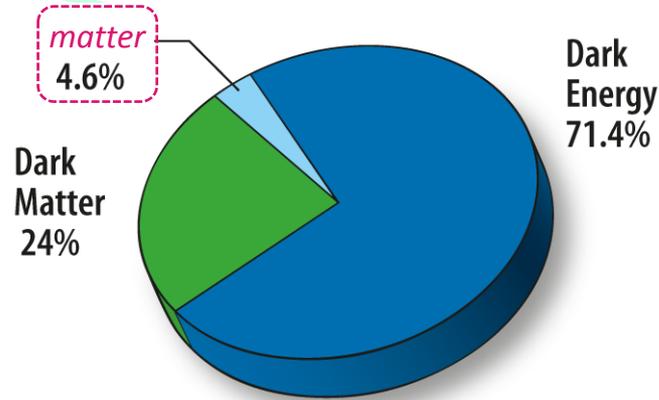
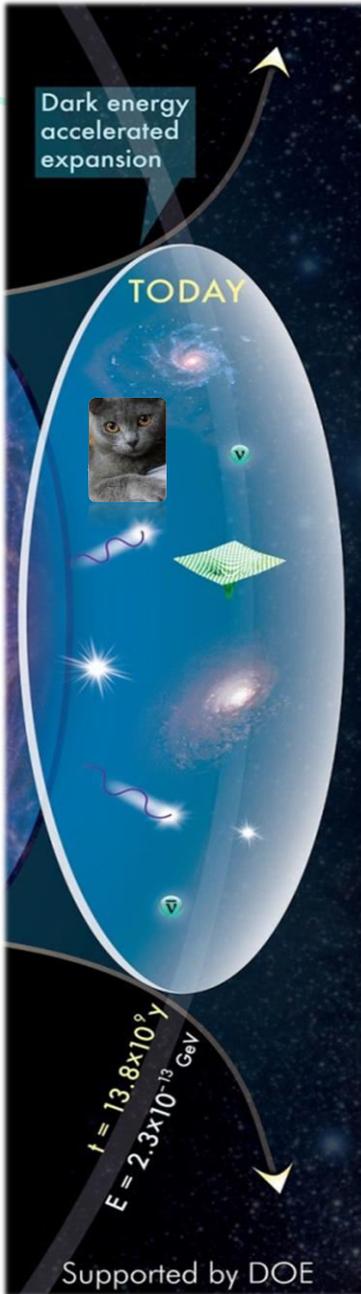
The Universe Today

- Today we live in an expanding, cold Universe – 2.73 K (10^{-13} GeV typical energy) – populated by normal matter, dark matter and dark energy



The Universe Today

- Today we live in an expanding, cold Universe – 2.73 K (10^{-13} GeV typical energy) – populated by **normal matter**, dark matter and dark energy



, you and I are made of normal matter



ELEMENT	%
Oxygen	65.0
Carbon	18.5
Hydrogen	9.5
Nitrogen	3.2
Calcium	1.5
Phosphorus	1.0
Potassium	0.4
Sodium	0.2
Chlorine	0.2
Magnesium	0.1
Sulfur	0.04

The normal matter puzzle is not solved yet!

The Normal Matter Puzzle

How do we get from

here: Standard Model

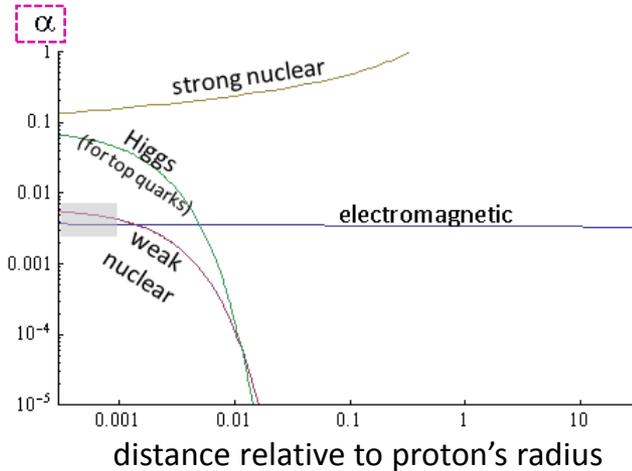
mass →	~2.3 MeV/c ²	~1.275 GeV/c ²	~173.07 GeV/c ²	0	~126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS					
	~4.8 MeV/c ²	~95 MeV/c ²	~4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	γ photon	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS					GAUGE BOSONS
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

to

here?



relative strength of forces acting on a top quark and top anti-quark

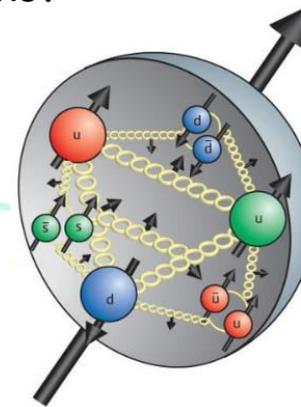
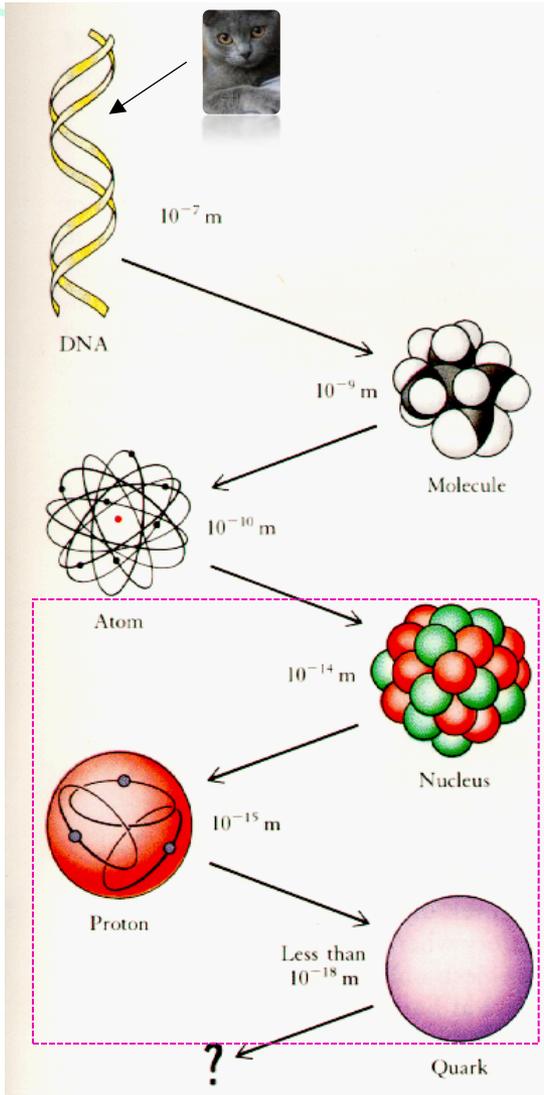


This is a quest across fields of physics, chemistry, biology...

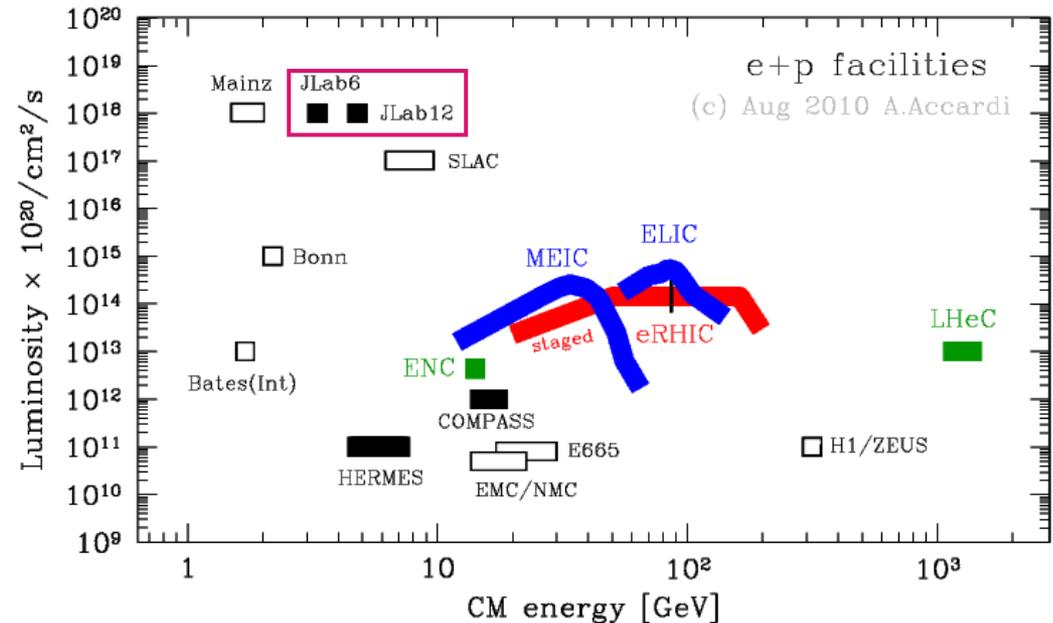


Essential Quest: The Strongly Interacting Normal Matter Puzzle

➤ **How** are nucleons - protons and neutrons - made of quarks and gluons?

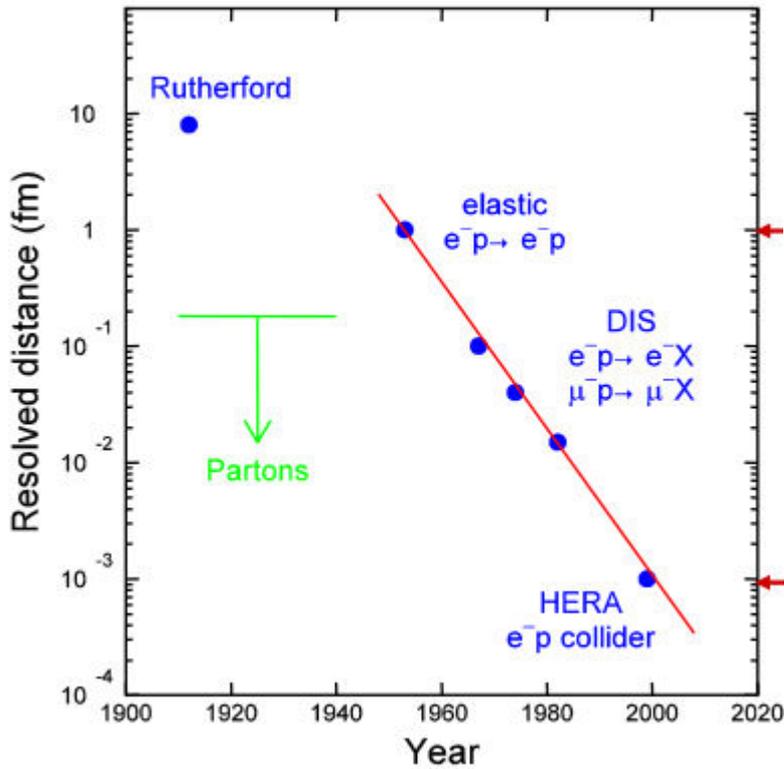


➤ **How** are protons and neutrons modified by the nuclear medium?



Nucleon Structure: Perspective

➤ **Nucleon structure** has been and is a very active field of research of **fundamental importance**

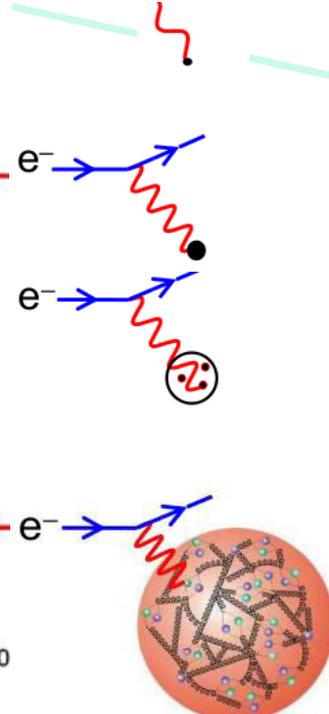


By probing nucleons with electrons we learned:

→ Nucleons have size

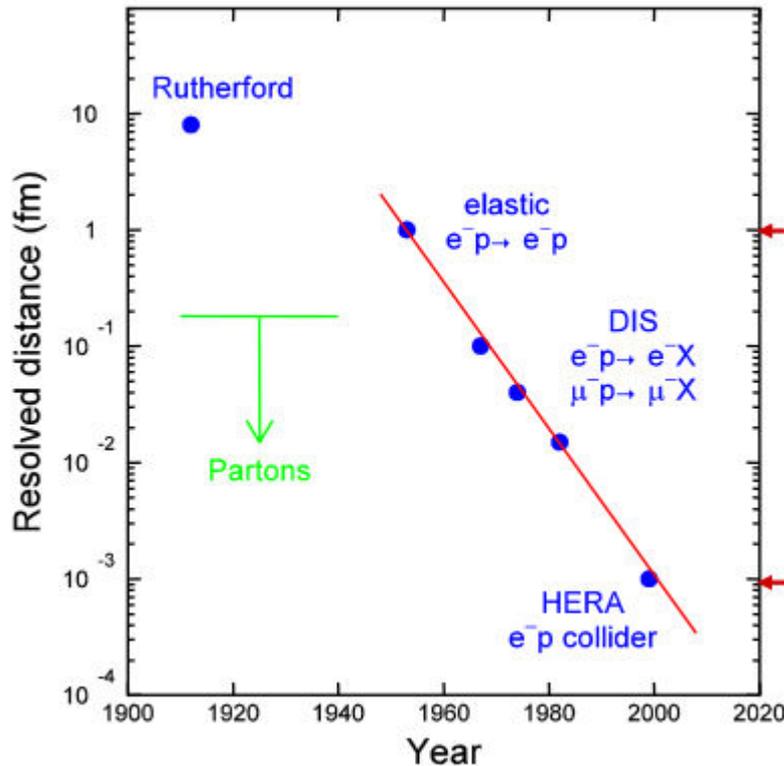
→ Nucleons are NOT elementary but are instead made of point-like particles, partons – quarks and gluons

→ Quarks and gluons inside nucleons are engaged in a dynamics governed by a force which manifests like nothing else out there – QCD's confinement and asymptotic freedom



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1. How did we get here?

2. What more there is to know?

1. How did we get here?

- The Geiger-Marsden-Rutherford Experiments – defining the structure of the atom
- Probing nucleons with leptonic probes – the “Geiger-Marsden-Rutherford Experiments” at a different scale
- Modeling the structure of the nucleon: Quark-Parton Model - a good start
- Modeling the dynamics of the nucleon: parton distribution functions and pQCD

2. What more there is to know?

- Parton distribution functions at large x
- Modeling non-perturbative dynamics of hadrons (just mentioned)
- Understand nuclear medium modifications of quarks and gluons (not covered)
- Structure of hadrons beyond longitudinal momentum distributions (just mentioned)
-

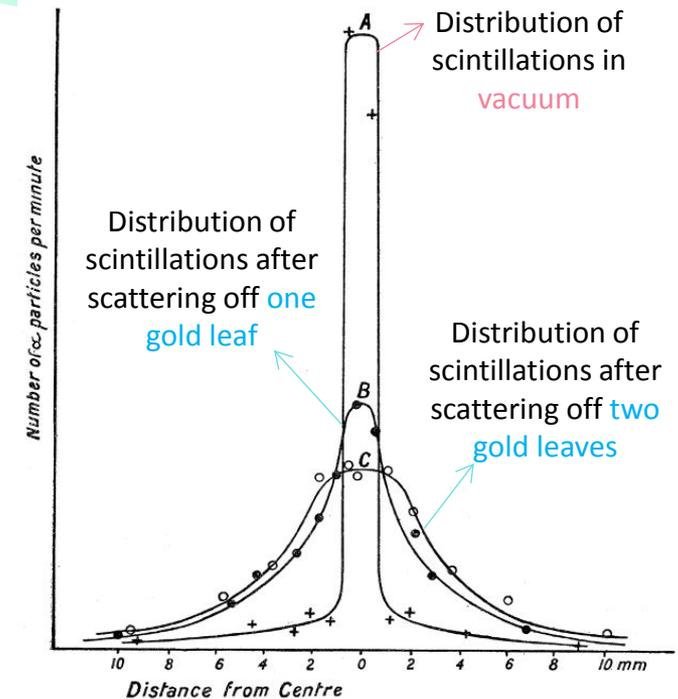
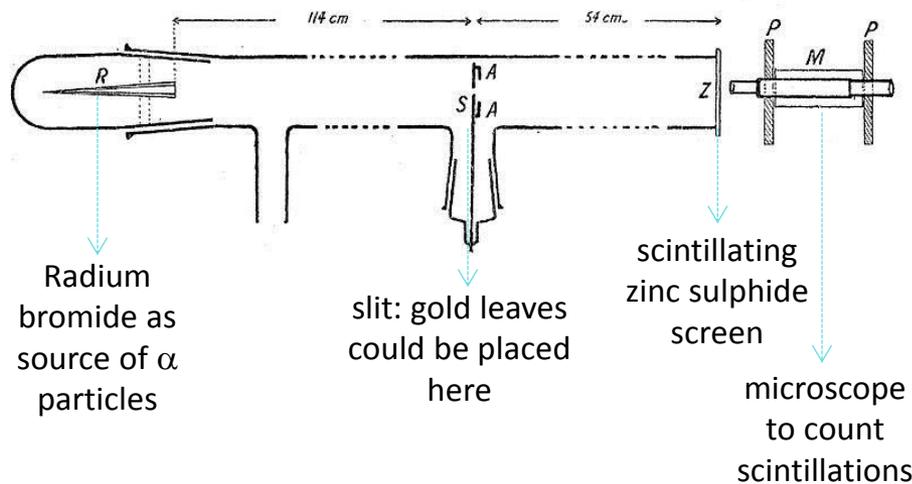
Not Too Long Ago: Atom NOT Like a Plum Pudding

➤ How did we get to our current understanding of matter?

A: By experimenting and by modeling what we observe – in an iterative manner.

nucleus 1908-1913: work by Geiger, Marsden, Rutherford points to the correct conclusion about the structure of the atom

• Experiment 1 → 1908



“there is a very marked scattering of α particles in passing through matter, whether gaseous or solid ... some of the α -particles after passing through very thin leaves were deflected through quite an appreciable angle”

Didn't quite have the acceptance for backward angle detection...

Not Too Long Ago: Atom NOT Like a Plum Pudding

➤ How did we get to our current understanding of matter?

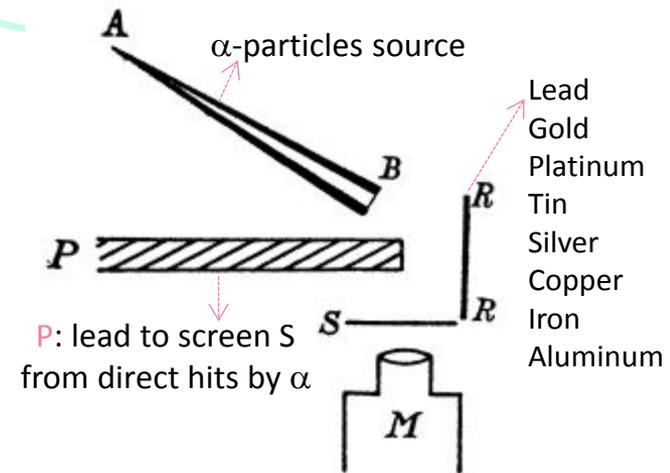
A: By experimenting and by modeling what we observe – in an iterative manner.

nucleus 1908-1913: work by Geiger, Marsden, Rutherford points to the correct conclusion about the structure of the atom

• Experiment 2 → 1909 **backward angle detection**

“surprising that the α -particles can be turned within a layer of 6×10^{-5} cm of gold through angles of 90° , and even more”

... this was a rare event... 1 in 8000 α particles deflected through a large angle by Pt



• Experiment 3 → 1910 **characterizes the most probable scattering**

→ Most probable scattering happens via small angles

→ Can be explained assuming compound scattering: total deflection of α results from very small deflections as many atoms are encountered

Compound scattering cannot explain large angle deflections observed in 1909

Not Too Long Ago: Atom NOT Like a Plum Pudding

➤ How did we get to our current understanding of matter?

A: By experimenting and by modeling what we observe – in an iterative manner.

nucleus 1908-1913: work by Geiger, Marsden, Rutherford points to the correct conclusion about the structure of the atom

- Theoretical prediction → 1911

Coulomb force, Newton's laws, conservation of linear and angular momentum

+

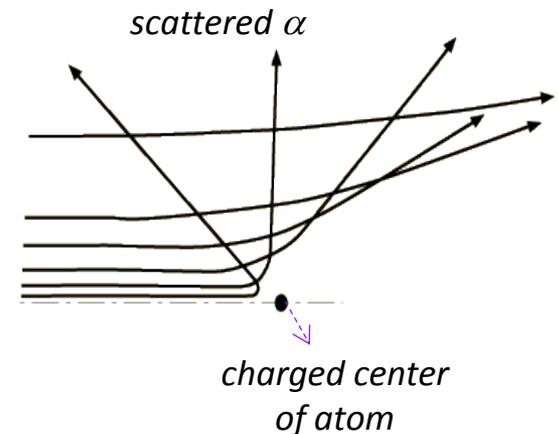
atoms consist of a + or – charge concentrated within a sphere of $< 10^{-14}$ m radius surrounded by charge of the opposite sign distributed throughout the remainder of 10^{-10} m

Large deflection scattering – **one single atomic encounter** when α sufficiently close to center of atom

number of α scattered at angle θ

$$y = \frac{ntb^2 Qcsc^4 \theta / 2}{16r^2}, \quad b = \frac{2NeE}{mu^2}$$

- Experiment 4 → 1913, verification of theoretical prediction and experimental determination of the “central charge” of Au: $197/2 \pm 20\% \rightarrow 98 \pm 20$



The Scientific Method

➤ How did we get to our current understanding of matter?

A: By experimenting and by modeling what we observe – in an iterative manner.

→ Observe a pattern, may point to existence of “discrete units”: Dalton’s atoms

→ Probe “discrete units” to investigate for substructure:

→ scattering a probe off object under investigation: Geiger, Marsden, Rutherford’s scattering

→ measure probability of interaction probe - object (cross section) and its dependence on experimental parameters: helps with modeling

→ Model observation, this may:

→ involve drastic departure from current understanding – photon as particle

→ lead to very different view of the world: fields are primary while particles are derived concepts appearing after quantization - quantum field theories

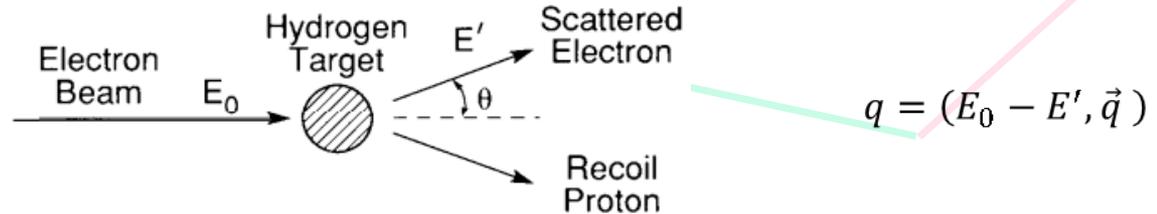
→ Integrate it all in the BIG PICTURE

... and iterate

Matter Puzzle: Is the Proton Point-Like?

➤ Do **protons** have size?

1948-50 – Schiff, Rosenbluth: use **elastic electron-proton scattering** to probe the proton

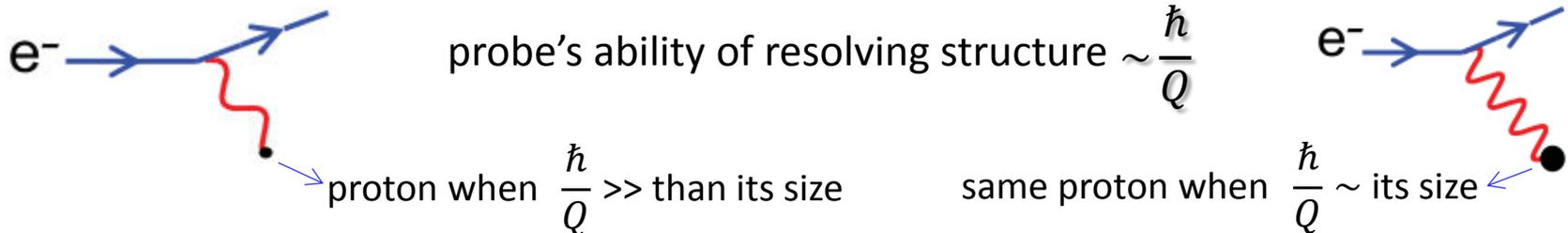


$$E' = \frac{E_0}{1 + \frac{2E_0}{M} \sin^2 \frac{\theta}{2}}$$

electron is left with less energy after meeting the proton

$$Q^2 = 4E_0 E' \sin^2 \frac{\theta}{2}$$

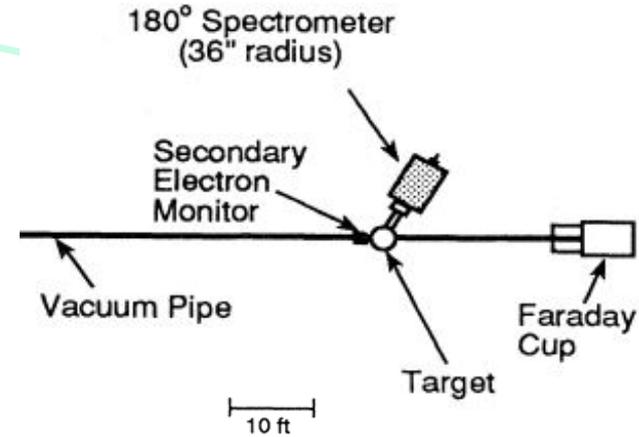
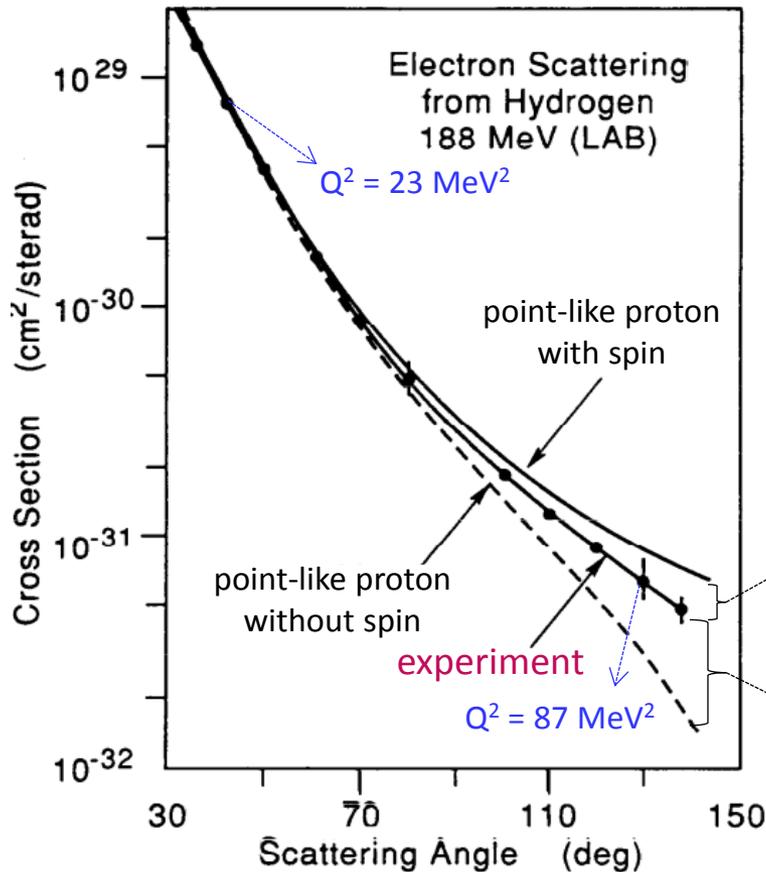
square of four-momentum transfer: connected to the probe's ability of resolving the structure of the proton



The Proton Is NOT Point-Like

➤ Do **protons** have size?

Yes! from experiments at High Energy Physics Laboratory Stanford, 1955



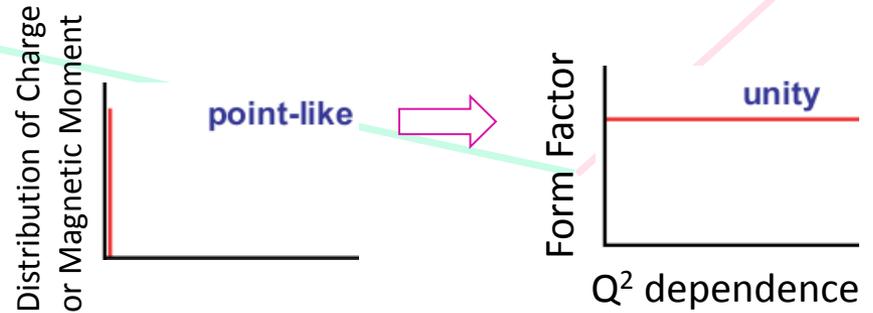
Probability of interaction less than expected from **point-like proton with spin**

Probability of interaction more than expected from **point-like proton without spin**

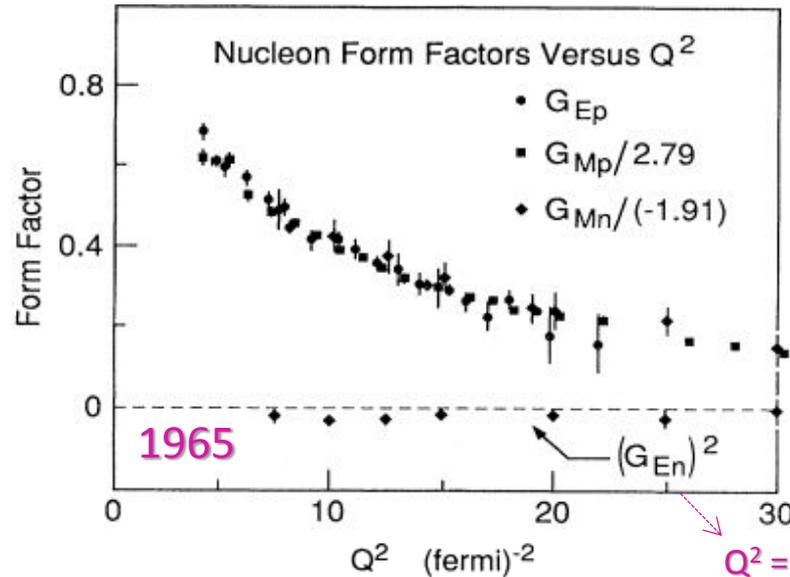
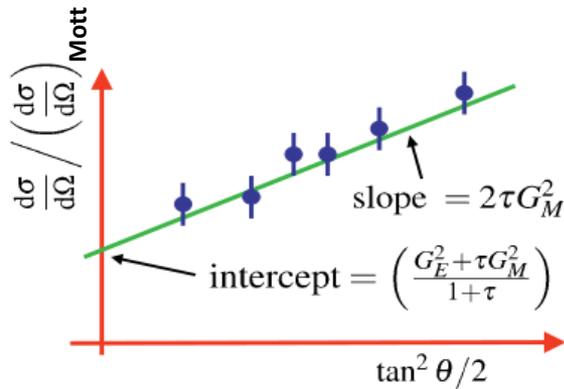
How Do the Charge and Magnetic Moment Distribute?

Probability of elastic interaction: $\frac{d\sigma}{d\Omega} / \left(\frac{d\sigma}{d\Omega} \right)_{point} = \left[\frac{G^2_E(Q^2) + \tau G^2_M(Q^2)}{1 + \tau} + 2\tau G^2_M(Q^2) \tan^2 \frac{\theta}{2} \right] \quad \tau = \frac{Q^2}{4M^2}$

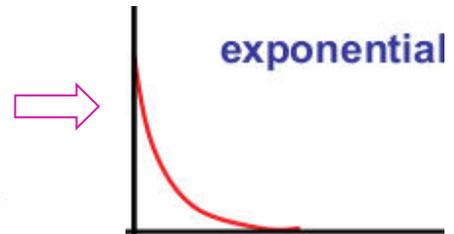
- Form Factors are (in some limit) Fourier transforms of charge and magnetic moment distributions



- And the Q^2 dependence of form factors was measured...



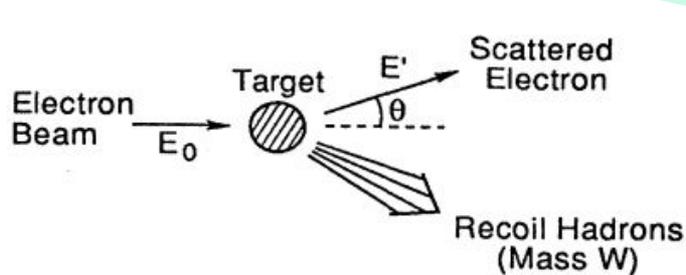
Distribution of Charge or Magnetic Moment



Matter Puzzle: What's Inside the Proton?

➤ Is the **proton elementary**?

To find out increase the probe's ability of resolving structure (decrease $\frac{\hbar}{Q}$)



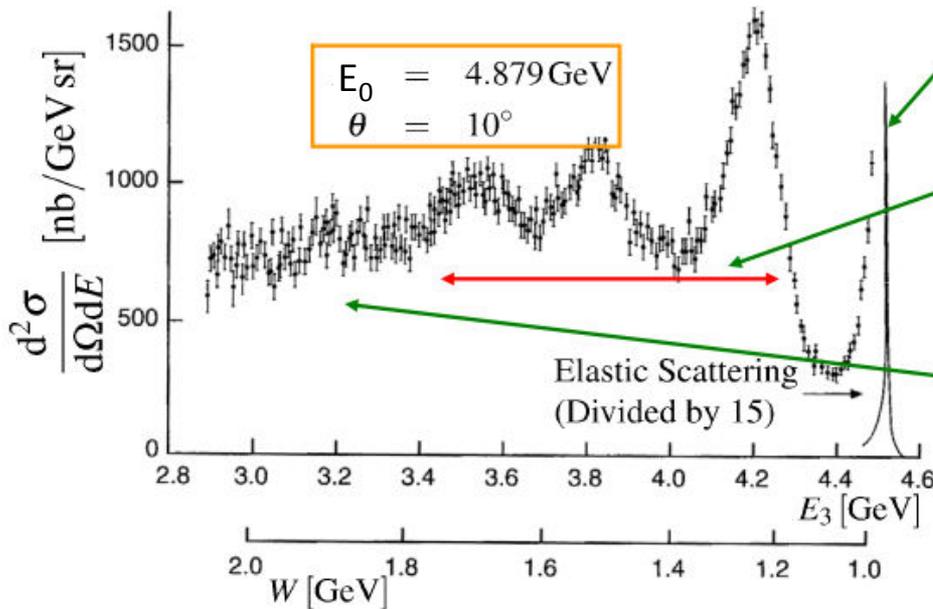
$$E' = \frac{E_0 - \frac{W^2 - M^2}{2M}}{1 + \frac{2E_0}{M} \sin^2 \frac{\theta}{2}}$$

$$v = E_0 - E'$$

$$y = \frac{v}{E_0}$$

$$x = \frac{Q^2}{2Mv}$$

$$W^2 = M^2 + 2Mv - Q^2$$



Elastic scattering: proton stays intact, $W = M$

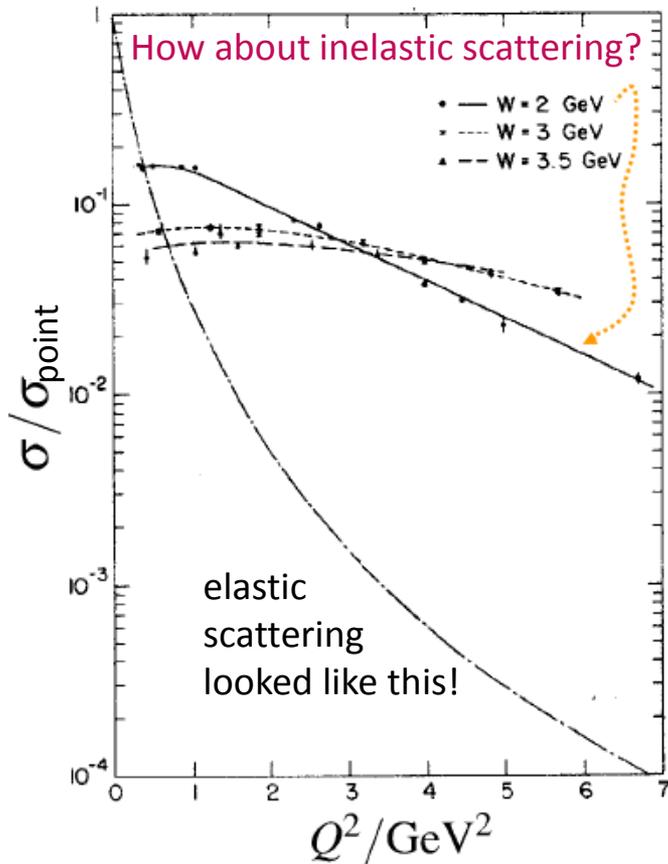
Inelastic scattering: proton gets excited, produce excited states or proton's resonances, $W = M_{resonance}$

Deep inelastic scattering: proton breaks up and we end up with a many particle final state, $W = large$

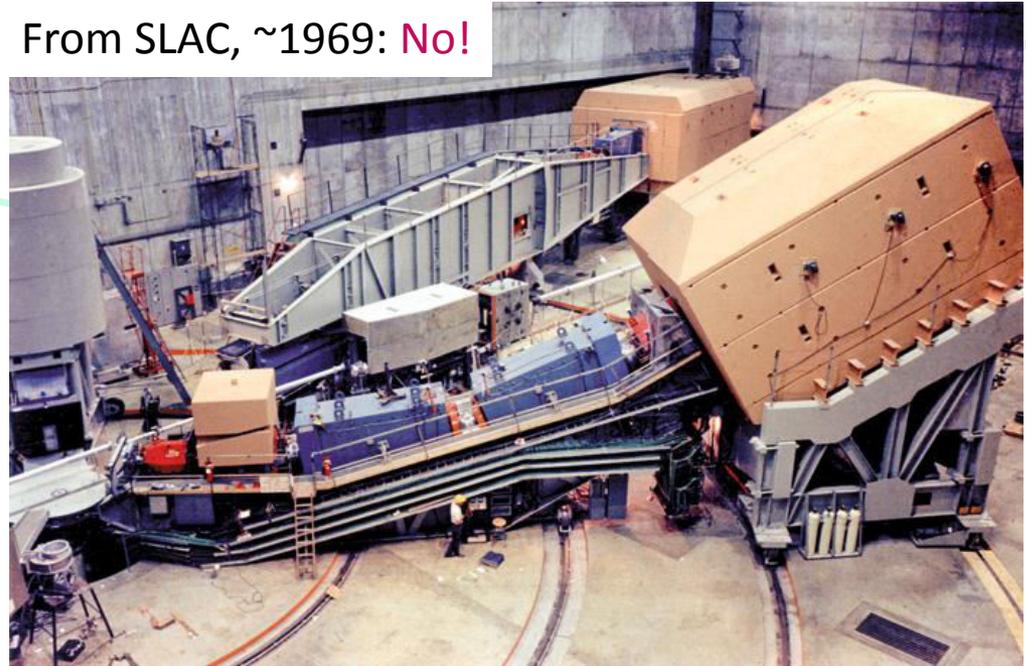
Point-Like Constituents Inside Proton

➤ Is the **proton elementary**?

Map cross section when proton is probed deeper resulting in final states with **higher W** ...

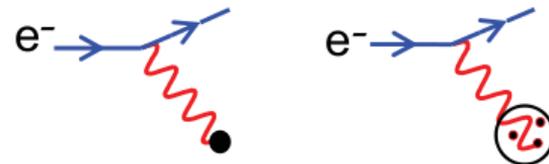


From SLAC, ~1969: **No!**



- Low- W Inelastic scattering: weakly dependent on Q^2
- Deep inelastic scattering: almost independent of Q^2 !

Scattering from point-like, charged objects in the proton



Point-Like Constituents Inside Proton: Formalism

➤ Probability of **inelastic** interaction:

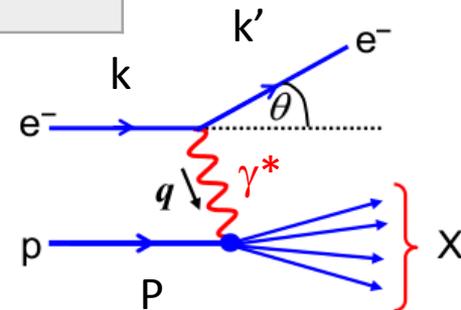
$Q^2 = -q^2 = -(k - k')^2$	four-momentum transfer squared	$4E_0 E' \sin^2 \frac{\theta}{2}$
$W^2 = (q + P)^2$	invariant mass of produced hadronic system squared	$M^2 + 2M(E_0 - E') - Q^2$
$x = \frac{Q^2}{2p \cdot q}$	Bjorken x	$\frac{Q^2}{2M(E_0 - E')}$
$y = \frac{q \cdot P}{k \cdot P}$	inelasticity	$\frac{E_0 - E'}{E_0}$

What we measure:

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma \left[\sigma_T(x, Q^2) + \varepsilon \sigma_L(x, Q^2) \right] = \Gamma \sigma_T (1 + \varepsilon \cdot R)$$

Cross section for photoabsorption of **Longitudinal γ^*** (helicity 0)

Cross section for photoabsorption of **Transverse γ^*** (helicity +/- 1)



Point-Like Constituents Inside Proton: Formalism

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Cross section for photoabsorption of **Longitudinal γ^*** (helicity 0)

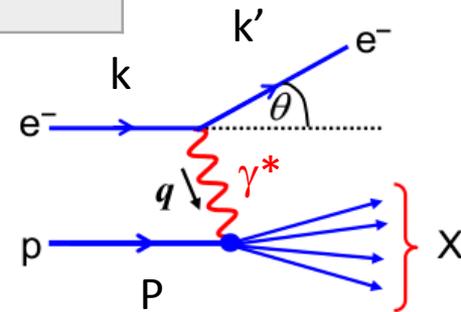
Cross section for photoabsorption of **Transverse γ^*** (helicity +/- 1)

Relative flux of longitudinal virtual photons

$$\Gamma = \frac{\alpha K}{2\pi^2 Q^2} \frac{E}{E'} \frac{1}{1 - \varepsilon}$$

$$\varepsilon = \left(1 + 2 \left(1 + \frac{Q^2}{4M^2 x^2} \right) \tan^2 \frac{\theta}{2} \right)^{-1}$$

Flux of transverse virtual photons



Point-Like Constituents Inside Proton: Formalism

➤ Probability of **inelastic** interaction:

In QFT we can calculate:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4} \frac{E'}{E_0} L_{\mu\nu} W^{\mu\nu} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2} \right) \frac{1}{x} F_2(x, Q^2) + y^2 F_1(x, Q^2) \right]$$

leptonic tensor

hadronic tensor

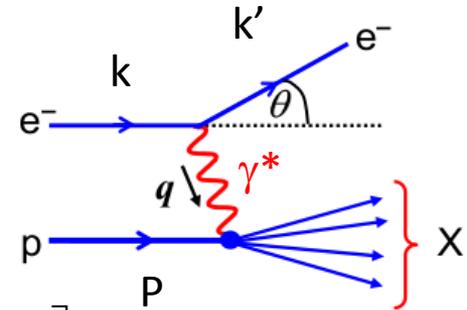
electromagnetic structure function

magnetic structure function

$$F_L(x, Q^2) = \left(1 + \frac{4M^2 x^2}{Q^2} \right) F_2(x, Q^2) - 2x F_1(x, Q^2)$$

Connection between photoabsorption cross sections that we can measure and structure functions:

$$F_1(x, Q^2) \approx \sigma_T(x, Q^2) \quad F_L(x, Q^2) \approx \sigma_L(x, Q^2) \quad F_2 \approx (\sigma_T + \sigma_L)$$



Point-Like Constituents Inside Proton: Formalism

→ The L & T contributions are separated by performing a fit of the reduced cross section dependence with ε at fixed x and Q^2

$$\frac{1}{\Gamma} \frac{d^2\sigma}{d\Omega dE'} = (\sigma_T + \varepsilon\sigma_L)$$

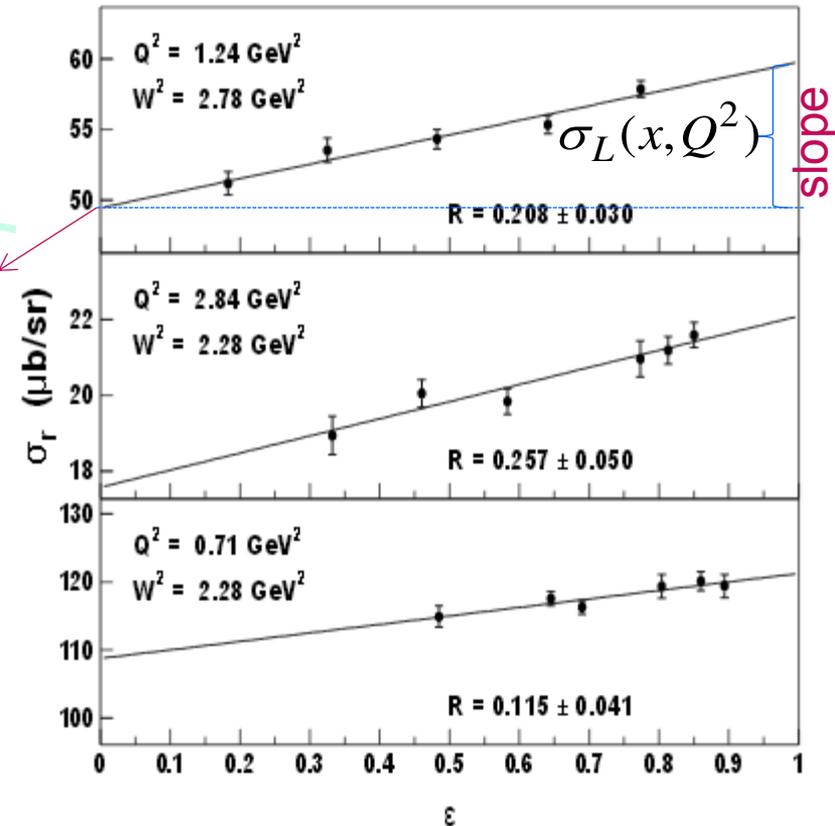
Γ = transverse flux

ε = relative longitudinal flux

→ Requirements for precise L/T extractions:

- As many ε points as possible spanning a large interval from 0 to 1 → as many (E, E', θ) settings as possible
- Very good control of point-to-point systematics → 1-2 % on the reduced cross section translates into 10-15 % on F_L

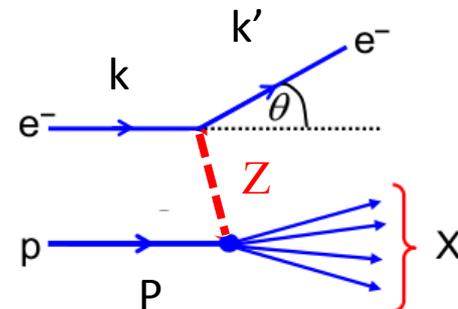
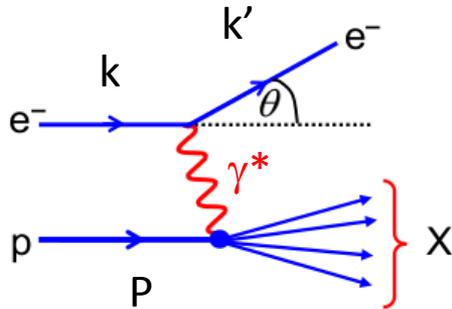
L/Ts needed in order to access F_L, F_1, F_2



Point-Like Constituents Inside Proton: Formalism

➤ Probability of **inelastic** interaction:

There is another boson that could couple to point-like constituents inside nucleons



$$\frac{d\sigma^{e^\pm}}{dx dQ^2} = \frac{2\pi\alpha^2(1 + (1 - y)^2)}{Q^4 x} \left[\tilde{F}_2 \mp \frac{1 - (1 - y)^2}{1 + (1 - y)^2} x \tilde{F}_3 - \frac{y^2}{1 + (1 - y)^2} \tilde{F}_L \right]$$

$$\tilde{F}_2 = F_2 - \kappa_Z v_e F_2^{\gamma Z} + \kappa_Z^2 (v_e^2 + a_e^2) F_2^Z$$

$$\tilde{F}_L = F_L - \kappa_Z v_e F_L^{\gamma Z} + \kappa_Z^2 (v_e^2 + a_e^2) F_L^Z$$

$$x \tilde{F}_3 = -\kappa_Z a_e x F_3^{\gamma Z} + \kappa_Z^2 2v_e a_e x F_3^Z$$

$$\kappa_Z = \frac{Q^2}{Q^2 + M_Z^2} \frac{1}{4 \sin^2 \theta_W \cos \theta_W^2}$$

Vector weak coupling
 $= -1/2 + 2 \sin^2 \theta_W$

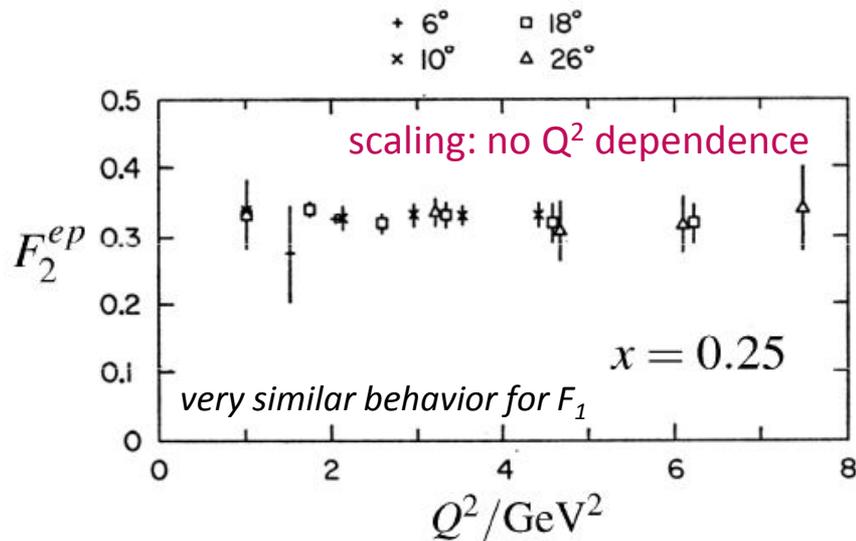
Axial-vector weak coupling
 $= -1/2$

Point-Like Constituents Inside Proton with Spin $\frac{1}{2}$

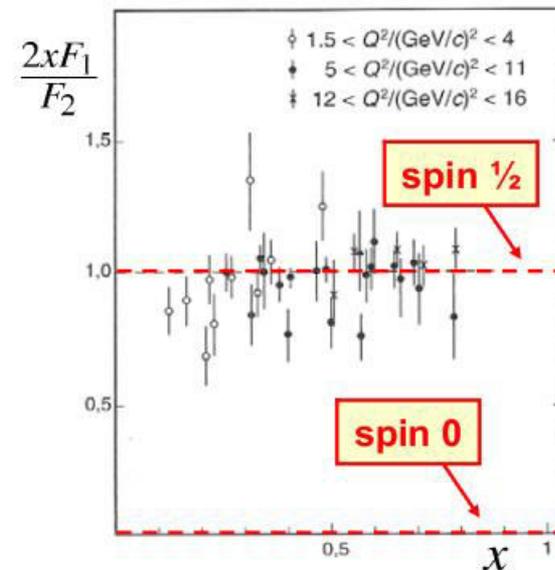
➤ Is the proton elementary? **No!**

Going back to the early SLAC data...

F_1, F_2 account for the sub-structure of the protons and neutrons – **structure functions**



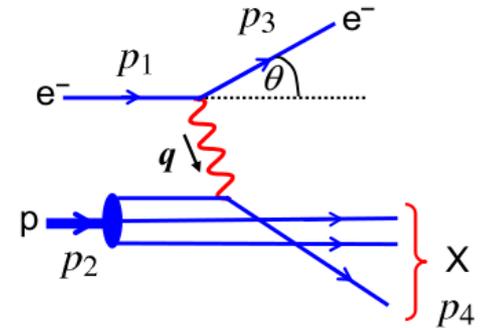
Structure “looks the same” even as the probe’s resolution is increased more and more



If point-like constituents were spin zero particles, we would expect F_1 to be zero

Model: Quarks/Partons Inside the Proton...

- 1969-1971, Feynman, Bjorken: Quark-Parton model interpreted the SLAC large momentum-transfer electron-nucleon scattering as scattering from quasi-free, point-like, spin $\frac{1}{2}$ constituents – **partons** (elastic scattering off partons)

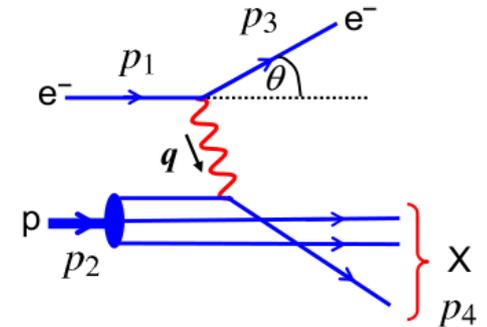


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Infinite Momentum Frame:

Proton moves with infinite momentum: time it takes for virtual photon to couple to partons much smaller than interaction time between partons



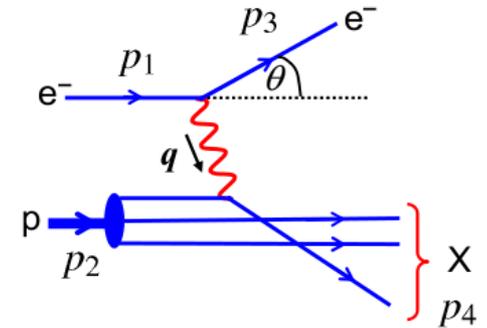
- We neglect the proton's mass
- We also neglect quark masses and any momentum that's transverse to the direction of the proton
- We can calculate the elementary cross section (QED) for elastic electron-quark scattering for a quark
- We then need to introduce a quark/parton momentum distribution function to account for scattering off any quark inside the proton carrying a momentum fraction x of proton's momentum
- Finally we sum over all quarks in the proton

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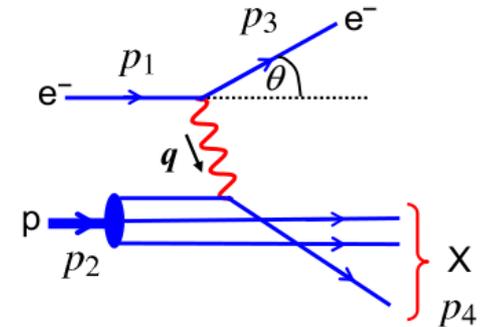
→ electron scatters elastically off a parton: $\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right]$

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→ electron scatters off any one particular parton carrying a fraction x of proton's total momentum

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right] \sum e_q^2 q(x)$$

Parton distribution
function

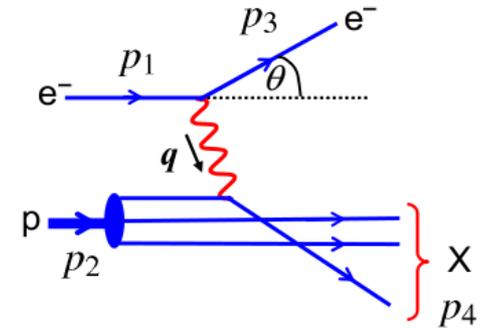
$q(x)dx$ – number of quarks of type q inside the proton with momenta fractions between x and $x + dx$

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Parton distribution function

$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1-y - \frac{M^2 y^2}{Q^2}\right) \frac{1}{x} F_2 + y^2 F_1 \right]$$

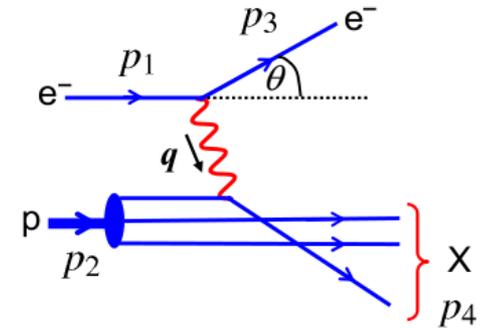
$Q^2 \gg M^2 y^2 \rightarrow F_2(x) = 2xF_1(x) = x \sum_q e_q^2 q(x)$

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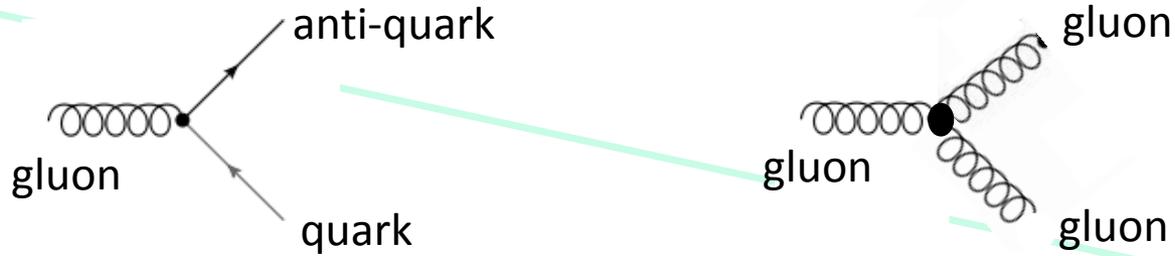
$Q^2 \gg M^2 y^2 \rightarrow F_2(x) = 2xF_1(x) = x \sum_q e_q^2 q(x)$

$F_L(x) = 0$

$$F_L(x, Q^2) = \left(1 + \frac{4M^2 x^2}{Q^2}\right) F_2(x, Q^2) - 2xF_1(x, Q^2)$$

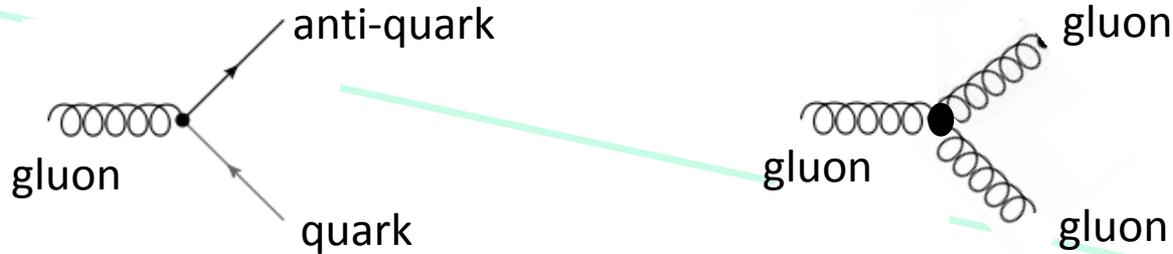
What Can We Learn from This Simple Picture?

➤ First of all, we KNOW there are gluons...



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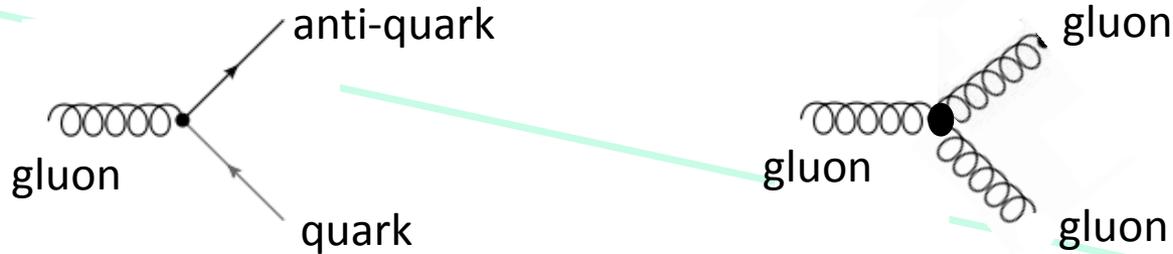
→ Simple equations from the Parton Model:

$$F_2^{e-proton} = x \left[\frac{4}{9} (u^p(x) + \bar{u}^p(x)) + \frac{1}{9} (d^p(x) + \bar{d}^p(x)) \right]$$

$$F_2^{e-neutron} = x \left[\frac{4}{9} (u^n(x) + \bar{u}^n(x)) + \frac{1}{9} (d^n(x) + \bar{d}^n(x)) \right]$$

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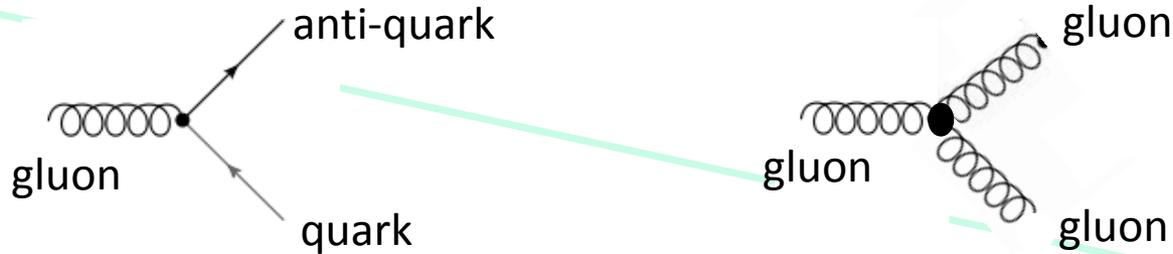


→ Simple equations from the Parton Model:

$$\begin{array}{ccc}
 F_2^{e-proton} = x \left[\frac{4}{9} (u^p(x) + \bar{u}^p(x)) + \frac{1}{9} (d^p(x) + \bar{d}^p(x)) \right] & \begin{array}{c} \text{isospin} \\ \text{symmetry} \end{array} & F_2^{e-proton} = x \left[\frac{4}{9} (u(x) + \bar{u}(x)) + \frac{1}{9} (d(x) + \bar{d}(x)) \right] \\
 F_2^{e-neutron} = x \left[\frac{4}{9} (u^n(x) + \bar{u}^n(x)) + \frac{1}{9} (d^n(x) + \bar{d}^n(x)) \right] & \begin{array}{c} u^p = d^n \equiv u \\ d^p = u^n \equiv d \end{array} & F_2^{e-neutron} = x \left[\frac{4}{9} (d(x) + \bar{d}(x)) + \frac{1}{9} (u(x) + \bar{u}(x)) \right]
 \end{array}$$

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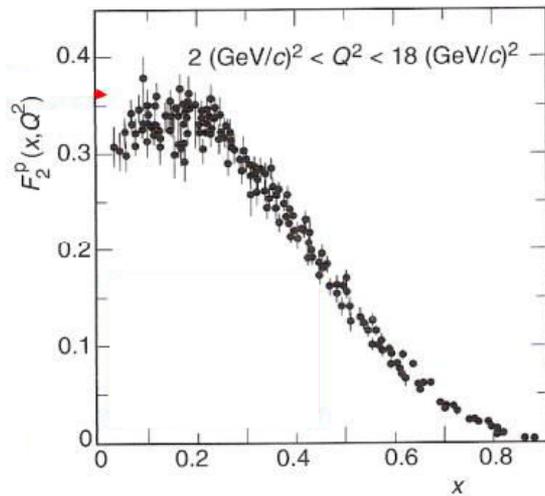
isospin symmetry

$$u^p = d^n \equiv u$$

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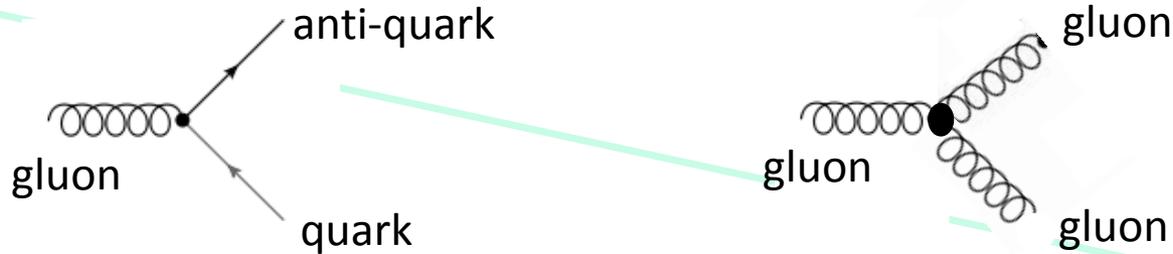
$$\int F_2^{e-proton}(x) dx \approx 0.18$$

$$\int F_2^{e-neutron}(x) dx \approx 0.12$$

from data

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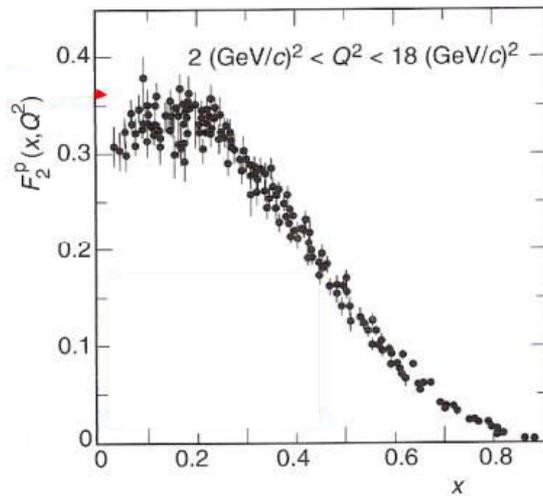
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$$\int F_2^{e-proton}(x) dx \approx 0.18$$

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from data

Up quarks carry twice the momentum of down quarks in the proton and gluons carry half of the total proton's momentum

This Simple Picture, Does It Check Out?

- This picture checks out when **probing with** a different probe: electrically neutral **neutrinos**

F_1, F_2, F_3 – proton structure functions

$$\frac{d^2\sigma^{\nu p}}{dx dy} = \frac{G_{FS}^2}{2\pi} \left[(1-y)F_2^{\nu p}(x, Q^2) + y^2 x F_1^{\nu p}(x, Q^2) + y \left(1 - \frac{y}{2}\right) x F_3^{\nu p}(x, Q^2) \right]$$

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$$\frac{1}{2}(F_2^{\nu p} + F_2^{\nu n}) = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$

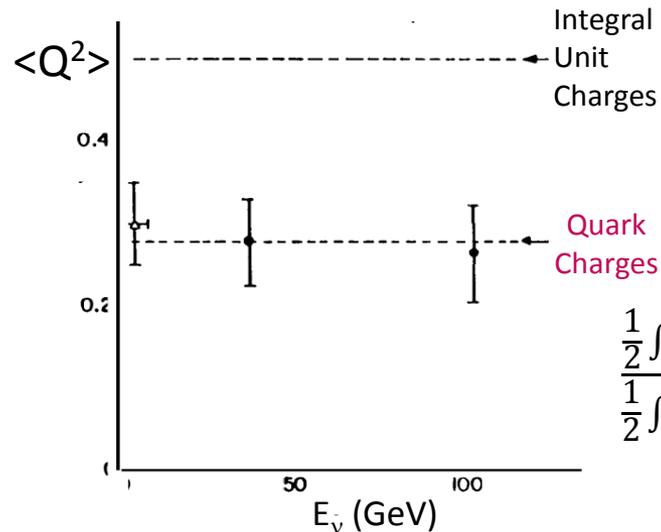
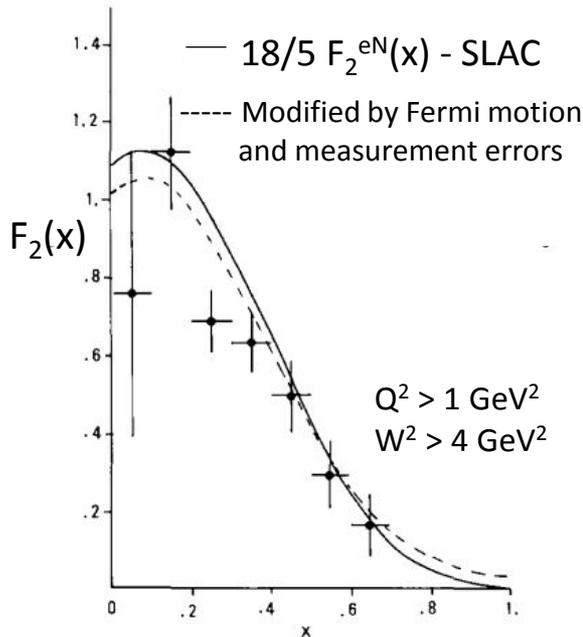
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$$\frac{\frac{1}{2} \int [F^{ep}(x) + F^{en}(x)] dx}{\frac{1}{2} \int [F^{\nu p}(x) + F^{\nu n}(x)] dx} = \frac{e_u^2 + e_d^2}{2}$$

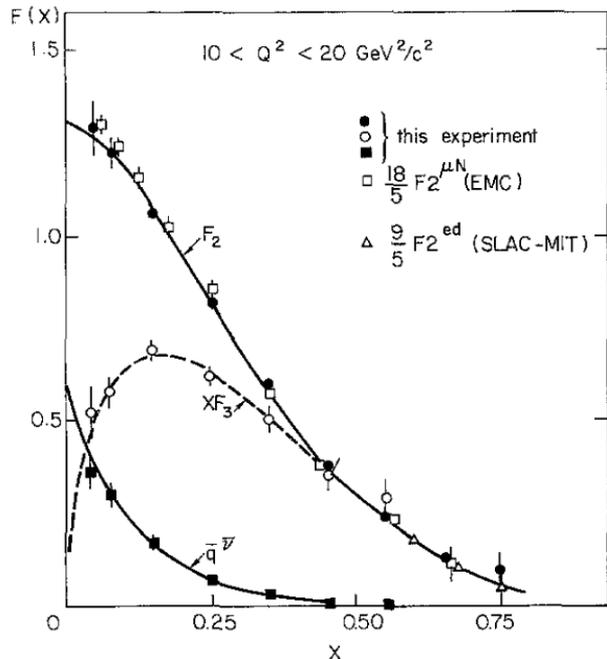
Sum of quarks charges can be extracted from combination of proton's structure functions in electron and neutrino scattering

This Simple Picture, Does It Check Out?

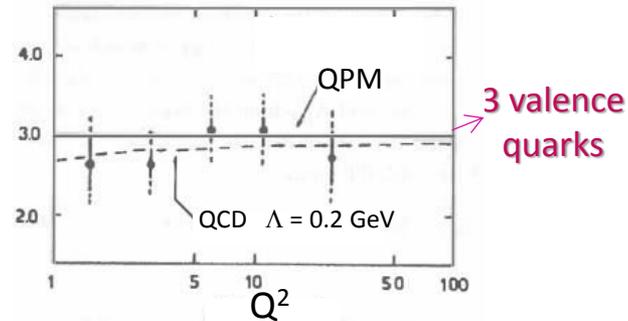
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$$\frac{1}{2}(F_2^{\nu p} + F_2^{\nu n}) = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)] \quad \frac{1}{2}(xF_3^{\nu p} + xF_3^{\nu n}) = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$$



- Subtract the two equations to get the anti-quark parton distribution functions
- Integrate F_3 to get the number of valence quarks



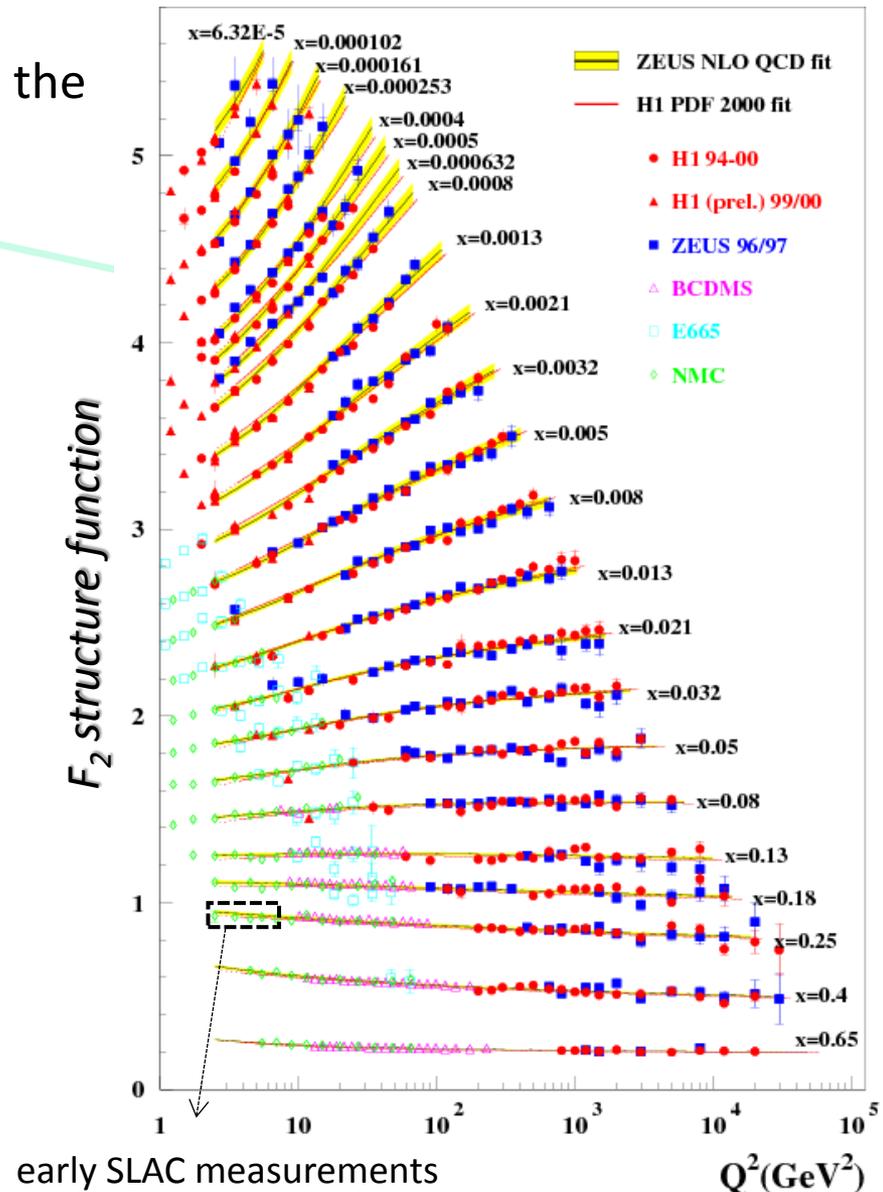
- xF_3 goes to zero at very low x as sea dominates and $q(x) = \bar{q}(x)$

Matter Puzzle: Proton's Rich Inner Life

➤ After 50 years of exploration we found that the proton has a very rich inner life...

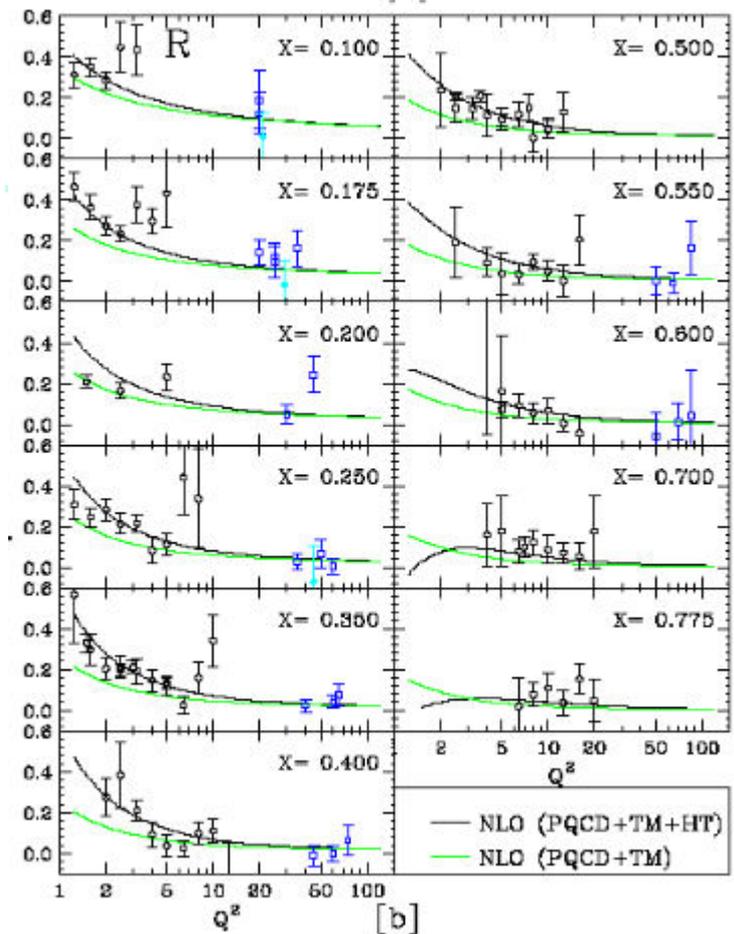
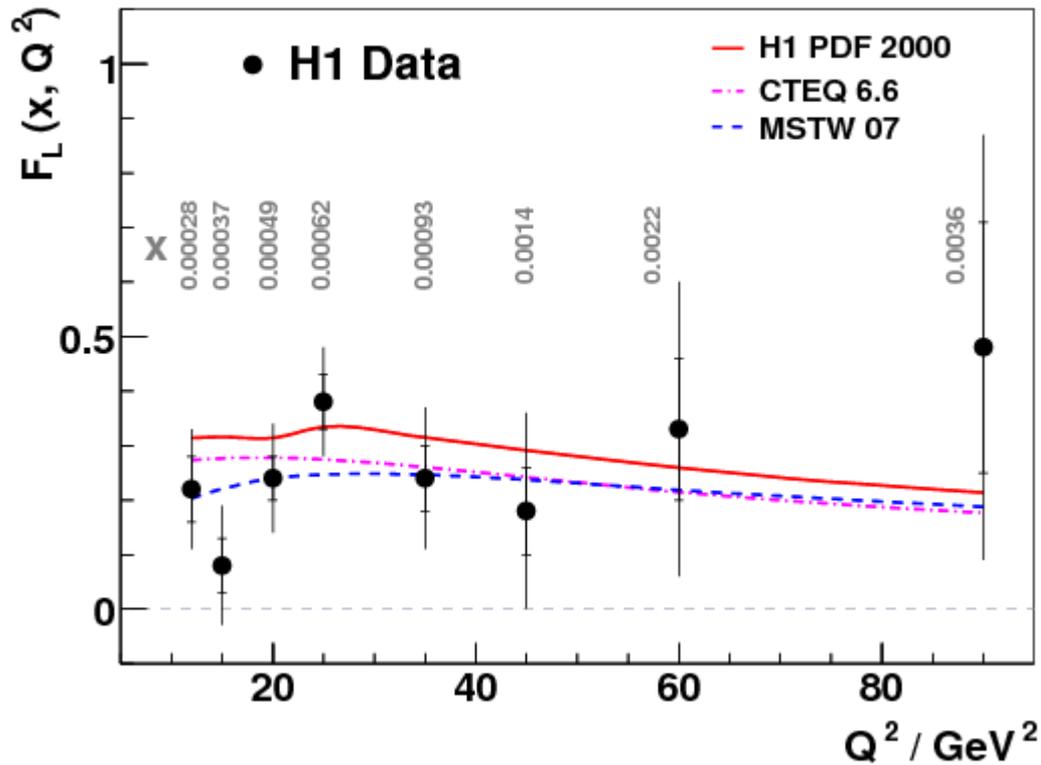
→ Structure functions do not scale with Q^2 after all but they exhibit a well defined pattern of Q^2 scaling violations

→ It is precisely this approach to exploring the proton that led to the development of the Quantum Field Theory of strong interactions – Quantum Chromodynamics



Matter Puzzle: Proton's Rich Inner Life

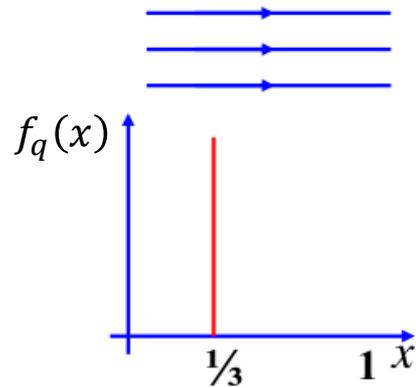
➤ And F_L and R are not zero!



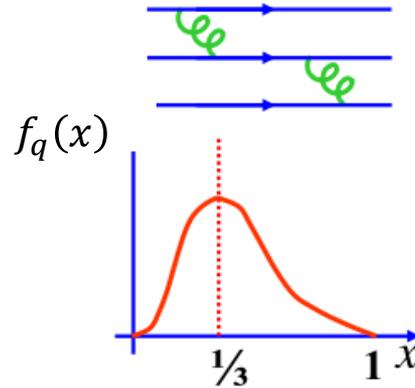
Matter Puzzle: Proton's Rich Inner Life

- Parton distribution functions keep track of the dynamics of quarks and gluons inside nucleons

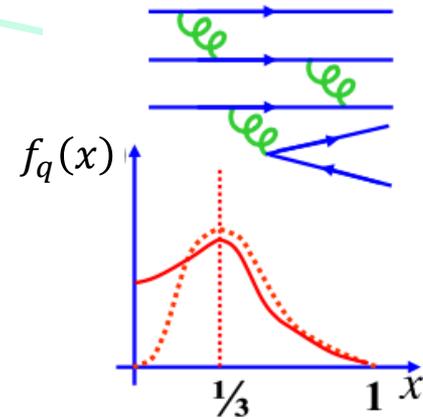
Three non-interacting quarks



Three interacting quarks



Three interacting quarks with sea of quarks, anti-quarks and gluons



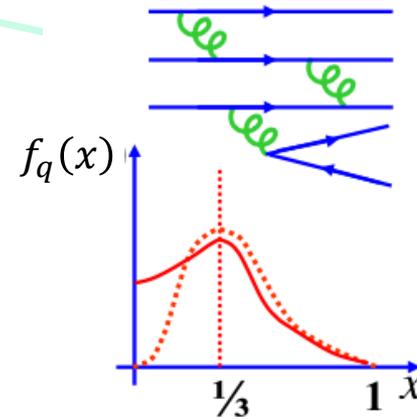
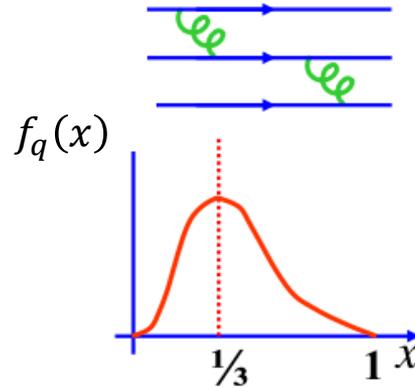
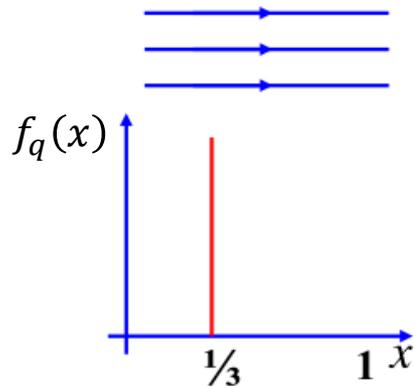
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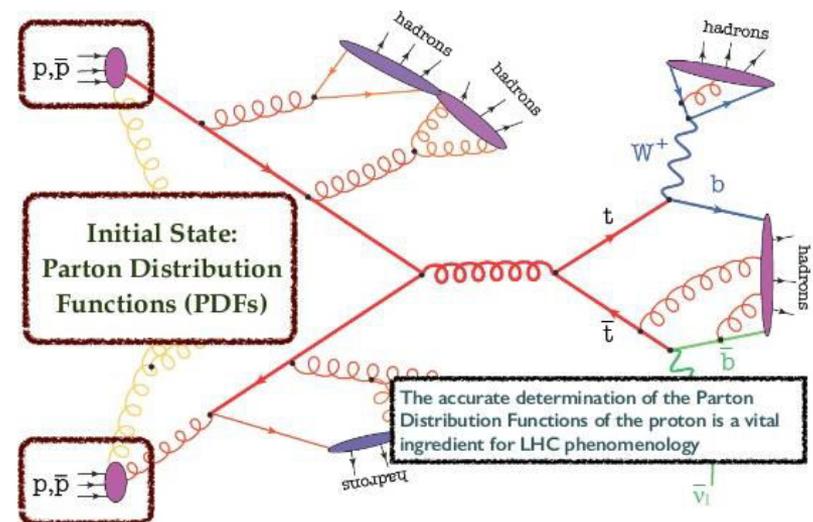
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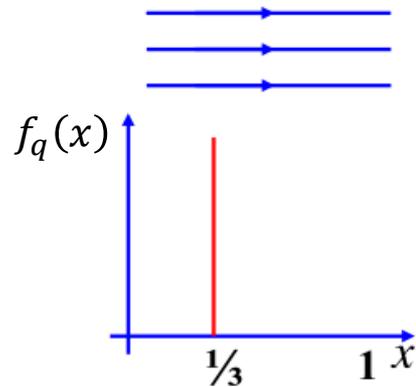
- Parton distribution functions are the initial state in searches for new physics within and beyond the Standard Model



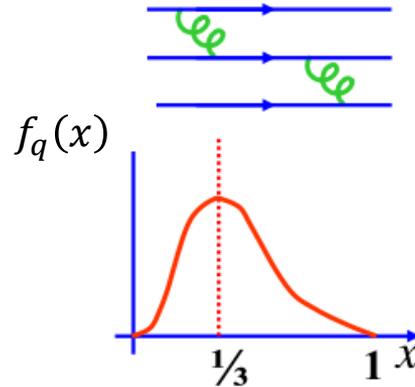
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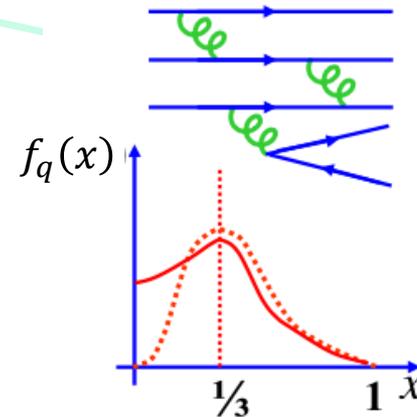
Three non-interacting quarks



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Three interacting quarks with sea of quarks, anti-quarks and gluons



- Quantum field theory of strong interactions – Quantum Chromodynamics (QCD) – models the dynamics of parton distribution functions

A decorative line starts at the top left, slopes downwards to the right, and then turns upwards to the right, ending at the top right. The first segment is light green and the second segment is light pink.

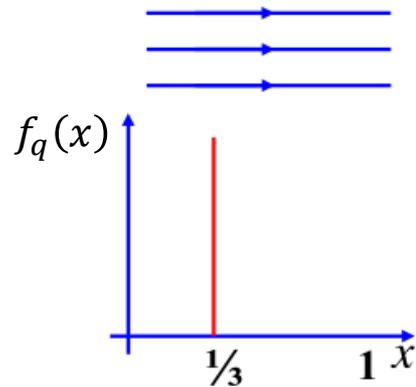
DIS Lectures at CTEQ 2017 Summer School- Lecture 2

Simona Malace
Jefferson Lab

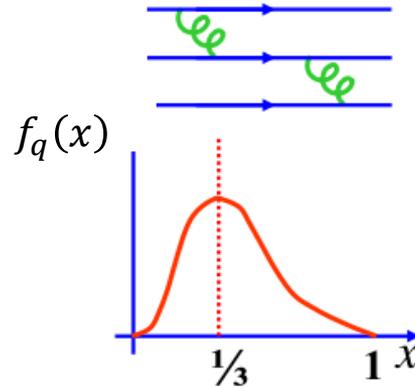
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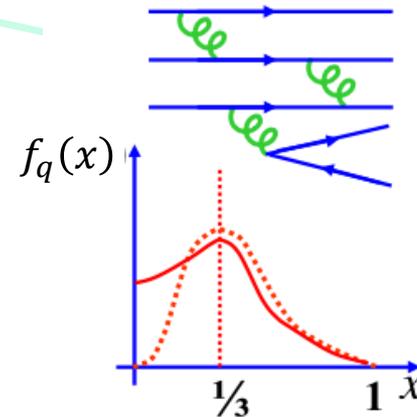
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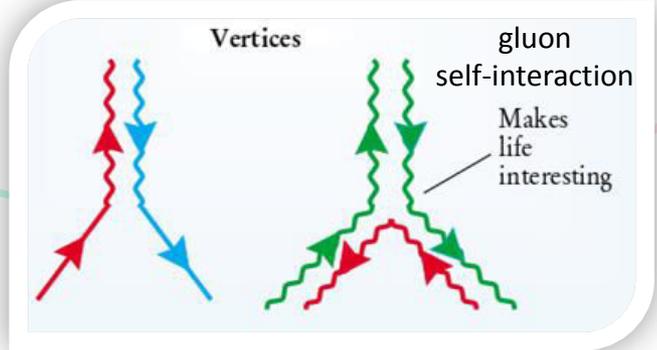
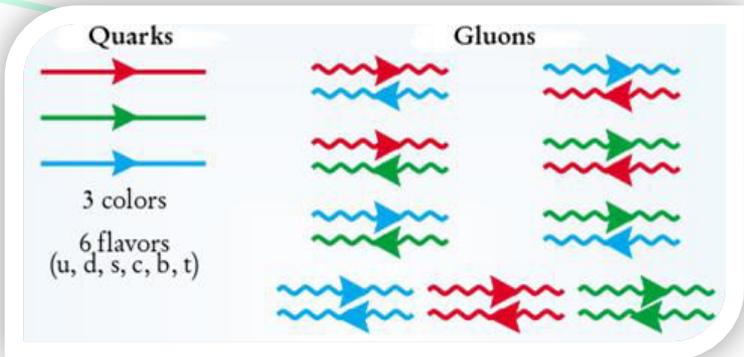
Three interacting quarks with sea of quarks, anti-quarks and gluons



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QCD Weaves the Story of Hadron's Rich Inner Life

➤ Quantum Chromodynamics? This is all there is to it... according to Frank Wilczek



→ **In principle** this Lagrangian gives a complete description of the strong interactions

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

coupling constant of strong interactions (points to g^2)

mass and quantum field of the quark of j^{th} flavor (points to m_j and q_j)

$$G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + if^a_{bc} A^b_\mu A^c_\nu$$

color indices (points to a, b, c)

$$D_\mu \equiv \partial_\mu + it^a A^a_\mu$$

SU(3) color (gauge) symmetry (points to the term $it^a A^a_\mu$)

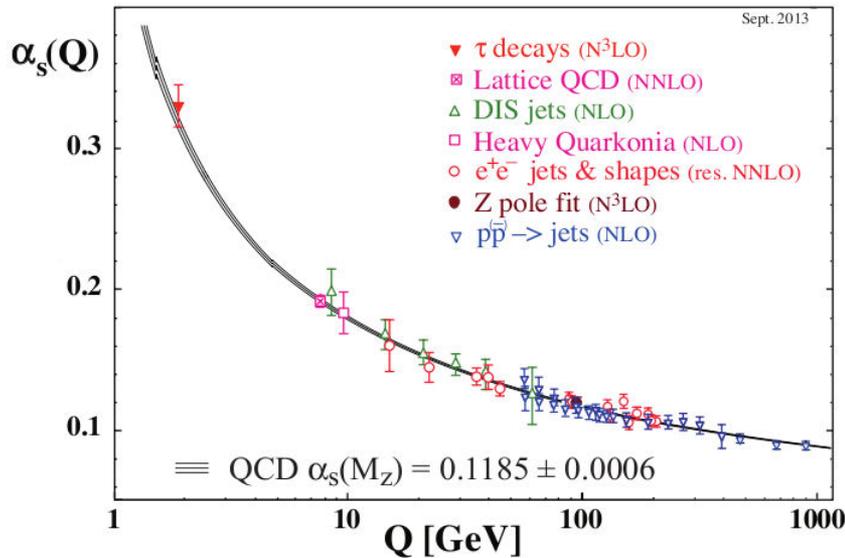
gluon field (points to A^a_μ)

... **in practice** this Lagrangian leads to equations that are very hard to solve.

Ways have been found: perturbative QCD, lattice QCD,...

The QCD Story of Asymptotically Free Quarks

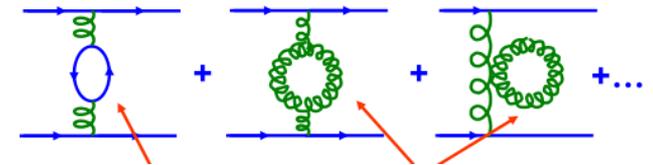
- Energy scale dependence of strong interaction as **predicted** by **R**enormalization **G**roup **E**quations (**p**erturbative **Q**CD) and **confirmed by measurements** is suggestive of barely interacting quarks at high energy scales - **asymptotic freedom**



➔ QCD describes variation as a **perturbative expansion** in the coupling constant, α_s

$$\frac{\mu^2}{\alpha_s(\mu^2)} \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = -\frac{\alpha_s(\mu^2)}{4\pi} \beta_0 - \left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2 \beta_1 + \dots$$

➔ The way the variation happens – decreases with increasing energy scale of interaction – is born by the fundamental properties of QCD: color, self-interacting gluon mediator



Just like QED if gluon replaced by photon

Propagator (gluon) self-interaction – unique to QCD

➔ QCD perturbative expansion valid for $\mu \gg \Lambda \sim 400 \text{ MeV}$, beyond QCD predicts non-negligible non-perturbative effects (QED stays perturbative for $\mu \ll 10^{90} \text{ GeV!!}$)

Confinement: An Even More Complicated Story...

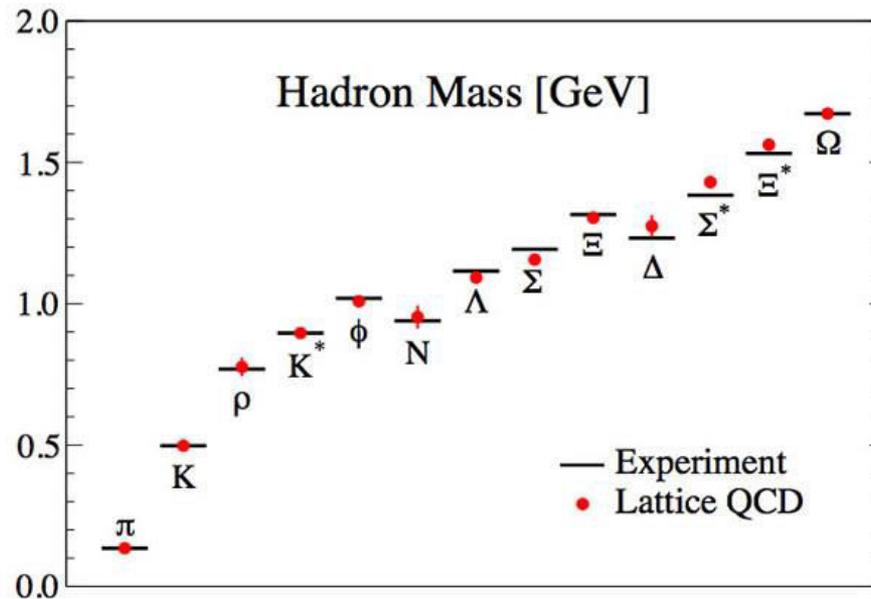
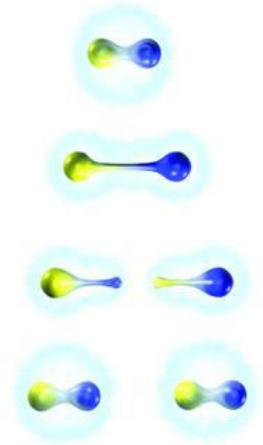
➤ Quarks and gluons are trapped in color-less hadrons which is all we can observe - **confinement**

→ Confinement is an experimental observation: so far we have not seen free quarks

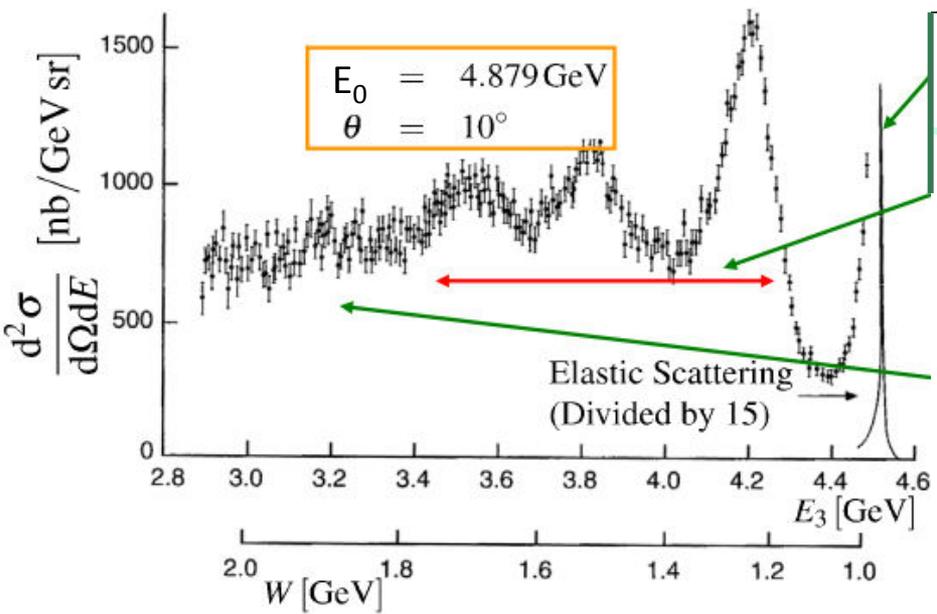
→ Confinement which arises from highly non-perturbative processes is “postulated” in QCD

One approach:

→ Solving of equations derived from the QCD Lagrangian with minimal fundamental input: quark masses and strength of interaction



Going Back to Scattering Off Nucleons

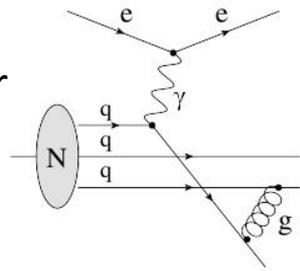


Elastic scattering: proton stays intact, $W = M$

Inelastic scattering: nucleon excited, we produce excited states or resonances, $W = M_{resonance}$

Elastic and Resonance regions: highly non-perturbative **quark-quark interactions** that lead to **confinement** are **dominant**

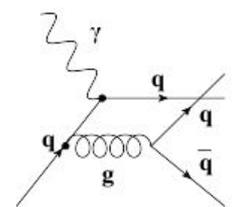
To fully understand the behavior of the proton here **we must understand confinement**



Deep inelastic scattering: proton reveals its point-like structure, $W = large$

Dynamics of nucleon that arises from gluon emission at various energy scales is encoded in **universal functions (parton distribution functions - PDFs)** extracted from data within the framework of perturbative QCD

Having well constrained PDFs is still a challenge



Parton Distribution Functions: Extraction

- In pQCD we can connect the DIS cross section to universal longitudinal momentum distributions of quarks and gluons inside nucleons **via factorization**

$$\sigma^{\text{DIS}}(p) = f_q \sigma^{\text{NLO}}(\hat{p}) = \tilde{f}_q(\mu) \hat{\sigma}(p, \mu)$$

$$\tilde{f}(\mu) \equiv f \otimes \left(\mathbb{1} + \frac{\alpha_s C_F}{2\pi} \ln \frac{\mu^2}{\lambda^2} P_{qq} \right)$$

$$\hat{\sigma}(p, \mu) \equiv \left(\mathbb{1} + \frac{\alpha_s C_F}{2\pi} \ln \frac{Q^2}{\mu^2} P_{qq} \right) \sigma^{(0)}(\hat{p})$$

- σ^{NLO} contains a collinear divergence which can be regulated by low-energy, non-perturbative physics
- σ^{NLO} can be made finite if PDFs are redefined to absorb these collinear singularities

→ The cutoff λ is an infrared scale that regulates the collinear divergence

→ μ is the factorization scale

Parton Distribution Functions: Extraction

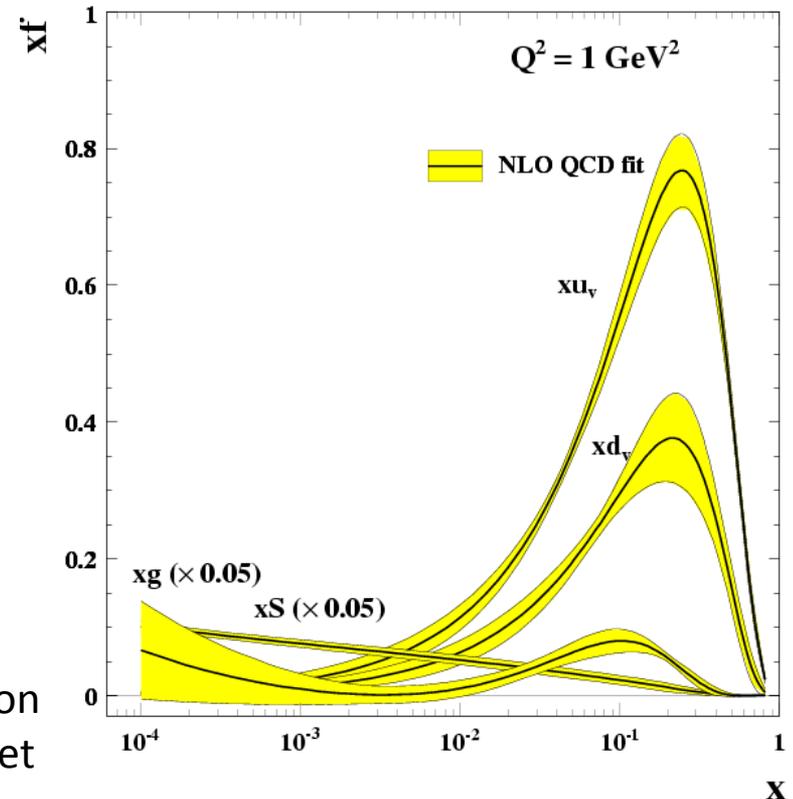
➤ PDFs evolution equations (DGLAP) describe their dependence on the factorization scale

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(\mu) = \sum_j P_{ij} \otimes f_j(\mu) \quad P_{ij}(y) = \frac{\alpha_S(\mu)}{2\pi} P_{ij}^{(0)}(y) + \left(\frac{\alpha_S(\mu)}{2\pi} \right)^2 P_{ij}^{(1)}(y) + \dots$$

- At large x u and d dominate but there is definitely gluon and some sea as well
- At low x gluon and sea dominate; $u \sim d$

As scale changes and gluons are radiated:

- Partons loose momentum because of gluon radiation so at **large x** quarks and gluons shift to the left
- Gluons create quarks-antiquarks and gluon-gluon pairs so at small x sea and gluon increase and get steeper



Parton Distribution Functions: Extraction

- Perturbative QCD gives the Q^2 dependence of PDFs, the x dependence must be extracted from fits to data

Example of data set used for PDFs extraction

Process	Partons	x range
$\ell^\pm \{p, n\} \rightarrow \ell^\pm X$	q, \bar{q}, g	$x \gtrsim 0.01$
$\ell^\pm n/p \rightarrow \ell^\pm X$	d/u	$x \gtrsim 0.01$
$pp \rightarrow \mu^+ \mu^- X$	\bar{q}	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \rightarrow \mu^+ \mu^- X$	\bar{d}/\bar{u}	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$	q, \bar{q}	$0.01 \lesssim x \lesssim 0.5$
$\nu N \rightarrow \mu^- \mu^+ X$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \rightarrow \mu^+ \mu^- X$	\bar{s}	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \rightarrow e^\pm X$	g, q, \bar{q}	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \rightarrow \bar{\nu} X$	d, s	$x \gtrsim 0.01$
$e^\pm p \rightarrow e^\pm c\bar{c} X$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \rightarrow \text{jet} + X$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \rightarrow \text{jet} + X$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \rightarrow (W^\pm \rightarrow \ell^\pm \nu) X$	u, d, \bar{u}, \bar{d}	$x \gtrsim 0.05$
$p\bar{p} \rightarrow (Z \rightarrow \ell^+ \ell^-) X$	d	$x \gtrsim 0.05$

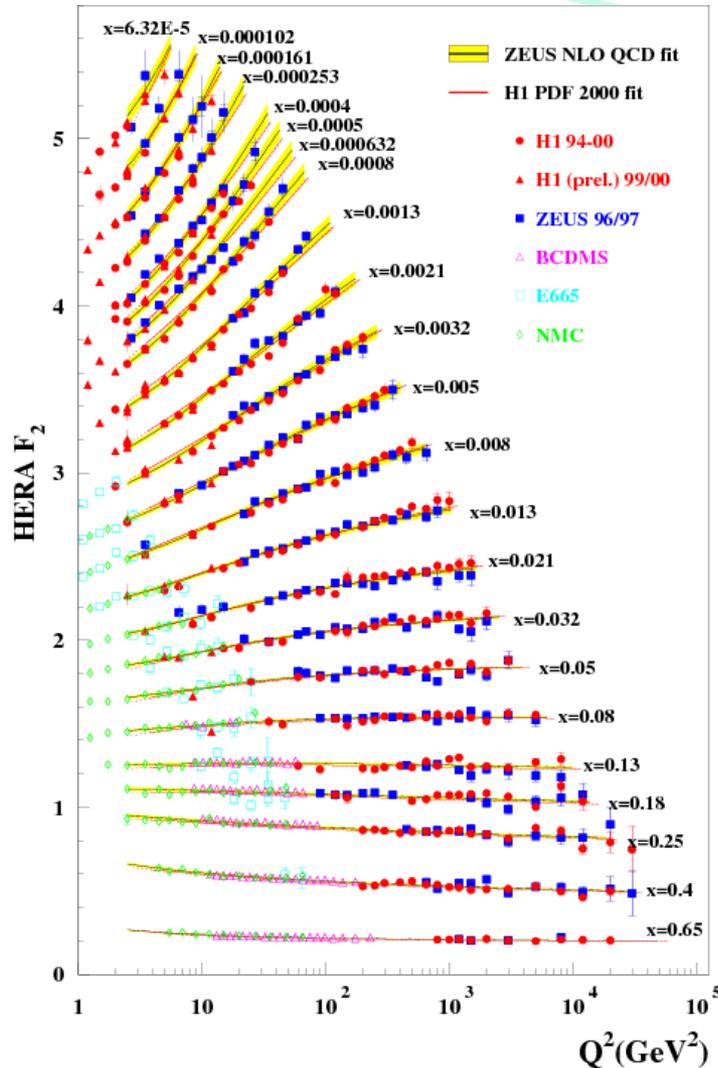
- Need to use a variety of processes to separate flavors

- Need data everywhere we want well-constrained parton distribution functions

Parton Distribution Functions: Indeed Universal

➤ Perturbative QCD gives the Q^2 dependence of PDFs, the x dependence must be extracted from fits to data

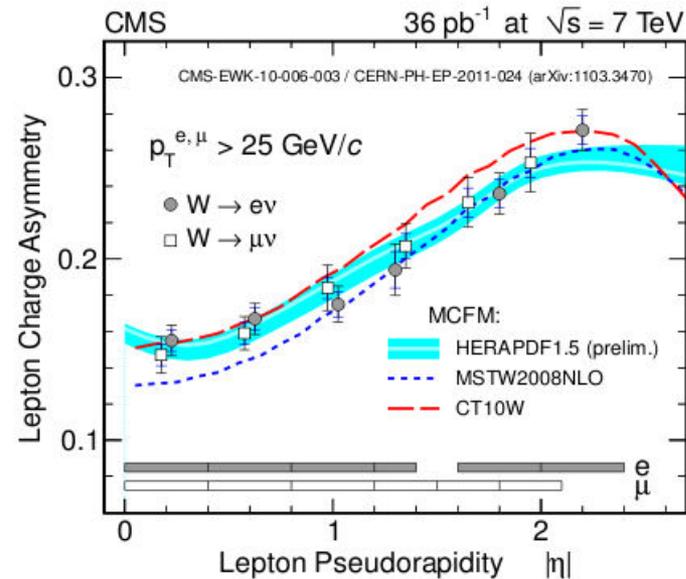
→ Deep Inelastic Scattering on proton



→ Production of $W^{+/-}$ bosons in proton-proton collisions at LHC

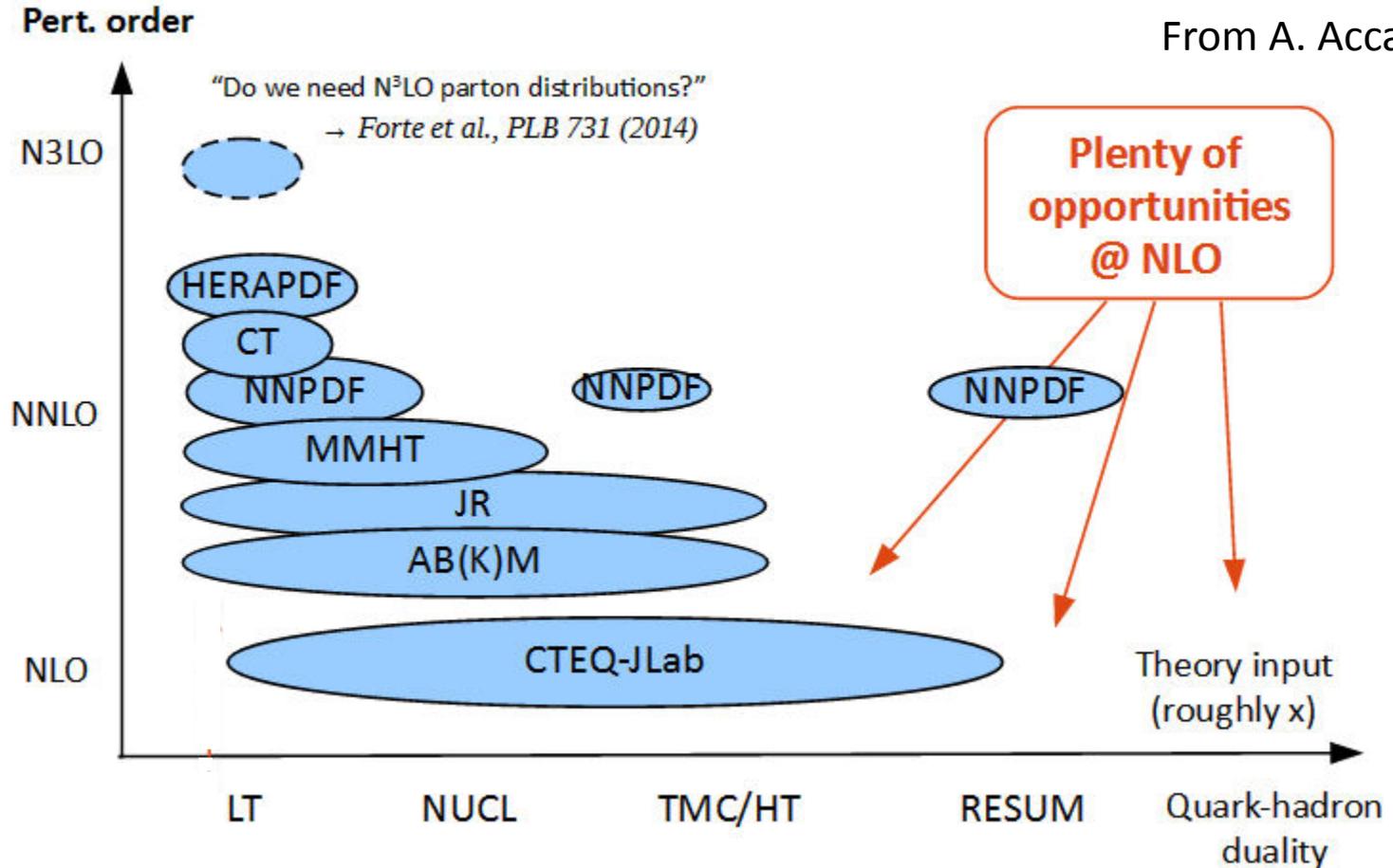
$$d\bar{u} \rightarrow W^-$$

$$u\bar{d} \rightarrow W^+$$



Parton Distribution Functions Extraction: Complicated Business

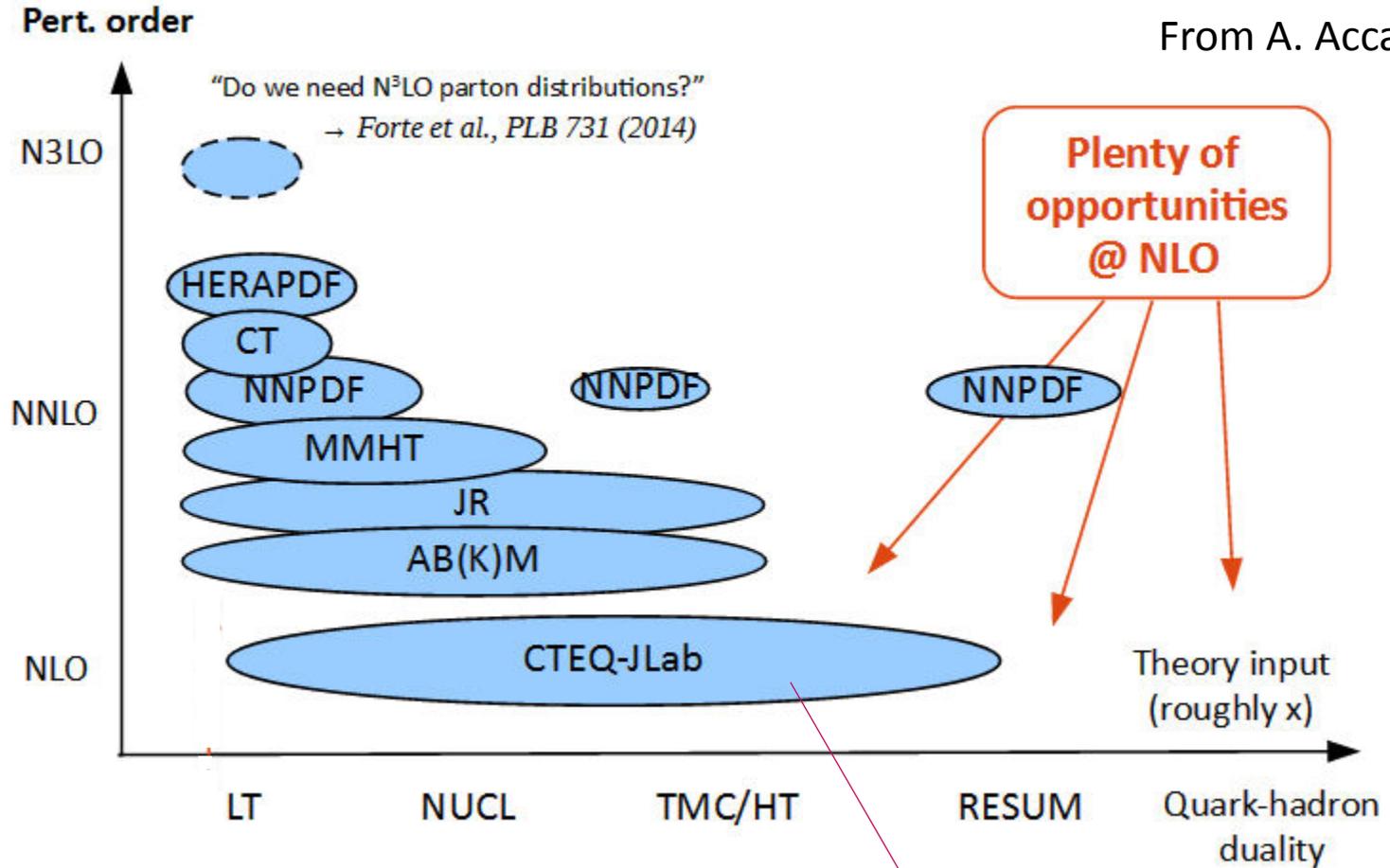
A PDF landscape



Parton Distribution Functions Extraction: Complicated Business

A PDF landscape

From A. Accardi



Will focus on this

Preamble: Parton Distribution Functions & Constraints from Data

- Perturbative QCD gives the Q^2 dependence of PDFs, the x dependence must be extracted from fits to data

Example of data set used for PDFs extraction

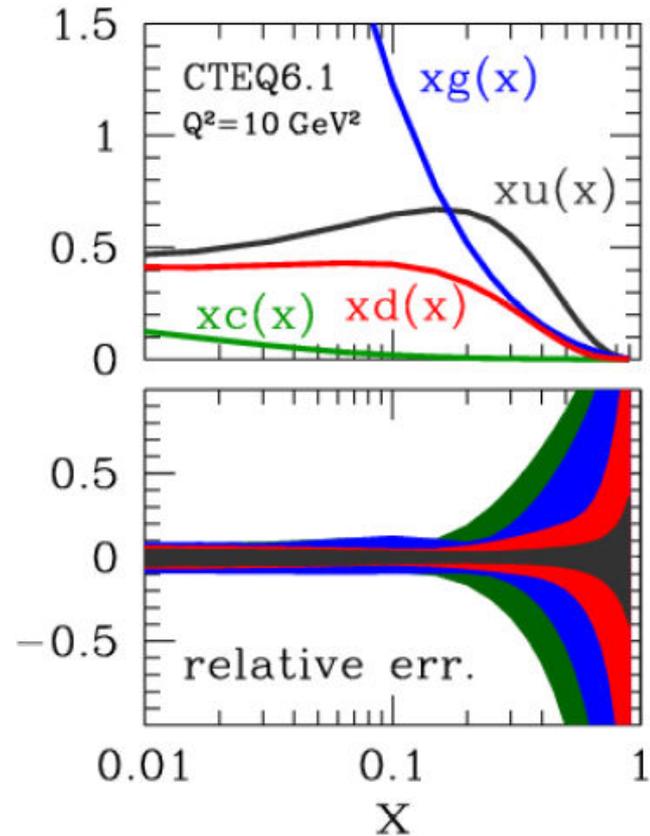
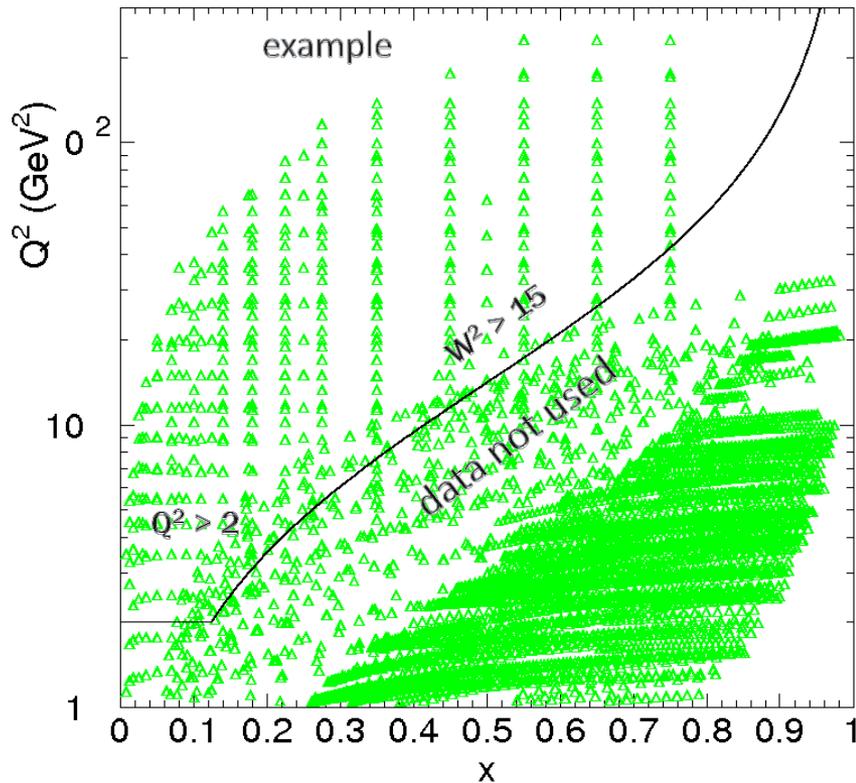
Process	Partons	x range
$\ell^\pm \{p, n\} \rightarrow \ell^\pm X$	q, \bar{q}, g	$x \gtrsim 0.01$
$\ell^\pm n/p \rightarrow \ell^\pm X$	d/u	$x \gtrsim 0.01$
$pp \rightarrow \mu^+ \mu^- X$	fixed target \bar{q} \bar{d}/\bar{u} q, \bar{q} s \bar{s}	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \rightarrow \mu^+ \mu^- X$		$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$		$0.01 \lesssim x \lesssim 0.5$
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$e^\pm p \rightarrow e^\pm c\bar{c} X$	collider c, g g g, q	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \rightarrow \text{jet} + X$		$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \rightarrow \text{jet} + X$		$0.01 \lesssim x \lesssim 0.5$
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$p\bar{p} \rightarrow (Z \rightarrow \ell^+ \ell^-) X$	d	$x \gtrsim 0.05$

Why stop here?

Parton Distribution Functions: Constraints from Data

- Most PDF extractions not well constrained at large x ! Why?

Typical kinematic coverage of data used in PDF fits (early 2000)



Parton Distribution Functions: Constraints from Data

- At least 2 complications with low W and low Q^2 kinematic regime
 - Non-perturbative dynamical higher-twist contributions become large; process dependent, no prescription to treat them in a unified way across various processes – inclusion would spoil PDF universality

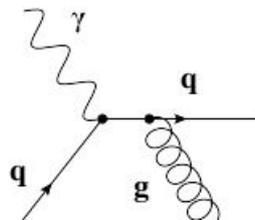
Operator Product Expansion: Expansion of F_2 moments in powers of $1/Q^2$

Twist = dimension - spin

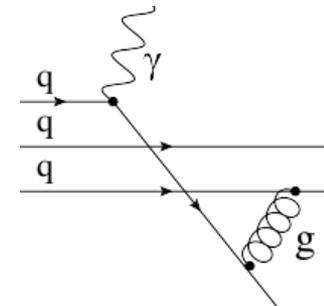
A 's – matrix elements of operators with specific twist

$$\int_0^1 dx F_2(x, Q^2) = A_2(\alpha_s(Q^2)) + \sum_{\tau=4,6,\dots}^{\infty} \frac{A_\tau(\alpha_s(Q^2))}{Q^{\tau-2}}$$

Leading twist: calculable in pQCD



higher twist



Parton Distribution Functions: Constraints from Data

- At least 2 complications with low W and low Q^2 kinematic regime
 - Non-perturbative kinematical higher-twist are large (Target Mass Corrections)

$$x = \frac{Q^2}{2p \cdot q} \quad \text{For massless quarks and targets (or } Q^2 \rightarrow \infty \text{) Bjorken scaling variable is the light-cone momentum fraction of target carried by parton}$$

Finite Q^2 , light-cone momentum fraction given by Nachtmann variable

$$\xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2 / Q^2}}$$

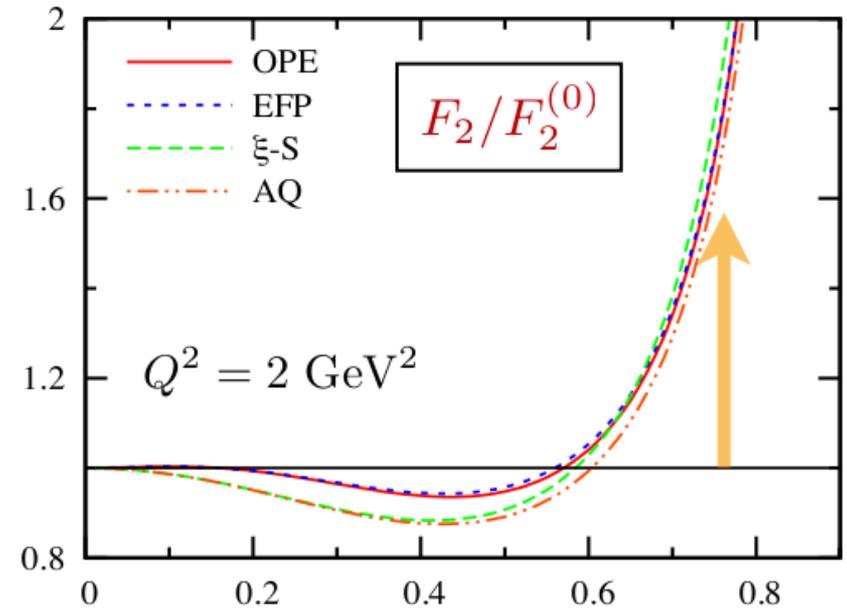
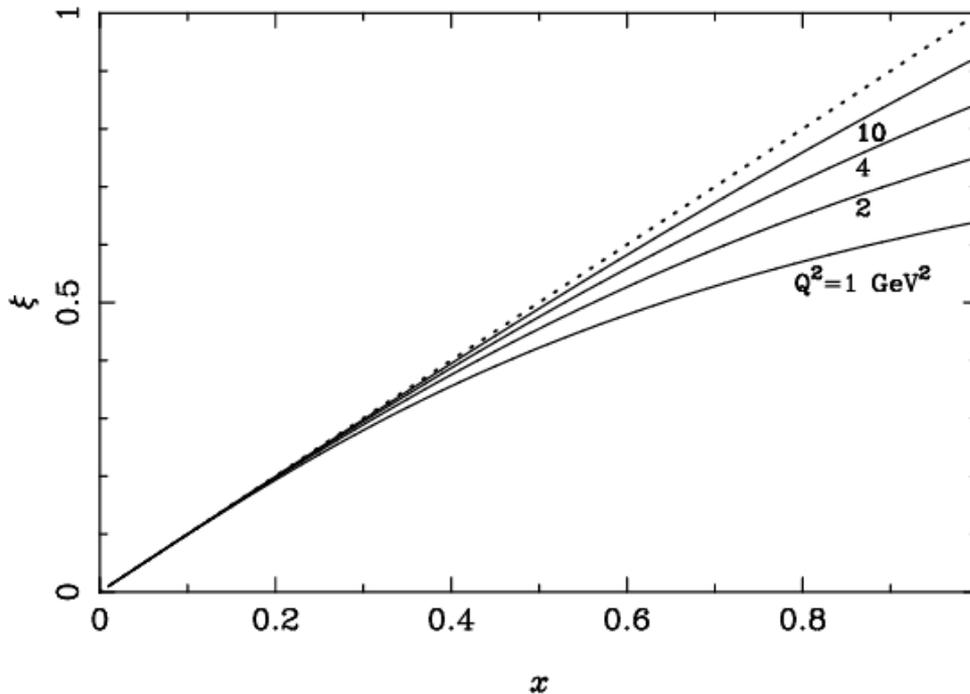
A prescription for target mass corrections can be derived in terms of the Operator Product Expansion and moments of the structure functions
 → Result is “master equation”

$$F_2^{TM}(x, Q^2) = \frac{x^2}{r^3} \frac{F_2^{(0)}(\xi, Q^2)}{\xi^2} + 6 \frac{M^2}{Q^2} \frac{x^3}{r^4} \int_{\xi}^1 dx' \frac{F_2^{(0)}(x', Q^2)}{x'^2} + 12 \frac{M^4}{Q^4} \frac{x^4}{r^5} \int_{\xi}^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'', Q^2)}{x''^2}$$

massless limit structure function calculated from PDFs

Parton Distribution Functions: Constraints from Data

- At least 2 complications with low W and low Q^2 kinematic regime
 - Non-perturbative kinematical higher-twist are large (Target Mass Corrections)



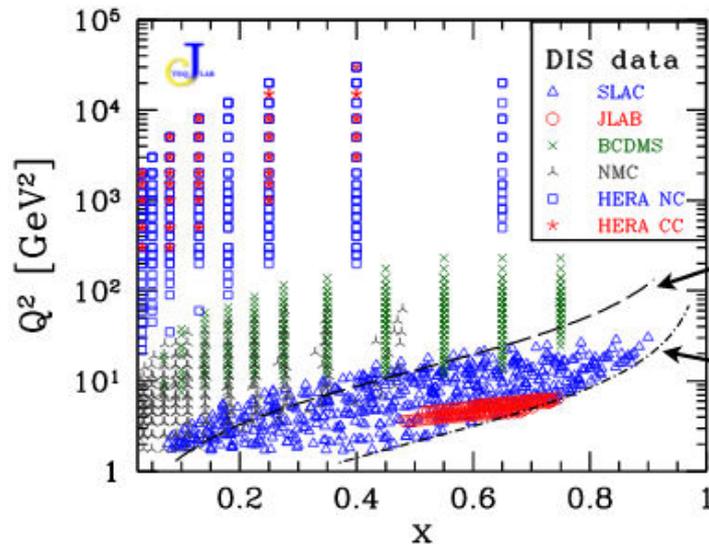
No universally agreed upon prescription to calculate TMCs

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

- Next-to-leading order (NLO) analysis of expanded data set on proton and deuterium

A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016

→ **Improve large- x precision of PDFs** with larger DIS data set on both **proton and deuterium** by relaxing kinematic cuts to push to larger x ; this leads to a factor of 2 increase in number of DIS data points used for fitting



$$W^2 = M^2 + Q^2 \frac{1-x}{x}$$

stringent cut:

$$Q^2 > 4 \text{ GeV}^2, W^2 > 12.25 \text{ GeV}^2$$

relaxed cut:

$$Q^2 > m_c^2 \text{ GeV}^2, W^2 > 3 \text{ GeV}^2$$

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

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A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016
- **Improve large- x precision of PDFs** with larger DIS data set on both **proton and deuterium** by relaxing kinematic cuts to push to larger x ; this leads to a factor of 2 increase in number of DIS data points used for fitting
 - Include all relevant large- x / small- Q^2 theory non-perturbative corrections: **dynamical and kinematic higher-twist (HT)**
 - Include nuclear corrections: use of deuterium data requires careful treatment of **nuclear corrections** -- off-shell effects and sensitivity to the deuteron wave function

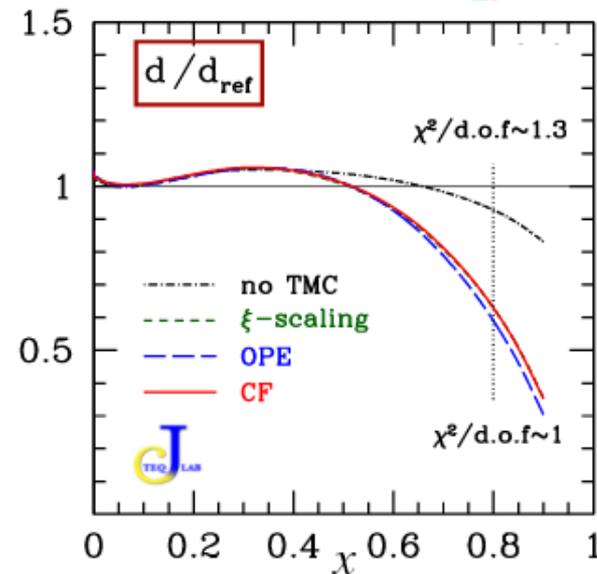
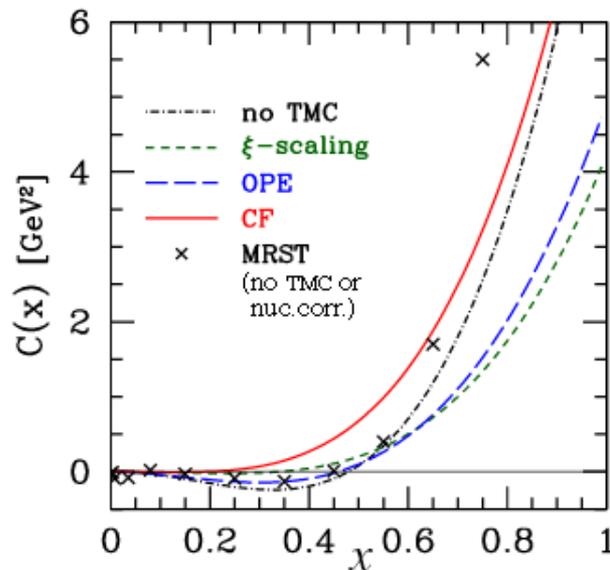
Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

- **Non-perturbative $1/Q^2$ corrections:** dynamical and kinematic higher-twist

$$F_2(x, Q^2) = F_2^{\text{LT}}(x, Q^2) \left(1 + \frac{C(x)}{Q^2} \right)$$

TMC here

$$C(x) = c_1 x^{c_2} (1 + c_3 x) \quad \text{dynamical HT}$$



1) The dynamical HT extraction depends on the TMC prescription used

2) Almost identical results for the d-quark distribution when different prescriptions of TMCs are **used in conjunction with the dynamical HT** → that's great! We don't want the PDF extraction to be affected by our imperfect knowledge of non-perturbative corrections

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

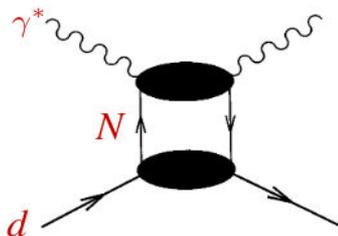
- **Nuclear corrections:** wave function & off-shell dependence

A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016

A. Accardi *et al.*, Phys. Rev. D 84 (2011) 014008

W. Melnitchouk DIS2015

Weak binding approximation:



$$F_2^d(x, Q^2) = \int_x^1 dy f(y, \gamma) F_2^N(x/y, Q^2) + \delta^{(\text{off})} F_2^d$$

\swarrow $N=p+n$
 \nearrow nucleon momentum distribution in d ("smearing function")
 \nearrow off-shell correction

$$f(y, \gamma) = \int \frac{d^3p}{(2\pi)^3} |\psi_d(p)|^2 \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right) \times \frac{1}{\gamma^2} \left[1 + \frac{\gamma^2 - 1}{y^2} \left(1 + \frac{2\varepsilon}{M} + \frac{\vec{p}^2}{2M^2} (1 - 3\hat{p}_z^2) \right) \right]$$

y = deuteron's momentum fraction carried by the struck nucleon

$\gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4M^2 x^2 / Q^2}$ in the nucleus rest frame, virtual photon relative velocity

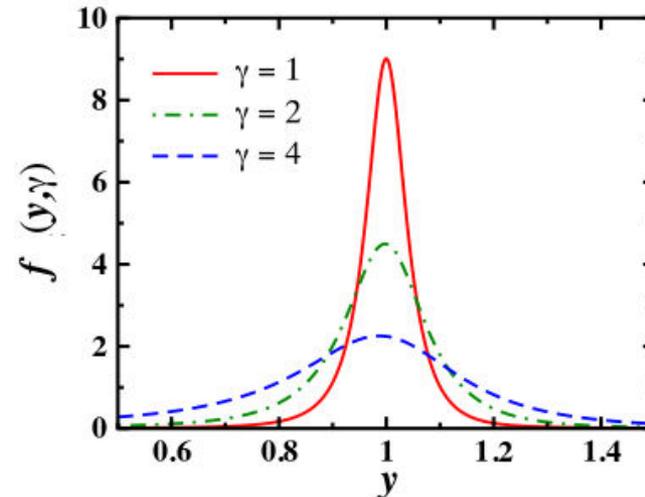
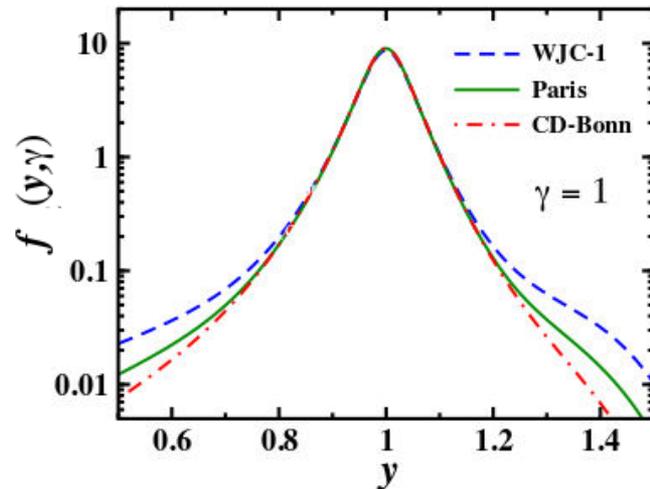
Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

- Nuclear corrections: wave function & off-shell dependence

A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016

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W. Melnitchouk DIS2015



→ greater wave function dependence at large y (and x)

→ more smearing for larger x and lower Q^2

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

- **Nuclear corrections:** wave function & off-shell dependence

A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016

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J. Owens *et al.*, Phys. Rev. D 87 (2013) 094012

W. Melnitchouk DIS2015

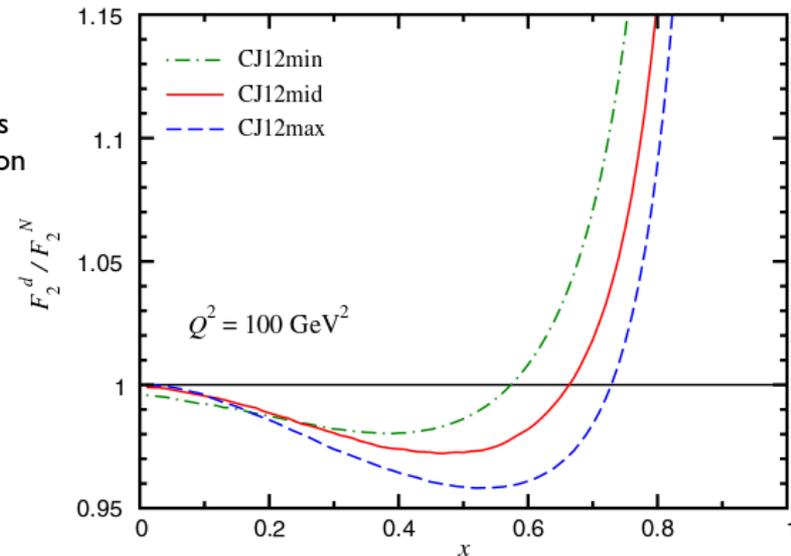
→ Representation of a quark q in an off-shell nucleon with invariant mass p^2 in the off-shell covariant quark “spectator” model

$$\tilde{q}(x, p^2) = \int d\hat{p}^2 \Phi_q(\hat{p}^2, \Lambda(p^2))$$

applied to q_v, \bar{q} & g

momentum distribution of quarks with virtuality \hat{p}^2 in bound nucleon

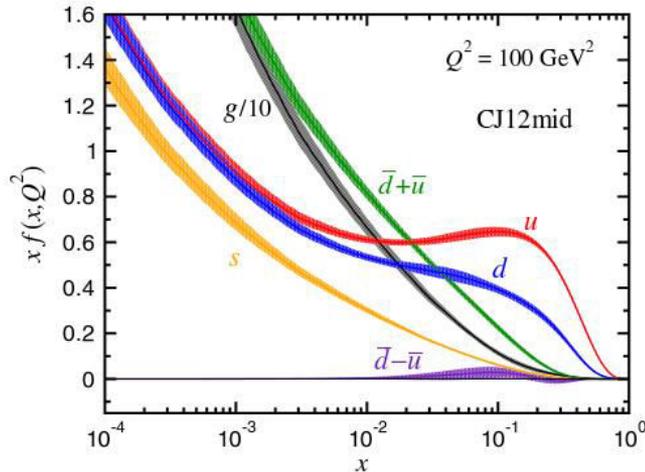
Scale parameter $\Lambda(p^2)$ which is related to the nucleon confinement radius (*swelling of nucleon in nuclear medium*) suppresses large- p^2 contributions



Off-shell rescaling parameter $\lambda = \partial \log \Lambda^2 / \partial \log p^2 \big|_{p^2=M^2}$ varied in fit to minimize χ^2

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

J. Owens *et al.*, Phys. Rev. D 87 (2013) 094012

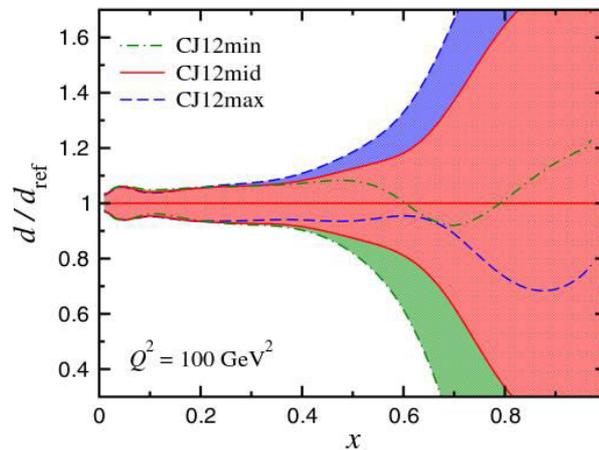
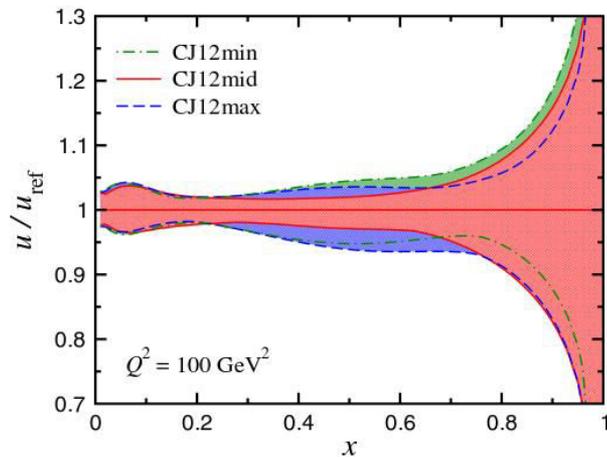


Different combinations of wave functions and size of nuclear corrections

CJ12min: WJC-1 + small off-shell ($\lambda = 0.3\%$)

CJ12mid: AV18 + medium off-shell ($\lambda = 1.2\%$)

CJ12max: CD-Bonn + large off-shell ($\lambda = 2.1\%$)

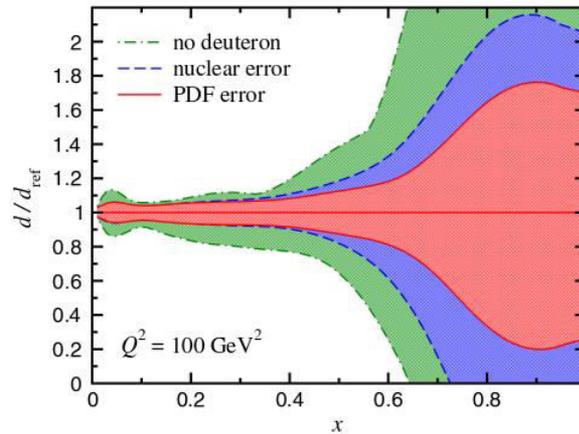
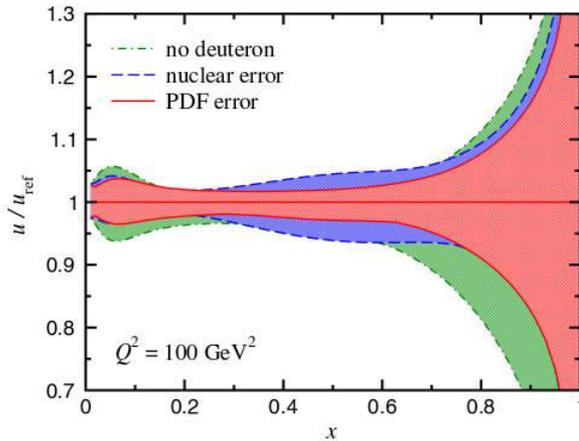


PDF uncertainties relative to the reference CJ12mid

→ **Is it worth using deuterium data** and dealing with nuclear corrections?

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

J. Owens *et al.*, Phys. Rev. D 87 (2013) 094012

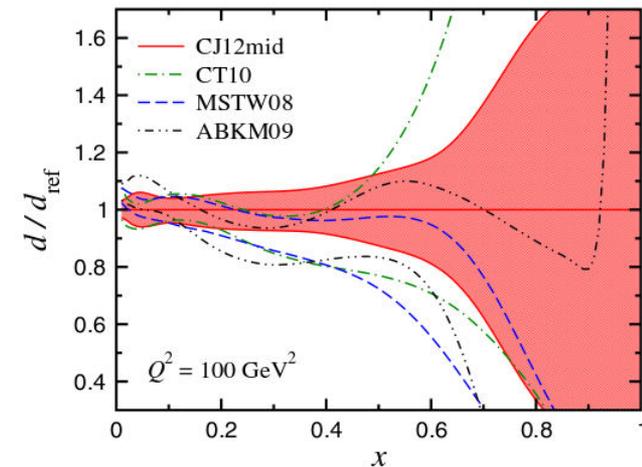
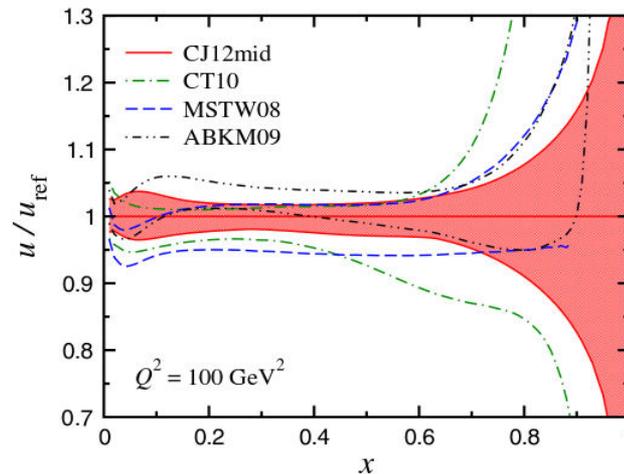


Global fits without deuterium data show:

- modest increase in the error band for **u** for $x > 0.7$
- significant increase in error band for **d**

→ smaller error band for **d** if deuteron hence nuclear corrections included

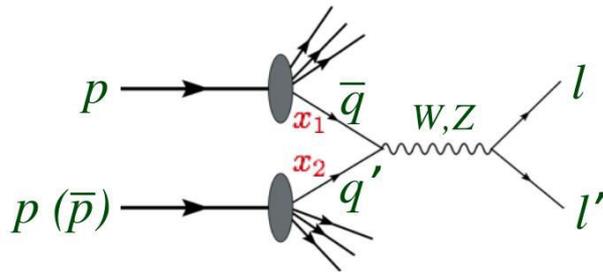
Inclusion of more data at large x leads to better constrained PDFs



Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

➤ Nuclear and high energy connection

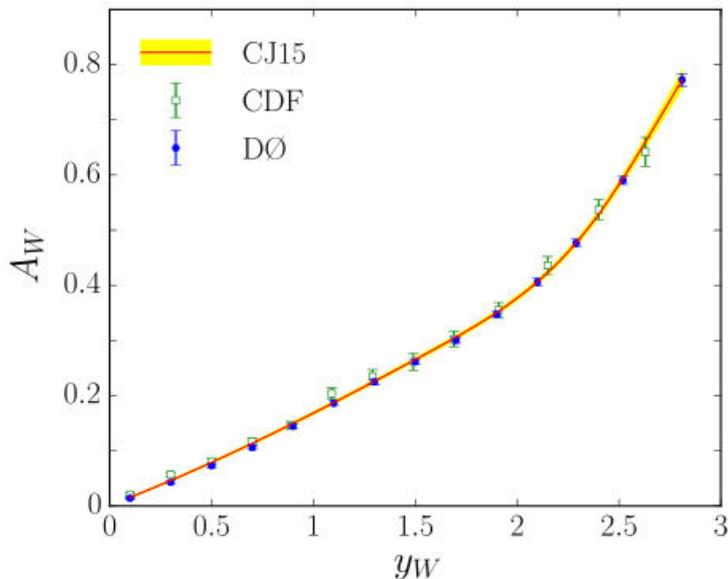
→ **W[±] asymmetries** at large W-boson rapidity are sensitive to **d/u** PDF ratio at large x



Example: W and decay lepton charge asymmetry at large rapidity

$$A_W(y) = \frac{\sigma(W^+) - \sigma(W^-)}{\sigma(W^+) + \sigma(W^-)} \approx \frac{d/u(x_2) - d/u(x_1)}{d/u(x_2) + d/u(x_1)} \quad [x_1 \gg x_2]$$

$$A_l(y) = A_W \otimes B_{W \rightarrow l}(y)$$



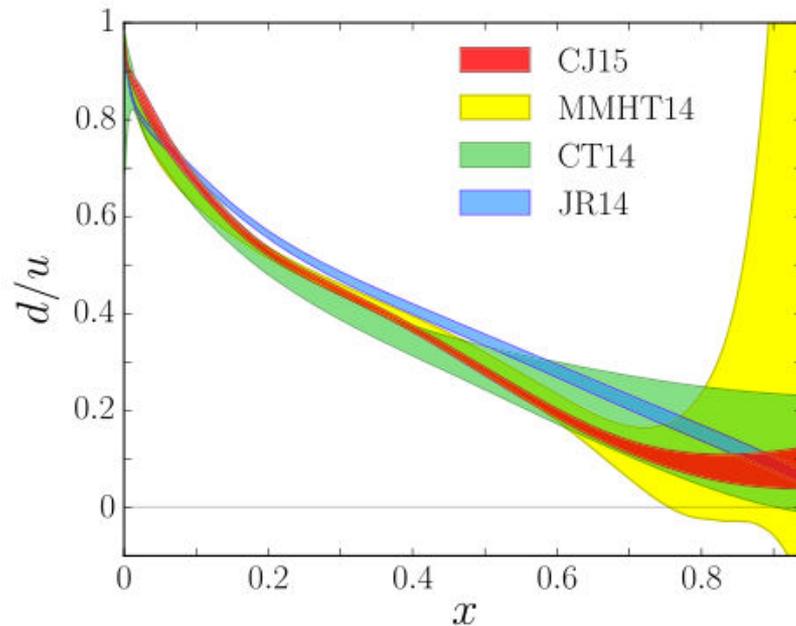
- Earlier CDF and more recent D0 W-asymmetry data “select” small but non-zero nuclear corrections

In CJ15 **D0 W**, lepton asymmetries constrain the **d-quark in a free nucleon** so that the **deuteron data** can be used to **constrain nuclear corrections**

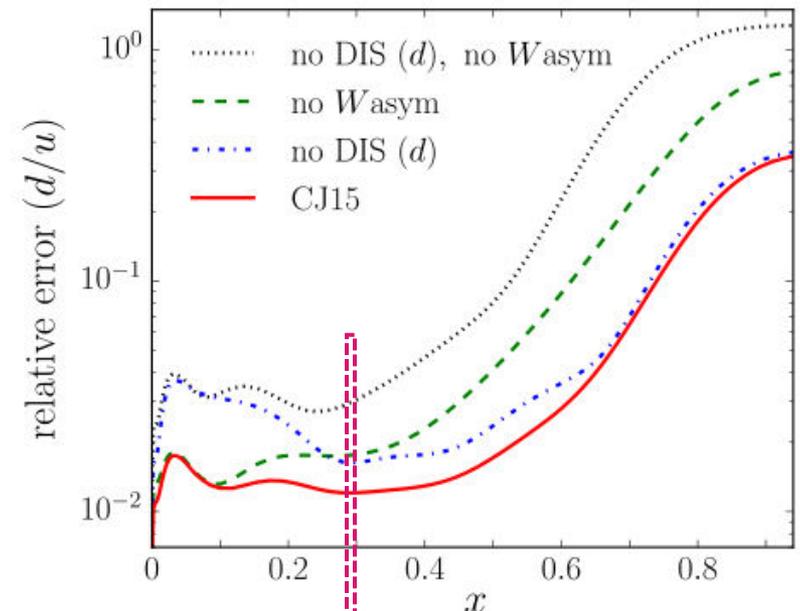
Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

➤ Nuclear and high energy connection

→ W^+ asymmetries at large W -boson rapidity are sensitive to d/u PDF ratio at large x



Marked improvement in the d/u uncertainty



Deuterium data allow for precise determination of d/u

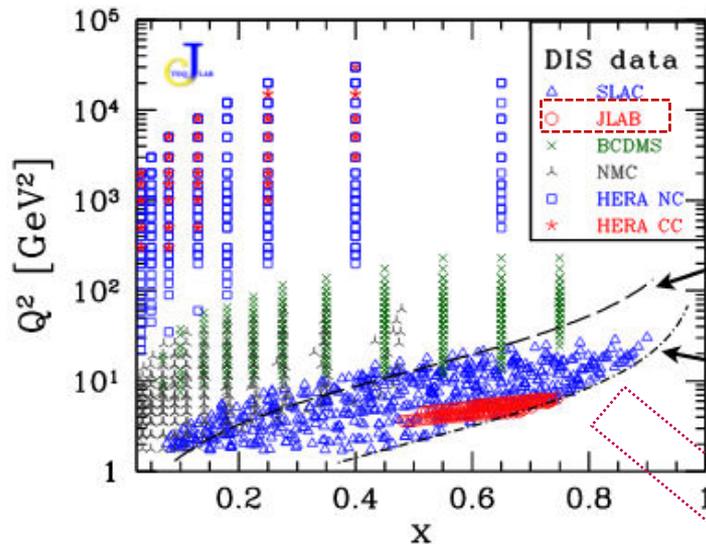
D_0 asymmetries determine the "free nucleon" d -quark AND Deuterium data determine the off-shell correction

Parton Distribution Functions from CTEQ-JLab: Beyond the Perturbative Regime

- Next-to-leading order (NLO) analysis of expanded data set on proton and deuterium

A. Accardi *et al.*, Phys. Rev. D 81 (2010) 034016

→ **Improve large- x precision** with larger DIS data set on both **proton and deuterium**: relaxing kinematic cuts to push to larger x leads to a factor of 2 increase in number of DIS data points used for fitting



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stringent cut:
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Can we push below $W^2 > 3 \text{ GeV}^2$?

Quark-Hadron Duality

➤ What is **Quark-hadron duality**?

Quark-hadron duality = complementarity between **quark** and **hadron** descriptions of observables

$$\sum_{\text{hadrons}} = \sum_{\text{quarks}}$$

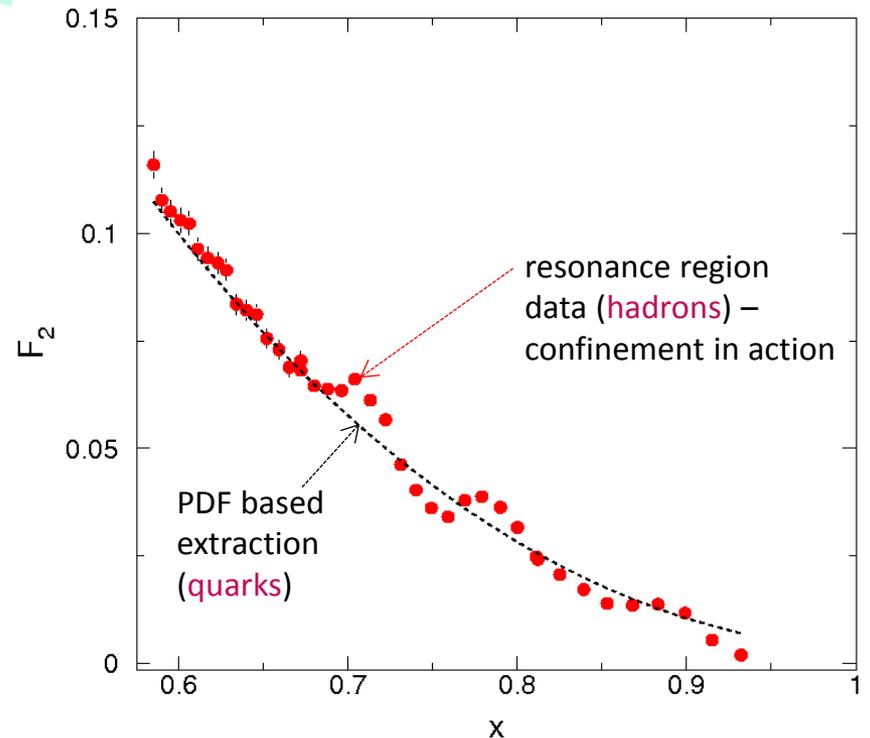
↙
Description suitable for
low-energy regime
(**confinement**)

↘
Description suitable for
high-energy regime
(**asymptotic freedom**)

→ We can use either set of **complete** basis states to describe physical phenomena

→ **In practice**, at finite energy we typically have access only to a **limited set of basis states**

→ **Even so**, **quark-hadron duality shown to hold** globally and locally in **many observables**

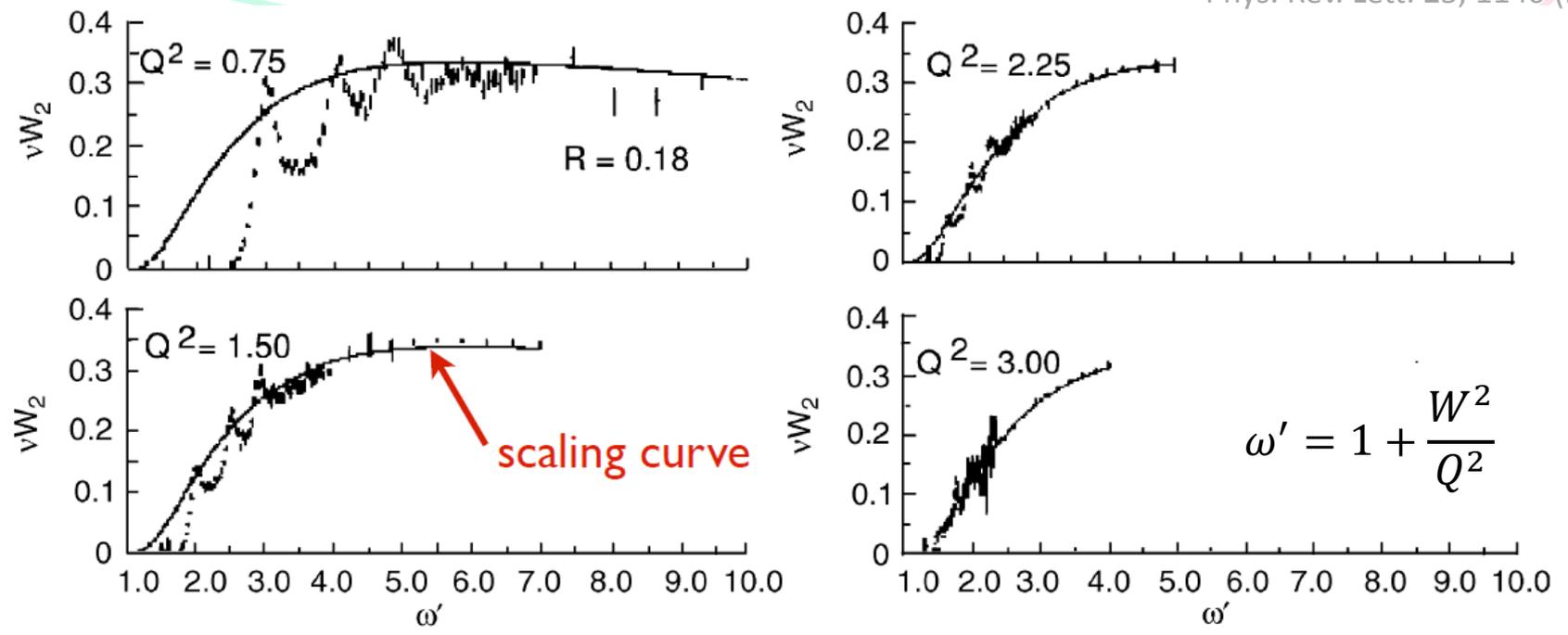


*Resonance region data average to PDF based curve:
 $1/Q^{2n}$ corrections small or cancel **on average***

Bloom-Gilman Duality

➤ Duality in inclusive **electron-proton scattering**: Bloom-Gilman duality

Phys. Rev. Lett. 25, 1140 (1970)



→ ω' allows comparison of high- W^2 , high- Q^2 curves (fit to DIS data) to low- W^2 , low- Q^2 resonance region data

The resonance region data:

- oscillate around and are on average equivalent to the scaling curve
- “slide” along the deep inelastic curve with increasing Q^2

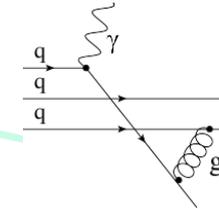
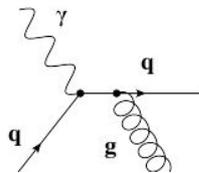
The Q^2 dependence of the proton’s resonances (hadrons) is strongly correlated with the dynamics of the proton in the DIS region where quark and gluon degrees of freedom take over

Quark-Hadron Duality in QCD

➤ Duality in QCD via the Operator Product Expansion

Twist (= dimension - spin) expansion of F_2 moments in QCD

Perturbative leading twist results in shallow Q^2 dependence of integral



Non-perturbative higher twist would induce a strong Q^2 dependence to the integral

$$\int_0^1 dx x^{n-2} F_2(x, Q^2) = A_2^{(n)}(\alpha_s(Q^2)) + \sum_{\tau=4,6,\dots}^{\infty} \frac{A_{\tau}^{(n)}(\alpha_s(Q^2))}{Q^{\tau-2}}$$

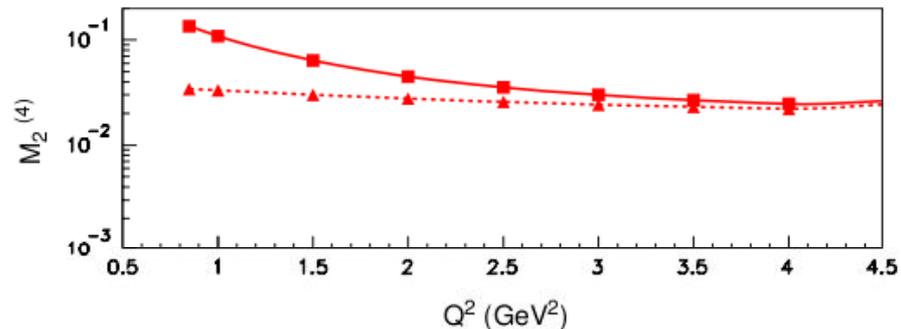
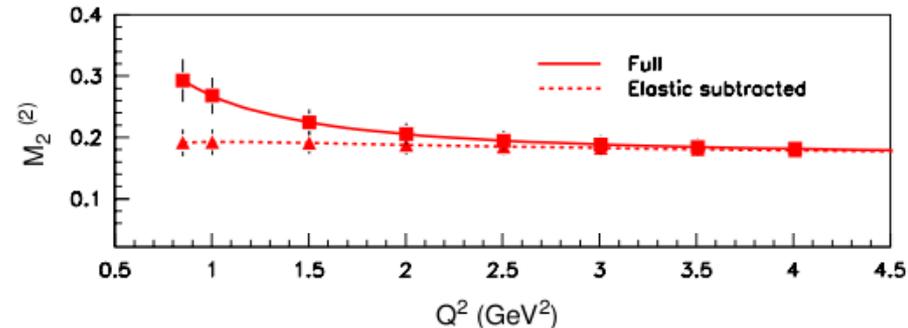
→ The total integral - moment of the structure function - exhibits a shallow Q^2 dependence down to a Q^2 value of $\sim 1 \text{ GeV}^2$!



Quark-hadron duality = higher twist (leading to confinement) are small or cancel on average

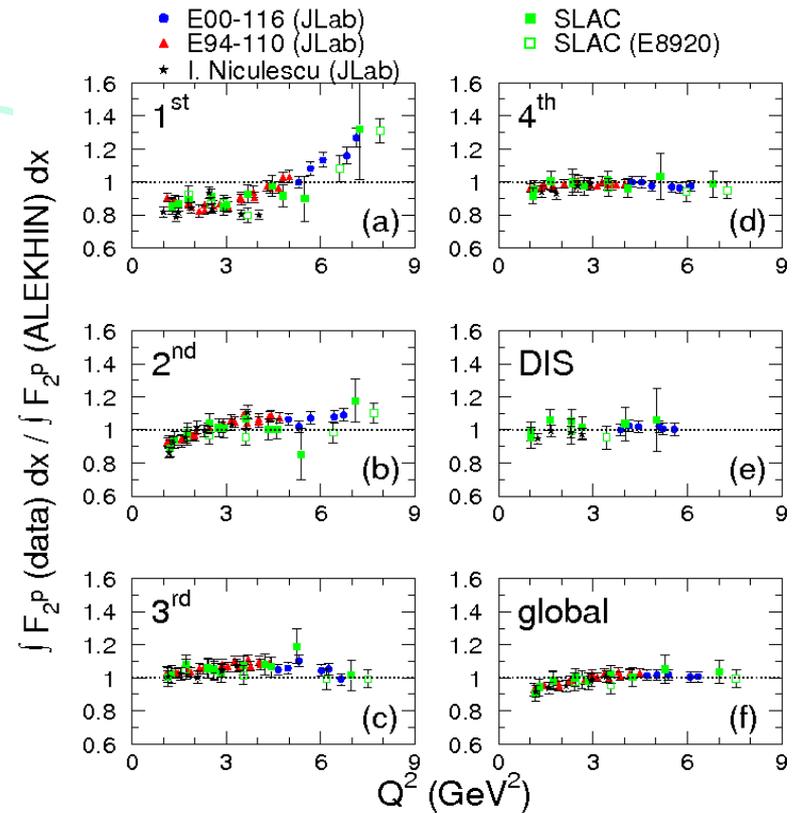
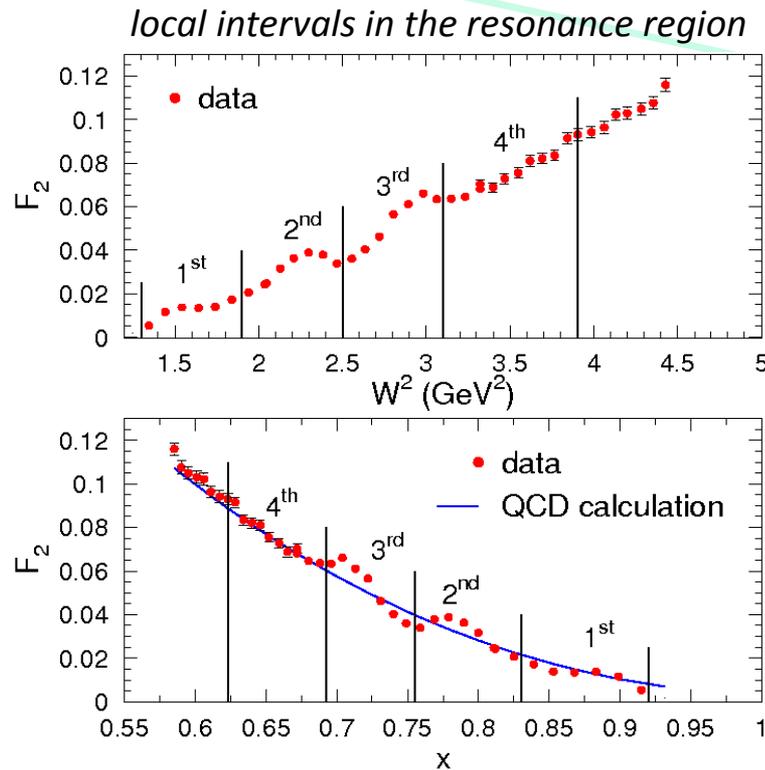
The more remarkable as:

- At fixed Q^2 resonances occupy the largest x
- At lower Q^2 resonances occupy a larger x region of the total x interval than at higher Q^2
- For higher moments ($n=4, \dots$) the contribution from larger x (resonances) is even more enhanced



Quark-Hadron Duality: How Well It Works?

- Global study of **global** and **local Quark-hadron duality** in F_2 structure function: averaged resonance region data vs PDF fits



→ We define local W^2 intervals in the resonance region to verify **local quark-hadron duality**

→ We calculate integral to verify quantitatively how well duality holds when compared to PDF fits constrained at large x

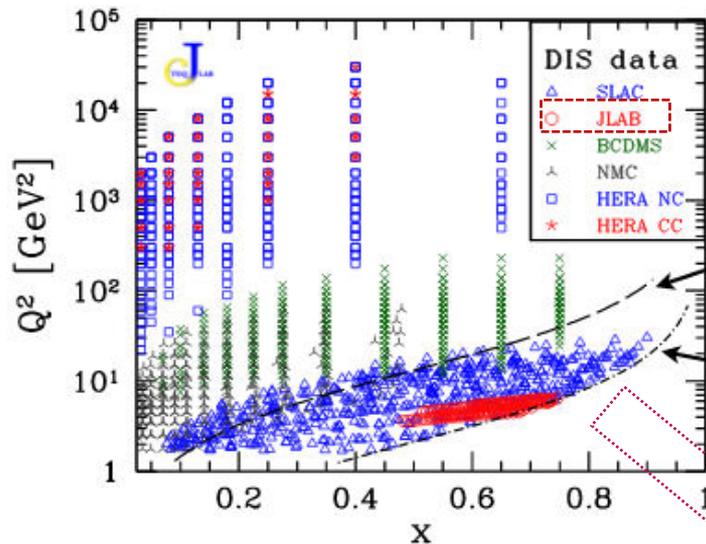
$$\int_{x_{min}}^{x_{max}} F^{data}(x, Q^2) dx / \int_{x_{min}}^{x_{max}} F^{param.}(x, Q^2) dx$$

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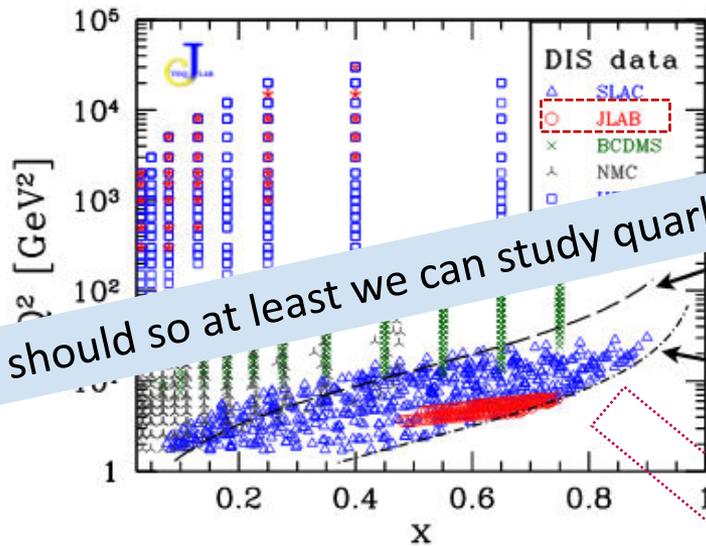
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We should so at least we can study quark-hadron duality in the framework of PDF fits!

Can we push below $W^2 > 3 \text{ GeV}^2$?

Don't Forget: Nucleon Structure Is Even More Complicated

- Partons inside nucleons can have specific positions and momenta w.r.t. a defined center of the nucleon; GTMD contain the most general one-body information of partons

Symmetric Infinite Momentum Frame:

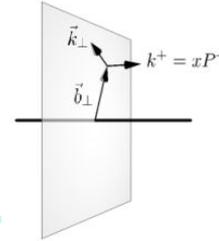
$$P = [P^+, P^-, \vec{0}_\perp],$$

$$k = [xP^+, k^-, \vec{k}_\perp],$$

$$\Delta = [-2\xi P^+, 2\xi P^-, \vec{\Delta}_\perp]$$

GTMD(x, k_\perp, Δ)

$\xi = 0$



theoretical objects

C. Lorce

$\Delta=0$

$$\int d^2 k_\perp$$

Transverse Momentum
Dependent Distributions

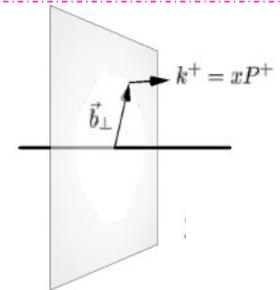
TMDs

$$f(x, k_\perp)$$

Generalized Parton
Distributions

GPDs

$$f(x, \Delta)$$



physical objects

$$\int d^2 k_\perp$$

$$\int dx$$

Parton Distribution Functions

PDFs

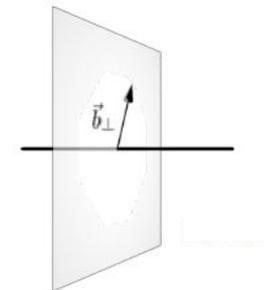
$$f(x)$$

I only touched on
this!

Form Factors

FFs

$$F(\Delta)$$



→ Knowledge of distribution functions implies knowledge of nucleon dynamics based on the unique features of QCD: asymptotic freedom and confinement, **factorization**, and **universality**

Perspective: There Is Still So Much We Don't Know

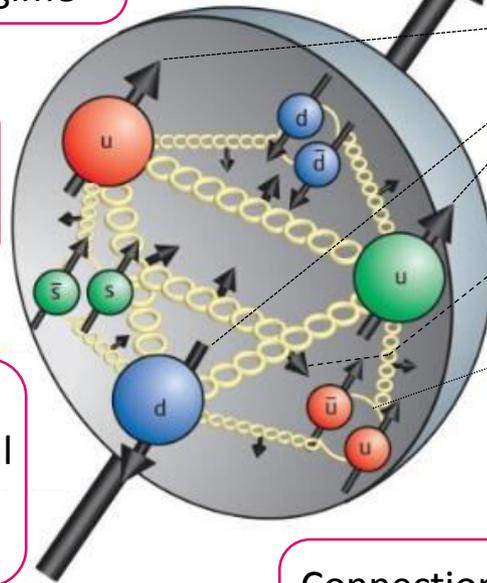
➤ **Nucleon structure** has been and is a very active field of research of **fundamental importance**

Structure and dynamics of the nucleon in its state of ultimate confinement – elastic regime

How do quarks and gluons distribute according to their longitudinal momentum?

How do partons distribute according to their longitudinal momentum AND their transverse localization?

How is the spin of the nucleon shared among its constituents?



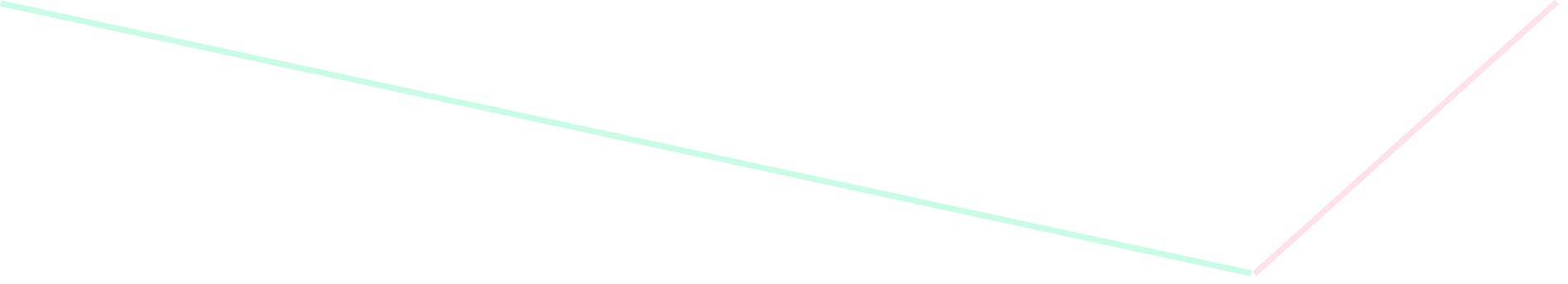
A **nucleon (e.g. proton)** with spin $1/2$ is made up of...

... three **valence quarks** which themselves have spin $1/2$ but also of...

... **gluons** with spin 1 that mediate the strong interaction and...

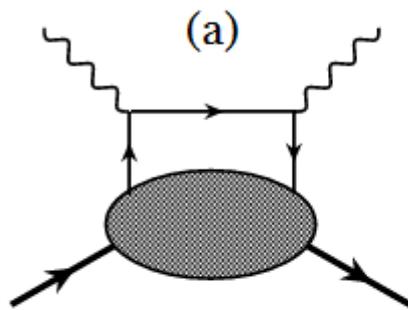
... a **sea** of quark-antiquark pairs and more gluons.

Connection between intrinsic motion of partons and their spin and the spin of the parent nucleon



THE END

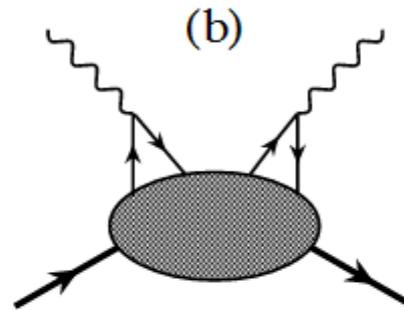
Higher twists



$$\tau = 2$$

single quark
scattering

e.g. $\bar{\psi} \gamma_\mu \psi$



$$\tau > 2$$

qq and qg
correlations

e.g. $\bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma_\nu \psi$
or $\bar{\psi} \tilde{G}_{\mu\nu} \gamma^\nu \psi$