

CTEQ School on
QCD Analysis and Electroweak Phenomenology

LECTURE 1

Introduction to the Parton Model and Perturbative QCD
Fred Olness (SMU)

University of Pittsburgh, PA
18-28 July 2017

Introduction:

Welcome to QCD:

3

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \\ &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \end{aligned}$$

Mozart: Inverted retrograde canon in G

Patterns, Symmetry (obvious & hidden), interpretation

Notes \Rightarrow themes \Rightarrow Melody/Harmony \Rightarrow interaction/counterpoint \Rightarrow structure

3

Welcome to QCD:

4

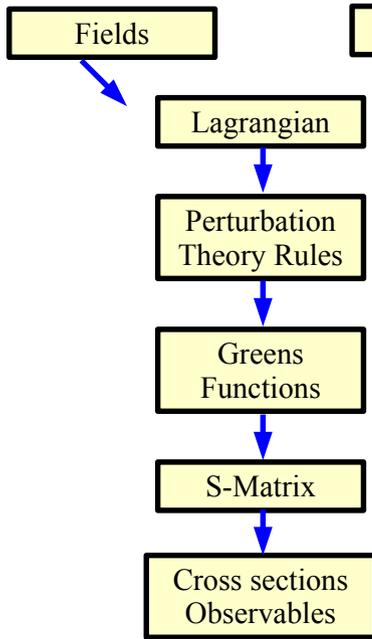
$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \\ &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \end{aligned}$$

Mozart: Inverted retrograde canon in G

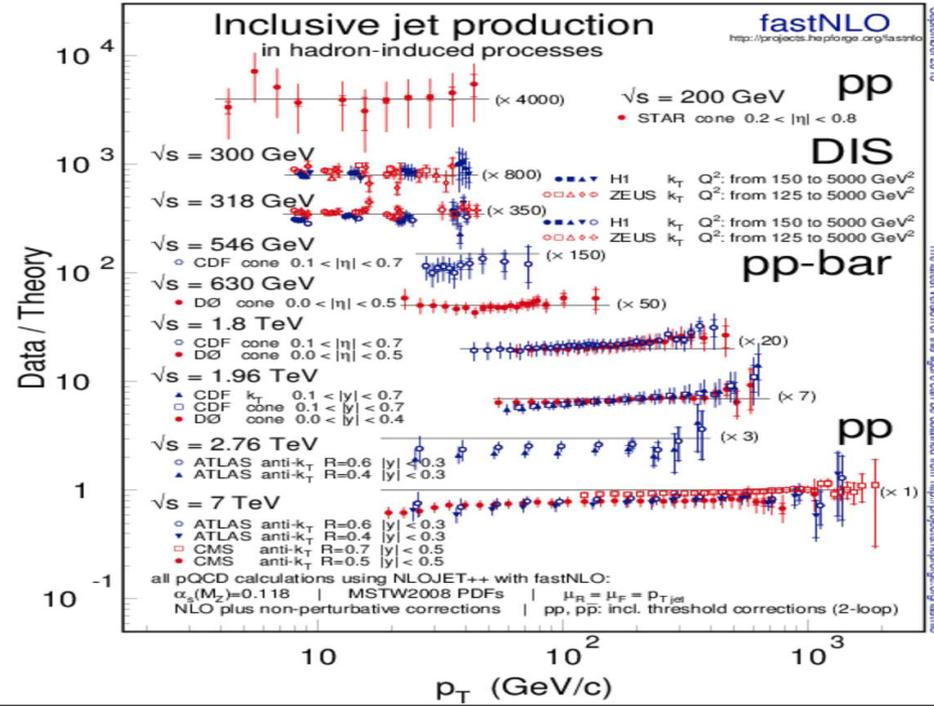
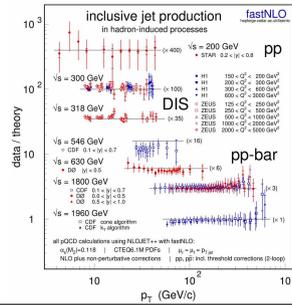
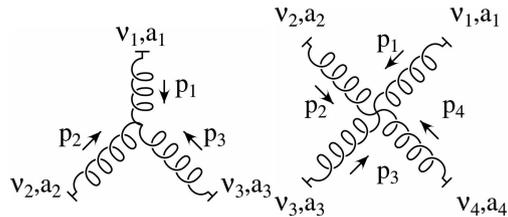
Patterns, Symmetry (obvious & hidden), interpretation

Notes \Rightarrow themes \Rightarrow Melody/Harmony \Rightarrow interaction/counterpoint \Rightarrow structure

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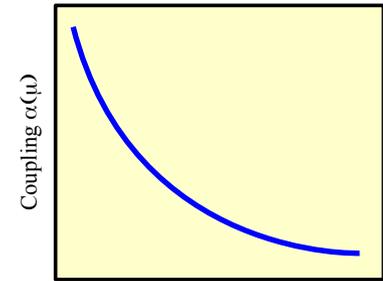
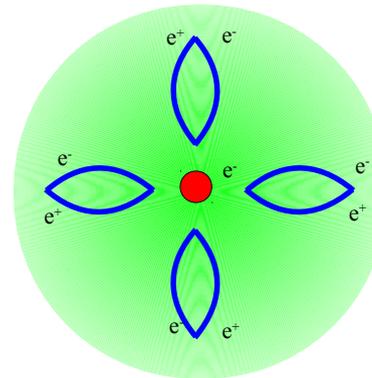
$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$



September 2013
The latest version of this figure can be obtained from <http://projects.hepforge.org/fastnlo>

QCD is just like QED,
.... *only different*

QED: Abelian U(1) Symmetry



Perturbation theory at large distance is convergent

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - 0$$

Abelian

$$\alpha(\infty) \sim \frac{1}{137}$$

$$\alpha(M_Z) \sim \frac{1}{128}$$

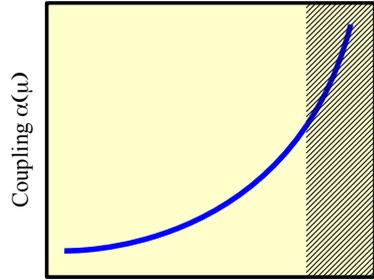
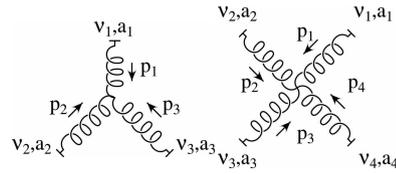
α is good expansion parameter

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

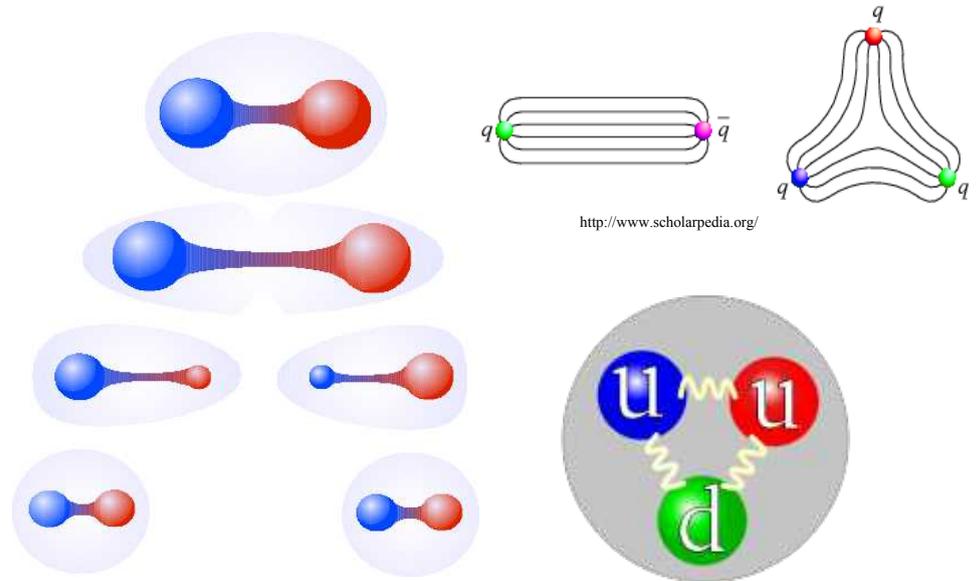
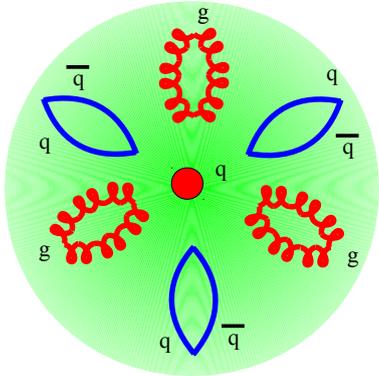
Non-Abelian

$$[T_a, T_b] = i \sum_{c=1}^8 f_{abc} T_c$$



Cannot perform Perturbation theory at large distance

$\alpha_s(M_Z) \sim 0.118$



<http://www.scholarpedia.org/>

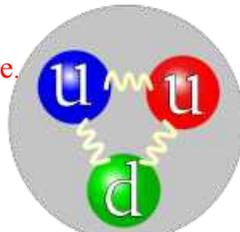
Thomas Lippert, NIC-ZAM, Jülich, for the SESAM Collaboration

<http://en.wikipedia.org/>

Quarks are confined

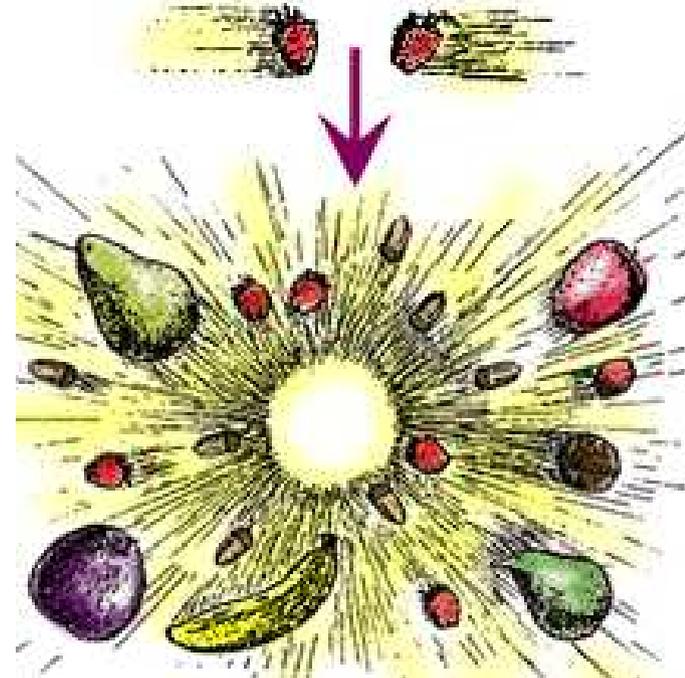
Statement of the problem

- Theorist #1: The universe is completely described by the symmetry group SO(10)
- Theorist #2: You're wrong; the correct answer is SuperSymmetric flipped SU(5)xU(1)
- Theorist #3: You've flipped! The only rational choice is E8xE8 dictated by SuperString Theology.
- Experimentalist: Enough of this speculative nonsense. I'm going to measure something to settle this question. What can you predict???
- Theorist #1: We can predict the interactions between fundamental particles such as quarks and leptons.
- Experimentalist: Great! Give me a beam of quarks and leptons, and I can settle this debate.
- Accelerator Operator: Sorry, quarks only come in a 3-pack and we can't break a set!

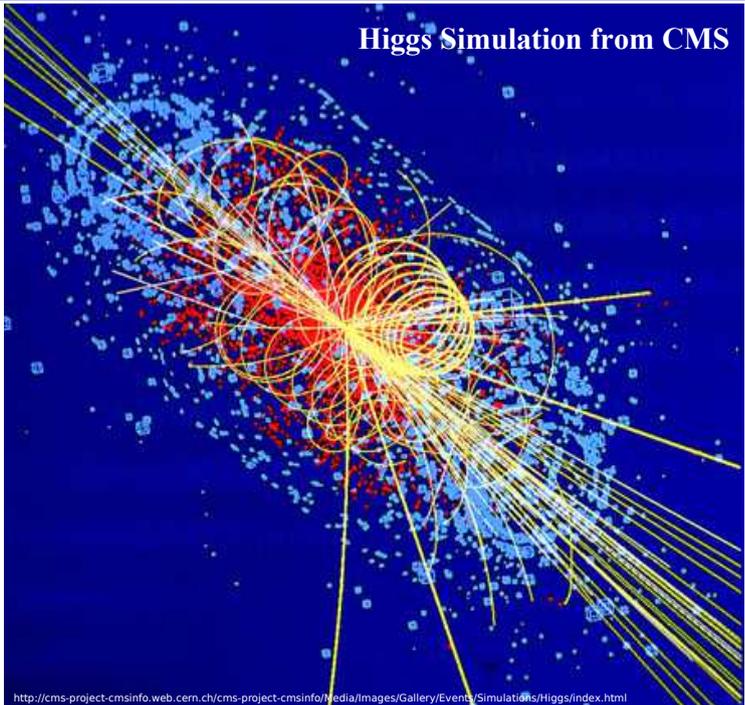


<http://en.wikipedia.org/>

One interpretation of a hadron-hadron collision



Did we find the Higgs?



OUTLINE



Working in the limit of a spherical horse ...

We are going to look at the essence of what makes QCD so different from the other forces.

As a consequence, we will need to be creative in how we study the properties, now we define observables, and interpret the results.

QCD has a history of more than 40 years, and we are still trying to fully understand its structure.

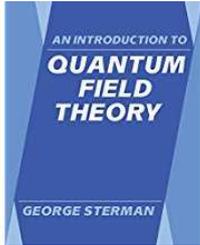
The goal of these lectures

Provide pictorial/graphical/heuristic explanations for everything that confused me as a student

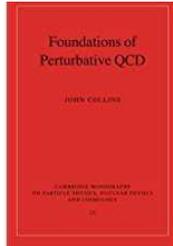


BEFORE

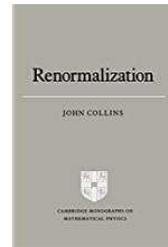
AFTER



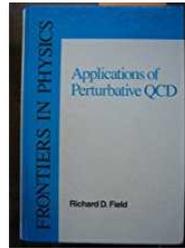
An Introduction to
Quantum Field Theory
George Serman



Foundations of
Perturbative QCD
John C. Collins



Renormalization:
John C. Collins



Applications of
Perturbative QCD,
Richard D. Field

The CTEQ Pedagogical Page

Linked from cteq.org

Everything you wanted to know about Lambda-QCD but were afraid to ask
Randall J. Scalise and Fredrick I. Olness

Regularization, Renormalization, and Dimensional Analysis:
Dimensional Regularization meets Freshman E&M

Fredrick Olness & Randall Scalise
e-Print: arXiv:0812.3578

Calculational Techniques in Perturbative QCD: The Drell-Yan Process.

Björn Pötter has prepared a writeup of the lecture given by Jack Smith.
This is a wonderful reference for those learning to do real 1-loop calculations.

Thanks to ...

Thanks to:

Dave Soper, George Serman,
John Collins, & Jeff Owens for ideas
borrowed from previous CTEQ
introductory lecturers

Thanks to Randy Scalise for the help on
the Dimensional Regularization.

Thanks to my friends at Grenoble who
helped with suggestions and corrections.

Thanks to Jeff Owens for help on
Drell-Yan and Resummation.

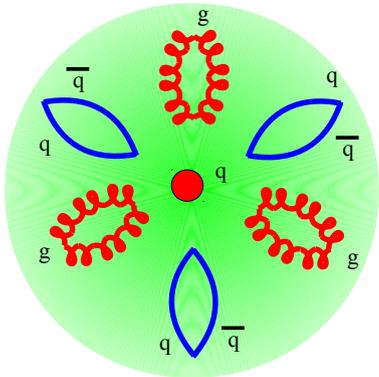
To the CTEQ and MCnet folks
for making all this possible.

The Strong Coupling, Scaling, and Stuff

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$

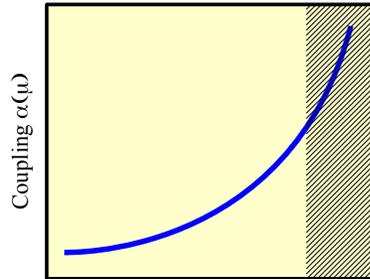
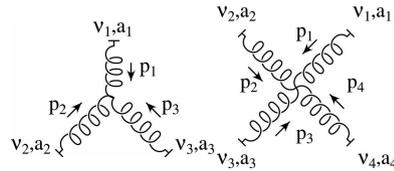
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$$

Non-Abelian



$\Lambda \sim 200 \text{ MeV} \sim 1 \text{ fm}$

$$[T_a, T_b] = i \sum_{c=1}^8 f_{abc} T_c$$



Cannot perform Perturbation theory at large distance

Consider a physical observable: $R(Q^2/\mu^2, \alpha_s)$

Q is the characteristic energy scale of the problem
 μ is an artificial scale we introduce to regulate the calculation (more later)

The Renormalization Group Equation (RGE) is:

$$\frac{dR}{d\mu^2} = 0$$

$$\left\{ \mu^2 \frac{d}{d\mu^2} \right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

Using the chain rule:

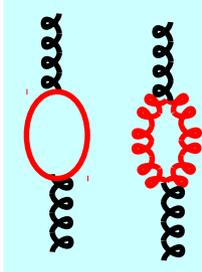
$$\left\{ \mu^2 \frac{\partial}{\partial \mu^2} + \underbrace{\left[\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} \right]}_{\beta(\alpha_s(\mu))} \frac{\partial}{\partial \alpha_s(\mu^2)} \right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$

$$\beta(\alpha_s(\mu))$$

beta function tell us how alpha_s changes with energy scale!!!

beta function tell us how α_s changes with energy scale!!!

$$\beta(\alpha_s(\mu)) = \mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = \frac{\partial \alpha_s(\mu^2)}{\partial \ln \mu^2}$$



We can calculate this perturbatively

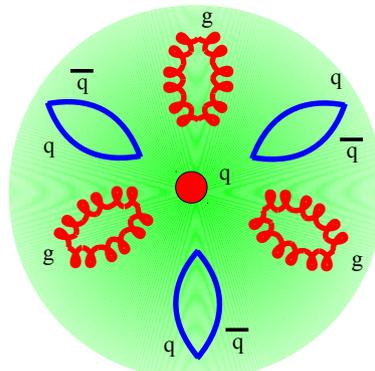
$$\beta(\alpha_s(\mu)) = - \left\{ b_0 \alpha_s^2 + b_1 \alpha_s^3 + b_2 \alpha_s^4 + \dots \right\}$$

$$b_0 = \frac{33 - 2 N_F}{12\pi}$$

$$\beta = -\alpha_s^2 \left[\frac{33 - 2 N_F}{12\pi} \right] + \dots$$

Note: b_0 and b_1 are scheme independent.

beta is negative; let's find the implications



Let: $t = \ln \mu^2$

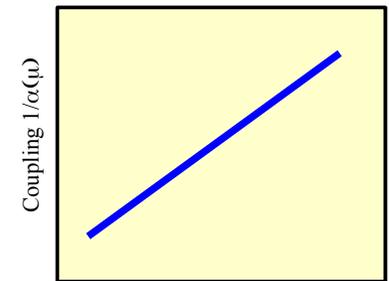
$$\beta = \frac{\partial \alpha_s}{\partial t} \simeq -b_0 \alpha_s^2 + \dots$$

$$\frac{\partial \alpha_s}{\alpha_s^2} = -b_0 \partial t$$

$$\frac{1}{\alpha_s} = b_0 t$$

$$b_0 = \frac{33 - 2 N_F}{12\pi}$$

$$\beta = -\alpha_s^2 \left[\frac{33 - 2 N_F}{12\pi} \right] + \dots$$



Observe $\beta_{\text{QCD}} < 0$ for $N_F < 17$ or for 8 generations or less.

Thus, in general $\beta_{\text{QCD}} < 0$ in the QCD theory

Contrast with QED: $\beta_{\text{QED}} > 0 = +\alpha^2/3\pi + \dots$

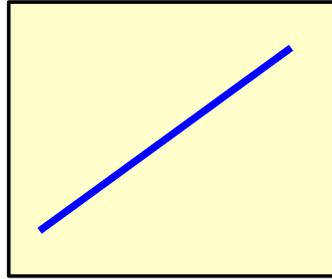
The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom"

Let: $t = \ln \mu^2$

$$\left. \frac{1}{\alpha_S} \right]_{\mu_0}^{\mu_1} = b_0 t]_{\mu_0}^{\mu_1}$$

$$\frac{1}{\alpha_S(\mu_1)} - \frac{1}{\alpha_S(\mu_0)} = b_0 \ln(\mu_1/\mu_0)$$

Coupling $1/\alpha(\mu)$



Energy Scale t

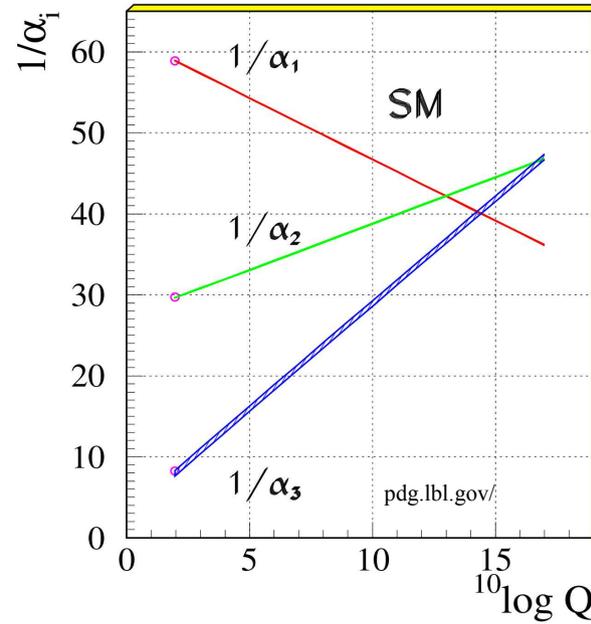
β functions gives us running, but we still need a reference

$$\alpha_s(\mu) = \frac{1}{b_0 \ln(\mu/\Lambda_{QCD})}$$

$$\Lambda = \mu e^{-1/(b_0 \alpha_s(\mu))}$$

Landau Pole
 $\Lambda = \mu$

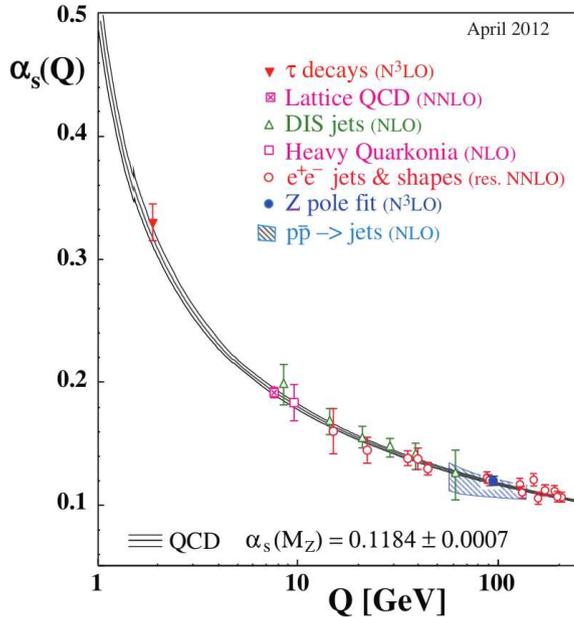
$$\Lambda_{QCD} \sim 200 \text{ MeV} \sim 1 \text{ fm}$$



$$b_1 = 0 + (2/3)N_F + \frac{1}{10}N_{Higgs}$$

$$b_2 = -22/3 + (2/3)N_F + \frac{1}{6}N_{Higgs}$$

$$b_3 = -11 + (2/3)N_F + 0$$



$$\alpha_s(M_Z) = 0.118$$

Low Q points have more discriminating power

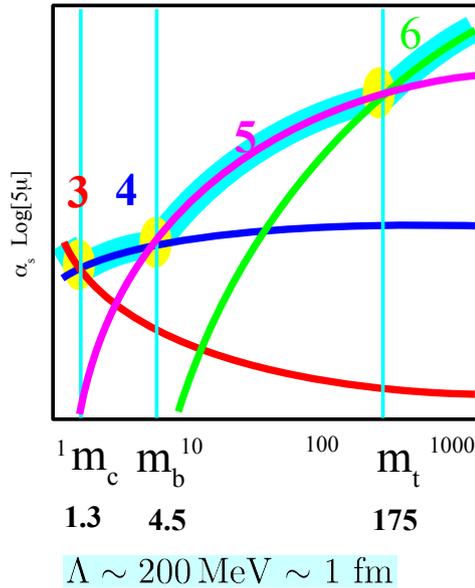
Caution: α_s is NOT a physical observable

BEYOND NLO

N_F Matters

$$\beta = -\alpha_S^2 \left[\frac{33 - 2N_F}{12\pi} \right] + \dots$$

At 1-loop and 2-loops,
continuous at thresholds



$$\alpha_S^{N_F+1}(\mu^2) = \alpha_S^{N_F}(\mu^2) \left[\begin{array}{l} 1 \\ + \alpha_S^1(c_{10} + c_{11}L^1) \\ + \alpha_S^2(c_{20} + c_{21}L^1 + c_{22}L^2) \\ + \dots \end{array} \right]$$

This is zero (pointing to the 1 term)
This is non-zero (pointing to the alpha_S^2 term)

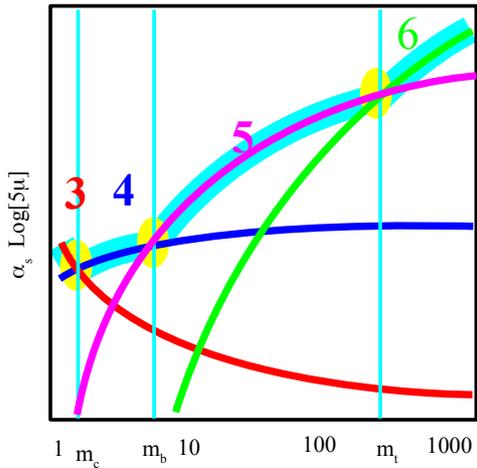
$$L = \ln(\mu^2/m^2)$$

$$c_0 = 0$$

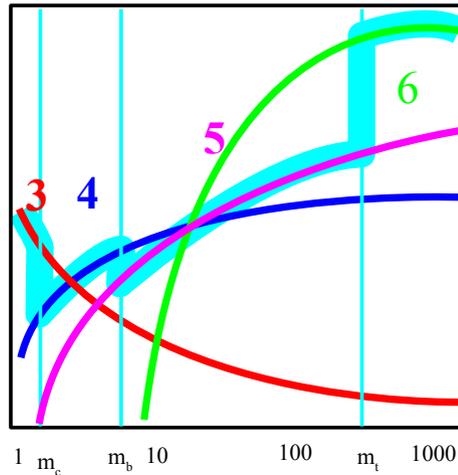
$$c_1 = \frac{-11}{72\pi} \neq 0$$

At $\mu = m$ $\alpha_S^{N_F+1}(\mu^2) = \alpha_S^{N_F}(\mu^2)[1 + 0 + c_{20}\alpha_S^2]$

At 1-loop and 2-loops,
continuous at thresholds

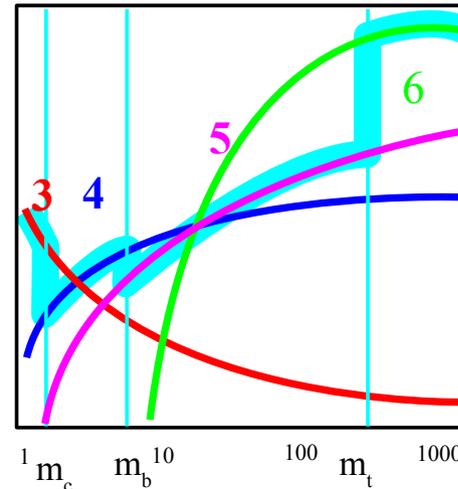


At $O(\alpha_s^3)$, not even
continuous at thresholds



$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2} \alpha_{(n_f-1)}^3(M) + \mathcal{O}(\alpha_{(n_f-1)}^4)$$

At $O(\alpha_s^3)$, not even
continuous at thresholds



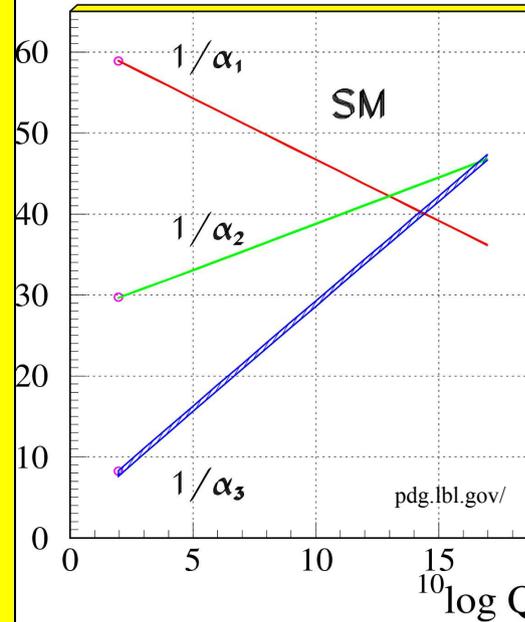
Un-physical theoretical constructs:
(E.g., as, PDFs, ...)

- Cannot be measured directly
- Depends on Schemes
- Renormalization Schemes:
MS, MS-Bar, DIS
- Renormalization Scale m
- Depends on Higher Orders

- Physical Observables**
- Measure directly
- Independent of Schemes/Definitions
- Independent of Higher Orders

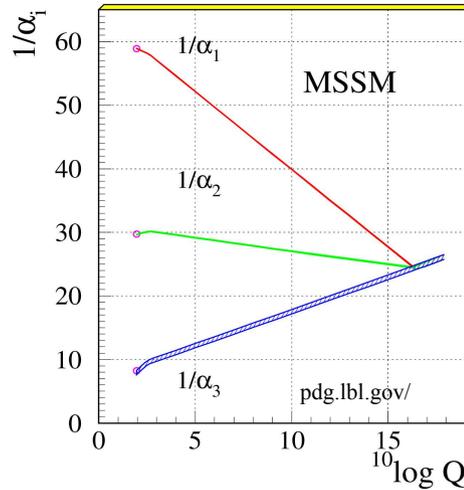
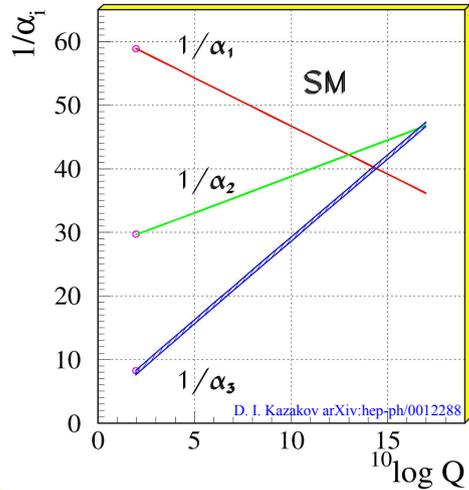
$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{11}{72\pi^2} \alpha_{(n_f-1)}^3(M) + \mathcal{O}(\alpha_{(n_f-1)}^4)$$

BEYOND SM



$$\begin{aligned}
 b_1 &= 0 + (2/3)N_F + \frac{1}{10}N_{Higgs} \\
 b_2 &= -22/3 + (2/3)N_F + \frac{1}{6}N_{Higgs} \\
 b_3 &= -11 + (2/3)N_F + 0
 \end{aligned}$$

Can we do better???

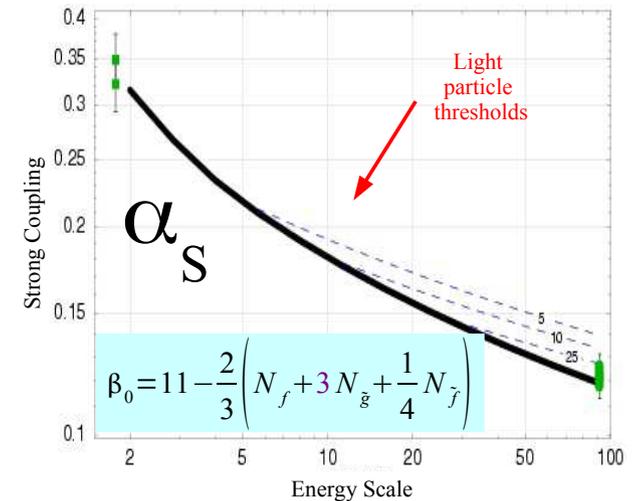
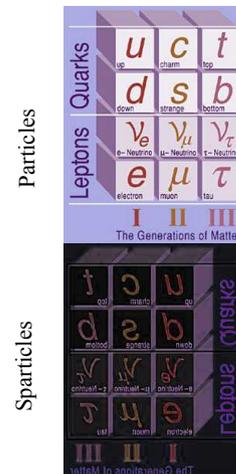


$$\begin{aligned}
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 \end{aligned}$$

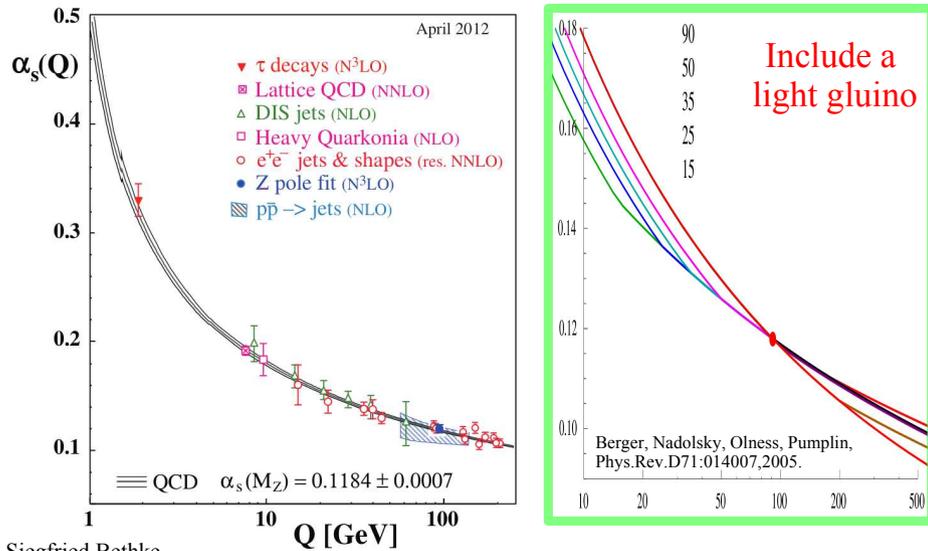
$$\beta_0 = 11 - \frac{2}{3} \left(N_f + 3 N_g + \frac{1}{4} N_j \right)$$

We've only discovered half the particles

New particles effects evolution of $\alpha_s(\mu)$

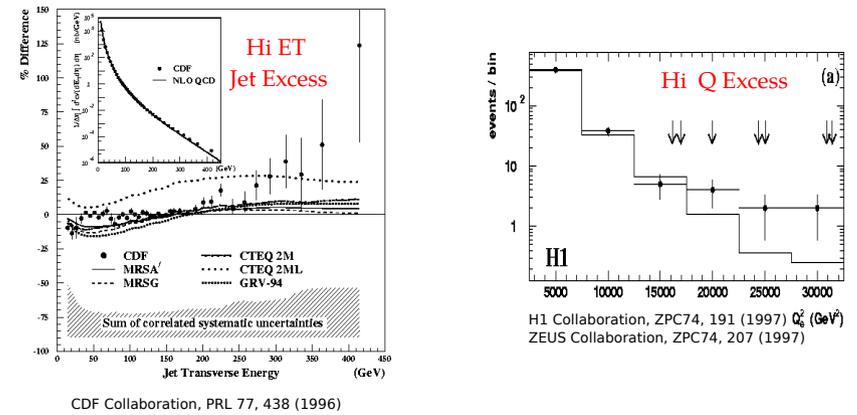


$$\beta_0 = 11 - \frac{2}{3} \left(N_f + 3 N_g + \frac{1}{4} N_j \right)$$



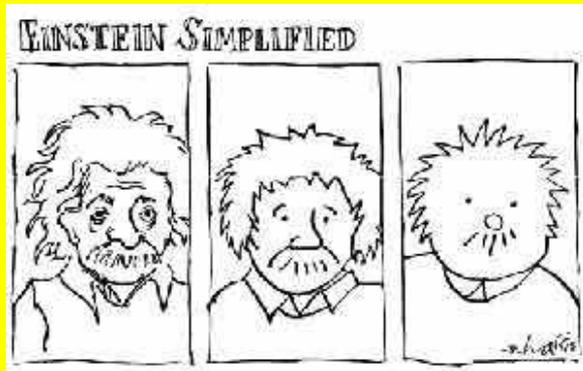
Siegfried Bethke
arXiv:1210.0325 [hep-ex]

$$b_0 = \frac{1}{12\pi} \left\{ 33 - 2N_F - 6N_g - \frac{1}{2}N_{\tilde{F}} \dots \right\}$$



Indispensable
for discovery of
"new physics"

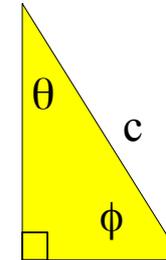
RESUMMATION



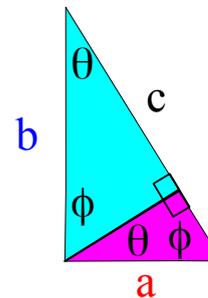
... over simplified

GOAL:
Pythagorean Theorem

METHOD:
Dimensional Analysis



$$A_c = c^2 f(\theta, \phi)$$



$$A_a + A_b = a^2 f(\theta, \phi) + b^2 f(\theta, \phi)$$

$$A_a + A_b = A_c$$

$$a^2 + b^2 = c^2$$

Two examples to come: 1) Resummation, and 2) Scaling

$$\left\{ \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s(\mu)) \frac{\partial}{\partial \alpha_s(\mu^2)} \right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) = 0$$



Logs can be large and spoil perturbation theory

If we expand R in powers of α_s , and we know β , we then know μ dependence of R.

$$R\left(\frac{\mu^2}{Q^2}, \alpha_s(\mu^2)\right) = R_0 + \alpha_s(\mu^2) R_1 \left[\ln(Q^2/\mu^2) + c_1 \right] + \alpha_s^2(\mu^2) R_2 \left[\ln^2(Q^2/\mu^2) + \ln(Q^2/\mu^2) + c_2 \right] + O(\alpha_s^3(\mu^2))$$

Since μ is arbitrary, choose $\mu=Q$.

$$R\left(\frac{Q^2}{Q^2}, \alpha_s(Q^2)\right) = R_0 + \alpha_s(Q^2) R_1 [0 + c_1] + \alpha_s^2(Q^2) R_2 [0 + 0 + c_2] + \dots$$

We just summed the logs

More Differential Quantities \Rightarrow More Mass Scales \Rightarrow More Logs!!!

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\frac{d\sigma}{dQ^2} \sim \ln\left(\frac{Q^2}{\mu^2}\right) \text{ and } \ln\left(\frac{q_T^2}{\mu^2}\right)$$

How do we resum logs? Use the Renormalization Group Equation

For a physical observable R:

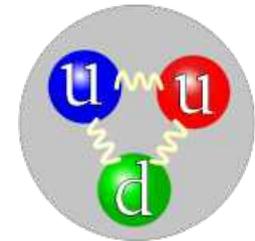
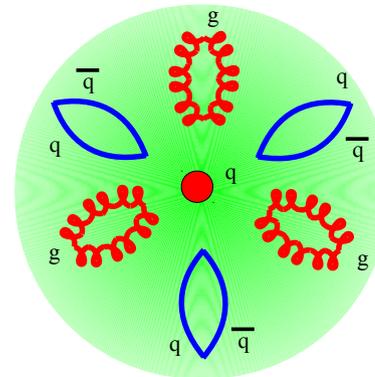
$$\mu \frac{dR}{d\mu} = 0$$

$$\mu \frac{dR}{d \text{ Gauge}} = 0$$

Applied to boson transverse momentum
CSS: Collins, Soper, Sterman
Nucl.Phys.B250:199,1985.

Interesting reference:
Peskin/Schroeder Text
(Renormalization ala Ken Wilson)

Scaling, and the proton structure



<http://en.wikipedia.org/>

Quarks confined, thus we must work with hadrons & mesons

E.g, proton is a "minimal" unit

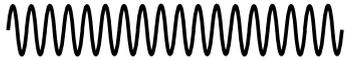
Highest energy (smallest distance) accelerators involve hadrons

E.g., HERA, TEV, LHC

We'd better learn to work with proton

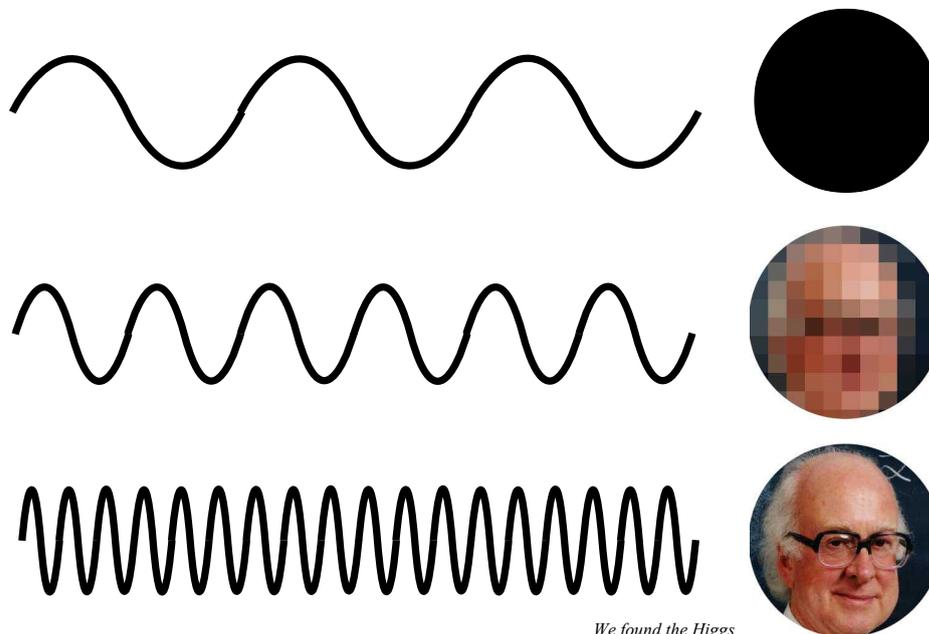
  $d\sigma \sim \frac{4\pi\alpha^2}{Q^2} \times 1$

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Dimensional considerations

Structure Function



We found the Higgs

Scaling, and the proton structure

Scale in m:

- 10^{-10} m **KeV atom** KeV $\sim 10^{-9}$
- 10^{-14} m **MeV nucleus** MeV $\sim 10^{-12}$
- 10^{-15} m **GeV proton** GeV $\sim 10^{-15}$
- $\leq 10^{-18}$ m **TeV quark** electron TeV $\sim 10^{-18}$

$hc = 1 \simeq 0.2 \text{ GeV fm}$

Going to smaller scale, we get to simpler, more fundamental objects

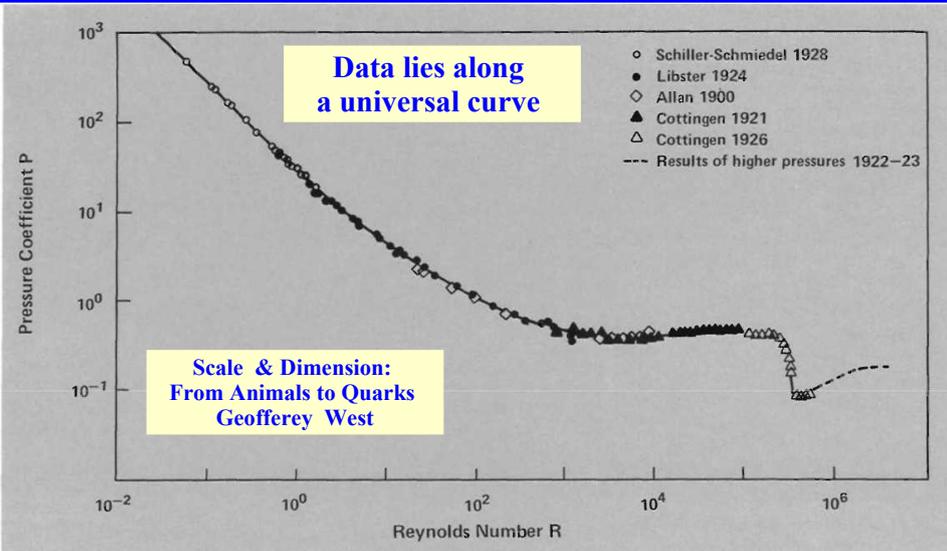
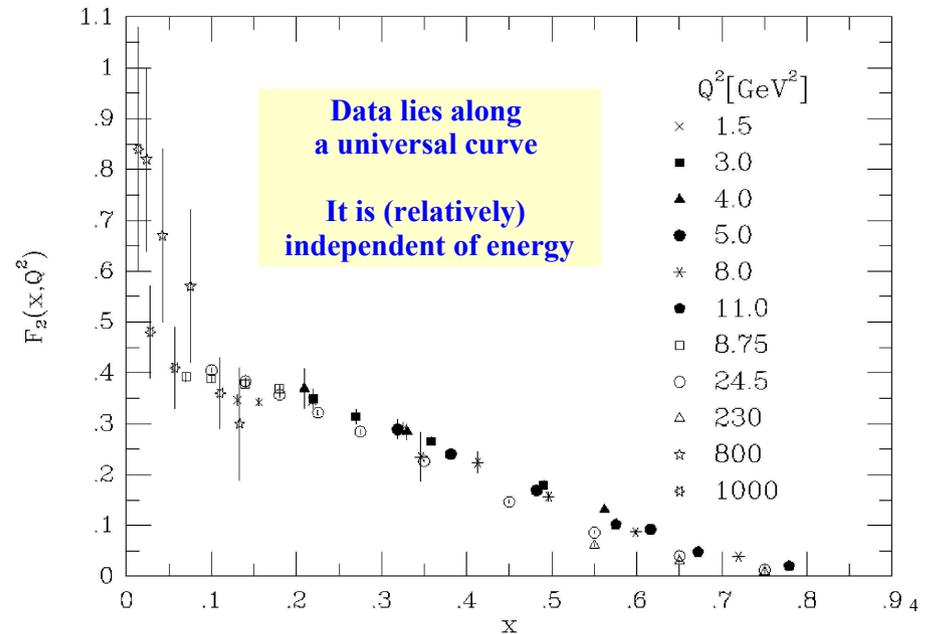
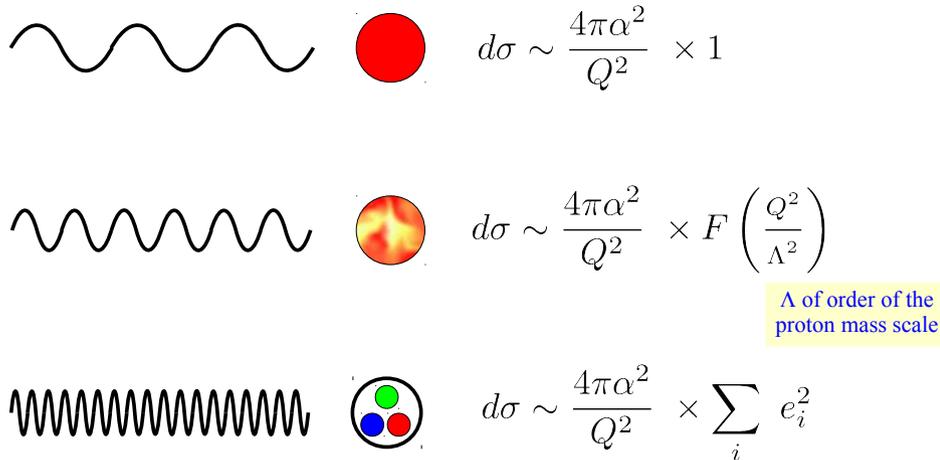


Fig. 5. The scaling curve for the motion of a sphere through a fluid that results when data from a variety of experiments are plotted in terms of two dimensionless variables: the

pressure or drag coefficient P versus Reynolds number R. (Figure adapted from AIP Handbook of Physics, 2nd edition (1963):section II, p. 253.)

- QCD is just like QED, ... only different
- QCD is non-Abelian, Quarks are confined,
- Running coupling $\alpha_s(\mu)$ tells how interaction changes with distance
- β -function: logarithmic derivative of $\alpha_s(\mu)$
- We can compute: Negative for QCD, positive for QED
- $\alpha_s(\mu)$ is **not** a physical quantity
- Discontinuous at NNLO
- New physics can influence $\alpha_s(\mu)$
- Unification of couplings at GUT scale
- Running of $\alpha_s(\mu)$ can help us “resum” perturbation theory
- Scaling and Dimensional Analysis are useful tools

END OF LECTURE 1