Higher-order corrections in Monte-Carlo event generators I

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Introduction

In this two-lecture course we will inspect the anatomy of NLO calculations and obtain a basic understanding of how to compute them.

We will also look at the anatomy of a parton shower to understand its resummation properties and the approximation to higher orders it represents.

Utilising this knowledge we will construct a mathing of a parton shower's resummation to the fixed order calculation.

Finally, we will look at how we can use these ingredients to construct a multijet merging LO and NLO.

Master equation

Expectation value of an observable at a hadron collider

$$\langle O\rangle = \sum_X \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int \mathrm{d}\Phi_X\; f_a(x_1,\mu_\mathrm{F})\, f_b(x_b,\mu_\mathrm{F})\, \hat\sigma_{ab\to X}(\Phi_X;\mu_\mathrm{R},\mu_\mathrm{F})\; O(\Phi_X)$$

The phase space element Φ_X is a set of initial and final state four momenta and corresponding flavours, obeying on-shell conditions and momentum conservation. X is an arbitrarily complex final state.

x_a, x_b are the Bjorken x momentum fractions of the initial state partons a and b. f_a and f_b are the associated parton distribution functions.

$\hat\sigma_{ab\to X}$ is the partonic transition matrix element for a and b reacting to produce X.

The observable O is our measurement function, it generally consist of a set of Θ-functions (cuts) etc., and takes the value 0 or 1.

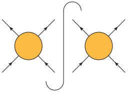
Fixed-order NLO corrections

Fixed-order LO calculations

$$\langle O\rangle = \sum_X \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \int \mathrm{d}\Phi_X\; f_a(x_1,\mu_\mathrm{F})\, f_b(x_b,\mu_\mathrm{F})\; \hat\sigma_{ab\to X}(\Phi_X;\mu_\mathrm{R},\mu_\mathrm{F})\; O(\Phi_X)$$

For the sake of clarity in the following, let us simply this a little bit. At leading order, the transition matrix element is given by its Born approximation

$$\langle O\rangle^\mathrm{LO} = \int \mathrm{d}\Phi_B\; \mathrm{B}(\Phi_B)\; O(\Phi_B)$$

Born term: $\mathrm{B} =$  $= \sum_\text{colour}\sum_\text{spin} |\mathcal{M}_\text{tree}|^2$

It further includes all PDFs, flux factors, symmetry factors, spin & helicity avaraging factors, etc.

Fixed-order LO calculations

$$\langle O\rangle^{\text{LO}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

Can be evaluated directly in 4 dimensions as everything is finite. All possible divergences must be regulated using phase space cuts

Example: $W + j$ production, there needs to be a minimal jet transverse momentum to be a well defined signature.

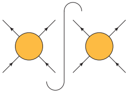
Monte-Carlo integration can proceed straight forwardly as discussed in Andrzej's lecture.

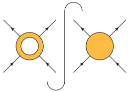
$\Rightarrow$ automation (MadGraph, Sherpa, Helac, . . .)

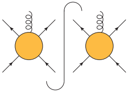
Fixed-order NLO calculations

General structure

$$\langle O \rangle^{\text{NLO}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_B\, \mathrm{V}(\Phi_B)\, O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\, O(\Phi_R)$$

Born term: $\mathrm{B} =$ 

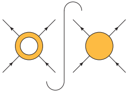
Virtual correction: $\mathrm{V} = 2\,\mathrm{Re}\Big\{$  $\Big\}$ loop integration

Real correction: $\mathrm{R} =$  $\mathrm{d}\Phi_R = \mathrm{d}\Phi_B\, \mathrm{d}\Phi_1$

Fixed-order NLO calculations

General structure

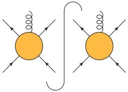
Virtual Corrections:



Contains a loop-integral $\mathrm{d}^4\ell$.

- UV divergences
  $\rightarrow$ can be absorbed in a redefinition of fields, couplings, masses, etc.
  $\rightarrow$ **renormalisation**
- IR divergences
  $\rightarrow$ related to the exchange of soft and/or collinear massless particles
  $\rightarrow$ regulate eg. in dimensional regularisation $D = 4 + 2\epsilon$ and express as poles in ϵ in limit $\epsilon \rightarrow 0$

Real Corrections:



Contains the emission of one potentially soft and/or collinear massless particle, leading to an IR divergence.

Fixed-order NLO calculations

General structure

$$\begin{aligned}\langle O\rangle^{\mathsf{NLO}} = &\int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)\\ &+\int \mathrm{d}\Phi_B\,\mathrm{V}(\Phi_B)\,O(\Phi_B)\\ &+\int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,O(\Phi_R)\end{aligned}$$

Problem: Although the sum of virtual and real corrections is guaranteed to be finite for infrared safe observables (KLN), V and R not finite themselves in $D = 4$ dimensions. Numerical integration, however, needs in integer number of dimensions.

Infrared safe observable: $O(\Phi_R) \to O(\Phi_B)$ if Φ_1 in collinear or soft region.

a) slicing

Split real emission integral into two regions, one region containing the divergence (δ), the other being free of divergences ($1-\delta$).

$$\begin{aligned}\langle O\rangle^{\text{NLO}} = & \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B) \\ & + \int \mathrm{d}\Phi_B \left[\mathrm{V}(\Phi_B) + \int_\delta \mathrm{d}\Phi_1 \mathrm{R}_{\text{approx}}(\Phi_B\cdot\Phi_1)\right] O(\Phi_B) \\ & + \int \mathrm{d}\Phi_B \int_{1-\delta} \mathrm{d}\Phi_1\, \mathrm{R}(\Phi_R)\, O(\Phi_R)\end{aligned}$$

We need an analytic integral of R in region δ. If δ very small, then we can find an approximation to R that we can analytically integrate and extract its divergences. They then can be explicitly cancelled before the intgral over Φ_B is taken numerically.

Exact as $\delta \to 0$, but large global cancellations as integrals $\propto \log\delta$.

a) slicing

Split real emission integral into two regions, one region containing the divergence (δ), the other being free of divergences ($1-\delta$).

$$\begin{aligned}\langle O\rangle^{\text{NLO}} = & \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B) \\ & + \int \mathrm{d}\Phi_B \quad \Delta^{\text{NLO}}_{\delta,\text{approx}}(\Phi_B) \quad O(\Phi_B) \\ & + \int \mathrm{d}\Phi_B \int_{1-\delta} \mathrm{d}\Phi_1\, \mathrm{R}(\Phi_R)\, O(\Phi_R)\end{aligned}$$

We need an analytic integral of R in region δ. If δ very small, then we can find an approximation to R that we can analytically integrate and extract its divergences. They then can be explicitly cancelled before the intgral over Φ_B is taken numerically.

Exact as $\delta \to 0$, but large global cancellations as integrals $\propto \log\delta$.

b) subtraction

Define an approximation D that coincides with R in its divergent regions.

$$\begin{aligned}\langle O\rangle^{\text{NLO}} = & \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)\\ & + \int \mathrm{d}\Phi_B \left[\mathrm{V}(\Phi_B) + \int \mathrm{d}\Phi_1 \mathrm{D}(\Phi_B\cdot\Phi_1)\right] O(\Phi_B)\\ & + \int \mathrm{d}\Phi_R \left[\mathrm{R}(\Phi_R)\,O(\Phi_R) - \mathrm{D}(\Phi_B\cdot\Phi_1)\,O(\Phi_B)\right]\end{aligned}$$

The subtraction terms D can be constructed universally as sum over sectors, dipoles, antennas, etc., and integrated once and for all.
Exact without the need of a regulator, still large cancellations locally in Φ_R integral in the infrared regions.

b) subtraction

Existing subtraction methods

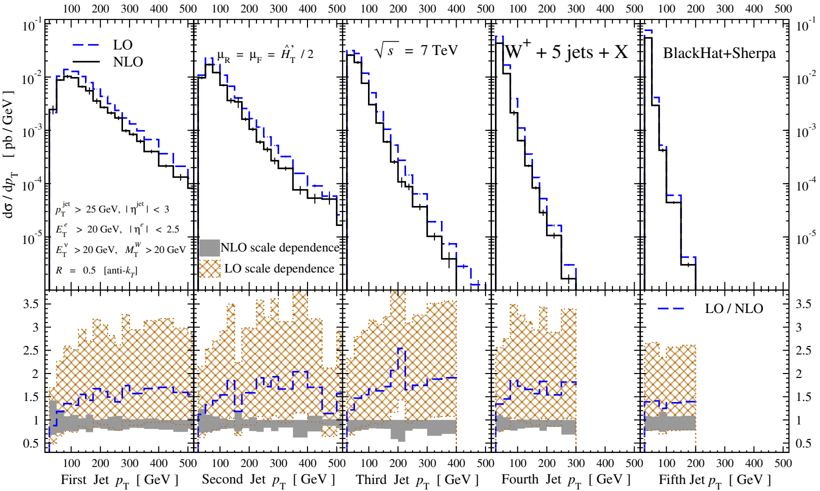
- Frixione-Kunszt-Signer subtraction
  implemented in MadGraph
- Catani-Seymour dipole subtraction
  implemented in Sherpa, ...
- Antenna subtraction
  implemented in Nnlojet
- Nagy-Soper subtraction
  implemented in Helac

Along with developments in the construction of one-loop matrix elements it enabled the automation of NLO calculation.

Results

$W + 5j$ production

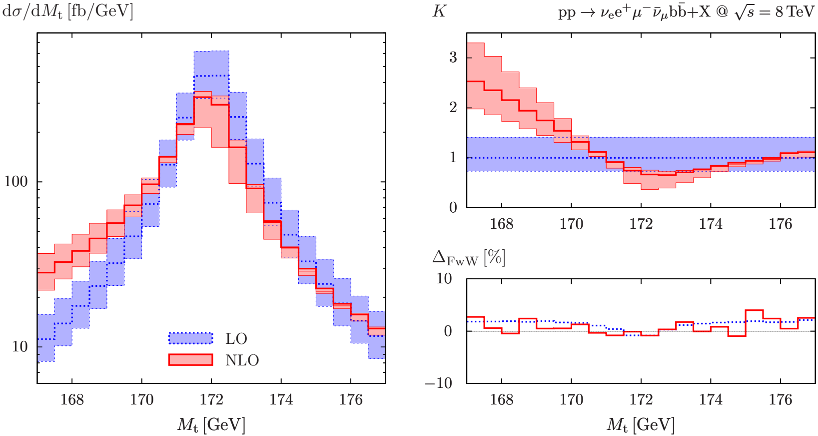
arXiv:1304.1253



Results

$\ell\bar{\ell}\nu\bar{\nu}b\bar{b}$ production

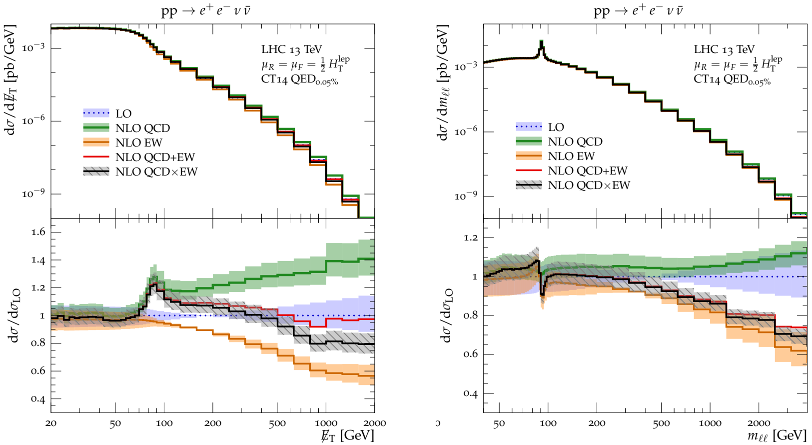
arXiv:1207.5018



Results

$2e + MET$ production (NLO EW)

arXiv:1705.00598



Fixed-order NNLO calculations

General structure

$$\begin{aligned}\langle O\rangle^{\text{NNLO}} = &\int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)\\ &+\int \mathrm{d}\Phi_B\,\mathrm{V}(\Phi_B)\,O(\Phi_B)+\int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,O(\Phi_R)\\ &+\int \mathrm{d}\Phi_B\,\mathrm{VV}(\Phi_B)\,O(\Phi_B)+\int \mathrm{d}\Phi_R\,\mathrm{RV}(\Phi_R)\,O(\Phi_R)\\ &+\int \mathrm{d}\Phi_{RR}\,\mathrm{RR}(\Phi_{RR})\,O(\Phi_{RR})\end{aligned}$$

new pieces:

double virtual corrections VV	two loops
real-virtual corrections RV	one loop, one emission
double real corrections RR	two emissions

$\mathrm{d}\Phi_R = \mathrm{d}\Phi_B\,\mathrm{d}\Phi_1$ and $\mathrm{d}\Phi_{RR} = \mathrm{d}\Phi_R\,\mathrm{d}\Phi_1 = \mathrm{d}\Phi_B\,\mathrm{d}\Phi_2$

Fixed-order NNLO calculations

Again, when attempting to numerically integrate all terms the same problems appear as at NLO, only more complex.

The singularity structure contains both one- and two-particle singularities as one or two parton may become soft or collinear.

The solutions follow also the same patterns as at NLO: slicing and subtraction.

a) slicing

Split real emission integral into two regions, one region containing the divergence (δ), the other being free of divergences ($1-\delta$).

$$\begin{aligned}
\langle O\rangle^{\text{NLO}} = & \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B) \\
& + \int \mathrm{d}\Phi_B \left[\mathrm{V}(\Phi_B) + \mathrm{VV}(\Phi_\mathrm{B}) + \int_\delta \mathrm{d}\Phi_1 (\mathrm{R}+\mathrm{RV})_{\text{approx}}(\Phi_\mathrm{B}\cdot\Phi_1)\right. \\
& \qquad\qquad \left. + \int_\delta \mathrm{d}\Phi_2\, \mathrm{RR}_{\text{approx}}(\Phi_B\cdot\Phi_2)\right] O(\Phi_B) \\
& + \int \mathrm{d}\Phi_B \int_{1-\delta} \mathrm{d}\Phi_1\, (\mathrm{R}+\mathrm{RV})(\Phi_R)\, O(\Phi_R) \\
& + \int \mathrm{d}\Phi_B \int_{1-\delta} \mathrm{d}\Phi_2\, \mathrm{RR}(\Phi_{RR})\, O(\Phi_{RR})
\end{aligned}$$

a) slicing

Split real emission integral into two regions, one region containing the divergence (δ), the other being free of divergences ($1-\delta$).

$$\begin{aligned}\langle O\rangle^{\text{NLO}} = & \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B) \\ & + \int \mathrm{d}\Phi_B \left[\Delta^{\text{NLO}}_{\delta,\text{approx}}(\Phi_B) + \Delta^{\text{NNLO}}_{\delta,\text{approx}}(\Phi_B)\right] O(\Phi_B) \\ & + \int \mathrm{d}\Phi_B \int_{1-\delta} \mathrm{d}\Phi_1 \Big[(\mathrm{B}_{+1} + \mathrm{V}_{+1})(\Phi_{B_{+1}})\, O(\Phi_{B_{+1}}) \\ & \qquad + \int \mathrm{d}\Phi_1\, \mathrm{R}_{+1}(\Phi_{R_{+1}})\, O(\Phi_{R_{+1}})\Big]\end{aligned}$$

In the unresolved region δ the NNLO coefficient emerges.
The resolved regions contains an NLO calculation with a process with one additional final state.

a) slicing

The calculation is exact as $\delta \to 0$. However, there are large cancellations between the coefficients in the δ-region and the exlicite NLO calculation outside this region, which are larger than in the NLO case.

Existing methogs:

- q_T slicing
  for colour-singlet production, use q_T of colour singlet to identify all two-particle singularities
  $\delta : q_T < q_{T,\text{cut}}$
- $N_\text{jettiness}$ slicing
  use $N_\text{jettiness}$ (event shape variable) to identify all two-particle singularities
  $\delta : \tau < \tau_\text{cut}$

Large numerical cancellations can be improved upon by including power corrections as non-local subtraction terms.

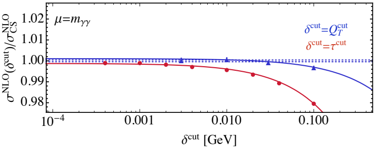
a) slicing

Independence of the result from the slicing parameters

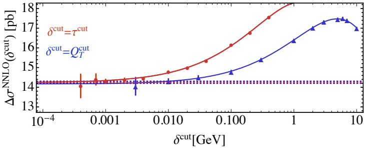
Check that the (N)NLO cros section converges as $\delta \to 0$. Determine the region of δ where

1) cross section corresponds to the one in $\delta \to 0$ limit
2) cancellations between cross-section in δ region and cross section outside are as small as possible

arXiv:1603.02663



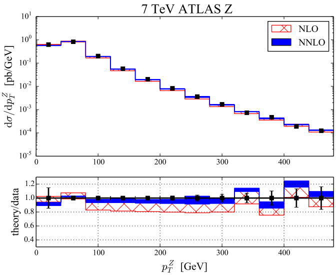
NLO



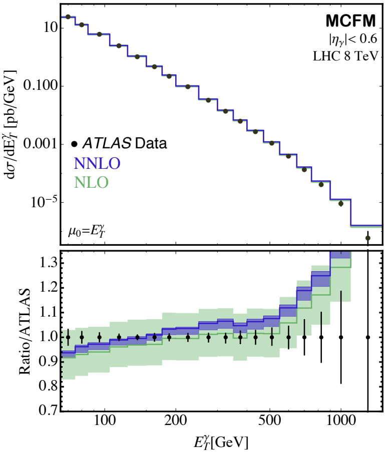
NNLO

a) slicing

Zj production arXiv:1602.05612



γj production arXiv:1612.04333



b) subtraction

$$
\begin{aligned}
\langle O\rangle^{\text{NNLO}} = \; & \int \mathrm{d}\Phi_B\, (\mathrm{B}+\mathrm{V})(\Phi_B)\; O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\; O(\Phi_R) \\
& + \int \mathrm{d}\Phi_B \left[\mathrm{VV}(\Phi_B) + \int \mathrm{d}\Phi_1\, \mathrm{DR}(\Phi_B\Phi_1) + \int \mathrm{d}\Phi_2\, \mathrm{DD}(\Phi_B\Phi_2) \right] O(\Phi_B) \\
& + \int \mathrm{d}\Phi_R \left[\left(\mathrm{RV}(\Phi_R) + \int \mathrm{d}\Phi_1\, \mathrm{DV}(\Phi_R\Phi_1) \right) O(\Phi_R) - \mathrm{DR}(\Phi_B\Phi_1)\; O(\Phi_B) \right] \\
& + \int \mathrm{d}\Phi_{RR} \left[\mathrm{RR}(\Phi_{RR})\; O(\Phi_{RR}) - \mathrm{DV}(\Phi_R\Phi_1)\; O(\Phi_R) - \mathrm{DD}(\Phi_B\Phi_2)\; O(\Phi_B) \right]
\end{aligned}
$$

One subtraction per singular structure (one- and two-particle singularities).

Can be constructed universally, and integrated once and for all.

Exact without the need of a regulator, still large cancellations locally in Φ_{RR} and Φ_R. NLO part treated as before.

b) subtraction

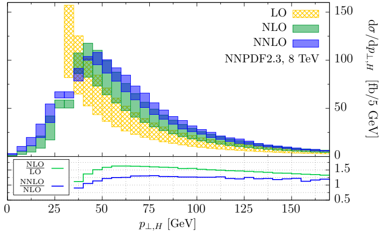
Existing subtraction methods:

- Antenna subtraction
  NNLOJET
- Dipole subtraction
  ColorFull
- Sector decomposition and subtraction
  Stripper
- Projection-to-Born

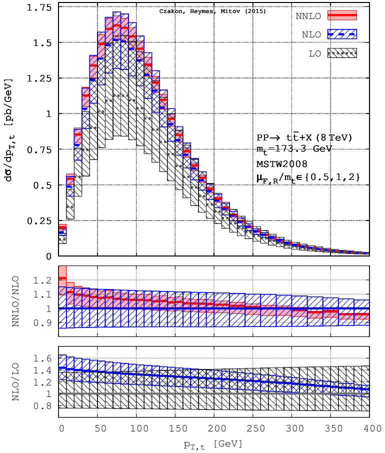
Enabled large number of NNLO now available.

b) subtraction

hj production   arXiv:1504.07922



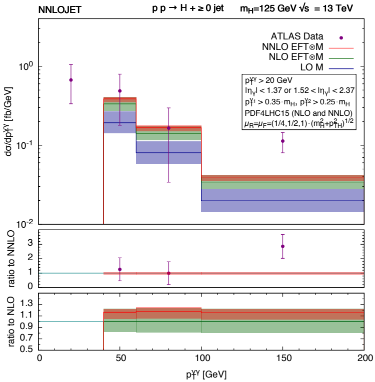
$t\bar{t}$ production   arXiv:1511.00549

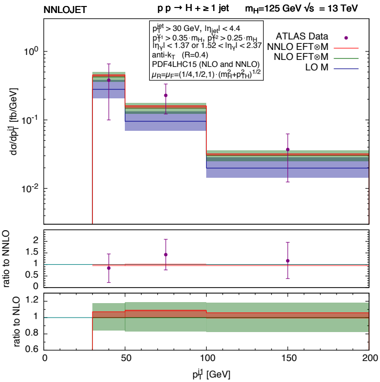


b) subtraction

hj production

arXiv:1607.08817

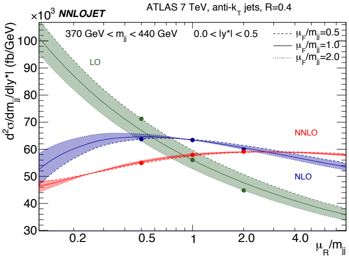


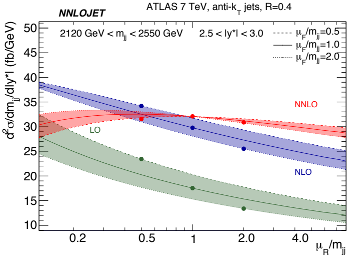


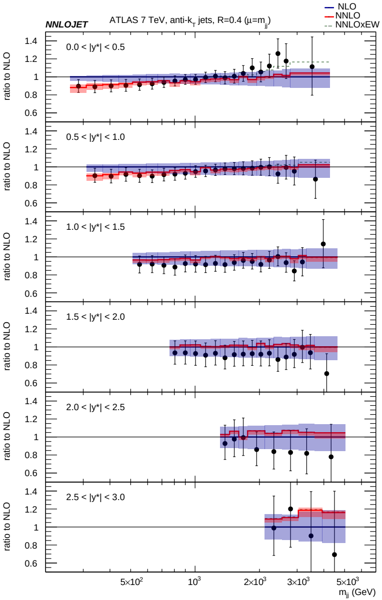
b) subtraction

Dijet production

arXiv:1705.10271







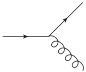
Parton showers

1. Introduction
2. Fixed-order NLO corrections
   - Structure and treatment of IR divergences in NLO calculations
3. Fixed-order NNLO calculations
   - Structure and treatment of IR divergences in NNLO calculations
4. Parton showers
   - Parton showers as approximate higher-order corrections
5. Summary

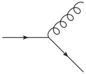
Parton showers

Parton showers were introduced in Andrzej's lecture today.
To summarise their features:

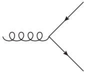
- universal collinear approximation is constructed using Altarelli-Parisi splitting kernels P_{ab} (or improvements thereupon)



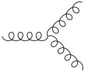
$$= C_F \, \frac{1+z^2}{1-z}$$



$$= C_F \, \frac{1+(1-z)^2}{z}$$



$$= T_R \, \left[z^2+(1-z)^2\right]$$



$$= C_A \left[\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right]$$

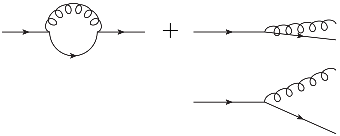
- universal collinear approximation

$$\mathrm{d}\sigma_{n+1} \propto \mathrm{d}\sigma_n \sum_{\text{partons}} \frac{\mathrm{d}t}{t} \, \mathrm{d}z \, \frac{\alpha_s}{2\pi} \, P_{ab}(z)$$

- t evolution variable, z splitting variable

Parton showers

- soft and/or collinear partons not separately resolvable
- introduce resolution criterion, e.g. $t > t_c$, also acts as infrared regulator



- Poisson statistics leads to no-emission probability

$$\mathrm{d}\mathcal{P}_{\text{em}}(t) = \frac{\mathrm{d}t}{t}\int \mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z) \quad\to\quad \mathcal{P}_{\text{no-em}}(t,t') = \exp\left\{-\int_t^{t'}\frac{\mathrm{d}\bar{t}}{\bar{t}}\int \mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\right\}$$

- Sudakov form factor $\Delta(t,t') := \mathcal{P}_{\text{no-em}}(t,t')$
- probability of a parton produced at t' to radiate/resolve another parton at t

$$\mathrm{d}\mathcal{P}(t) = \mathrm{d}\mathcal{P}_{\text{em}}(t)\,\mathcal{P}_{\text{no-em}}(t,t') = \mathrm{d}t\,\frac{\mathrm{d}\Delta(t,t')}{\mathrm{d}t}$$

Parton showers

The **generating functional** acts iterative, terminating in case of no emission

$$\mathrm{PS}_n(t, O) = \Delta_n(t_c, t)\, O(\Phi_n) + \int_{t_c}^{t} \mathrm{d}t' \int \mathrm{d}z \int \mathrm{d}\phi\, \mathrm{K}_n(\Phi_1(t', z, \phi))\, \Delta_n(t', t)\, \mathrm{PS}_{n+1}(t', O)$$

with

$$\Delta_n(t', t) = \exp\left\{ -\int_{t'}^{t} \mathrm{d}\tilde{t} \int \mathrm{d}z \int \mathrm{d}\phi\, \mathrm{K}_n(\Phi_1(\tilde{t}, z, \phi)) \right\}$$

The kernel K_n of the n parton configuration $\propto \alpha_s \prod_i P_{ab_i}$.
It determines radiation pattern and, through unitarity, the Sudakov form factor.

The phase space element Φ_1 for the next emission is determined through t, z and ϕ.

Parton showers

Effect of the parton shower on a Born calculation: LOPS.

$$\langle O\rangle^{\text{LOPS}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, \mathrm{PS}_B(\mu_Q^2, O)$$

$$= \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[\Delta_B(t_c,\mu_Q^2)\, O(\Phi_B) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_B(\Phi_1)\Delta_B(t,\mu_Q^2)\, \mathrm{PS}_{B+1}(t,O)\right]$$

The scale μ_Q^2 determines the starting scale of the parton shower.

Let's expand this to $\mathcal{O}(\alpha_s)$

$$\langle O\rangle^{\text{LOPS}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[\left(1 - \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_B(\Phi_1)\right) O(\Phi_B) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_B(\Phi_1)\, O(\Phi_R)\right]$$

Parton showers

$\mathcal{O}(\alpha_s)$ expansion

$$\langle O\rangle^{\text{LoPs}} = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[\left(1 - \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_B(\Phi_1)\right) O(\Phi_B) + \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{K}_B(\Phi_1)\, O(\Phi_R) \right]$$

Properties:

- $\langle O\rangle^{\text{LoPs}} = \langle O\rangle^{\text{LO}}$ for $O = 1$ (inclusive cross section)
  $\Rightarrow$ **unitarity**
- contains approximation for real and virtual corrections in soft-collinear limit
  - $\mathrm{R}_{\text{approx}}(\Phi_B\Phi_1) = \mathrm{B}(\Phi_B) \cdot \mathrm{K}_B(\Phi_1)$
  - $\mathrm{V}_{\text{approx}}(\Phi_B) = -\int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1\, \mathrm{R}_{\text{approx}}(\Phi_B\Phi_1)$
- IR divergences regulated through cutoff t_c

Summary

In NLO and NNLO calculation individual contributions exhibit UV and IR divergences.

- UV divergences are absorbed by redefining fields, couplings, etc.
  $\rightarrow$ renormalisation
- IR divergences are physical, but must cancel in any infrared safe observable (KLN theorem)
  - regulate divergences through dimensional regularisation, the introduction of particle masse, or other schemes
    $\rightarrow$ expressed in terms $\frac{1}{\epsilon}$-poles, $\log m$, etc.
  - use slicing or subtraction to render the expressions numerically integrable
- parton showers are approximations to higher-order corrections in the soft-collinear limit, and are thus closely related to subtraction schemes

Thank you for your attention!