

# Higher-order corrections in Monte-Carlo event generators II

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## ① Recap

## ② Matching

Matching parton showers to fixed-order calculations

## ③ Merging

Multijet Merging at leading and next-to-leading order

## ④ Summary

## Recap – Master equation

### Expectation value of an observable at a hadron collider

$$\langle O \rangle = \sum_X \int_0^1 dx_a \int_0^1 dx_b \int d\Phi_X f_a(x_a, \mu_F) f_b(x_b, \mu_F) \hat{\sigma}_{ab \rightarrow X}(\Phi_X; \mu_R, \mu_F) O(\Phi_X)$$

The phase space element  $\Phi_X$  is a set of initial and final state four momenta and corresponding flavours, obeying on-shell conditions and momentum conservation.  $X$  is an arbitrarily complex final state.

$x_a$ ,  $x_b$  are the Bjorken  $x$  momentum fractions of the initial state partons  $a$  and  $b$ .  $f_a$  and  $f_b$  are the associated parton distribution functions.

$\hat{\sigma}_{ab \rightarrow X}$  is the partonic transition matrix element for  $a$  and  $b$  reacting to produce  $X$ .

The observable  $O$  is our measurement function, it generally consist of a set of  $\Theta$ -functions (cuts) etc., and takes the value 0 or 1.

## Recap – Fixed-order calculations

At **leading order** approximation

$$\langle O \rangle^{\text{LO}} = \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

At **next-to-leading order**

$$\begin{aligned} \langle O \rangle^{\text{NLO}} = & \int d\Phi_B \left[ B(\Phi_B) + V(\Phi_B) + \int d\Phi_1 D(\Phi_B \cdot \Phi_1) \right] O(\Phi_B) \\ & + \int d\Phi_R \left[ R(\Phi_R) O(\Phi_R) - D(\Phi_B \cdot \Phi_1) O(\Phi_B) \right] \end{aligned}$$

in any subtraction scheme

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in any subtraction scheme

## Recap – Parton showers

### LOPs

$$\begin{aligned}
 \langle O \rangle^{\text{LOPs}} &= \int d\Phi_B B(\Phi_B) PS_B(\mu_Q^2, O) \\
 &= \int d\Phi_B B(\Phi_B) \left[ \Delta_B(t_c, \mu_Q^2) O(\Phi_B) \right. \\
 &\quad \left. + \int_{t_c}^{\mu_Q^2} d\Phi_1 K_B(\Phi_1) \Delta_B(t, \mu_Q^2) PS_{B+1}(t, O) \right]
 \end{aligned}$$

Parton showers give an estimate of higher order corrections in the soft-collinear limit.

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## Matching at LO

Let's start with the LO expression for the expectation value

$$\langle O \rangle^{\text{LOPs}} = \int d\Phi_B B(\Phi_B) O(\Phi_B)$$

**Can we simply replace  $O(\Phi_B)$  with the parton shower  $\text{PS}_n(t_B, O)$ ?**

Yes.

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There is no overlap in the terms included in the parton shower and the leading order matrix element.

The parton shower provides all higher order corrections to the given Born configuration in LOPs.

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splitting kernels, starting conditions, etc.

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There is no overlap in the terms included in the parton shower and the leading order matrix element.

The parton shower provides all higher order corrections to the given Born configuration in LOPS.

LOPS still contains many interesting problems:  
splitting kernels, starting conditions, etc.

## Matching at NLO

Let's start with the NLO expression for the expectation value

$$\begin{aligned} \langle O \rangle^{\text{NLO}} &= \int d\Phi_B \left[ B + V + I_D \right] (\Phi_B) O(\Phi_B) \\ &\quad + \int d\Phi_R \left[ R(\Phi_R) O(t_R) - D(\Phi_B \cdot \Phi_1) O(t_B) \right] \end{aligned}$$

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R and D receive different parton shower corrections,  
this spoils the subtraction in the IR limit

Additionally, as we have seen in the last lecture,  $B \cdot \text{PS}_B(t_B, O)$  creates  $V_{\text{approx}}$  and  $R_{\text{approx}}$ . They interfere with the proper V and R and spoil the NLO accuracy. **Let's try again.**

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## Matching at NLO

$$\langle O \rangle^{\text{NLO}} = \int d\Phi_B [B + V](\Phi_B) O(\Phi_B) + \int d\Phi_R R(\Phi_R) O(\Phi_R)$$

- Let's examine what happens when we effect the PS on  $\Phi_B$  to  $\mathcal{O}(\alpha_s)$ .

$$B(\Phi_B) = B(\Phi_B) + \mathcal{O}(\alpha_s) = B(\Phi_B)$$

$$V(\Phi_B) = -P(\Phi_B) + \mathcal{O}(\alpha_s) = -P(\Phi_B)$$

$$R(\Phi_R) = R(\Phi_R) + \mathcal{O}(\alpha_s) = R(\Phi_R)$$

- Still, NLO accuracy is spoiled, however, except for the sign, this looks very much like a subtraction at NLO.

## Matching at NLO

$$\langle O \rangle^{\text{NLO}_{\text{PS}}} = \int d\Phi_B [B + V](\Phi_B) \text{PS}(t_B, O) + \int d\Phi_R R(\Phi_R) O(\Phi_R)$$

- 1 Let's examine what happens when we effect the PS on  $\Phi_B$  to  $\mathcal{O}(\alpha_s)$ .  
→ we generate approximate real and virtual corrections

$$R_{\text{approx}}(\Phi_B \Phi_1) = B(\Phi_B) \cdot K_B(\Phi_1) = D_K(\Phi_B \Phi_1)$$

$$V_{\text{approx}}(\Phi_B) = -B(\Phi_B) \int_{t_c}^{\mu_0^2} d\Phi_1 K_B(\Phi_1) = -I_K(\Phi_B)$$

$$B(\Phi_B) \text{PS}_B(t_B, O)|_{\mathcal{O}(\alpha_s)} = [B - I_K](\Phi_B) O(\Phi_B) + D_K(\Phi_B \Phi_1) O(\Phi_R)$$

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- To  $\mathcal{O}(\alpha_s)$  the exact NLO expression is recovered

In order for both  $[B + V + I_K](\Phi_B)$  and  $[R - D_K]$  to be finite in the soft-collinear limit and, thus, integrable in 4 dimensions puts requirements on the accuracy of the parton shower  $\rightarrow \widetilde{\text{PS}}_B$



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## POWHEG/MC@NLO

$$\langle O \rangle^{\text{NLOps}} = \int d\Phi_B \left[ B + V + I_K \right] (\Phi_B) \widetilde{\text{PS}}_B(\mu_Q^2, O) \quad \text{S-event}$$

$$+ \int d\Phi_R \left[ R - D_K \right] (\Phi_R) \text{PS}_R(t_R, O) \quad \text{H-event}$$

There are still a few choices possible

- 1) choice of kernel  $D_K$  in the matched emission in  $\widetilde{\text{PS}}$
- 2) choice of parton shower resummation phase space, defined through  $\mu_Q^2$

	MC@NLO	POWHEG
$D_K$	$B \cdot \widetilde{K}_{\text{PS}}$	$R$
$\mu_Q^2$	$\mu_F^2$	$S_{\text{had}}$

- MC@NLO preserves parton shower resummation, corrected to  $\mathcal{O}(\alpha_s)$
- POWHEG eliminates the second line, the source of negative weights

## POWHEG/MC@NLO

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## POWHEG/MC@NLO

Description of emission spectrum:

$$\langle O_{\text{em}} \rangle^{\text{NLOps}} = \int d\Phi_B d\Phi_1 \bar{B}(\Phi_B) \frac{D_K(\Phi_B \cdot \Phi_1)}{B(\Phi_B)} O(\Phi_R) + \int d\Phi_R [R - D_K](\Phi_R) O(\Phi_R)$$

In the resummation region  $D_K \neq 0$  the emission spectrum is enhanced with  $\bar{B}/B$

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$D_K$	$B \cdot \bar{K}_{\text{PS}}$	$R$
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Powheg:

split  $R = R_{\text{soft}} + R_{\text{hard}}$ 

with

$$R_{\text{soft}} = \frac{h^2}{p_{\perp}^2 + h^2} R$$

and  $D_K = R_{\text{soft}}$



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$$\langle O_{\text{em}} \rangle^{\text{NLOPS}} = \int d\Phi_R \left[ R + \left( \frac{\bar{B}(\Phi_B)}{B(\Phi_B)} - 1 \right) D_K \right] (\Phi_R) O(\Phi_R)$$

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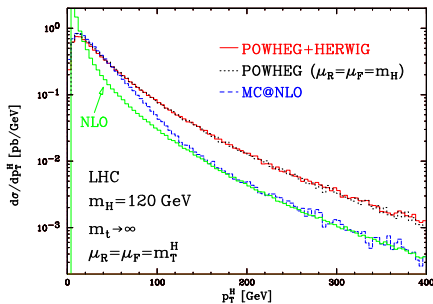
	MC@NLO	POWHEG
$D_K$	$B \cdot \tilde{K}_{\text{PS}}$	$R$
$\mu_Q^2$	$\mu_F^2$	$S_{\text{had}}$

Powheg:

split  $R = R_{\text{soft}} + R_{\text{hard}}$   
with

$$R_{\text{soft}} = \frac{h^2}{p_{\perp}^2 + h^2} R$$

and  $D_K = R_{\text{soft}}$



## POWHEG/MC@NLO

Description of emission spectrum:

$$\langle O_{\text{em}} \rangle^{\text{NLOPS}} = \int d\Phi_R \left[ R + \left( \frac{\bar{B}(\Phi_B)}{B(\Phi_B)} - 1 \right) D_K \right] (\Phi_R) O(\Phi_R)$$

In the resummation region  $D_K \neq 0$  the emission spectrum is enhanced with  $\bar{B}/B$

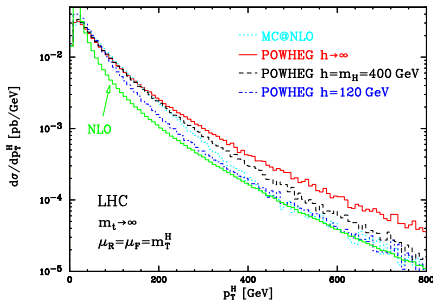
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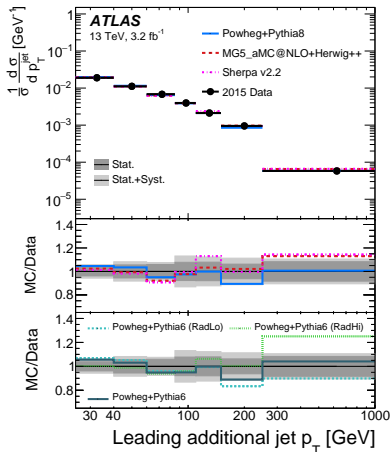
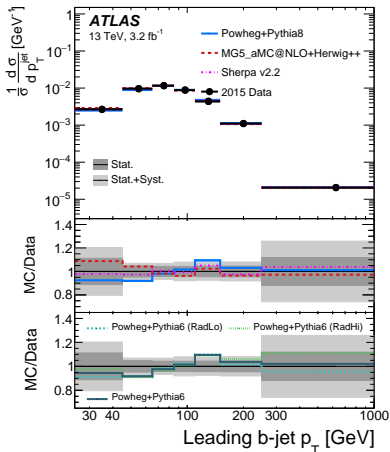
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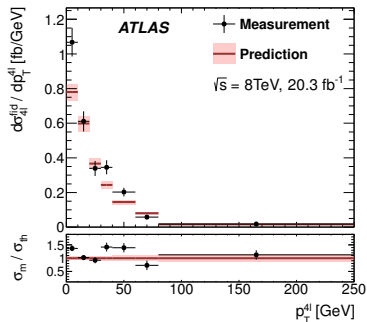
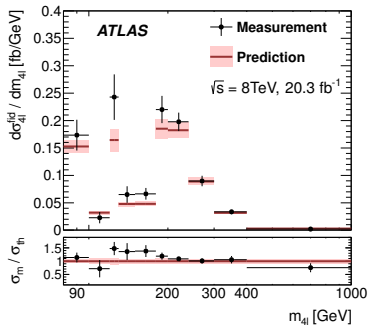
## POWHEG/Mc@NLO

 $t\bar{t}$  production

arXiv:1610.09978



## POWHEG/Mc@NLO

4 $l$  production [arXiv:1509.07844](https://arxiv.org/abs/1509.07844)

# POWHEG/Mc@NLO

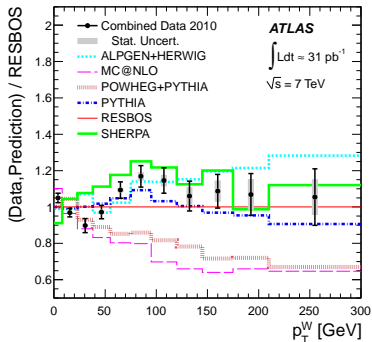
arXiv:1108.6308

## $W$ production

The  $W$  transverse momentum in different regions is dominated by contributions from different final states:

$W$ ,  $Wj$ ,  $Wjj$ , ...

NLOPS for incl.  $W$  production describe  $Wj$  at LO,  $Wjj$  with PS only



# POWHEG/Mc@NLO

## POWHEG

- + almost only positive weights
- modifies resummation
- may need modification of resummation region

implemented in

- POWHEG-BOX
- HERWIG7

→ need combination with PS, loop provider and/or ME provider

## Mc@NLO

- + retains exact PS resummation
- possibility of negative weights

implemented in

- Mc@NLO
- SHERPA
- aMc@NLO
- HERWIG7

**What else is there?**

UNLOPS

[arXiv:1211.4827](https://arxiv.org/abs/1211.4827)



# Higher-order corrections in Monte-Carlo event generators II

## ① Recap

## ② Matching

Matching parton showers to fixed-order calculations

## ③ Merging

Multijet Merging at leading and next-to-leading order

## ④ Summary

## Multijet Merging

LOPS, NLOPS and NNLOPS describe observables dominated by topologies of a single multiplicity very well.

However, many observables receive contributions from many final state multiplicities. Examples:  $H_T$ ,  $p_{\perp}$ , etc.

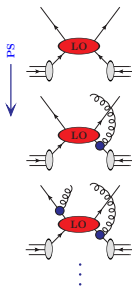
NLOPS, for example, will describe the low end at NLO accuracy, an intermediate region at LO accuracy, and the high end at PS accuracy only

We want to describe these observables as uniformly as possible

⇒ **multijet merging**

At the same time, multijet merged samples provide the LHC experiments with largest freedom of projecting these samples onto observables without the loss of accuracy.

# Multijet merging at LO

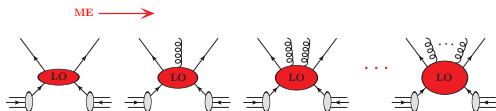


## Parton showers

resummation of (soft-)coll. limit  
→ intrajet evolution

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- MEPS combines multiple LOPs – keeping either accuracy
- NLOPS elevate LOPs to NLO accuracy
- MENLOPS supplements core NLOPS with higher multiplicities LOPs

# Multijet merging at LO

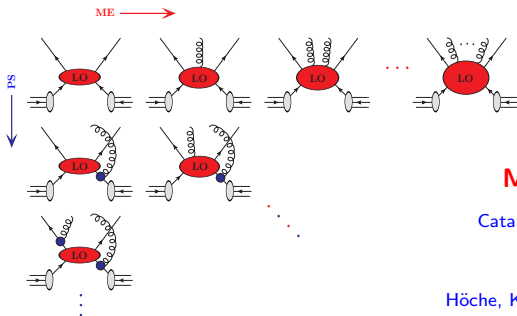


## Matrix elements

- fixed-order in  $\alpha_s$
- hard wide-angle emissions
- interference terms

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
  - MEPS combines multiple LOPS – keeping either accuracy
  - NLOPS elevate LOPS to NLO accuracy
  - MENLOPS supplements core NLOPS with higher multiplicities LOPS

# Multijet merging at LO



## MEPS (CKKW, MLM)

Catani, Krauss, Kuhn, Webber JHEP11(2001)063

Lönnblad JHEP05(2002)046

Mangano, Moretti, Pittau NPB632(2002)343

Höhe, Krauss, Schumann, Siegert JHEP05(2009)053

Lönnblad, Prestel JHEP03(2012)019

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## Multijet merging at LO

$$\begin{aligned}
 \langle O \rangle^{\text{MEPS}} = & \int d\Phi_n B_n \text{PS}_n(O) \Theta(Q_{\text{cut}} - Q_{n+1}) \\
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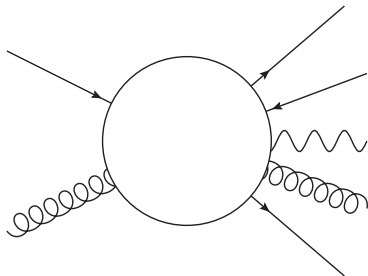
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## Determination of scales $t_i$

### Example: Drell-Yan production in association with jets

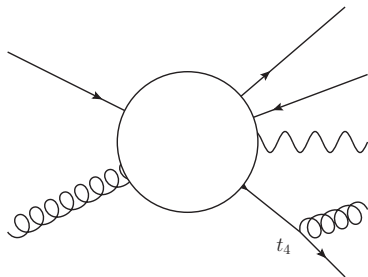


- cluster external particles using inverse parton shower  
→ flavour conscious, initial state aware, probability determined through splitting kernels
- identify a shower history (probabilistically), determine scale  $t_i$  up to predefined  $t_l$
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$$\alpha_s^{n+k}(\mu_R^2) = \alpha_s^k(\mu_{\text{core}}^2) \prod_{i=1}^n \alpha_s(t_i)$$

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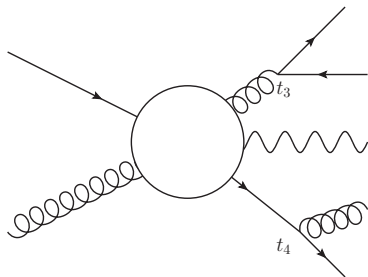


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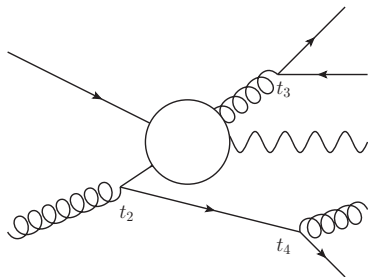


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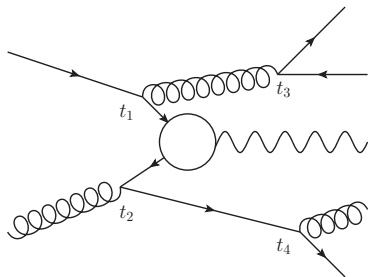


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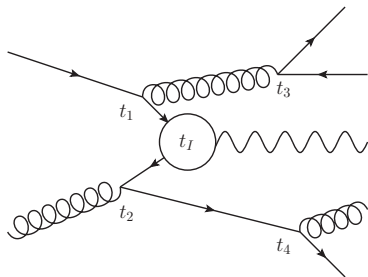


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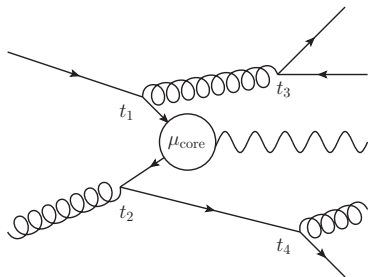
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## Multijet merging at LO

- PS: higher order real emission corrections in (soft-)collinear limit  
→ identify hard region, replace kernel with LO matrix element

$$\begin{aligned}
 \langle O \rangle^{\text{MEPS}} &= \int d\Phi_n B \left[ \Delta_n(t_c, t_{\text{max}}) + \int_{t_c}^{t_{\text{max}}} d\Phi_1 K_n \Delta_n(t, t_{\text{max}}) \Theta(Q_{\text{cut}} - Q_{n+1}) \right] \\
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- replace shower kernels in hard region by ratio of matrix elements  
→ contains correct description of hard emissions & interference

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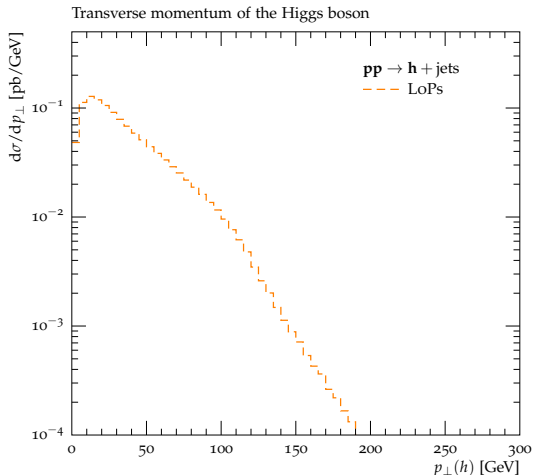
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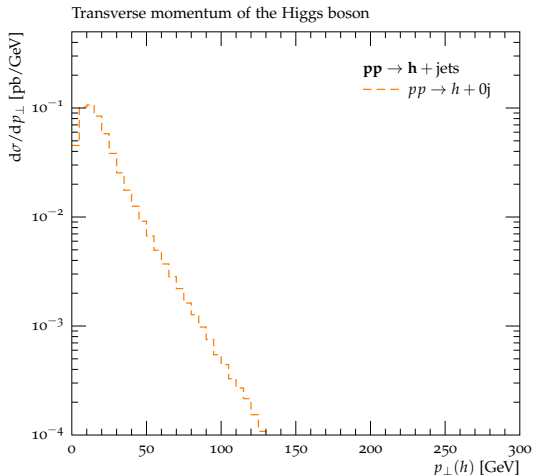
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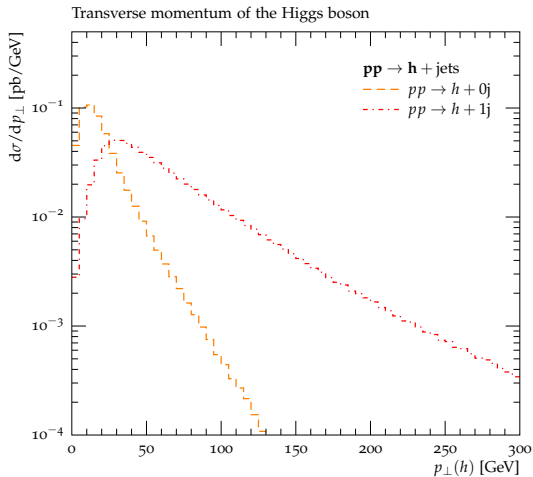
- first emission by PS, restrict to  $Q_{n+1} < Q_{\text{cut}}$
- LOPs  $pp \rightarrow h + \text{jet}$  for  $Q_{n+1} > Q_{\text{cut}}$
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- iterate
- sum all contributions

# Multijet merging at LO



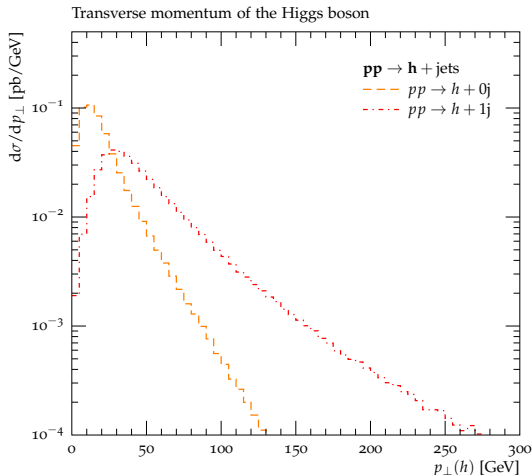
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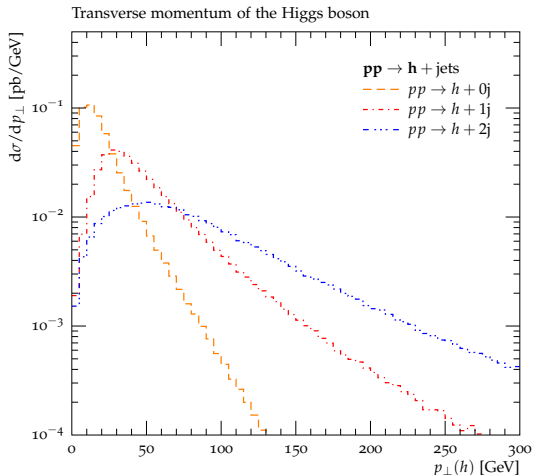
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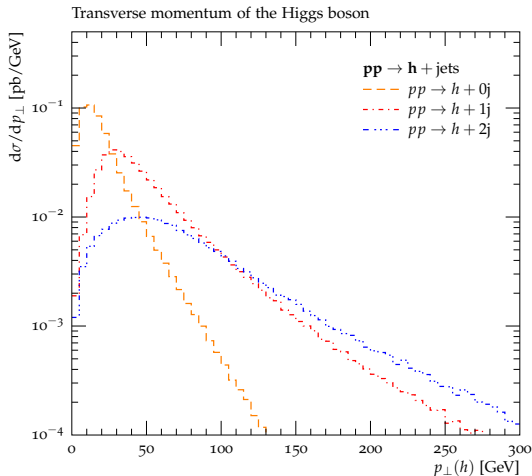


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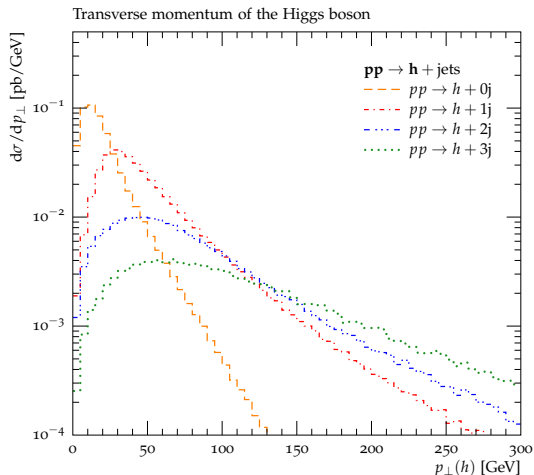
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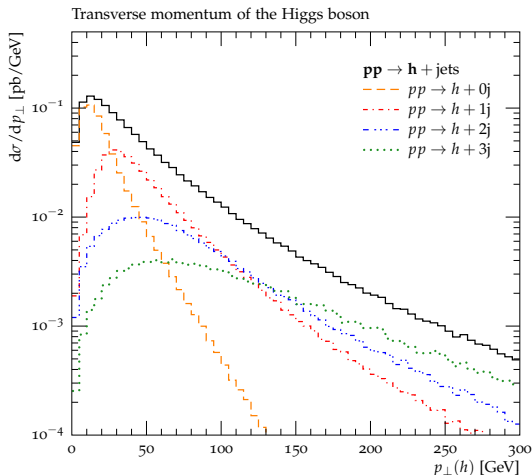
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## MLM

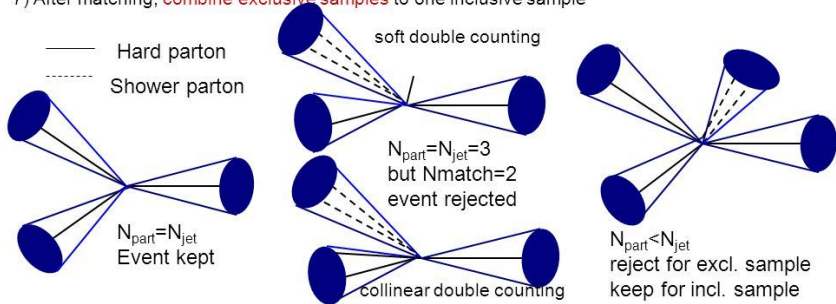
Mangano (2002)

Used in Alpgen

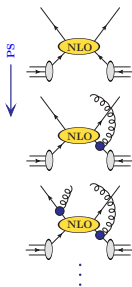
Prevent parton shower harder than any emission by ME using cone algo:

- 1) **Generate hard parton** configuration for given  $n=N_{\text{part}}$  with ME, imposing
- 2) **Define tree branching structure** using  $K_T$ -algo  
allowing only pairing consistent with color flow  
 $|\eta^i| < \eta^{\text{max}}, E_T^i > E_T^{\text{min}}, \Delta R_{ij} > R^{\text{min}}$
- 3) **Compute  $\alpha_s$**  at the nodal values, but do not apply Sudakov factors
- 4) **Shower the hard event** without any veto using Herwig/Pythia  
when done, find  $N_{\text{jet}}$  jet with cone algorithm with  $E_T, R$   
if  $N_{\text{part}} < N_{\text{jet}}$  reject event
- 5) **Matched jets to hard partons** using  $\min \Delta R_{i, \text{jets}}$   
Only keep events, if each hard parton is uniquely contained in jets  
Events with  $N_{\text{part}} < N_{\text{jet}}$  are rejected except for highest multiplicities
- 6) **Define exclusive N-jet sample** by requiring  $N_{\text{part}} = N_{\text{jet}}$
- 7) After matching, **combine exclusive samples** to one inclusive sample

This is equivalent to  
tSudakov reweighting  
in CKKW (external lines)



# Multijet merging at NLO



**NLOs** (Mc@NLO, POWHEG)

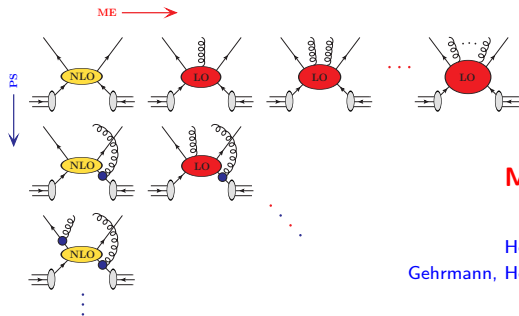
Frixione, Webber JHEP06(2002)029

Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

Höche, Krauss, MS, Siebert JHEP09(2012)049

- matrix elements (ME) and parton showers (PS) are approximations in different regions of phase space
- Multijet merging at LO combines multiple LOPs
- NLOs elevate LOPs to NLO accuracy
  - First step supplements core NLOs with higher multiplicities LOPs
  - Multijet merging at NLO combines multiple NLOs

# Multijet merging at NLO



## Multijet merging

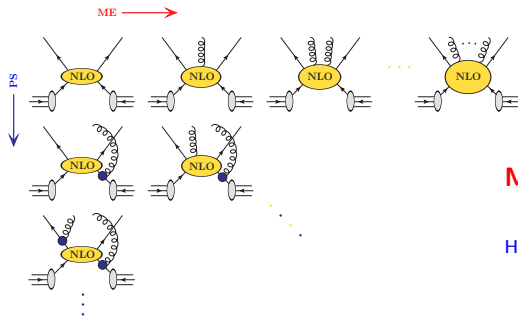
Hamilton, Nason JHEP06(2010)039

Höche, Krauss, MS, Siebert JHEP08(2011)123

Gehrmann, Höche, Krauss, MS, Siebert JHEP01(2013)144

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# Multijet merging at NLO



## Multijet merging at NLO

Lavesson, Lönnblad JHEP12(2008)070

Höhe, Krauss, MS, Siebert JHEP04(2013)027

Fredrerix, Frixione JHEP12(2012)061

Lönnblad, Prestel JHEP03(2013)166

Plätzer JHEP08(2013)114

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## Multijet merging at NLO

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 \langle O \rangle^{\text{MEPS@NLO}} = & \int d\Phi_n \bar{B}_n \widetilde{\text{PS}}_n(O) \Theta(Q_{\text{cut}} - Q_{n+1}) \\
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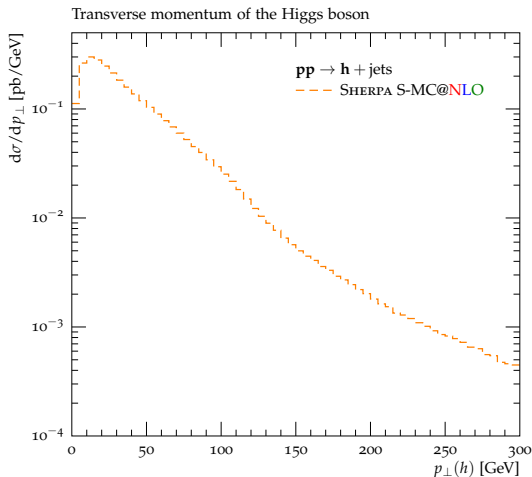
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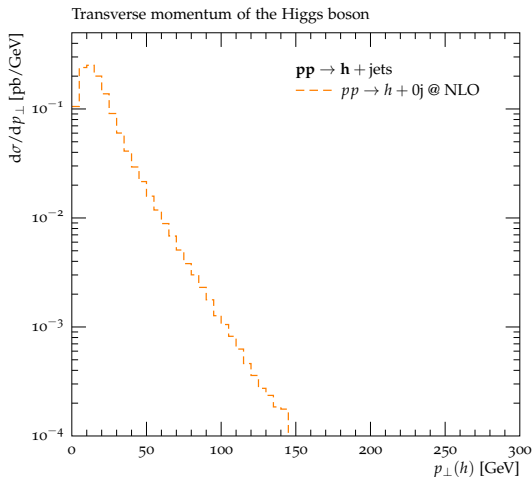
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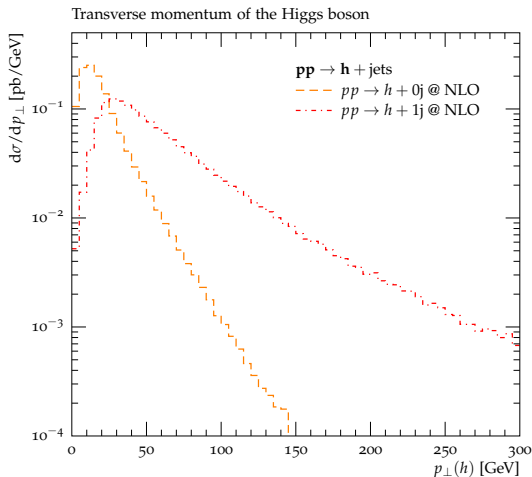
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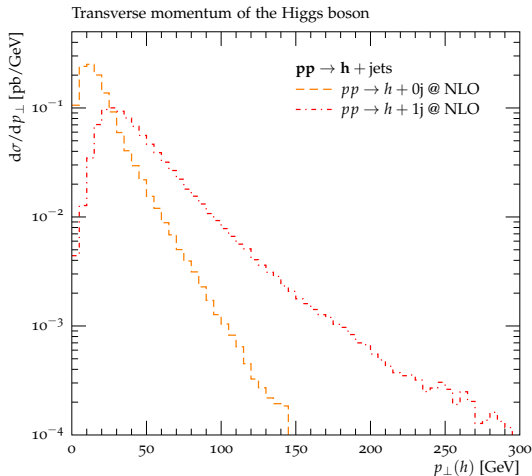


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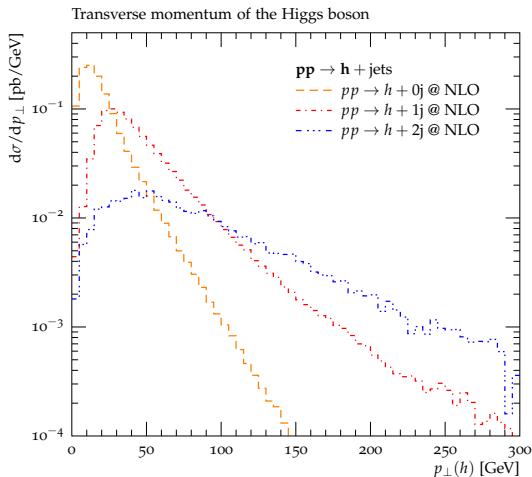
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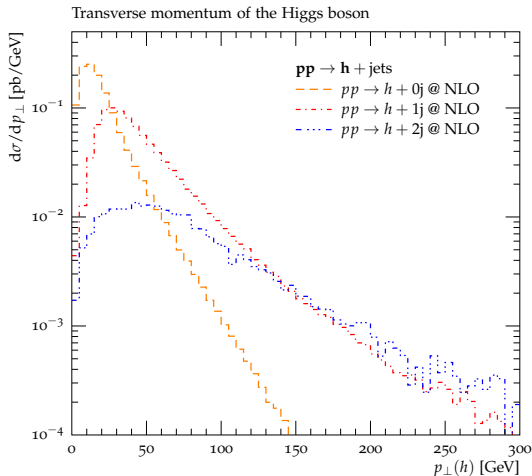
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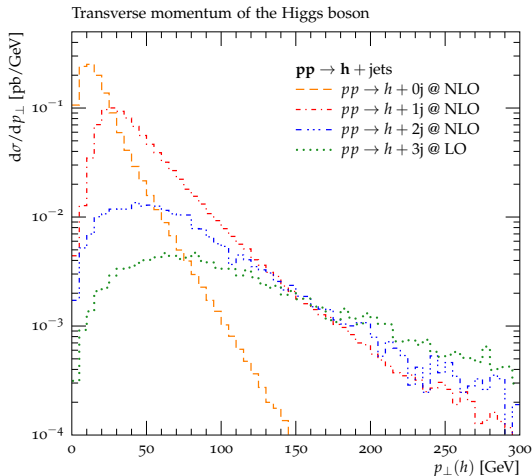
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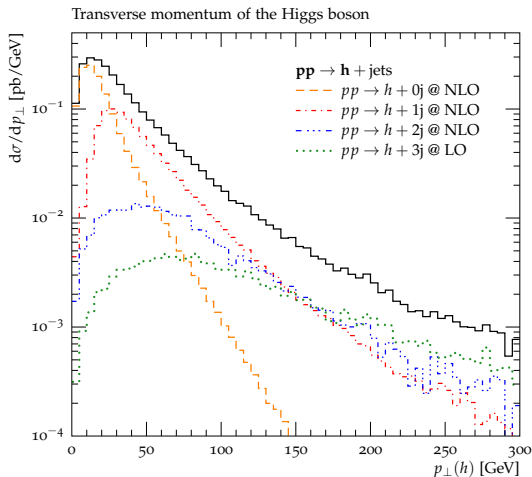
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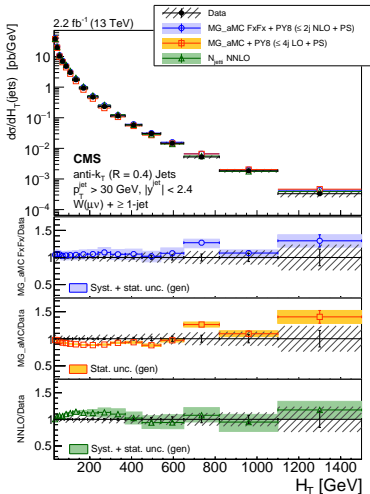
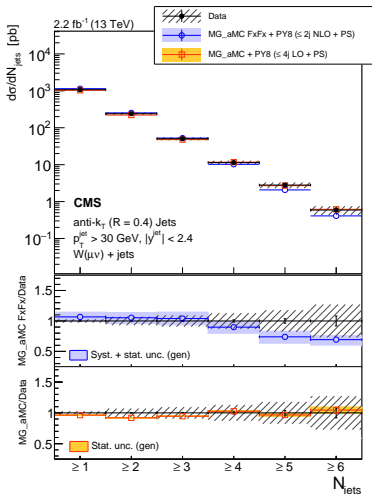
# Multijet merging at NLO

## Available implementations

- ALPGEN+HERWIG/PYTHIA  
MLM (LO)
- MADGRAPH/LOOPPROVIDER+HERWIG  
UNLOPs (NLO), UMEPs (LO)
- MADGRAPH+PYTHIA  
FxFx (NLO), UNLOPs (NLO), MLM (LO), UMEPs (LO)
- SHERPA+LOOPPROVIDER  
MEPs@NLO (NLO), MEPs (LO)

# Multijet merging at NLO

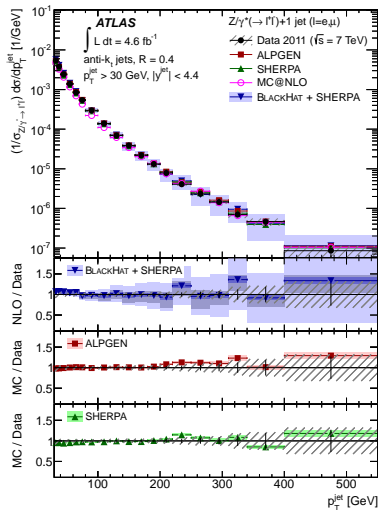
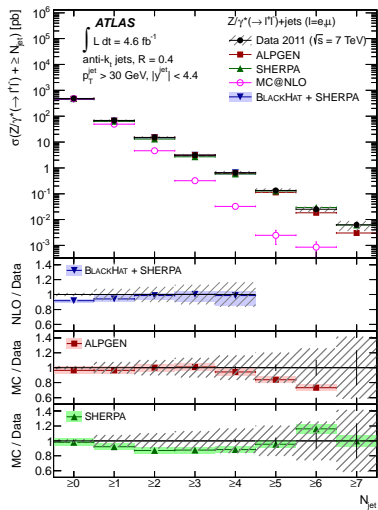
## lepton + MET production





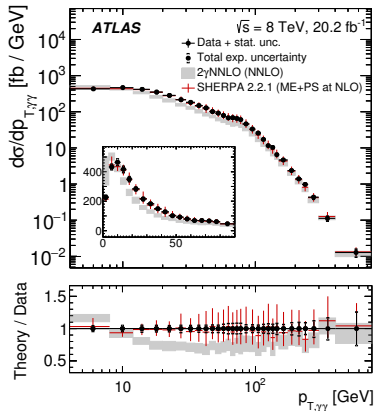
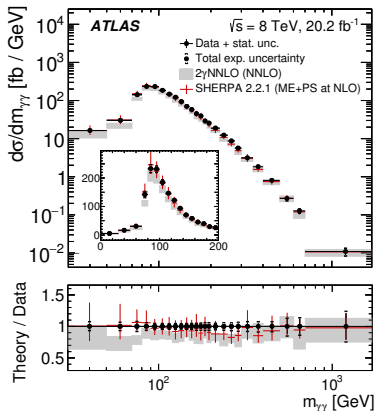
# Multijet merging at NLO

## lepton pair production



# Multijet merging at NLO

## diphoton production



## Summary

- NLOPS matching is automated and is a standard input for any LHC analysis
  - standard schemes (POWHEG/MC@NLO) closely related, but predictions can differ substantially
  - accuracy of the prediction restricted to observables which are described by a single multiplicity
  - extensions to NNLOPS exist for the simplest cases
- multijet merging improves the accuracy for the emission of additional jets
  - #emissions limited by CPU resources
  - NLO accuracy can be reached for the lowest few multiplicities
- computational complexity  
LO < LOPS < Multijet merging at LO  
NLO < NLOPS < Multijet merging at NLO

Thank you for your attention!