Jets

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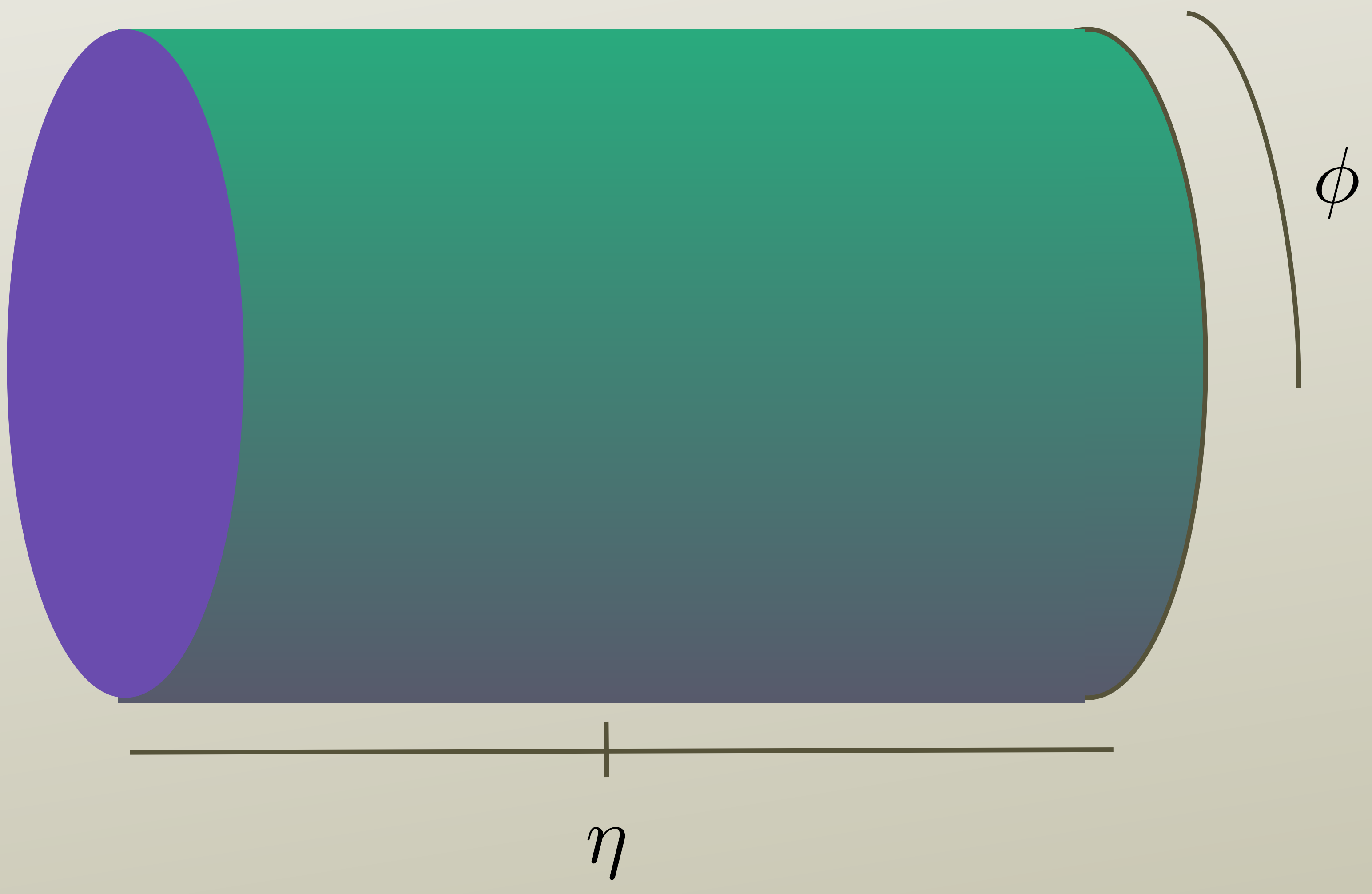
What are jets?

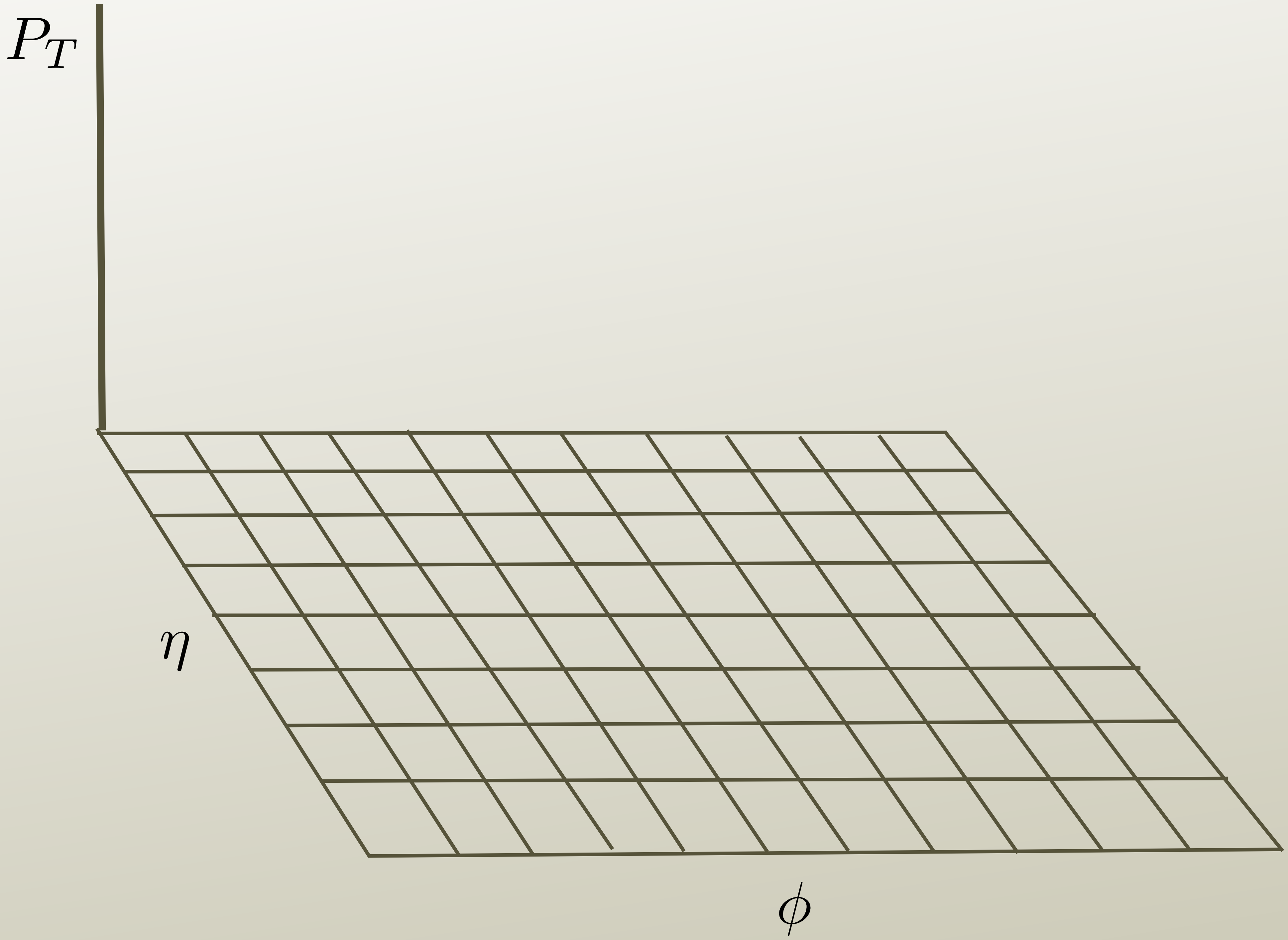
- Denote angles in the detector by

  azimuthal angle ϕ

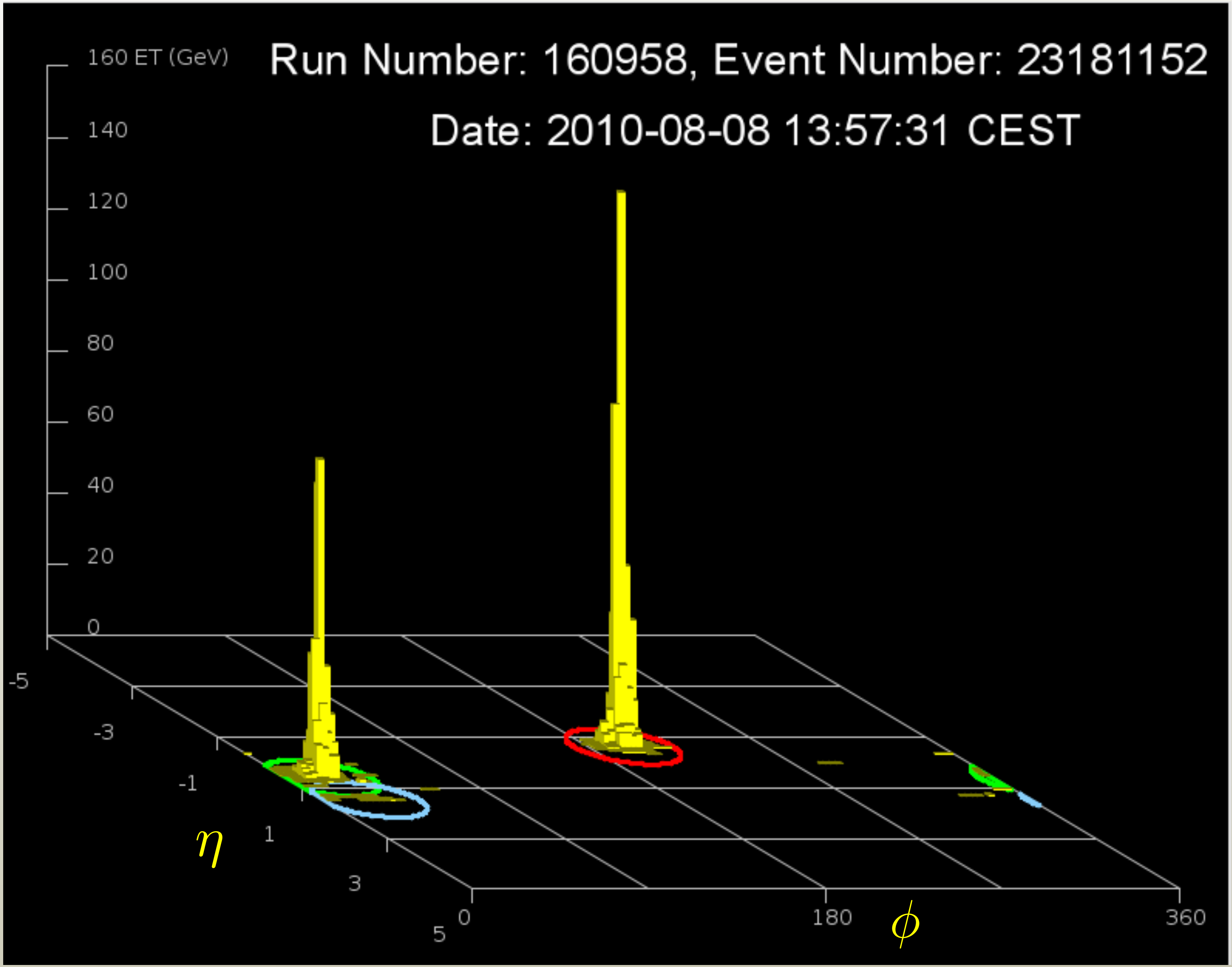
  pseudorapidity $\eta = -\log\tan(\theta/2)$



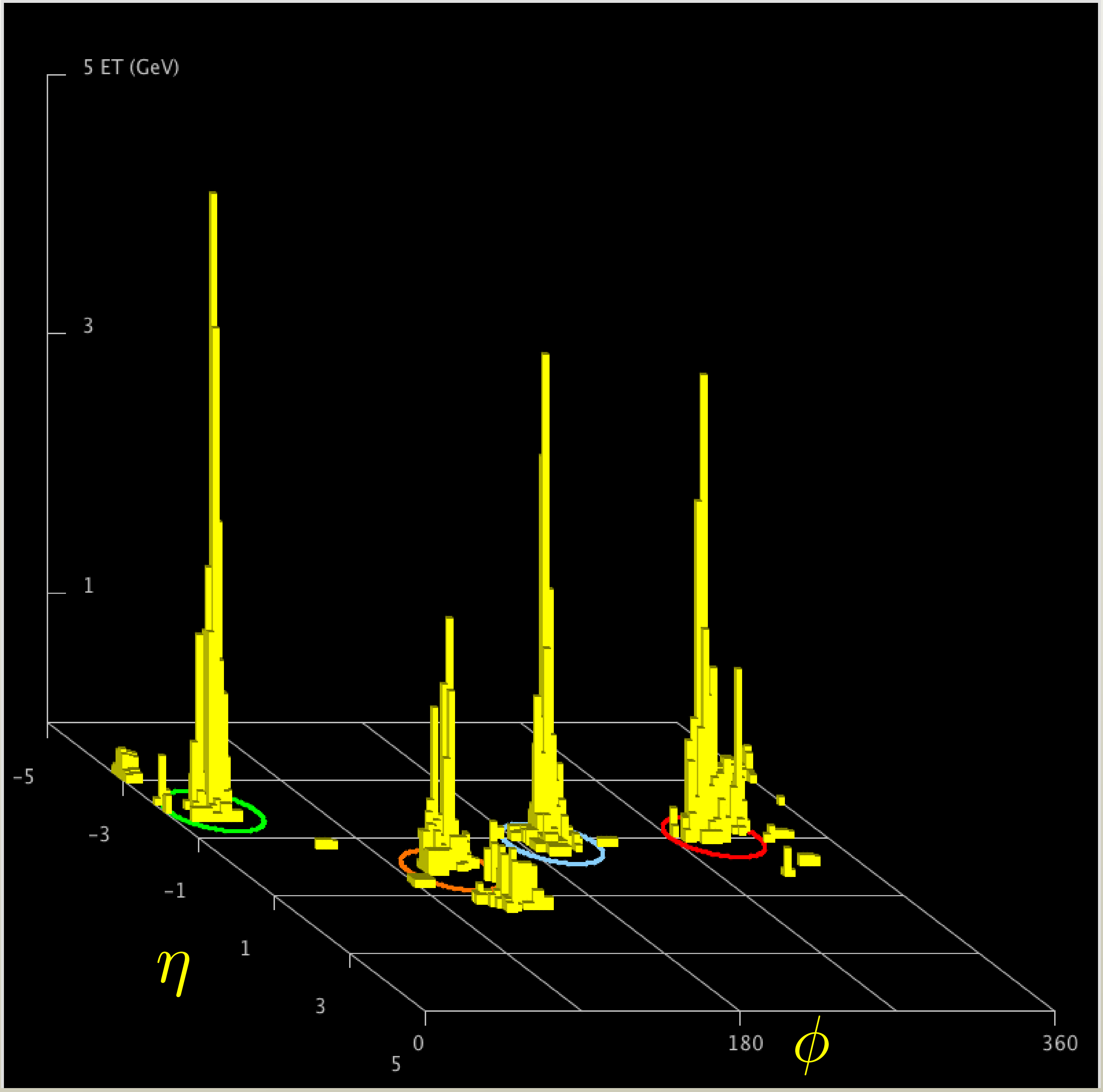
- Lego plot event display.
- Plot $P_T = |\vec{P}_T|$ in each calorimeter cell versus η and ϕ.



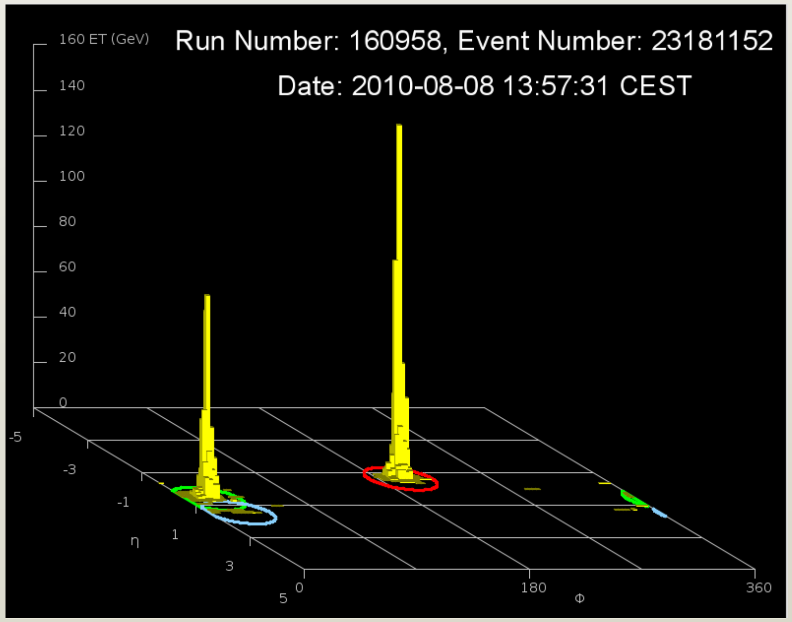
- An Atlas event.



- Sometimes there are more jets.



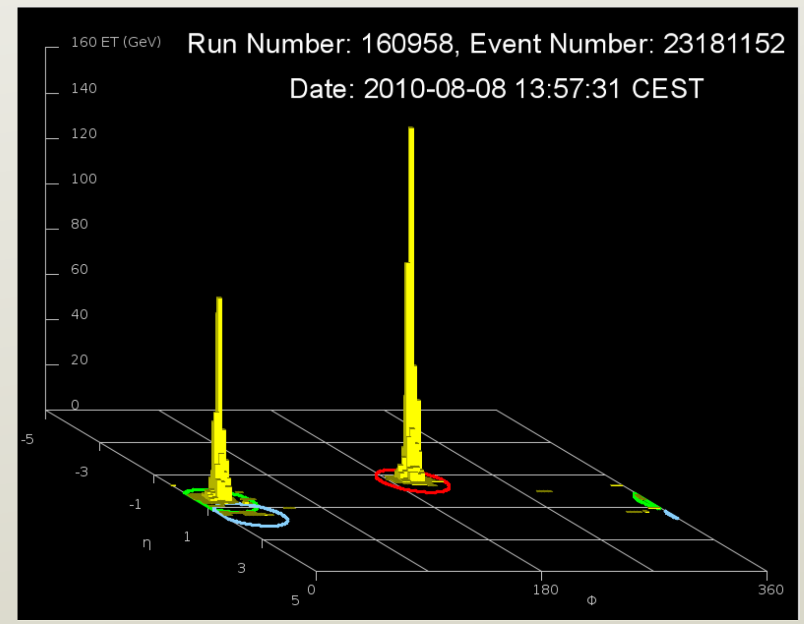
- The P_T is concentrated in a few narrow sprays of particles.
- These sprays are called jets.
- In this event, there are two jets.
- Events with big $\sum_i P_{T,i}$ are rare.

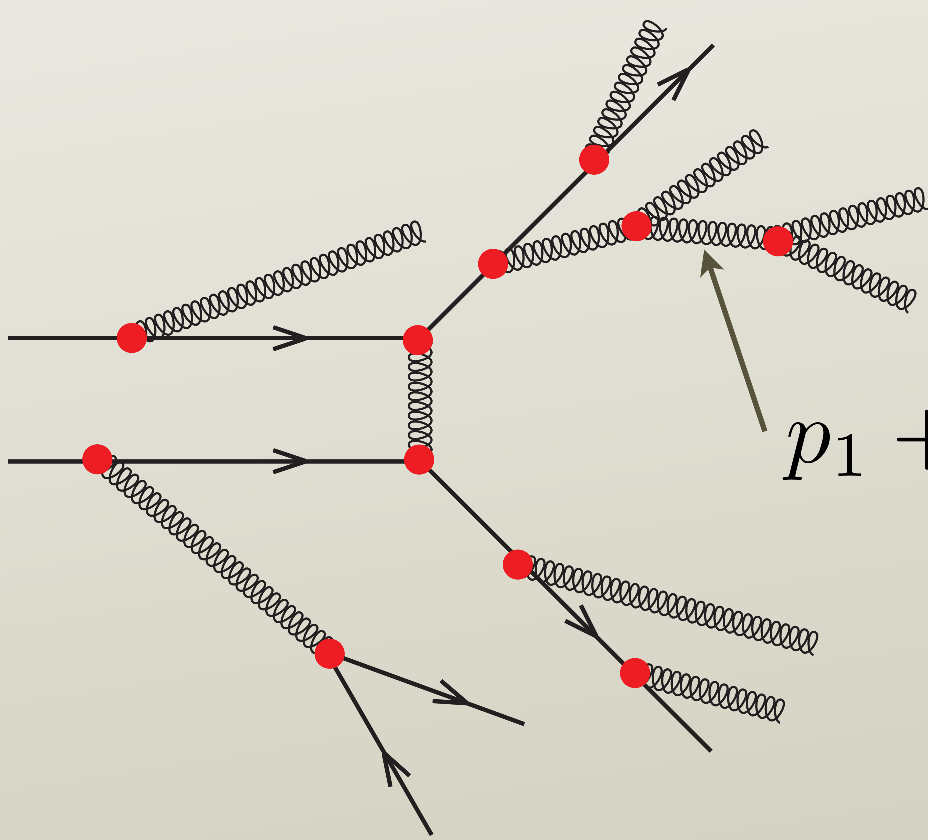


- When they happen, the P_T is always in jets.

Why are there jets?

- Here is a Feynman diagram for quark-quark scattering, with additional radiation.
- Diagram has a factor $1/(p_1 + p_2)^2$.





- If $p_1 \to 0$, then $1/(p_1 + p_2)^2 \to \infty$.
- If $p_2 \to 0$, then $1/(p_1 + p_2)^2 \to \infty$.
- If $p_2 \to \lambda p_1$, then $1/(p_1 + p_2)^2 \to \infty$.

- So probability is big to get a spray of collimated particles plus some low momentum particles at wide angles.

Prediction pre-QCD

- Bjorken, Berman and Kogut (1971) had it figured out before jets were seen and before QCD.
- "... the isolated high P_T partons will communicate with the 'wee' partons by cascade emission of partons."

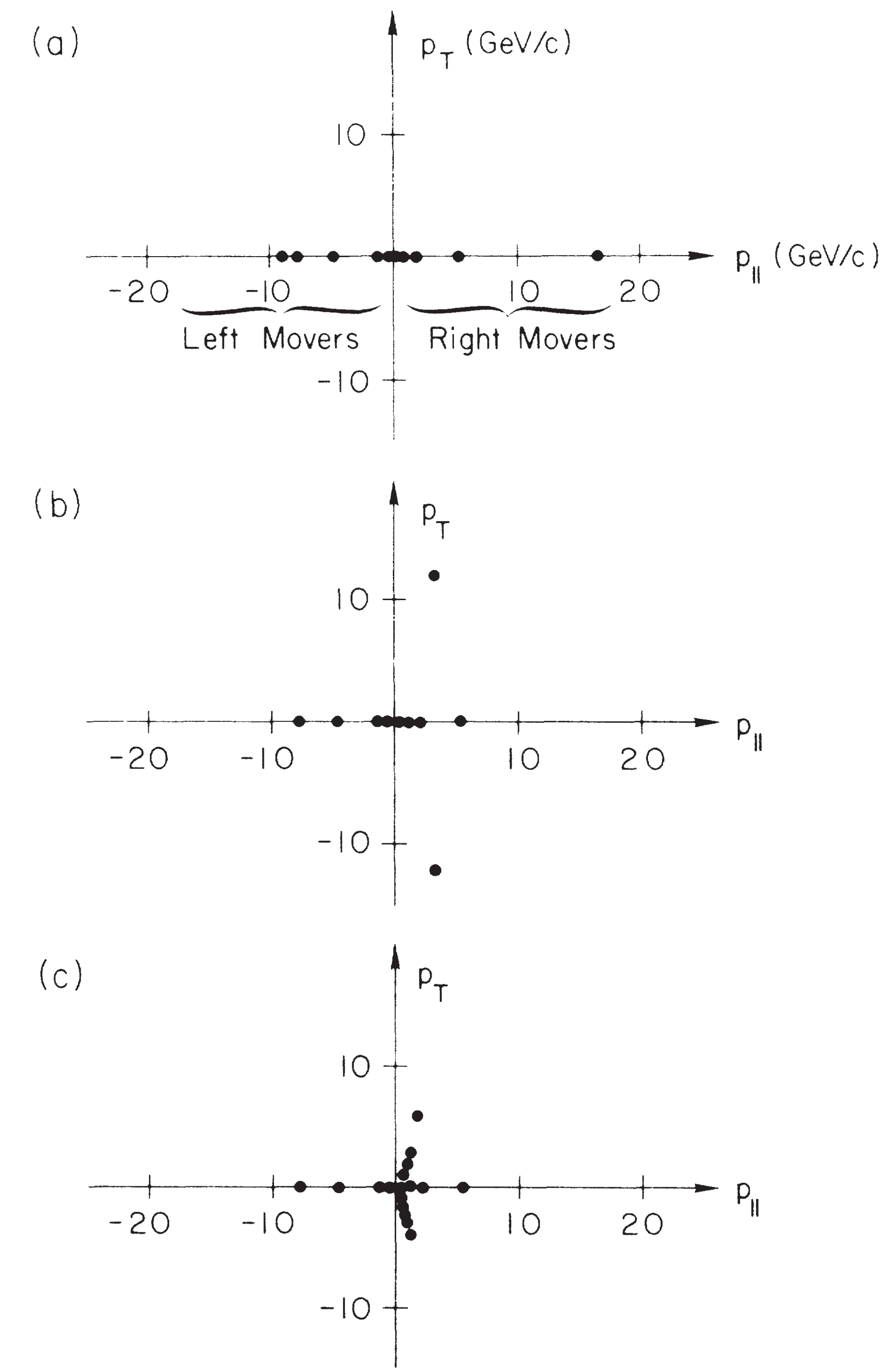


FIG. 4. A momentum-space visualization of hadron-hadron deep-inelastic scattering occurring in three steps.

Electron-positron to hadrons provided early evidence

- The PETRA accelerator (DESY) had enough energy to make jets clearly visible.
- The PETRA experiments had 4π detectors, so that one could be convinced that two and three jet events existed with single event displays.

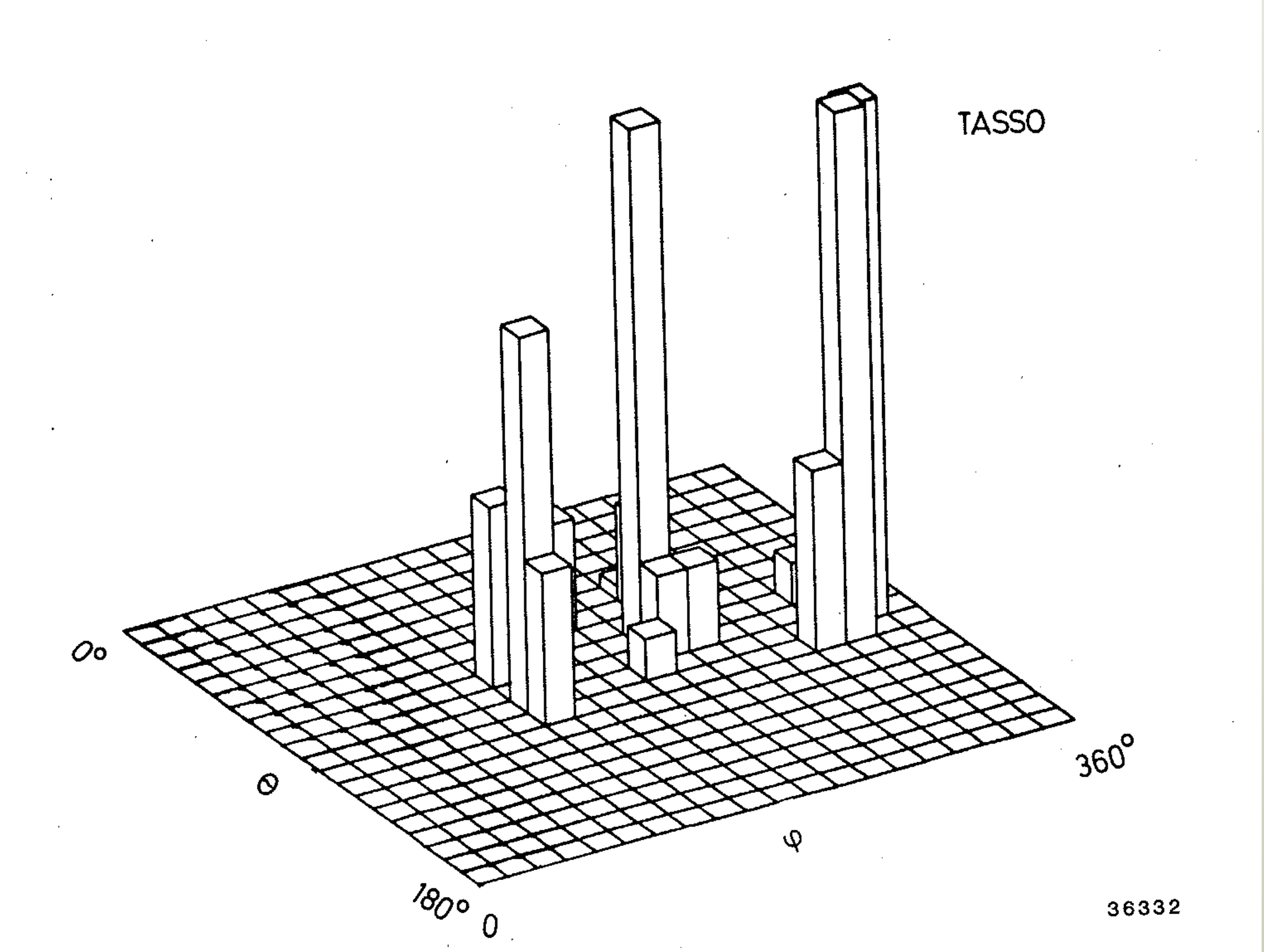
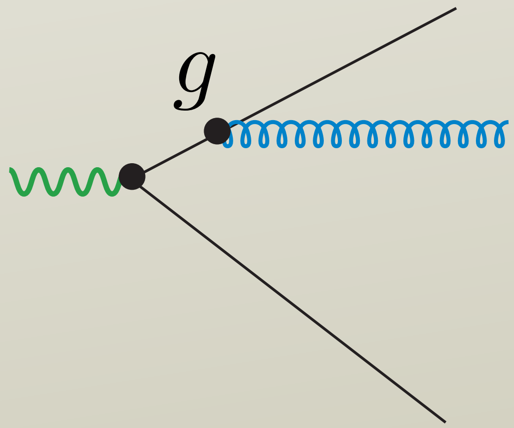


Fig.9 A typical three jet event

from G. Wolf, Multiparticle Conference, 1983

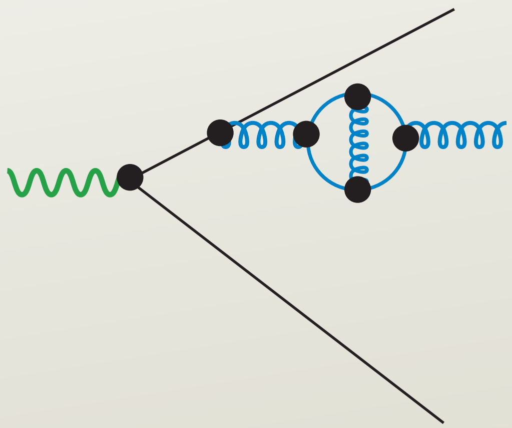
Renormalization of the QCD coupling

- At each vertex in a diagram, there is a factor g.
- $g^2/(4\pi) = \alpha_s$.
- Consider a diagram in which momenta are of order Q.

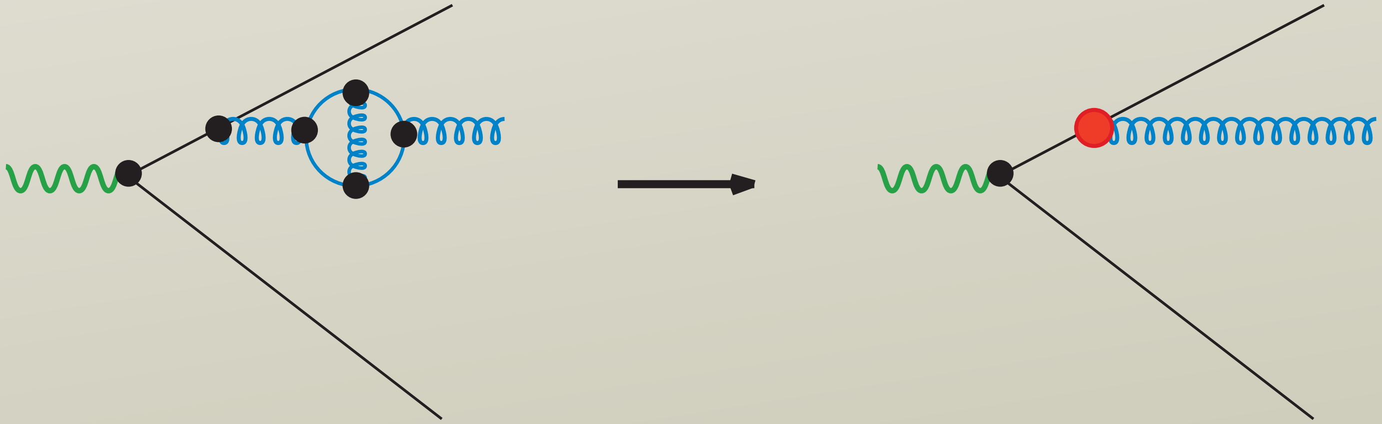


- We can add loop diagrams with $\int d^4k \cdots$.

- For some loop diagrams, $k^2 \gg Q^2$ is important.



- Surprisingly, we can (approximately) omit these loop diagrams if we simply adjust α_s.



- Then the value of the effective α_s depends on Q.

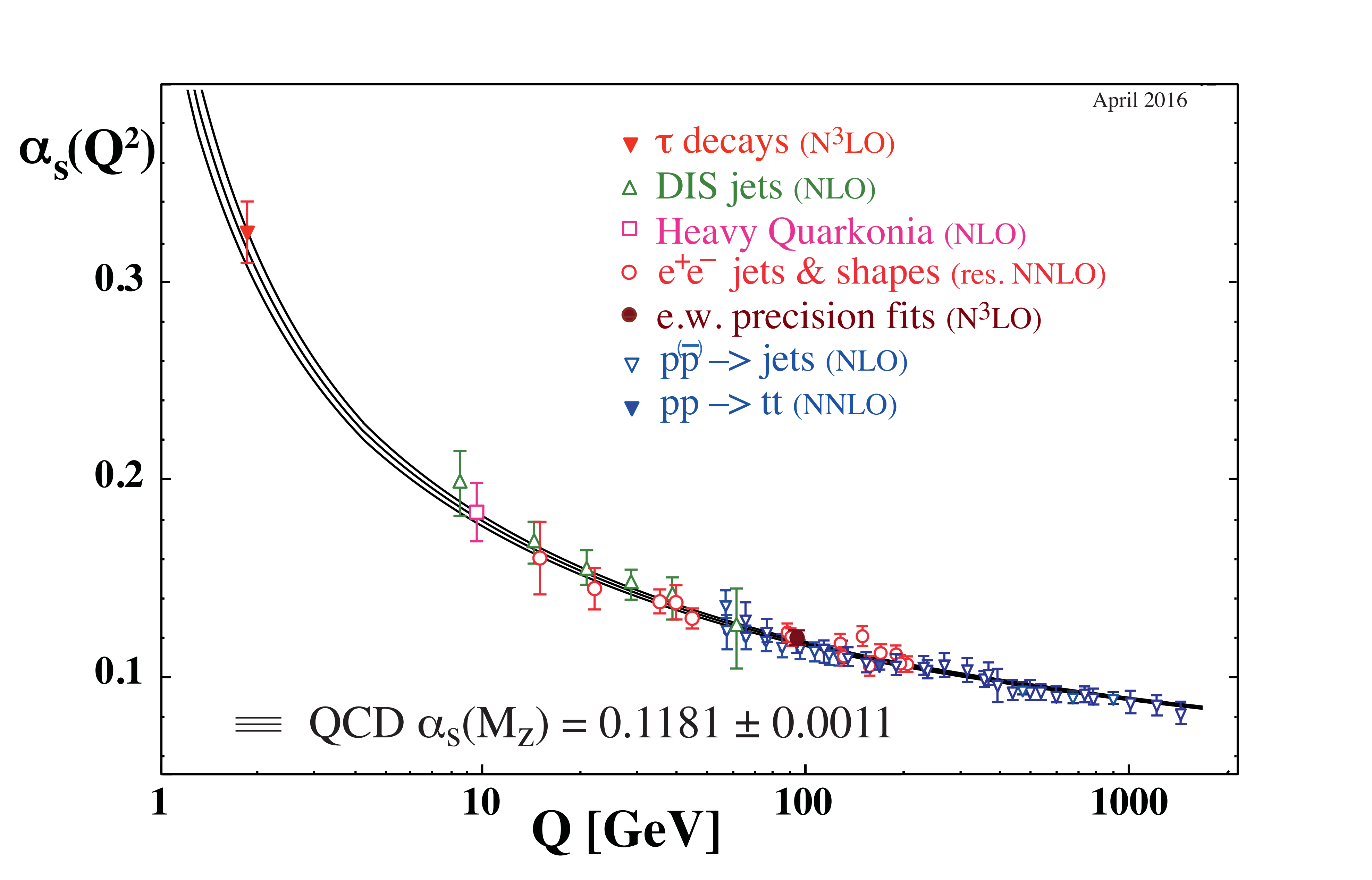
- Result of renormalization group analysis (M = cutoff scale).

$$
\begin{aligned}
\alpha_s(Q) &\sim \alpha_s(M) - (\beta_0/4\pi)\log(Q^2/M^2)\ \alpha_s^2(M) \\
&\qquad + (\beta_0/4\pi)^2 \log^2(Q^2/M^2)\ \alpha_s^3(M) + \cdots \\
&= \frac{\alpha_s(M)}{1 + (\beta_0/4\pi)\alpha_s(M)\log(Q^2/M^2)} \\
&= \frac{\alpha_s(M_Z)}{1 + (\beta_0/4\pi)\alpha_s(M_Z)\log(Q^2/M_Z^2)}
\end{aligned}
$$

- Note that $\alpha_s(Q)$ decreases as Q increases.

Running can be tested

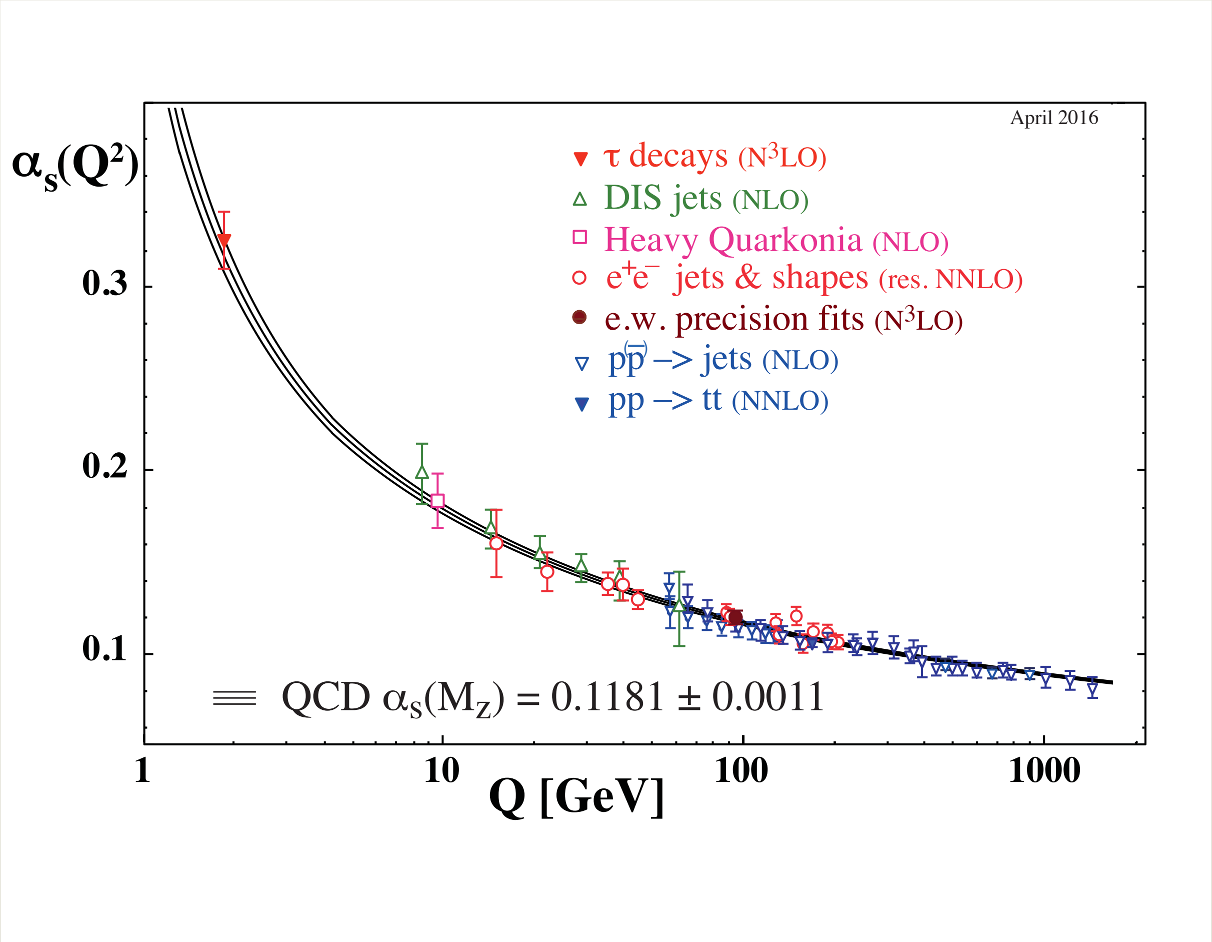
- Plot value of $\alpha_s(Q)$ determined from experiments at scale Q.



Plot from Review of Particle Physics (2016)

A lesson

- Perturbation theory, an expansion in powers of $\alpha_s(Q)$, is not reliable unless Q is large.



Jet cross sections

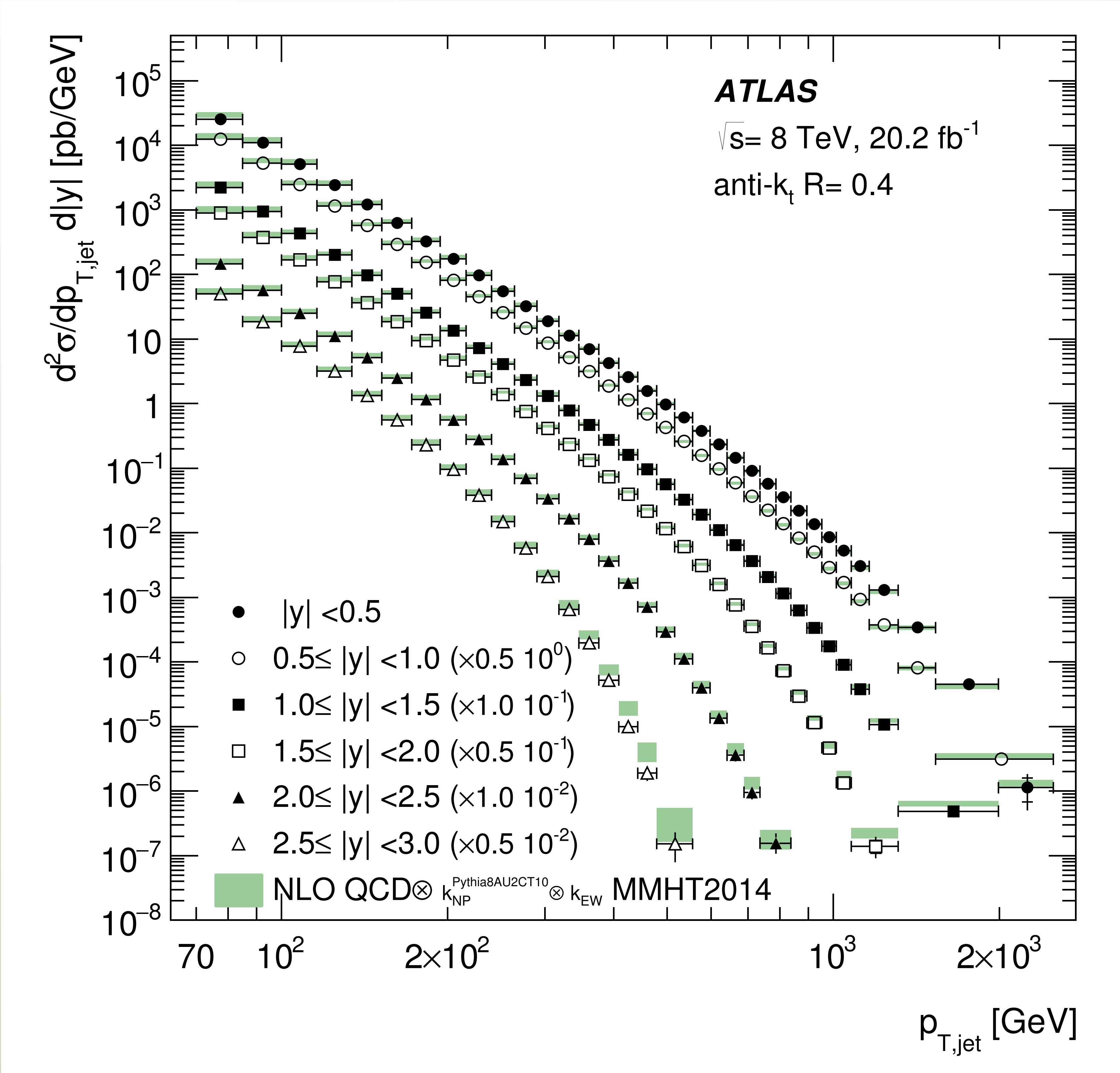
- Consider, for example, the one jet inclusive cross section

$$\frac{d\sigma}{dP_T\,dy}$$

$$P_T = \text{transverse momentum of the jet}$$
$$y = \text{rapidity of the jet} \approx -\log\tan(\theta/2)$$

- Here "inclusive" means that the event has one jet with P_T, y plus anything else.
- One can also look at Z-boson + two jets, missing P_T plus jet, ...

- A result from Atlas.



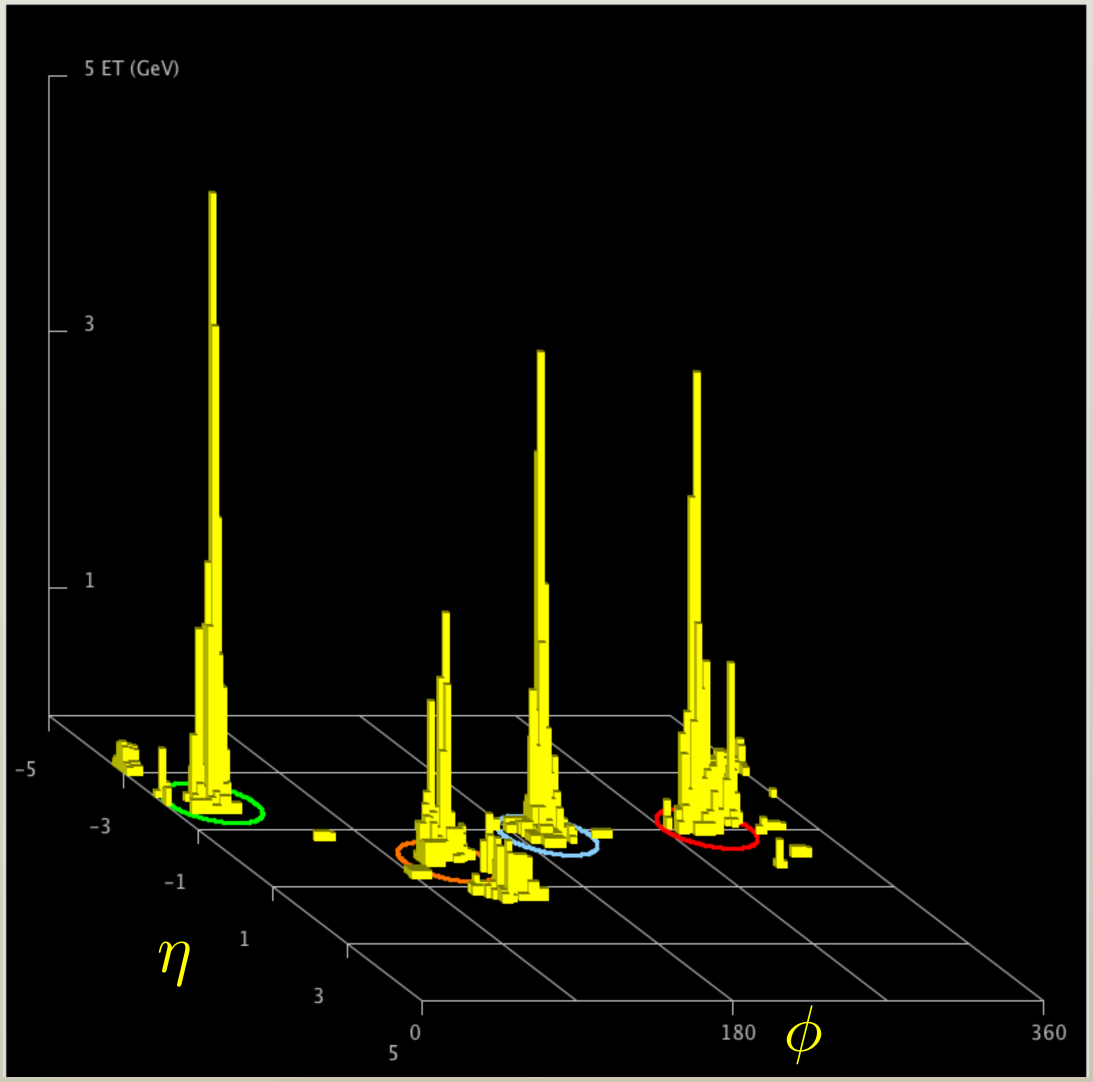
- Note ten orders of magnitude variation in cross section at one $|y|$.

- Approximate agreement of theory and experiment means no new-physics effects are seen to 2 TeV.

We need a jet definition

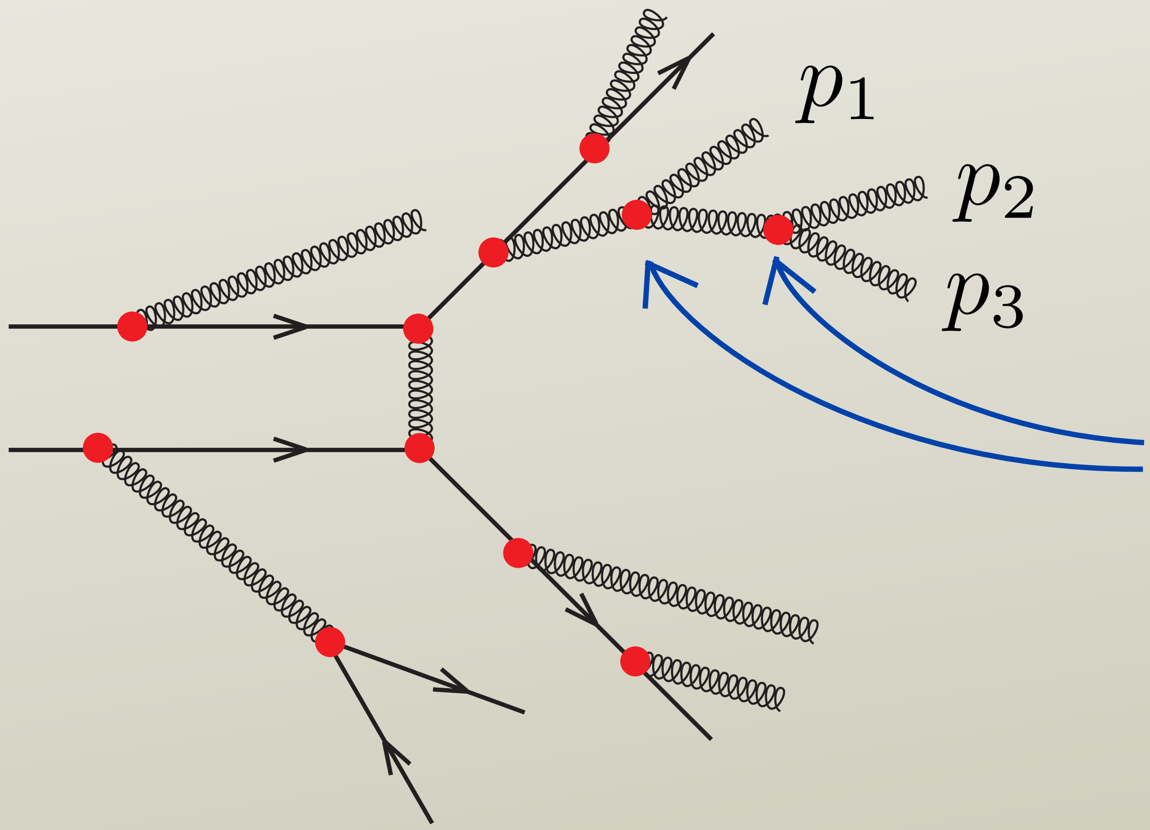
- Otherwise, jets are ambiguous and we cannot define a cross section.

- How many jets are there here?



The definition must be infrared safe

- What jets we measure must not depend on small Q^2 physics.

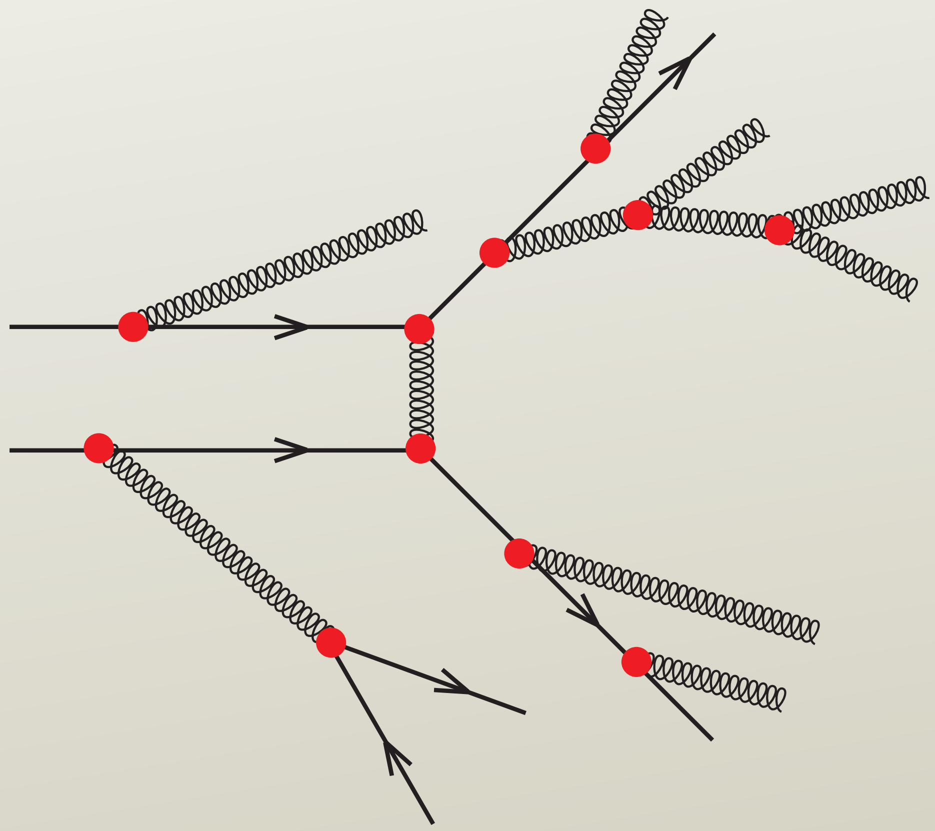


- Suppose $p_2 \approx 0$ and p_1 is almost collinear with p_3
- Then the Q^2 for the two splittings shown is small.
- The corresponding $\alpha_s(Q)$ couplings are large.

- Perturbation theory breaks down.

Infrared safety

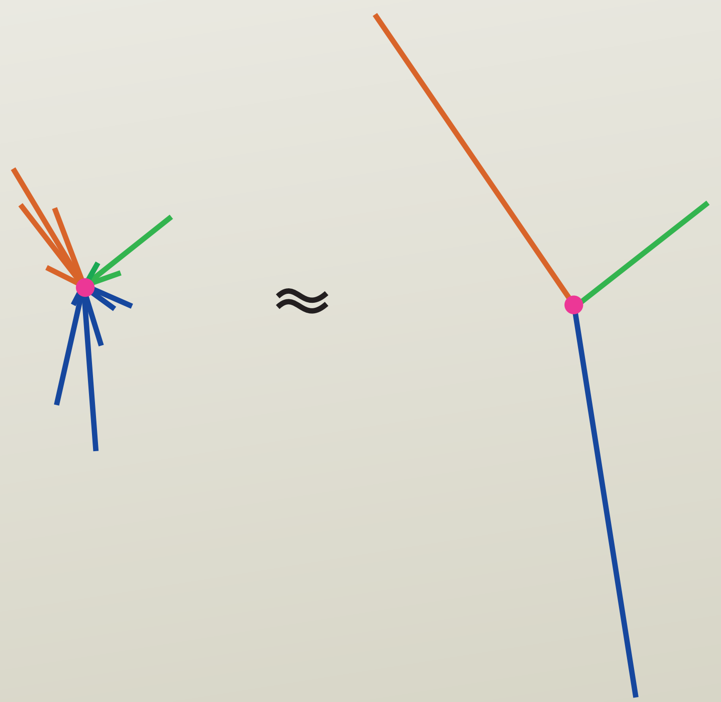
- We construct jets from particle momenta $\{p_1, p_2, \ldots, p_n\}$.
- We get N jets with momenta $\{P_1, P_2, \ldots, P_N\}$.



- If any p_i becomes very small, we should get the same jets by leaving particle i out.
- If any two momenta p_i and p_j become collinear, we should get the same jets by replacing the particles by one with momentum $p_i + p_j$.

What does IR safety mean?

- The physical meaning is that for an IR-safe quantity, the physical event with hadron jets should give approximately the same measurement as a parton event.

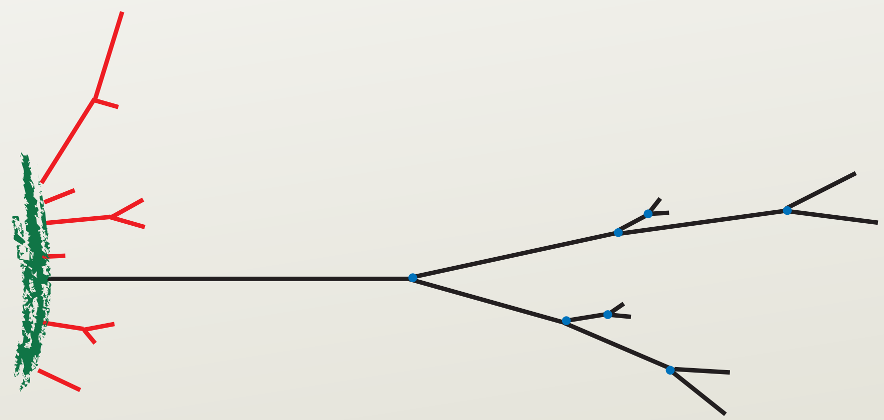


- It also means that in a Monte Carlo simulation (to be discussed later) the hadronization model and the underlying event model should not much matter.

Two kinds of jet algorithms

- There are two kinds of algorithms for defining jets:
  - ✳cone algorithms
  - ✳successive combination algorithms
- Both can be infrared safe.
- I will discuss just the successive combination algorithms.
- These trace back to the JADE collaboration at DESY.

The k_T jet algorithm



- Choose a resolution parameter R.
- Start with a list of protojets, specified by their p_j^μ.
- Start with an empty list of finished jets.
- Result is a list of finished jets with their momenta.
- Many are low p_T debris; just ignore these.

1. For each pair of protojets define

$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

For each protojet define

$$d_i = p_{T,i}^2$$

2. Find the smallest of all the d_{ij} and the d_i. Call it $d_{\min}$
3. If $d_{\min}$ is a d_{ij}, merge protojets i and j into a new protojet k with

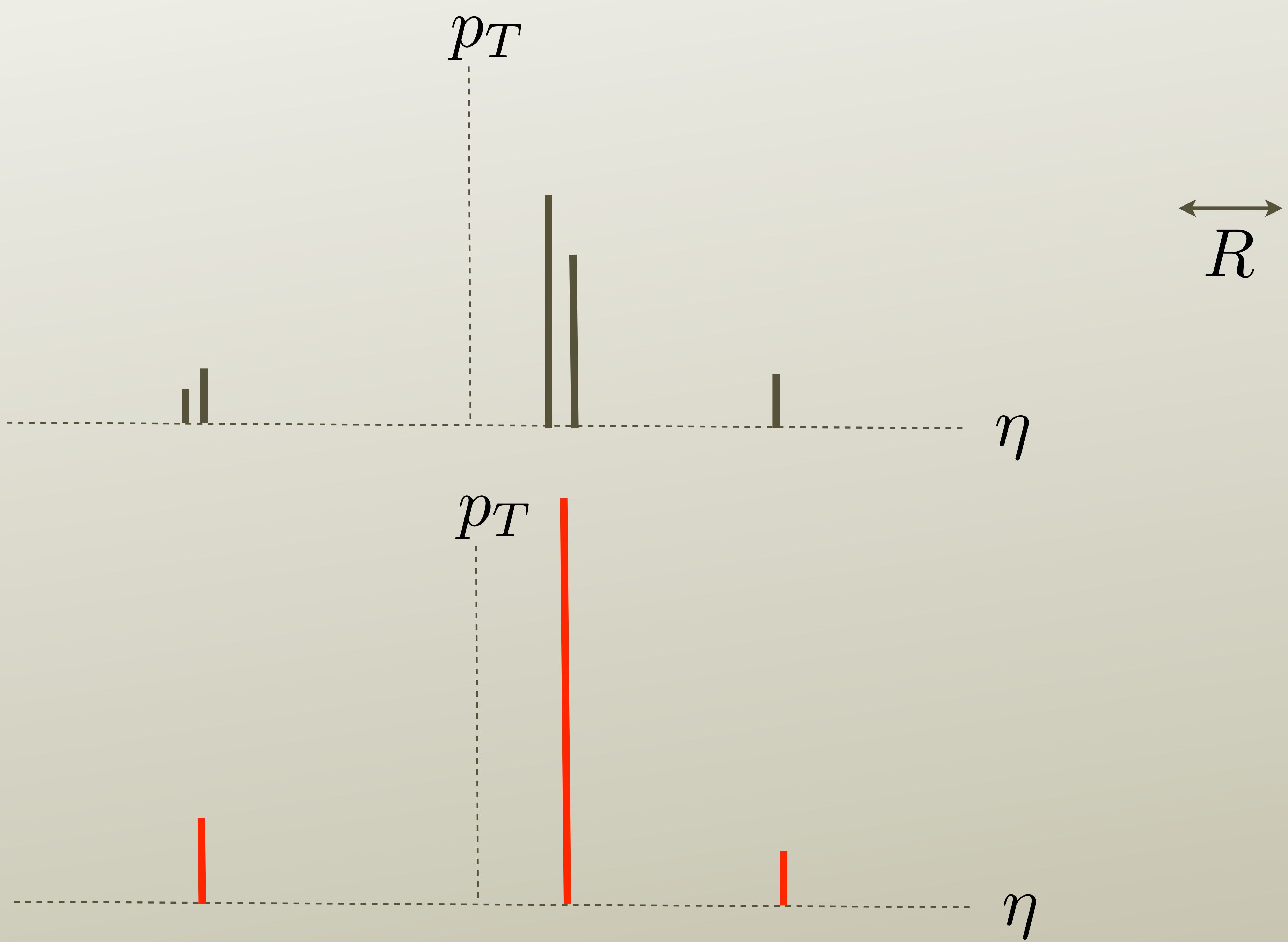
$$p_k^\mu \quad = \quad p_i^\mu + p_j^\mu$$

4. If $d_{\min}$ is a d_i, then protojet i is "not mergable." Remove it from the list of protojets and add it to the list of jets.
5. If protojets remain, go to 1.

Example

$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2) \left[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2\right]/R^2$$

$$d_i = p_{T,i}^2$$



Infrared safety of this

- Suppose $p_j \to 0$.
- Then when it merges with another protojet,

$$p_k = p_i + p_j \to p_i$$

- If it never merges, then it just remains as a low p_T jet at the end.
- Suppose $p_i = \lambda p_j$.
- Then protojets i and j are merged at the start to

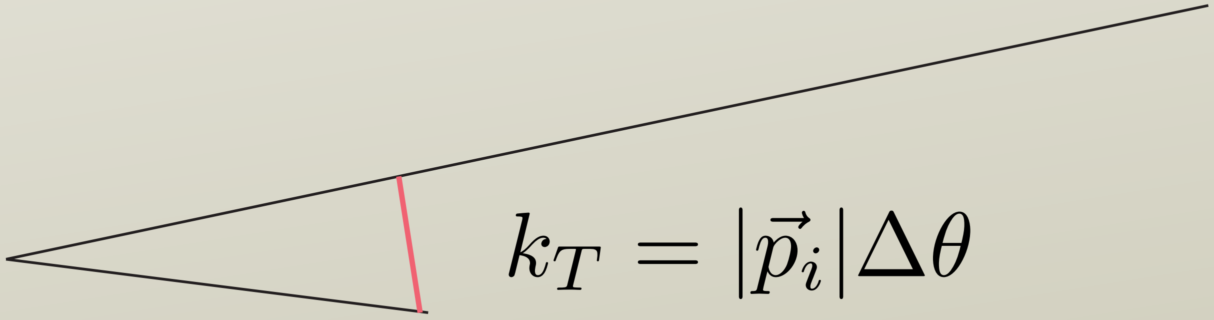
$$p_k = p_i + p_j$$

Why the name?

$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

is essentially

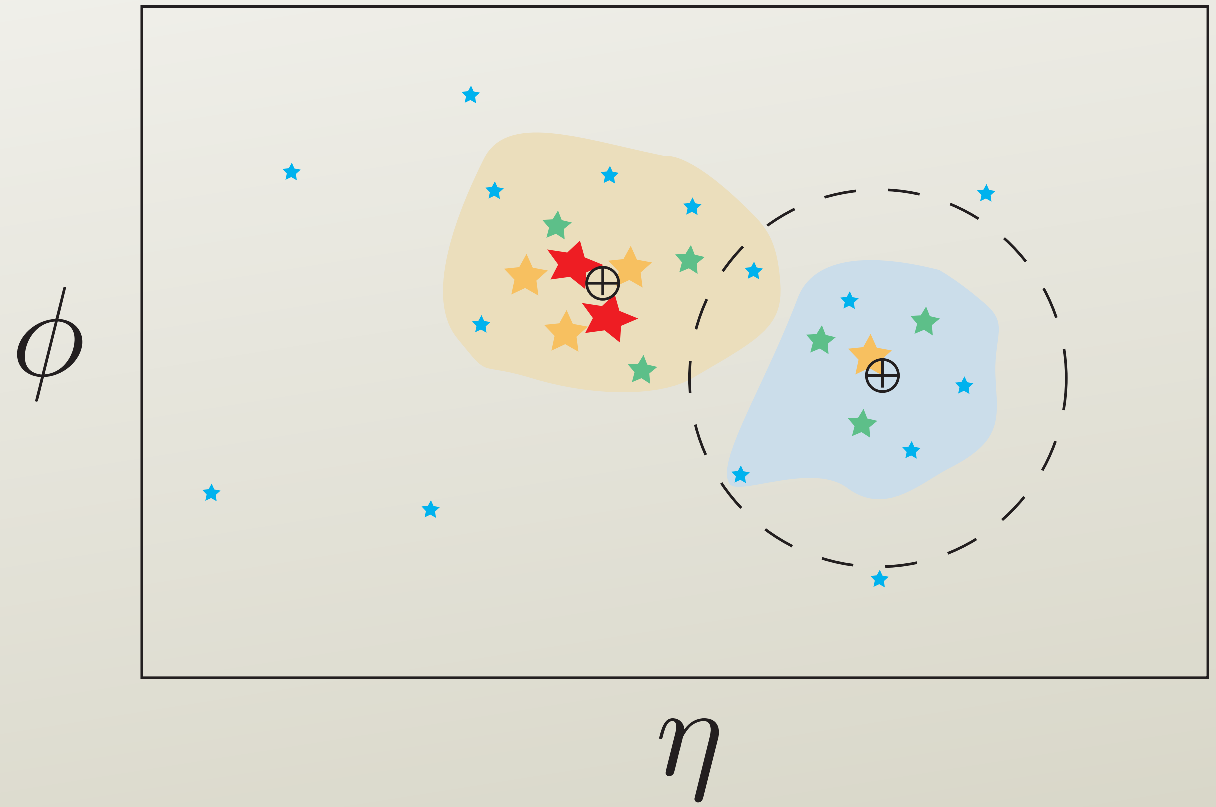
$$d_{ij} = k_T^2/R^2$$



$$k_T = |\vec{p}_i|\Delta\theta$$

The "no merge" condition

- Suppose $p_{T,i}^2 < p_{T,j}^2$.

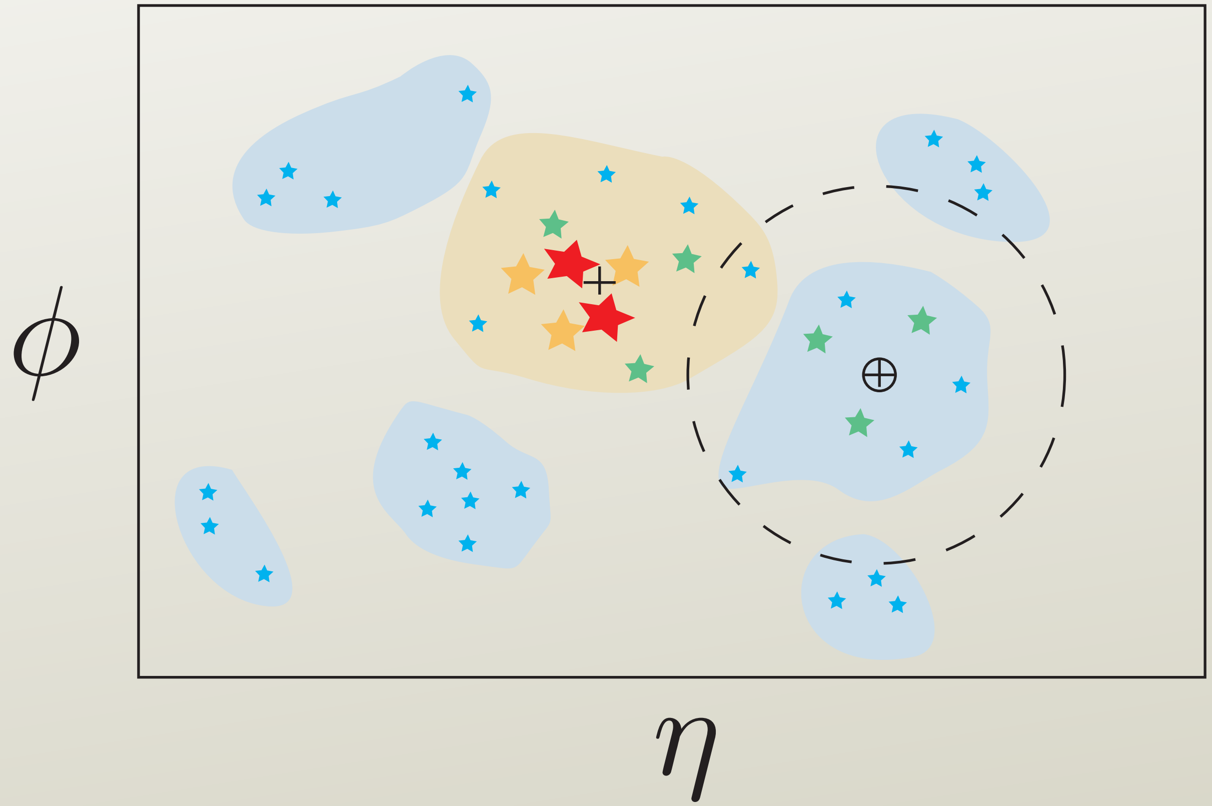


$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$
$$d_i = p_{T,i}^2$$

- Protojet i is not mergable with parton j if $d_{ij} > d_i$. That is if

$$[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2] > R^2$$

Why the no merge condition

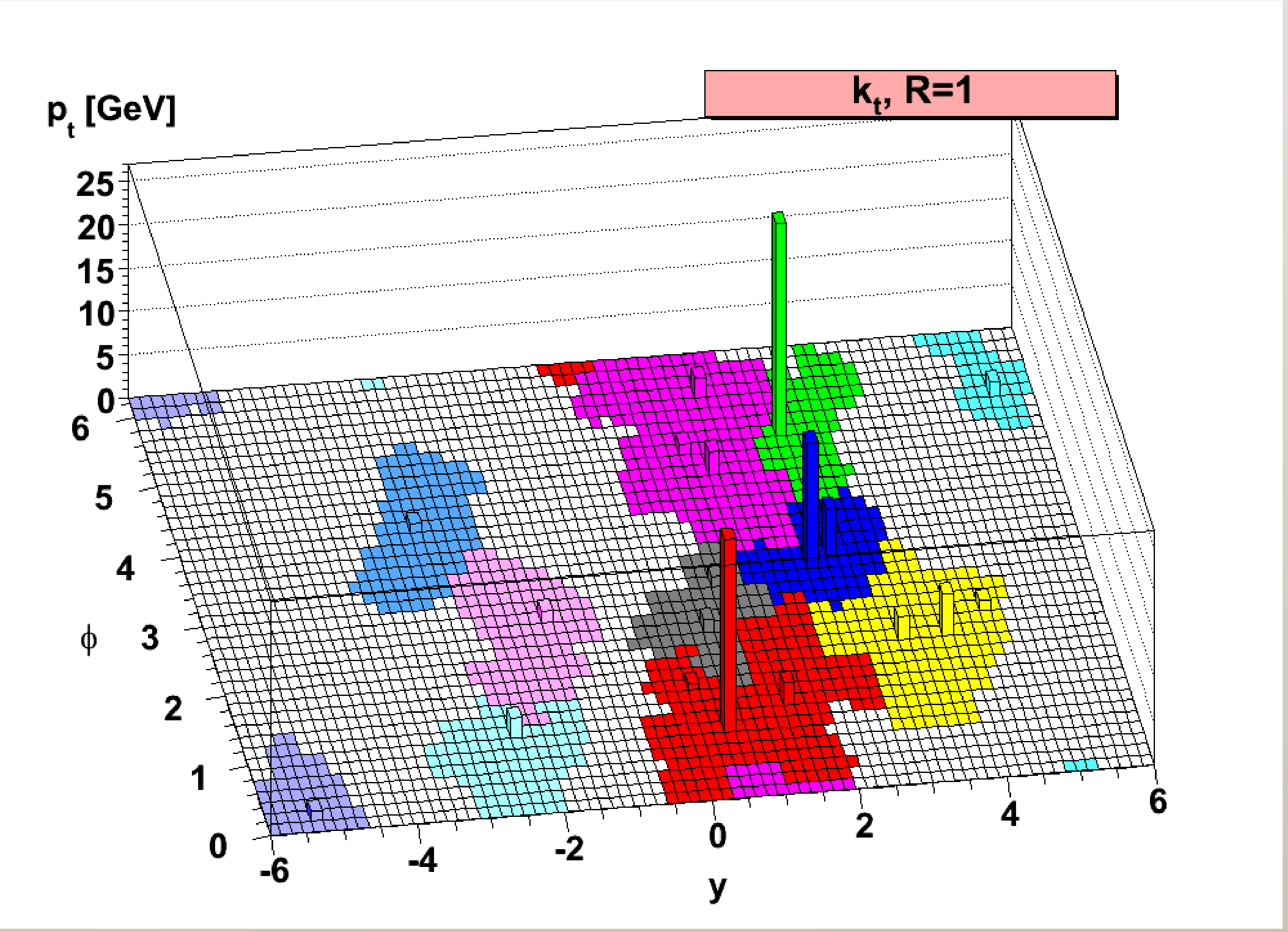


- There will be many soft jets.
- They should not merge into a few giant jets.

Example with k_T

- Here is an example event from Cacciari, Salam, and Soyez (2008).

- With the k_T algorithm, we see what detector area goes into each jet. The area is irregular.



Shower histories

- The graph of parton joinings (read right to left) can be thought of as a graph of parton splittings (read left to right) in a parton shower.
- If we use the k_T jet algorithm, then the parton splittings go from harder (high k_T) to softer (low k_T).
- Beware: the same final state can be generated in many different ways in a parton shower.

The Cambridge-Aachen algorithm

- This is a variation on the general successive combination plan.
- Use

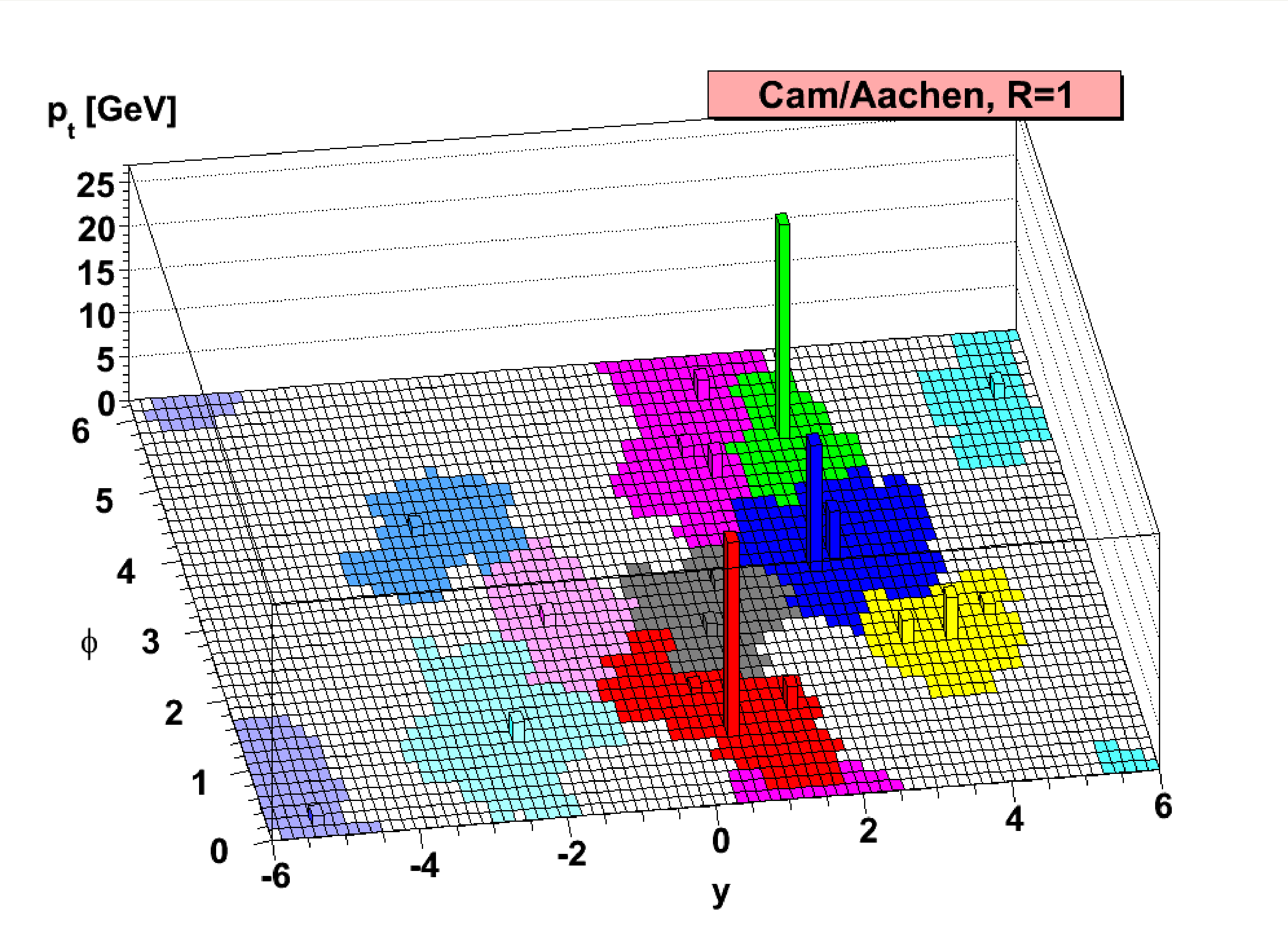
$$d_{ij} = [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

$$d_i = 1$$

- Thus only angles count.
- Keep everything else the same.

Example with *C-A*

- Here is the same example event from Cacciari, Salam, and Soyez (2008).
- With the Cambridge-Aachen algorithm, we see what detector area goes into each jet. Jets are irregular.



The anti-k_T algorithm

- This is another variation on the general successive combination plan.
- Use

$$d_{ij} = \min\left(\frac{1}{p_{T,i}^2}, \frac{1}{p_{T,j}^2}\right)[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

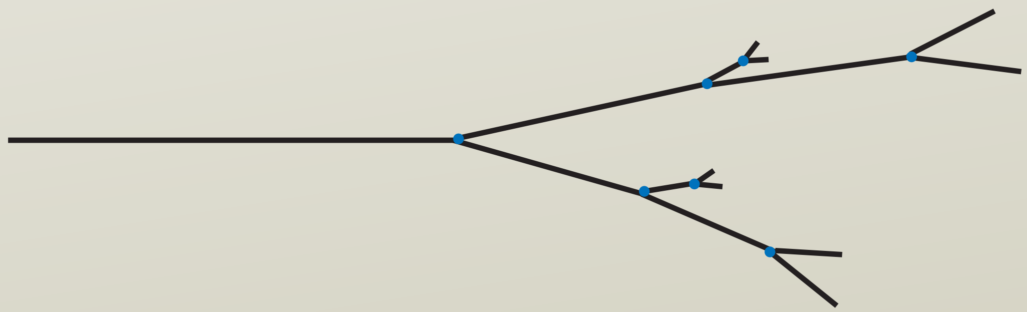
$$d_i = \frac{1}{p_{T,i}^2}$$

- Keep everything else the same.

$$d_{ij} = \min\left(\frac{1}{p_{T,i}^2}, \frac{1}{p_{T,j}^2}\right) [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

$$d_i = \frac{1}{p_{T,i}^2}$$

- This puts protojets together in an order that is nothing like the order that any shower Monte Carlo would generate splittings.

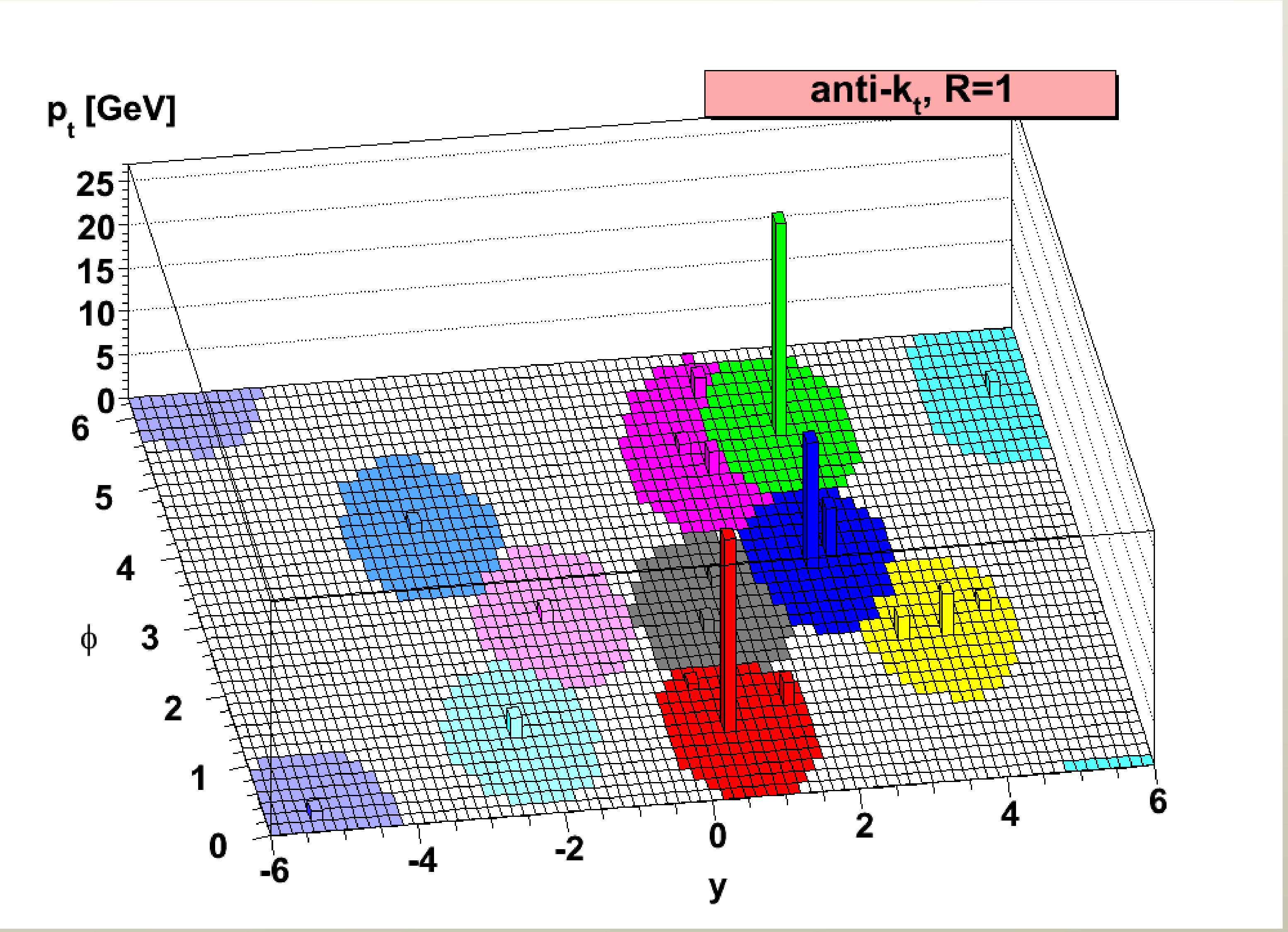


- The highest P_T protojet has priority to absorb nearby softer protojets (out to radius R).

Example with anti-k_T

- Here is the same example event from Cacciari, Salam, and Soyez (2008).

- With the anti-k_T algorithm, we see what detector area goes into each jet. High P_T jets are round.



Conclusions

- QCD gives us jets.
- Jets are seen in experiments.
- To measure jet cross sections, you need a careful definition of a jet.
- The definition needs to be infrared safe.
- Definitions typically use an angular size parameter R.
- The conceptually simplest kind of definition successively combine small protojets into bigger ones.