This is a very brief, and schematic, discussion of the cancellation of IR divergences in QED, using a photon mass as a regulator, showing the role of the energy resolution. – George

IR divergences in photon cross sections at order α_{EM}

Relative to a lowest-order cross section, the IR divergence from one *virtual* photon is of the schematic form

$$\frac{d\sigma_{\mathrm{virt}}^{(1,\mathrm{IR})}}{d\Omega} \sim -\frac{d\sigma_0}{d\Omega} \times \frac{\alpha_{\mathrm{EM}}}{\pi} \sum_e \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{E_e}{m_\gamma}\right) \tag{1}$$

where the sum over "e" is over all the electrons and positrons involved in the lowest-order scattering process. In the limit $m_\gamma \to 0$, this is a negatively infinite correction to lowest order.

Relative to a lowest-order cross section, the IR divergence from one *real* photon is of the schematic form

$$\frac{d\sigma_{\mathrm{real}}^{(1,\mathrm{IR})}}{d\Omega} \sim +\frac{d\sigma_0}{d\Omega} \times \frac{\alpha_{\mathrm{EM}}}{\pi} \sum_e \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{\epsilon E_{\mathrm{tot}}}{m_\gamma}\right) \tag{2}$$

where $\epsilon E_{\mathrm{tot}}$ is the "resolution energy", below which we sum over all photon radiation. This is a positively infinite correction to lowest order. Here "ϵ" is a small number (nothing to do with dimensional regularization) and E_{tot} is just the overall energy scale of the process.

In the sum of the two corrections, dependence on the photon mass cancels, and we get

$$\frac{d\sigma_{\mathrm{tot}}^{(1,\mathrm{IR})}}{d\Omega} \sim -\frac{d\sigma_0}{d\Omega} \times \frac{\alpha_{\mathrm{EM}}}{\pi} \sum_e \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{E_e}{\epsilon E_{\mathrm{tot}}}\right) \tag{3}$$

Because α_{EM} is so small, the sum of these two "infinite" contributions is typically a very small correction to the lowest order cross section.

IR divergences in photon cross sections at all orders

The leading infrared behavior for n-photon emission with *all orders* in virtual corrections is just the exponential of the virtual correction times $1/n!$ time the nth power of the order α_{EM} result. Leaving out the explicit sum over electrons, this is

$$\frac{d\sigma_n^{(\mathrm{IR})}}{d\Omega} \sim \frac{d\sigma_0}{d\Omega} \times \frac{1}{n!} \left(\frac{\alpha_{\mathrm{EM}}}{\pi} \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{\epsilon E_{\mathrm{tot}}}{m_\gamma}\right)\right)^n \exp\left[-\frac{\alpha_{\mathrm{EM}}}{\pi} \ln\left(\frac{E_e}{m_e}\right) \ln\left(\frac{E_e}{m_\gamma}\right)\right] \tag{4}$$

summing over n (i.e. any number of soft photons) gives the exponential of the order α_{EM} result, which again is typically a small correction.

Notice that at very, very high energy, $E_e/m_e \to \infty$, the corrections need not be small. This is the case of QCD, where the coupling is also much larger. In essence, in perturbative QCD jet cross sections, a logarithm of the "resolution" parameter R replaces the logs of E_q/m_q for quarks (and gluons) in (IR safe) jet cross sections at all orders.