

Higgs I : Fundamentals

CTEQ Summer School 2017

Ciaran Williams

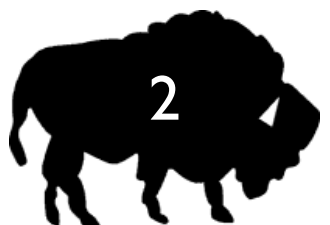


Lecture 1 :

- Fundamentals of the Higgs mechanism, SSB gauge invariance, masses of bosons and fermions

Lecture 2 :

- Higgs at the LHC: Higgs production, EFT, Higgs decays, Current and future studies of the Higgs.





- These slides can be downloaded from :
- Additionally a longer PDF writeup of this lecture:
- Also there are 4 very detailed CERN yellow reports on the Higgs boson :
https://inspirehep.net/search?ln=en&ln=en&p=Handbook+of+LHC+Higgs+Cross+Sections&of=hb&action_search=Search&sf=&so=d&rm=&rg=25&sc=0



On July 4th CERN announced the discovery of a new particle.



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‘Physics world celebrates Higgs boson discovery’

**CBS
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“Higgs boson-like particle discovery claimed at LHC”

**BBC
NEWS**



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Scientists at the Fermilab in Batavia, Ill., on Wednesday watched the presentation about the discovery of the Higgs boson, which was shown from Geneva.



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The New York Times



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Global Symmetries and Gauge Theories

We begin by refreshing our notion of Gauge invariance.

We make the following transformation on our Dirac Spinor.

$$\psi \rightarrow e^{i\alpha(x)} \psi \qquad \bar{\psi} \rightarrow \bar{\psi} e^{-i\alpha(x)}$$



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We make the following transformation on our Dirac Spinor.

$$\psi \rightarrow e^{i\alpha(x)} \psi \qquad \bar{\psi} \rightarrow \bar{\psi} e^{-i\alpha(x)}$$

Then investigate what happens to the Dirac Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Under this transformation.



$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

The mass term is trivially gauge invariant since

$$\bar{\psi}\psi \rightarrow \bar{\psi}e^{-i\alpha(x)}e^{i\alpha(x)}\psi = \bar{\psi}\psi$$

However the spacetime derivative is a bit trickier

$$\partial_\mu\psi \rightarrow e^{i\alpha(x)}\partial_\mu\psi + ie^{i\alpha(x)}\psi\partial_\mu\alpha$$

So the kinetic term is not gauge invariant on its own.



We would like to change the derivative such that it transforms as follows

$$D_{\mu}\psi \rightarrow e^{i\alpha(x)} D_{\mu}\psi$$

This “covariant derivative” will be gauge invariant.

We can achieve this transformation by defining the covariant derivative as follows

$$D_{\mu} = \partial_{\mu} - ieA_{\mu}$$

And we demand the following transformation of A.

$$A_{\mu} \rightarrow A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha$$



The magic of gauge invariance is that we just mandated the inclusion of a gauge boson to our free Lagrangian. That is we recovered the interaction part of the QED Lagrangian

$$\begin{aligned}\mathcal{L} &= i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi \\ &= i\bar{\psi}\gamma^\mu\partial_\mu\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - m\bar{\psi}\psi\end{aligned}$$

We can get the full QED Lagrangian by including the Field Strength Tensor (which is gauge invariant)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

So

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

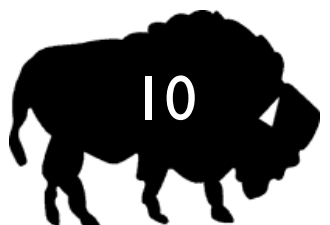


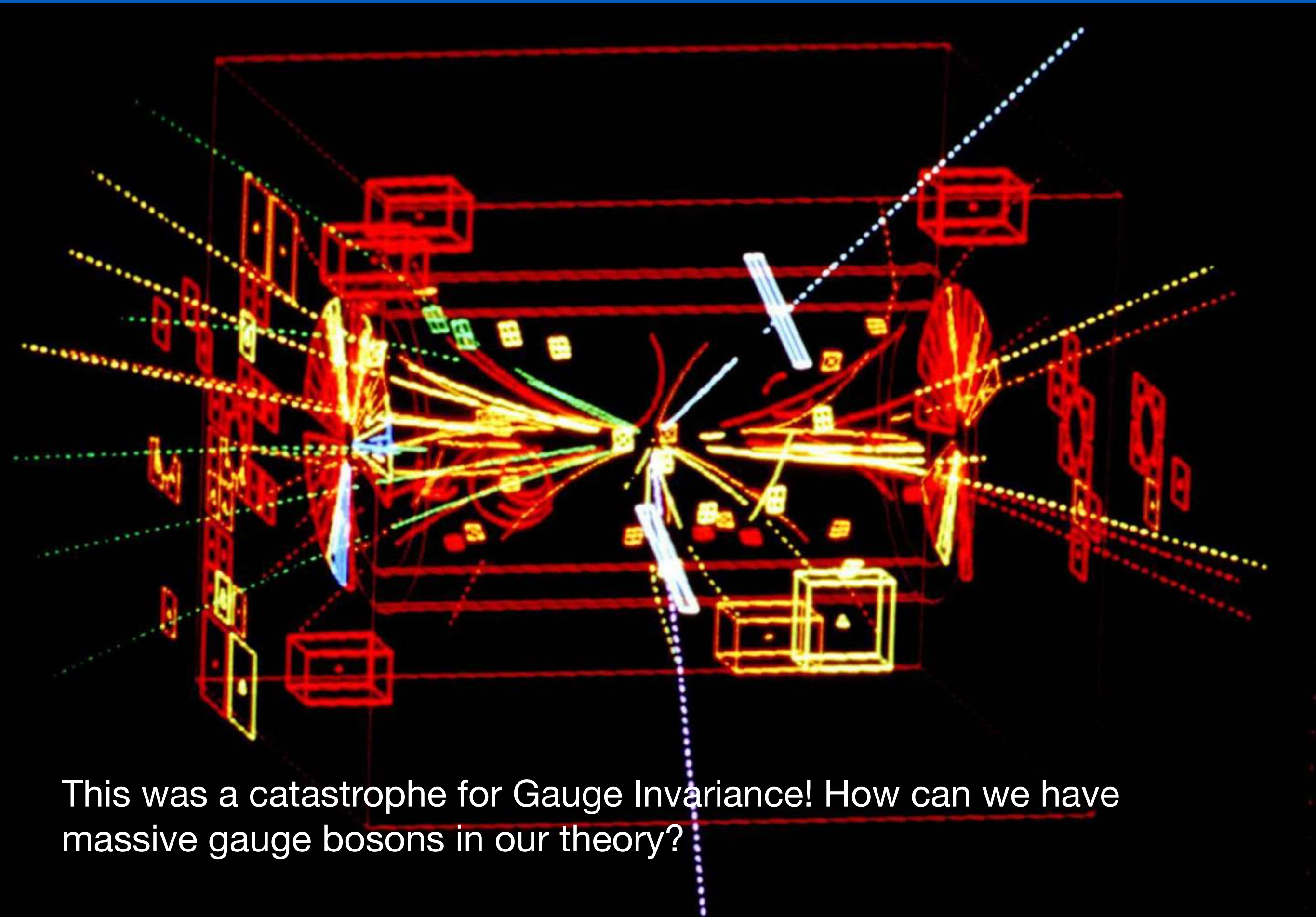
Now, what's important for our discussion today is that it's impossible to write down a mass term which respects gauge invariance since under our U(1) transformation the quadratic term

$$\mathcal{L}_m = \frac{1}{2} m^2 A^\mu A_\mu$$

behaves as follows

$$\frac{1}{2} m^2 A^\mu A_\mu \rightarrow \frac{1}{2} m^2 (A^\mu + \partial^\mu \lambda)(A_\mu + \partial_\mu \lambda) \neq \frac{1}{2} m^2 A^\mu A_\mu$$





This was a catastrophe for Gauge Invariance! How can we have massive gauge bosons in our theory?

Spontaneous Symmetry Breaking of a Gauge Theory

We start with a toy example, which actually contains nearly all of the physics we will need.

We have a Lagrangian

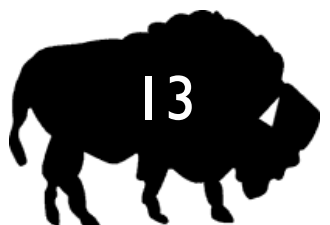
$$\mathcal{L} = T - V = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \left(\frac{1}{2}\mu^2 \phi^2 + \frac{1}{4}\lambda \phi^4 \right)$$

This is invariant under the following (discrete) symmetry

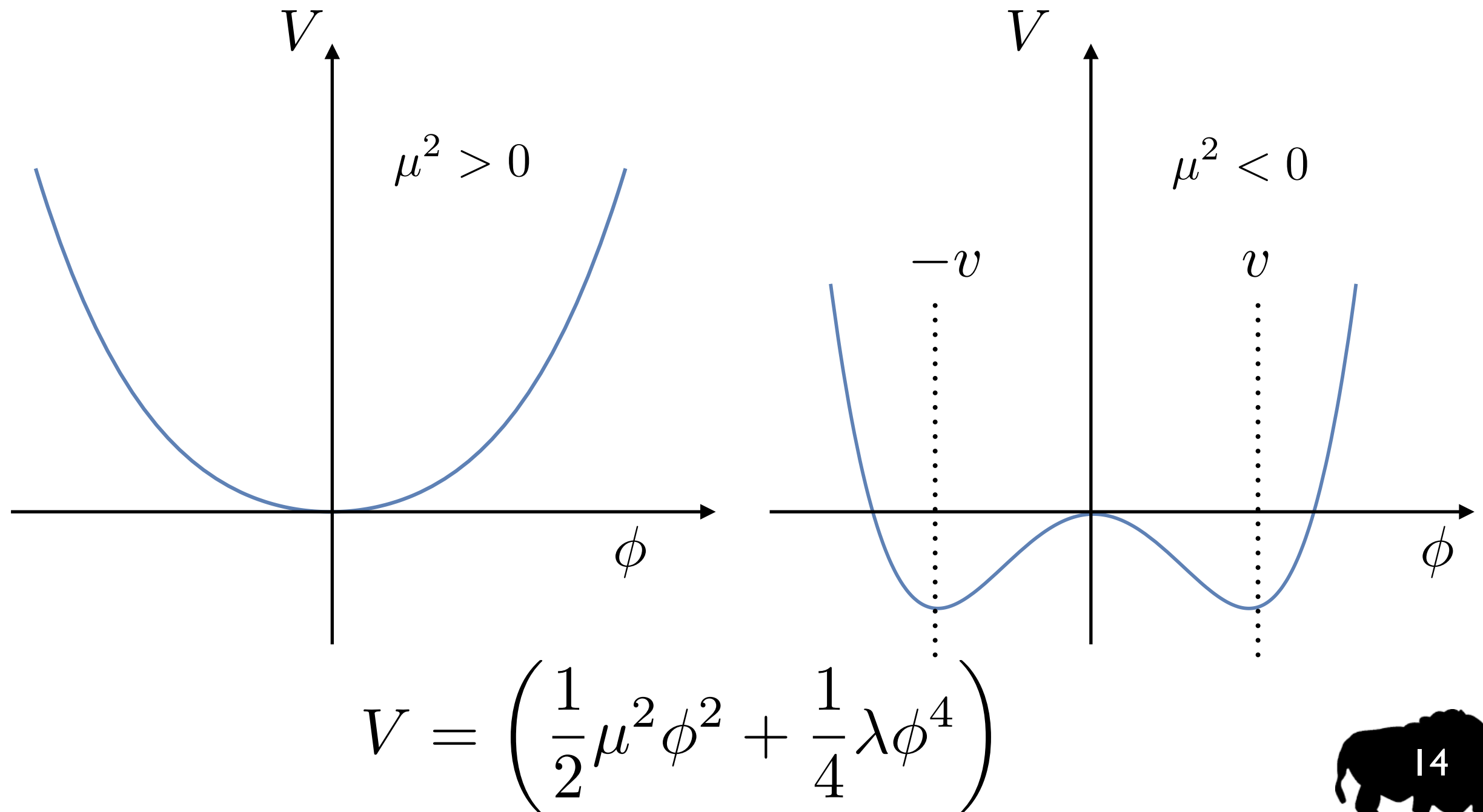
$$\phi \rightarrow -\phi$$

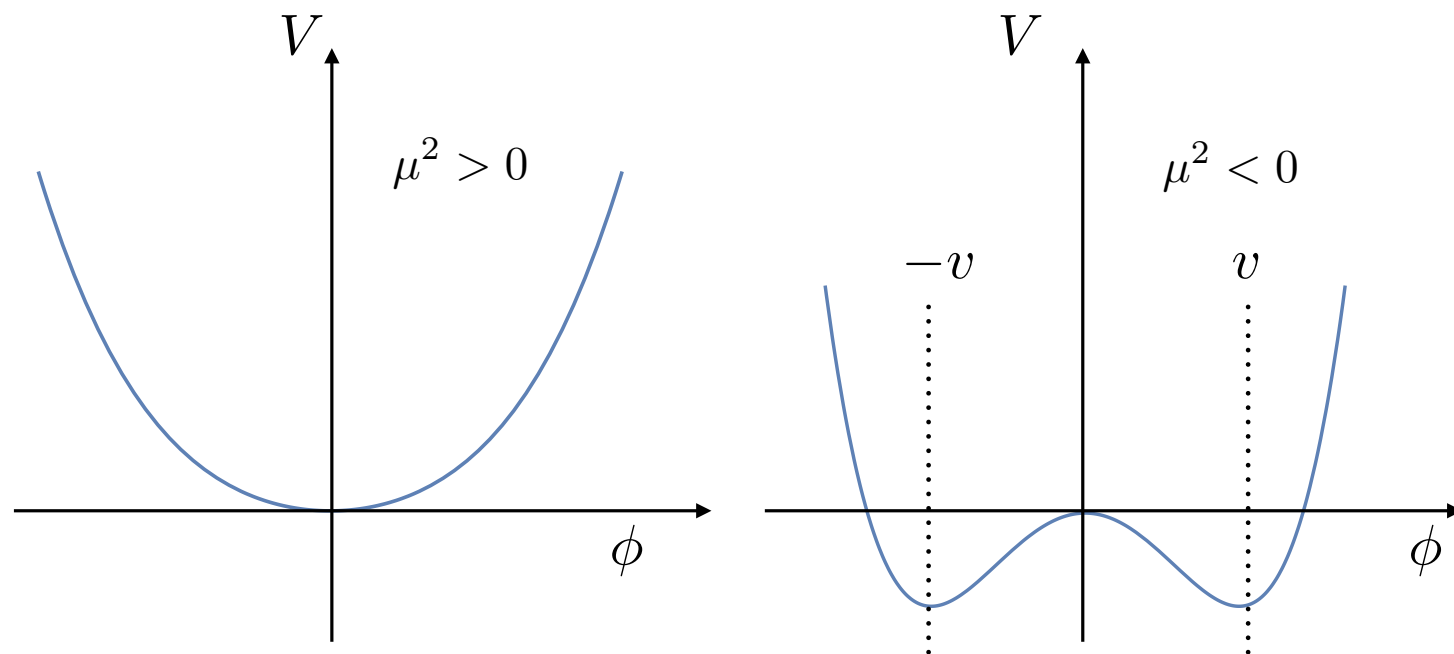
Lets look at the potential term

$$V = \left(\frac{1}{2}\mu^2 \phi^2 + \frac{1}{4}\lambda \phi^4 \right)$$



The potential has different types of structures depending on the sign of μ^2





$$V = \left(\frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \right)$$

We are interested in the minima of the potential which we find as follows

$$\frac{\partial V}{\partial \phi} = 0 \implies \phi(\mu^2 + \lambda \phi^2) = 0$$

Which we define as

$$\phi = \pm v \quad v = \sqrt{\frac{-\mu^2}{\lambda}}$$



In order to have a stable perturbation theory we want to expand our fields around the minimum. We can choose either +/- v. Lets choose +v.

$$\phi(x) = v + \eta(x)$$

Writing our Lagrangian in the new coordinates we see that

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4} \lambda \eta^4 + \text{const}$$



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Comparing the Lagrangian to that of a free scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

We see that the scalar field has a physical mass

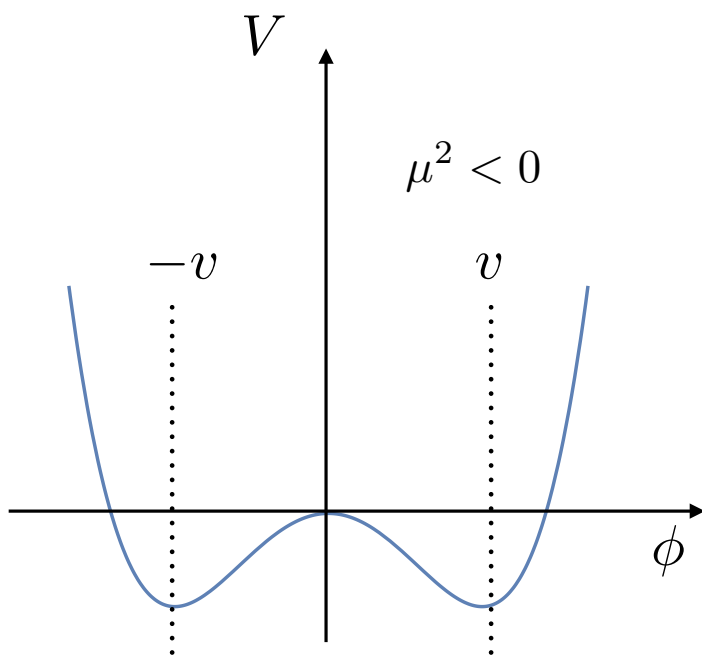
$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$



Writing our Lagrangian in the new coordinates we see that

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4} \lambda \eta^4 + \text{const}$$

$$\eta \rightarrow -\eta$$



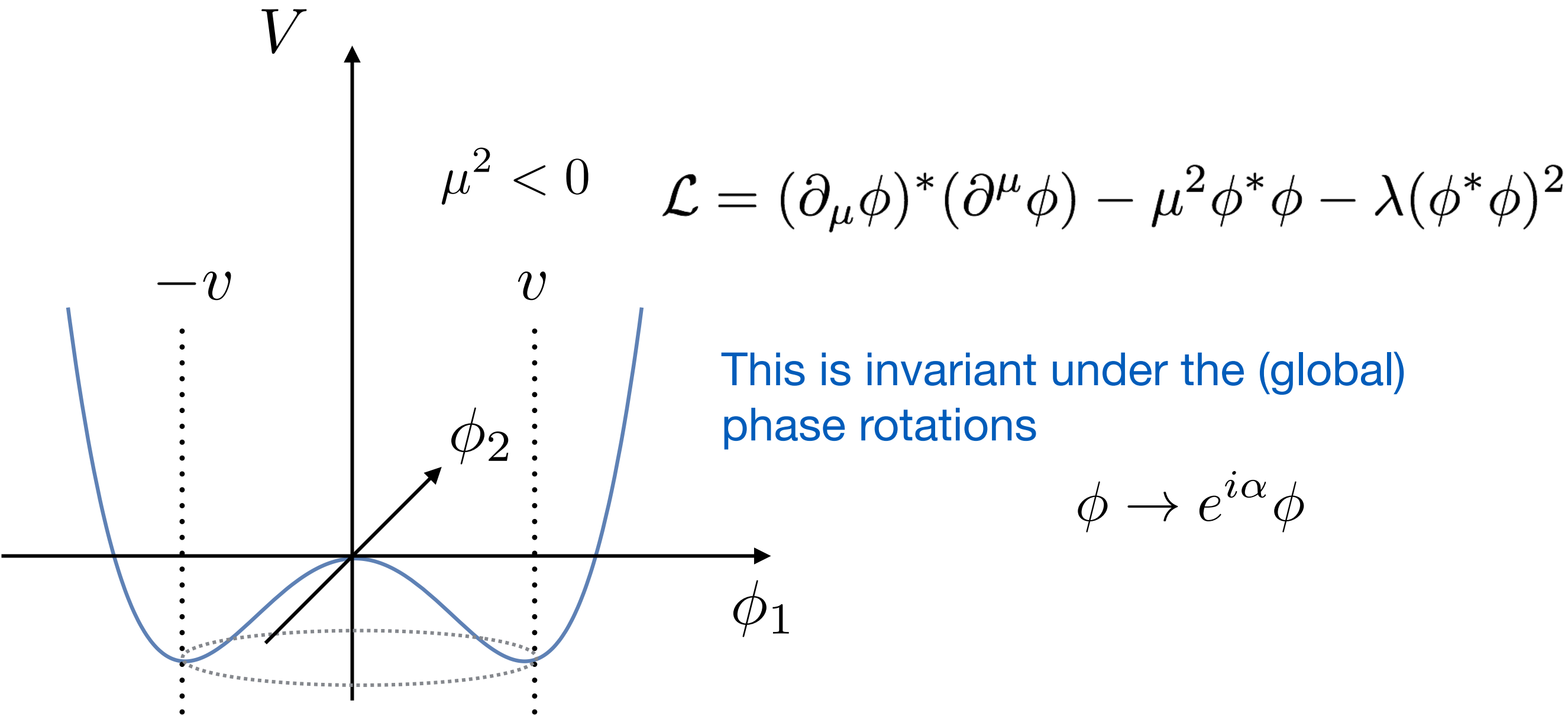
In the new coordinates the symmetry is gone

Note that we didn't change the Lagrangian, and the symmetry is still present globally. Our choice of $+v$ versus $-v$ breaks the symmetry.

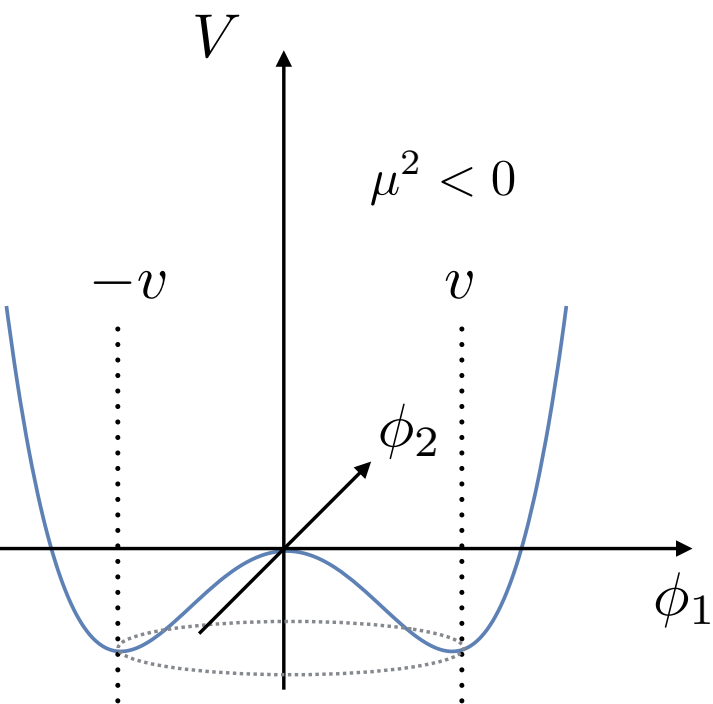
A perturbative expansion around 0 would be inherently unstable.



Lets consider a similar, but more complicated example. A complex scalar field with the following Lagrangian



Writing the complex scalar field as the combination of two real fields



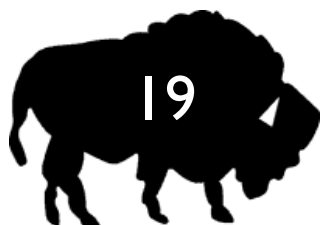
$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

The Lagrangian can then be written as follows

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^4 + 2\phi_1^2\phi_2^2 + \phi_2^4)$$

The minimum of the potential is a circle in the (ϕ_1, ϕ_2) plane

$$\phi_1^2 + \phi_2^2 = v^2 \quad v^2 = -\frac{\mu^2}{\lambda}$$



We spontaneously choose to expand our fields around the point

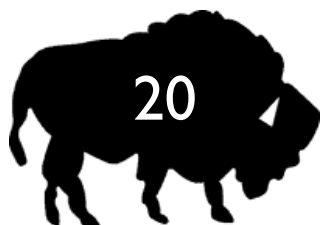
$$\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

So that
$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

And the Lagrangian becomes

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2 \eta^2 + \text{const} + \text{cubic} + \text{quartic}$$

We see one massless field and one with mass $m_\eta = \sqrt{-2\mu^2}$



The appearance of one massive and one massless boson is no accident. It is an example of **Goldstone's theorem**, which states that.

For every continuous symmetry of a physical system which is spontaneously broken there exists a massless boson

In our example we lost one symmetry (rotations in (ϕ_1, ϕ_2) plane) so as predicted we see one massless boson

So if SSB generates masses? Where are all the Goldstone bosons?



The Higgs mechanism



I aint afriad of no ghosts



We reconsider our previous Lagrangian,

$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$$\phi \rightarrow e^{i\alpha(x)} \phi$$

Which means we have to introduce a gauge field to ensure gauge invariance.

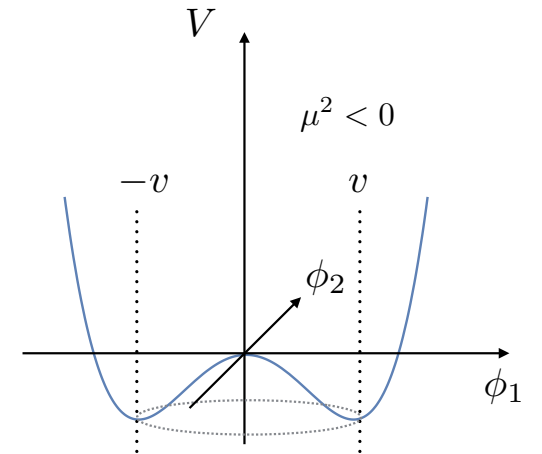
Our Lagrangian is thus

$$\mathcal{L} = (\partial^\mu + ieA^\mu) \phi^* (\partial_\mu - ieA_\mu) \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$



Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

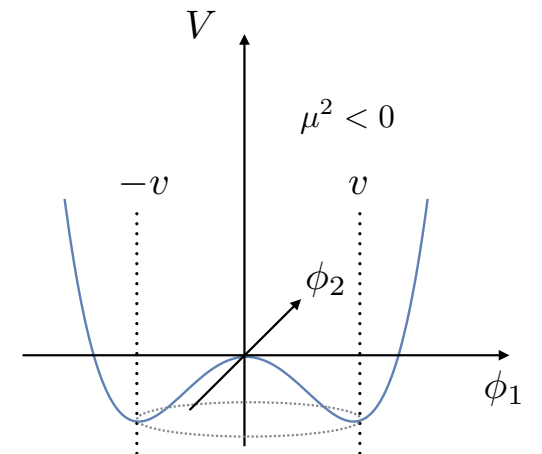
And
$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$



Our Lagrangian is then

Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$



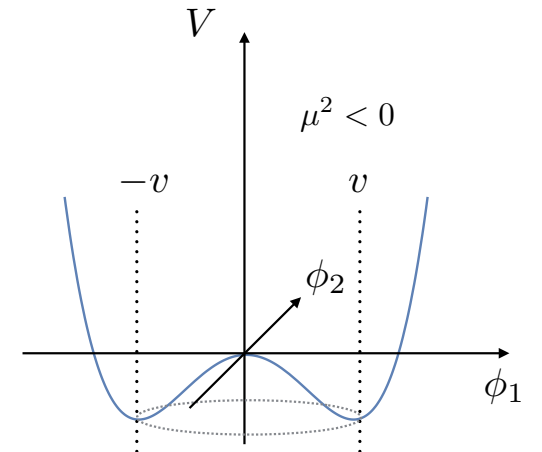
Our Lagrangian is then

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2 + \frac{1}{2}e^2 v^2 A_\mu A^\mu - ev A_\mu \partial^\mu \xi + \text{Interaction terms}$$

Lots to see here!

Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$



Our Lagrangian is then

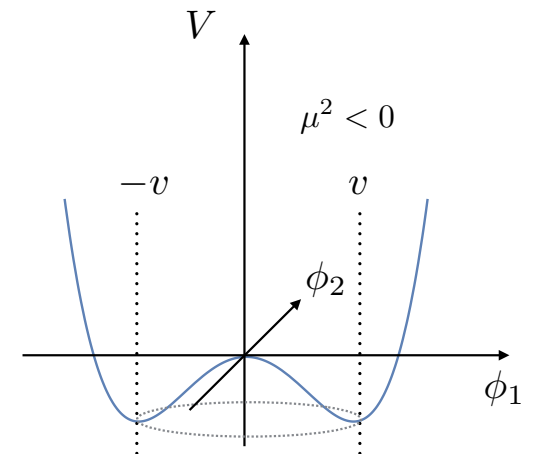
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Lots to see here!

Goldstone boson

Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And
$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$



Our Lagrangian is then

$$\mathcal{L}' = \underbrace{\frac{1}{2}(\partial_\mu \xi)^2}_{\text{Goldstone boson}} + \underbrace{\frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2}_{\text{Massive scalar}} + \frac{1}{2}e^2 v^2 A_\mu A^\mu - ev A_\mu \partial^\mu \xi + \text{Interaction terms}$$

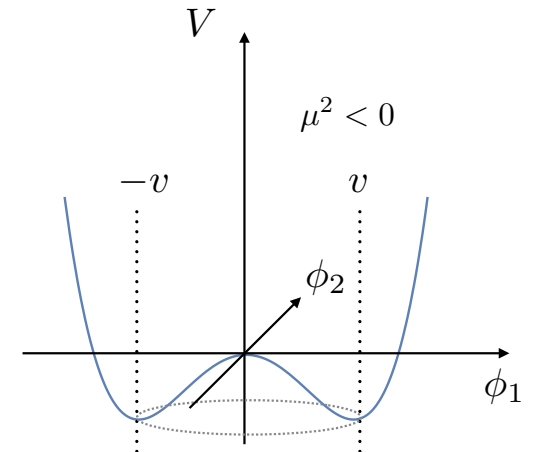
Lots to see here!

Goldstone boson

Massive scalar

Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$



Our Lagrangian is then

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Lots to see here!

Goldstone boson

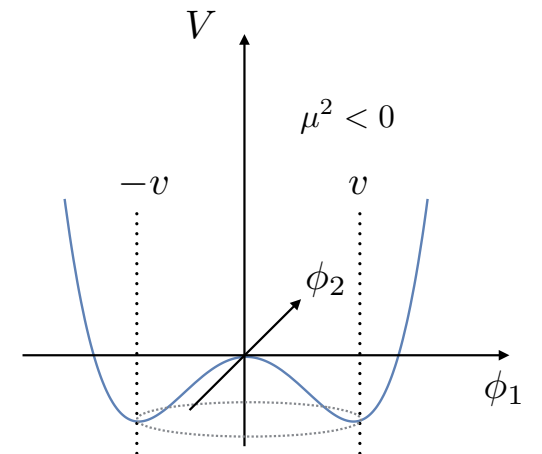
Massive scalar

Mass term for the gauge boson!



Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$



Our Lagrangian is then

$$\mathcal{L}' = \underbrace{\frac{1}{2}(\partial_\mu \xi)^2}_{\text{Goldstone boson}} + \underbrace{\frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2}_{\text{Massive scalar}} + \underbrace{\frac{1}{2}e^2 v^2 A_\mu A^\mu}_{\text{Mass term for the gauge boson!}} - ev A_\mu \partial^\mu \xi + \text{Interaction terms}$$

Lots to see here!

Goldstone boson

Massive scalar

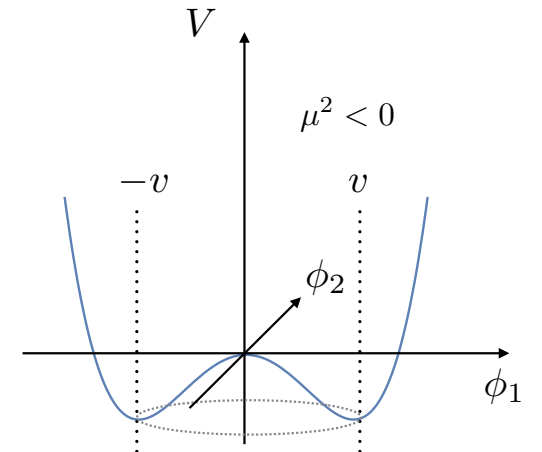
Mass term for the gauge boson!

$$m_\xi = 0 \quad m_\eta = \sqrt{2\lambda v^2} \quad m_A = ev$$



Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And $\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$



Our Lagrangian is then

?????

$$\mathcal{L}' = \underbrace{\frac{1}{2}(\partial_\mu \xi)^2}_{\text{Goldstone boson}} + \underbrace{\frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2}_{\text{Massive scalar}} + \underbrace{\frac{1}{2}e^2 v^2 A_\mu A^\mu}_{\text{Mass term for the gauge boson!}} + \underbrace{ev A_\mu \partial^\mu \xi + \text{Interaction terms}}_{\text{Interaction terms}}$$

Lots to see here!

Goldstone boson

Massive scalar

Mass term for the gauge boson!

$$m_\xi = 0 \quad m_\eta = \sqrt{2\lambda v^2} \quad m_A = ev$$



$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2 + \frac{1}{2} e^2 v^2 A_\mu A^\mu - ev A_\mu \partial^\mu \xi + \text{Interaction terms}$$

This term is nasty, and corresponds to a mixing between the Goldstone and gauge bosons.

Its appearance is related to the longitudinal degree of symmetry of the (massive) gauge boson.

Since we didn't redefine A , it still has only transverse degrees of freedom

We can write the Lagrangian in a nicer (more physical) framework by applying a gauge transformation.



Firstly we note that our choice of writing the scalar field was not unique, the following definition

$$\phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi) \implies \phi = \frac{1}{\sqrt{2}}(v + \eta)e^{i\xi/v}$$

Is equally valid, (and equivalent to lowest order in the fields)



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Is equally valid, (and equivalent to lowest order in the fields)

We therefore write out fields as follows

$$\begin{aligned}\phi &\rightarrow \frac{1}{\sqrt{2}}(v + h(x))e^{i\theta(x)/v} \\ A_\mu &\rightarrow A_\mu + \frac{1}{ev}\partial_\mu\theta\end{aligned}$$

This is equivalent to choosing a favorite gauge to work in. We call it the unitary gauge.



In terms of these fields our Lagrangian becomes

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 v^2 A_\mu A^\mu - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2 A_\mu A^\mu h^2 + v e^2 A_\mu A^\mu h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- There are no physical Goldstone bosons in the theory (absorbed into the longitudinal degree of freedom of the gauge boson)
- One physical massive scalar (the Higgs boson)
- One massive gauge boson.

The old Lagrangian wasn't wrong, but it would have been extremely tedious to work with, we would carry around a lot of spurious ghosts in our calculation.



SSB of a local $SU(2)$

We are nearly in a position to construct the SM Higgs. Our two remaining problems are 1) Generating two different masses for the W and Z bosons and 2) Generating masses for fermions.

We start by promoting our basic Lagrangian

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

To be symmetric under local SU(2) transformations of the form

$$\phi \rightarrow e^{i\alpha_a \tau_a / 2} \phi$$

Sums over the 3 (2x2 matrices) which generate SU(2)

$$\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$



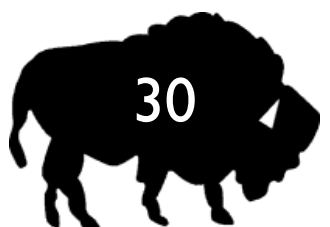
Since there are 3 generators we introduce the following covariant derivative.

$$D_\mu \rightarrow \partial_\mu + ig \frac{\tau_a}{2} W_\mu^a$$

Which sums over 3 gauge bosons.

Under an SU(2) transformation the fields transform as follows.

$$\vec{W}_\mu \rightarrow \vec{W}_\mu - \frac{1}{g} \partial_\mu \vec{\alpha} - \vec{\alpha} \times \vec{W}_\mu$$



Putting this all together we obtain the following Lagrangian

$$\mathcal{L} = \left(\partial_\mu \phi + ig \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \phi \right)^\dagger \left(\partial_\mu \phi + ig \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \phi \right) - V(\phi) - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu}$$

With our usual Higgs potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

Which has minima at

$$\phi^\dagger \phi = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda}$$



We (spontaneously) choose the following vacuum state

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

And expand around it as follows (in the unitary gauge and suppressing

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$



From our past experience we know that the the mass terms for the gauge bosons come from the $|D_\mu \phi|^2$ part of the Lagrangian

So for simplicity we focus on that

$$\begin{aligned}
 \left| ig \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \phi \right|^2 &= \frac{g^2}{8} \left| \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\
 &= \frac{g^2 v^2}{8} [(W_\mu^1)^2 + (W_\mu^2)^2 + (W_\mu^3)^2]
 \end{aligned}$$

So we have 3 massive gauge bosons, of equal mass.



The Standard Model

In order to generate the mass spectrum of the SM, we need to break a more complicated structure than just a single $SU(2)$.

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$$

Where we define hypercharge as follows

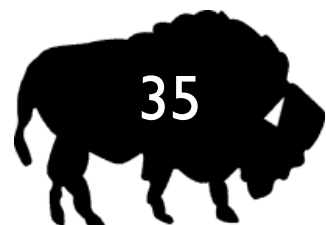
$$Q = T^3 + \frac{Y}{2}$$

Left-handed and right handed matter transforms as follows

$$\begin{aligned}\chi_L \rightarrow \chi'_L &= e^{i\alpha(\vec{x}) \cdot \vec{\tau}/2 + i\beta(x)Y} \chi_L \\ \psi \rightarrow \psi' &= e^{i\beta(x)Y} \psi\end{aligned}$$

Quarks and Leptons are represented by

$$\chi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \psi_R = u_R \quad \text{or} \quad d_R \qquad \chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \psi_R = e_R$$



For instance, the leptonic part of the Lagrangian

$$\mathcal{L}_1 = \bar{\chi}_L \gamma^\mu \left[i\partial_\mu - g\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu - ig' \left(-\frac{1}{2} \right) B_\mu \right] \chi_L + \bar{e}_R \gamma^\mu [i\partial_\mu - g'(-1)B_\mu] e_R - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

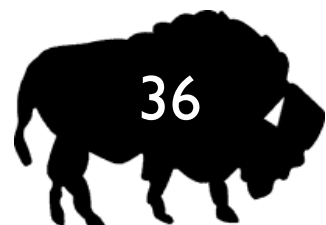
Is manifestly gauge invariant.

Note the absence of any mass term in the Lagrangian, in fact the mass term

$$\begin{aligned} -m\bar{e}e &= -m\bar{e}\left[\frac{1}{2}(1 - \gamma^5) + \frac{1}{2}(1 + \gamma^5)\right]e \\ &= -m(\bar{e}_R e_L + \bar{e}_L e_R) \end{aligned}$$

Is **NOT** invariant under SU(2) rotations.

The Higgs will help us here too.



Next we need to work out the Higgs sector, The Higgs is an SU(2) doublet with hypercharge $Y=1$.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$Q=1/2-1/2$ → $Q=1/2+1/2$

Expanding our complex scalars

$$\phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$\phi^0 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)$$

The Higgs part of the Lagrangian is then

$$\mathcal{L}_2 = \left| \left(i\partial_\mu - g\frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - g'\frac{Y}{2}B_\mu \right) \phi \right|^2 - V(\phi)$$

Covariant derivative →

Higgs potential →

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$



Again we expand around the vacuum given by

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Now we get to see something really beautiful, consider the following linear combination of generators

$$Q = T^3 + \frac{Y}{2}$$

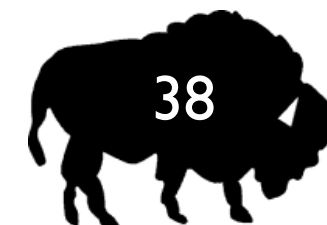
Acting on the vacuum state, i.e.

$$Q\phi_0 \propto \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

So Q annihilates the vacuum, i.e.

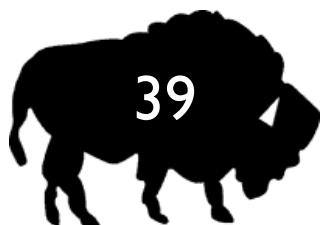
$$\phi_0 \rightarrow \phi'_0 = e^{i\alpha(x)Q} \phi_0 = \phi_0$$

The symmetry of the vacuum wrt Q , will give us EM (and the photon)!



Getting back to our massive bosons we see that the piece we are interested in is

$$\left| \left(-ig\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu - i\frac{g'}{2}B_\mu \right) \phi \right|^2 = \frac{g^2}{8} \left| \begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

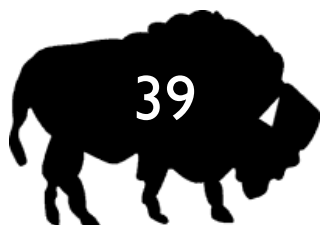


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Expanding this out yields

$$\left| \left(-ig\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu - i\frac{g'}{2}B_\mu \right) \phi \right|^2 = \frac{1}{8}v^2g^2 [(W_\mu^1)^2 + (W_\mu^2)^2] + \frac{1}{8}v^2(g'B_\mu - gW_\mu^3)(g'B^\mu - gW^{3\mu})$$



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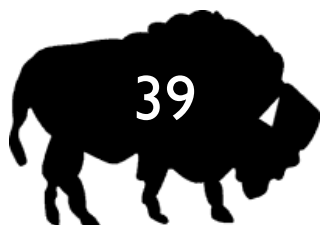
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Or alternatively in terms of the physical W^+, W^- physical states

$$\left| \left(-ig\frac{1}{2}\vec{\tau} \cdot \vec{W}_\mu - i\frac{g'}{2}B_\mu \right) \phi \right|^2 = \left(\frac{1}{2}vg \right)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2(W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$



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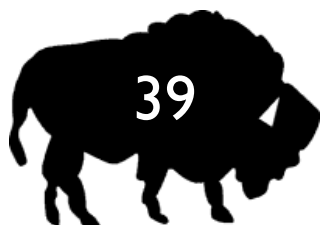
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We have our first big result

$$M_W = \frac{1}{2} v g$$



We can tidy up the last term by re-packing the fields into a diagonal form

$$\frac{1}{8}v^2 \left[g^2 (W_\mu^3)^2 - 2gg' W_\mu^3 B_\mu + g'^2 B_\mu^2 \right] = \frac{1}{8}v^2 \left[gW_\mu^3 - g'B_\mu \right]^2 + 0 \left[gW_\mu^3 + g'B_\mu \right]^2$$

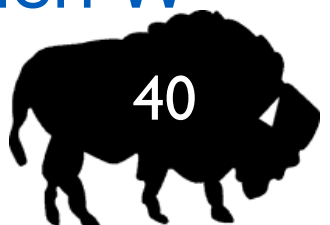
Which we interpret as

$$\frac{1}{2}M_Z Z^\mu Z_\mu + \frac{1}{2}M_A A^\mu A_\mu$$

With

$$\begin{aligned} A_\mu &= \frac{gW_\mu^3 + g'B_\mu}{\sqrt{g^2 + g'^2}} & M_A &= 0 \\ Z_\mu &= \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} & M_Z &= \frac{1}{2}v\sqrt{g^2 + g'^2} \end{aligned}$$

So we predict a massless photon and a Z which is **heavier** than then W :-) Happy days.



Finally we turn our attention to the fermion mass terms, note that the following combination is gauge invariant

$$\mathcal{L}_3 = -G_e [\bar{\chi}_L \phi \psi_R + \text{h.c.}]$$

Doublets

singlet

For instance for the leptons we have

$$\mathcal{L}_3 = -G_e \left[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \text{h.c.} \right]$$



In our time honored tradition we expand around the new vacuum

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

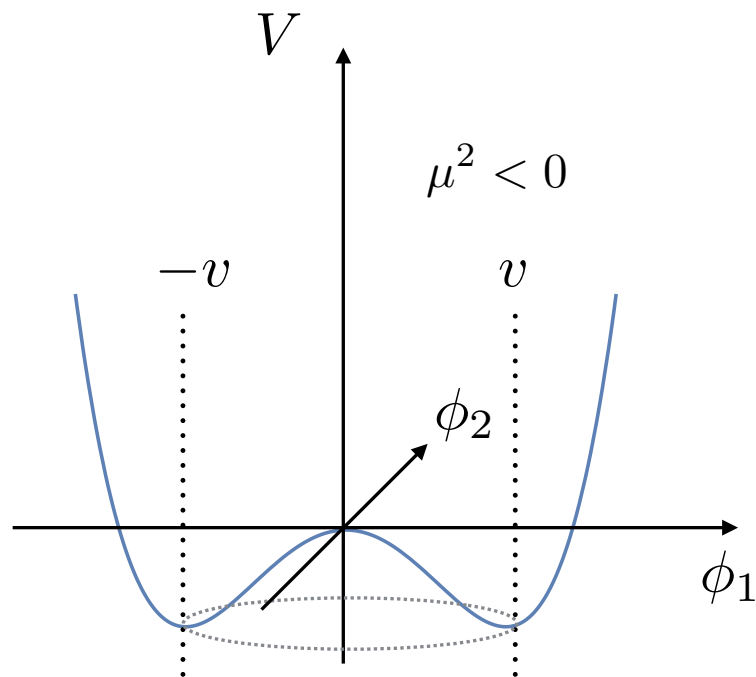
Finding

$$\mathcal{L}_3 = -\frac{G_e}{\sqrt{2}}v(\bar{e}_Le_R + \bar{e}_Re_L) - \frac{G_e}{\sqrt{2}}(\bar{e}_Le_R + \bar{e}_Re_L)h$$

I.e.

$$\mathcal{L}_3 = -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh$$





$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

- Potentials with multiple non-zero global minimum, lead to spontaneous symmetry breaking.
- By breaking $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$ we are able to explain the generation of mass terms for both gauge bosons and fermions in a theoretically robust manner.

