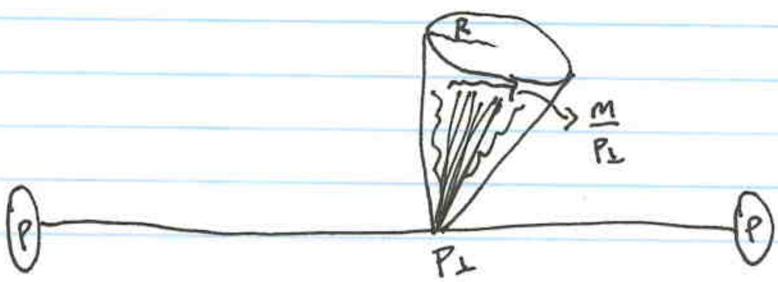
CTEQ Lecture 2 Jets Beyond LO

Last lecture, we developed a very simple, elegant picture of jets. Starting from the simple assumption of scale invariance of QCD at high energies, we were able to make a robust prediction for the jet mass to all-orders in the coupling α_s. With one emission dominating the mass, all other emissions were forbidden to exceed the contribution from this leading emission. This produced an exponentiation of a no-emission probability, which is called the Sudakov factor. At this leading order, differences between quark and gluon initiated jets arise from their different color charges; gluons have a larger color charge, and so emit more than quarks. This leads to a larger suppression of the mass distribution from the Sudakov factor for gluons, and a technique for discriminating quark- from gluon-intiated jets.

In this lecture, we will work to understand features of jets beyond this first approximation. This will necessitate the introduction of jet grooming as a technique to clean up up jets produced at the LHC and simplify their structure.

Let's first visualize* what our simple picture of a jet is, produced at the LHC:

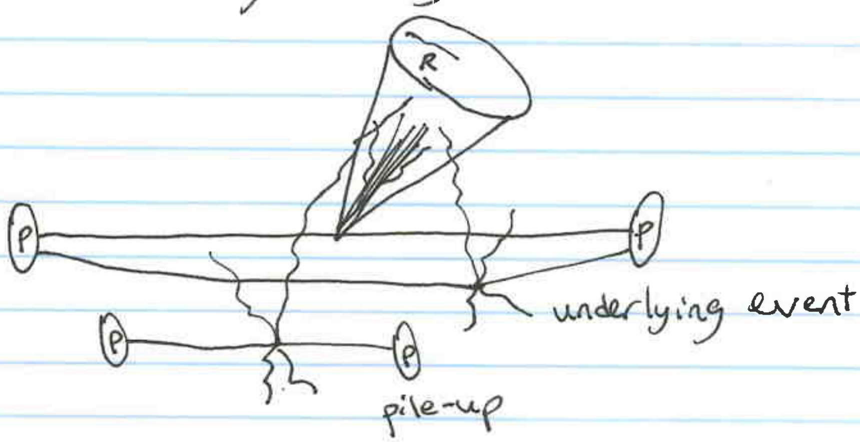


A jet is some restricted region of our detector whose size is fixed by a radius R. This region is identified by a large transverse momentum $p_\perp$ of a collection of collimated particles produced from individual parton scattering from proton collisions. The angular size of the collimated particles is approximately the ratio of the jet's invariant mass m to its $p_\perp$:

$\theta \sim \frac{m}{p_\perp}$. So, with this simple picture of

a jet as a collection of final state, collimated emissions the distribution of the mass of the jet can only depend on the dimensionless quantities $m/p_\perp$ and R. We saw the $m/p_\perp$ dependence yesterday, while jet radius dependence R only first appears at higher orders.

At the LHC, however, this simple picture is woefully incomplete. Protons are composite particles, consisting of numerous quarks and gluons, which are the particles that ultimately interact. Multiple parton interactions can occur in each proton collision, contributing to what is referred to as underlying event. Additionally, the LHC doesn't collide individual pairs of protons, but rather bunches containing billions of protons. Many of these protons can collide in each bunch crossing, which is referred to as pile-up. Including underlying event and pile-up, our picture of jet production muddies significantly.



Now, not only is the mass of the jet sensitive to the ~~s~~ angular size of collinear emissions, but it is also sensitive to the energy of underlying event and pile-up! We can estimate this contribution to the squared jet mass, m^2. We can safely assume that the characteristic underlying event and pile-up transverse momentum scales are much less than the jet $p_\perp$:

$$p_\perp \gg p_{\perp UE}, p_{\perp PU}$$

Also, note that the direction of the jet is (essentially) uncorrelated with the underlying event and pile-up radiation. Therefore, on average, this radiation just is uniformly distributed over the area of the jet. A single UE or PU emission with transverse momentum $p_{\perp U/P}$ therefore affects the jet mass as:

$$\Delta m^2 \cong p_\perp p_{\perp U/P} R^2,$$ or, for a transverse momentum

density $\Lambda_{U/P}$ (transverse momentum per unit angular area) the mass is affected as

$$\Delta m^2 \cong p_\perp \Lambda_{U/P} R^4.$$

The linear mass is affected as:

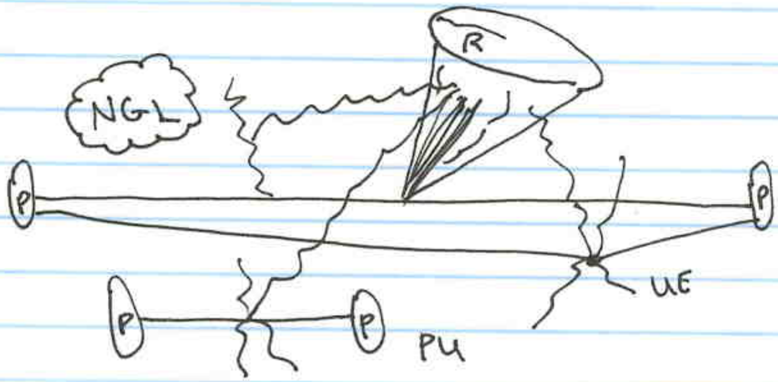
$$m \cong (m^2 + \Delta m^2)^{1/2} = m\left(1 + \frac{\Delta m^2}{2m^2} + \ldots\right) = m + \frac{\Delta m^2}{2m}$$
$$\cong m + \frac{p_\perp \Lambda_{U/P} R^4}{2m}.$$

To get a sense of the size of this effect, take $p_\perp = 1\text{ TeV}$, $m = 100\text{ GeV}$ and typically $\Lambda_{U/P} \sim 1\text{ GeV}$. For a jet radius $R = 1.0$, the change to the mass is

$$\frac{\Delta m^2}{2m} \approx \frac{1000 \cdot 1}{2 \cdot 100} \approx 5 \text{ GeV}$$

So, this affects the jet mass by about 5%. However, this is a conservative estimate. As the luminosity of the LHC increases, the rate of pile-up collisions increases and correspondingly so does the average transverse momentum density. Depending on the process and nominal jet mass, the effect of UE + PU could be 20% or more, which is significant.

There's another type of radiation that can affect the jet. There can be emissions that land in the jet that arise from re-radiation from outside the jet:



The leading contribution from these re-emissions is called non-global logarithms, or NGLs. NGLs introduce a correlation between physics in and out of the jet. Inside the jet, scales are set by the jet mass, while out of the jet, there is no restriction on radiation. That is, we only identify and measure the jet, so out-of-jet scales are completely unconstrained, and as high as the jet $p_\perp$ or even the scattering energy Q. The leading contribution comes from two strongly-ordered gluon emissions, one of which lands in the jet. Just like UE or PU, the direction of these NGL emissions is uncorrelated with the jet direction. Therefore, the leading NGLs are proportional to the area of the jet:

$$\mathrm{NGL} \cong R^2 \cdot \alpha_s^2 \ln^2 \frac{m}{p_\perp}$$

In general, systematically calculating NGLs is extremely non-trivial though progress has been made understanding them recently. Nevertheless, along with UE and PU, NGLs present a challenge to a systematic theoretical understanding of a jet.

All of these sources of contamination had two things in common: the radiation was relatively uniformly distributed over the area of the jet and it is low energy. This is distinct from the high energy, collinear radiation that forms the main structure of the jet (and that was our simple picture of a jet last lecture). So, if we have a procedure to systematically remove soft, wide angle radiation in the jet, we can eliminate all of these sources of contamination! Let's see how to do this.

First, let's identify wide-angle emissions in the jet. To do this, we will recluster the particles in the jet with the Cambridge/Aachen algorithm. That is, for all pairs i,j of particles in the jet calculate their angular separation:

$$d_{ij} = \frac{\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2}{R^2}, \quad \text{where } \Delta\eta_{ij} = \eta_i - \eta_j, \quad \Delta\phi_{ij} = \phi_i - \phi_j$$

Cluster the pair of particles with the smallest d_{ij}; that is, sum their four-vectors and replace them in the list of particles by their sum:

$$p_{(ij)} = p_i + p_j$$

Continue this process until all particles in the jet are clustered into the total jet four-vector. This defines an angular-ordered branching history, starting with the widest angle particles:

where $\theta_1 > \theta_2 > \theta_3 > \ldots$

We can then systematically march through the clustering history and ask if the emission/particle is sufficiently high energy. For a branching with particles i, j we test:

$$\frac{\min[p_{\perp i}, p_{\perp j}]}{p_{\perp i} + p_{\perp j}} > z_{cut} \left(d_{ij} \right)^{\beta}$$

Here, z_{cut} and β are parameters: z_{cut} defines the relative energy cut, typically taken to be about 10%. β defines the aggressiveness of the test. If $\beta \to \infty$, then $(d_{ij})^{\beta} \to 0$, which always is true. If $\beta \to 0$, then it is just a relative energy/$p_\perp$ cut. If the clustering fails this requirement, the softer branch is eliminated from the jet, and the procedure steps to the next smaller angle branch. When a branching passes, the procedure terminates, and the remaining particles constitute the groomed jet.

This procedure is called "soft drop grooming" and when $\beta = 0$, the modified mass drop tagger groomer (mMDT). Because of the angular ordering and elimination of soft radiation, it indeed removes the radiation from UE, PU, or NGLs that contaminate the jet. Only the hard, collinear core remains, which is just what we want.

To end this lecture, let's calculate the ~~groo~~ jet mass distribution ~~of~~ in the presence of grooming. We'll go back to our familiar Lund plane and identify the regions of that phase space that are no longer accessible when groomed. For simplicity, we'll just restrict the discussion to $\beta = 0$ / mMDT grooming.

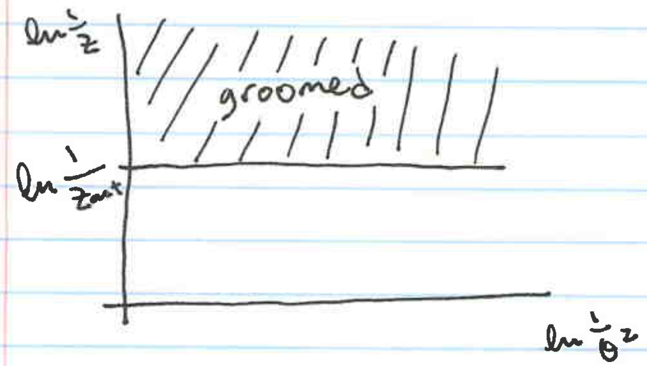
The requirement on the particles in a branching is:

$$\frac{\min[p_{\perp i}, p_{\perp j}]}{p_{\perp i} + p_{\perp j}} > z_{cut}.$$

In the Lund plane, we restrict to the soft and collinear limit, so one of these particles carries the entire jet energy. That is, the picture is:

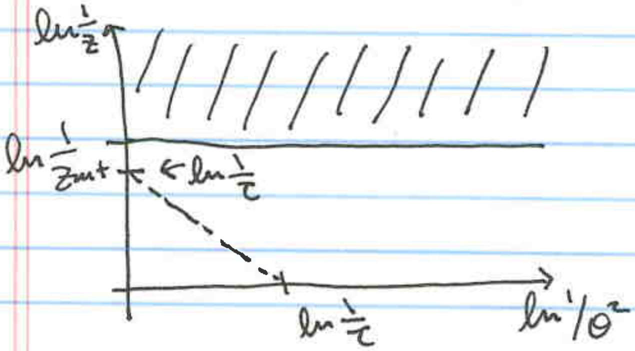
and we ask if

$p_{\perp i} > z_{cut}\, p_\perp$. If this is not true, the emission is removed. So, on the Lund plane, this eliminates a semi-infinite region from consideration:



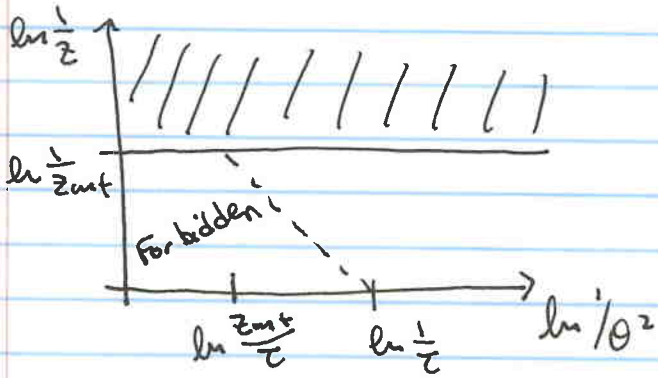
Let's calculate the Sudakov factor for this groomed phase space

There are two distinct regions to consider. First, if $\tau > z_{cut}$ ($\tau = m^2/p_\perp^2$), we just find the Sudakov factor we derived yesterday:



$$\Rightarrow \quad \Theta(\tau - z_{cut}) \exp\left[-\frac{\alpha_s}{\pi}\frac{C_F}{2}\ln^2\tau\right]$$

This successfully resums double logarithms of τ. However, if $\tau < z_{cut}$, we find something very different. The Lund diagram is:



Note that the angle of emission when the energy fraction $z = z_{cut}$ is

$$\tau = z_{cut}\theta^2 \Rightarrow \theta^2 = \frac{\tau}{z_{cut}}$$

The area of the forbidden region is now

$$\ln\frac{1}{z_{cut}}\ln\frac{z_{cut}}{\tau} + \frac{1}{2}\ln\frac{1}{z_{cut}}\left(\ln\frac{1}{\tau} - \ln\frac{z_{cut}}{\tau}\right) =$$

$$= -\frac{1}{2}\ln^2 z_{cut} + \ln z_{cut}\ln\tau =$$

Therefore, the Sudakov factor is:

$$\Theta(z_{cut} - \tau)\exp\left[-\frac{\alpha_s}{\pi}C_F\left(-\frac{\ln^2 z_{cut}}{2} + \ln z_{cut}\ln\tau\right)\right]$$

There are no double logarithms of τ! Soft drop removes soft logs of τ and replaces them with $\ln z_{cut}$.

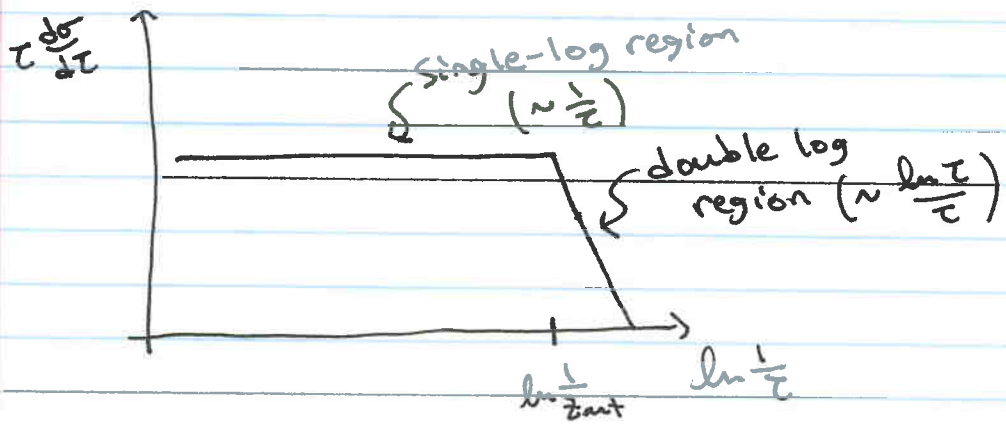
Putting it together, the total Sudakov factor is

$$\Sigma(\tau) = \Theta(\tau - z_{cut}) \exp\left[-\frac{\alpha_s}{\pi}\frac{C_F}{2}\ln^2\tau\right] + \Theta(z_{cut}-\tau)\exp\left[-\frac{\alpha_s C_F}{\pi}\left(-\frac{1}{2}\ln^2 z_{cut} + \ln z_{cut}\ln\tau\right)\right]$$

The differential cross section is therefore:

$$\frac{d\sigma}{d\tau} = \frac{d}{d\tau}\Sigma(\tau) = \Theta(\tau - z_{cut})\frac{\alpha_s}{\pi}C_F\frac{\ln\frac{1}{\tau}}{\tau}\exp\left[-\frac{\alpha_s}{\pi}\frac{C_F}{2}\ln^2\tau\right] + \Theta(z_{cut}-\tau)\frac{\alpha_s C_F}{\pi}\frac{\ln\frac{1}{z_{cut}}}{\tau}\exp\left[-\frac{\alpha_s C_F}{\pi}\left(-\frac{1}{2}\ln^2 z_{cut}+\ln z_{cut}\ln\tau\right)\right]$$

A plot of this in the logarithmic plane is:



Below $\tau = z_{cut}$, the distribution is effectively a straight line! Thus, it is very sensitive (through its slope) to the value of α_s, for example. This groomer points the way to a precision jet physics program!

Stay tuned...