Lectures on QCD in the Standard Model

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U. of Puerto Rico
Mayaguez, PR
George Sterman

See: "Partons, Factorization and Resummation", hep-ph-9606312
"Handbook of Perturbative QCD", Rev. Mod. Phys. 67 (1995) 157.

I. The Parton Model and Deep-inelastic Scattering

II. From the Parton Model to QCD

III. Factorization and Evolution

The Context of QCD: "Fundamental Interactions"

- Electromagnetic
- + Weak Interactions $\Rightarrow$ Electroweak
- + Strong Interactions (QCD) $\Rightarrow$ Standard Model
- + ... = Gravity and the rest?
- QCD: A theory "off to a good start". Think of ...

  – $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2 \Rightarrow$ elliptical orbits
  ... 3-body problem ...

  – $L_{\rm QCD} = \bar{q}\ \not{D} q - (1/4)F^2 \Rightarrow$ asymptotic freedom
  ... confinement ...

I. The Parton Model and Deep-inelastic Scattering

IA. Nucleons to Quarks

IB. DIS: Structure Functions and Scaling

IC. Getting at the Quark Distributions

ID. Classic Parton Model Extensions: Fragmentation and Drell-Yan

Introduce concepts and results that predate QCD, led to QCD and were incorporated and explained by QCD.

IA. From Nucleons to Quarks

- **The pattern: nucleons, pions and isospin:**

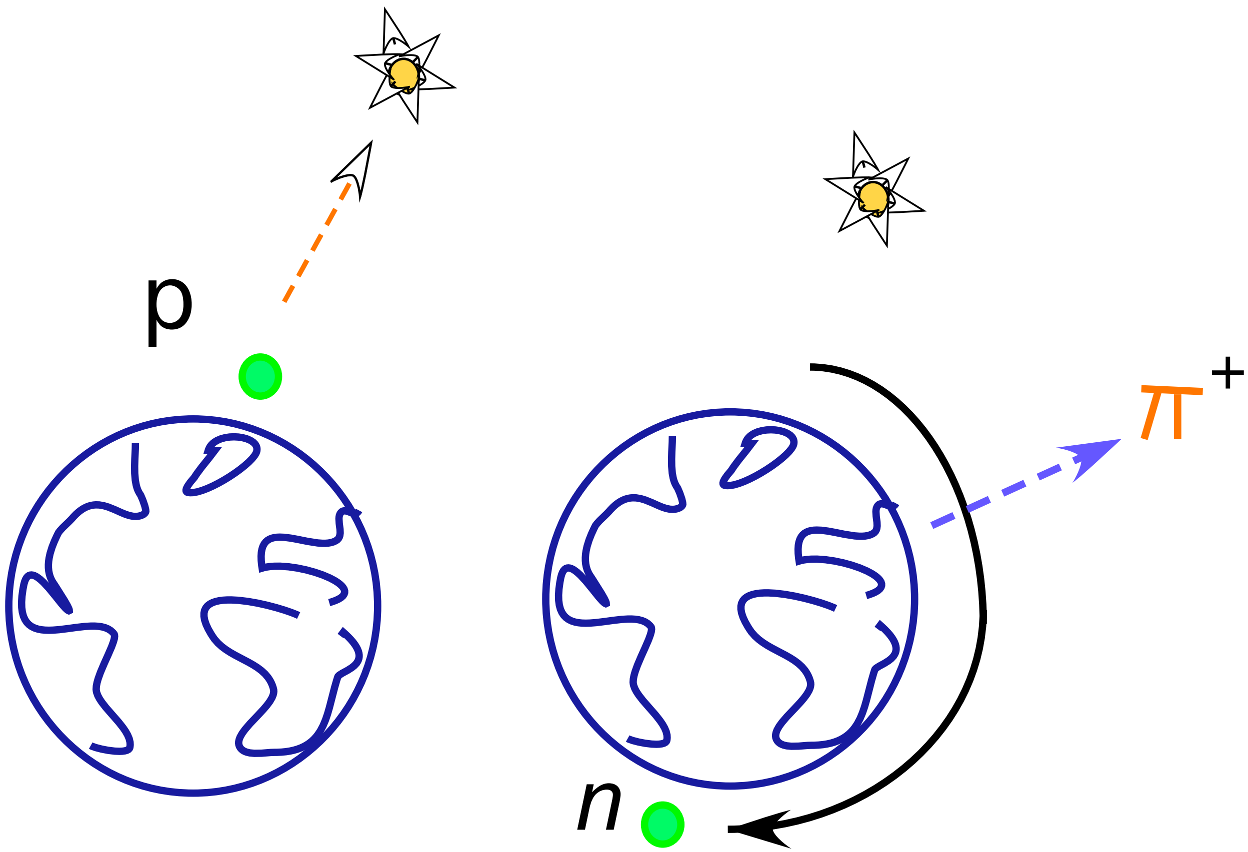
$$\begin{pmatrix} p \\ n \end{pmatrix}$$

  - **p: m=938.3 MeV,** $S = 1/2$, $I_3 = 1/2$
  - **n: m=939.6 MeV,** $S = 1/2$, $I_3 = -1/2$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

  - $\pi^{\pm}$**: m=139.6 MeV,** $S = 0$, $I_3 = \pm 1$
  - π^{0}**: m=135.0 MeV,** $S = 0$, $I_3 = 0$

- **Isospin space ...**
- **Globe with a "north star" set by electroweak interactions:**



Analog: the rotation group (more specifically, $SU(2)$).

- **Explanation: π, N common substructure: quarks**

  (Gell Mann, Zweig 1964)

- **spin $S = 1/2$,**
  $I = 1/2$ (u, d) & $I = 0$ (s)
  with approximately equal masses (s heavier);

$$\begin{pmatrix} u \; (Q = 2e/3, I_3 = 1/2) \\ d \; (Q = -e/3, I_3 = -1/2) \\ s \; (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d}) \; , \quad \pi^- = (\bar{u}d) \; , \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \; ,$$

$$p = (uud) \; , \quad n = (udd) \; , \quad K^+ = (u\bar{s}) \ldots$$

This is the quark model

- **Quark model nucleon has symmetric spin/isospin wave function (return to this later) ⇒ many predictions.**

- **Early success: μ_p/μ_n = -3/2 ; good to %** **(from $S = 1/2$, $I = 1/2$ (uud), (ddu) wave functions.)**
- **And now, six: 3 'light', u, d, s, 3 'heavy': c, b, t.**

- **Of these all but t form bound states of quark model type:**

$$(q_1\,\bar{q}_2) \qquad (q_1\,q_2\,q_3)$$

- **Special interest: "heavy charmonia": $(c\,\bar{c}), (b\,\bar{b})$ "$J\psi$" (1974).**

- New excitement for quark model! "Exotic" hadrons with additional combinations involving c and b.
- Made possible by new, high-luminosity experiments at B-factories and colliders
- Terminology:
  - hadronic molecules: $([c,\bar{u}]\,[\bar{c},u])$ – with different quantum nos. than quarkonium $(c\bar{c})$: the "X(3872)" (2003).
  - pentaquarks: $(q\,q\,q\,q\,\bar{q})$: $(u\,u\,d\,c\,\bar{c})$: the "P_c" (2015).
  - tetraquarks: $(\bar{q}_1\,\bar{q}_2\,[Q,Q])$. Not seen yet, but:
    high hopes for $(\bar{u}\,\bar{d}\,[b,b])$ at LHCb.

    Here, $[b,b]$ may be a "diquark", acting like a single anti-quark, in the color of QCD.

For a recent review: M. Karliner, J.L. Rosner, T. Skwarnicki, arXiv:1711.10626.

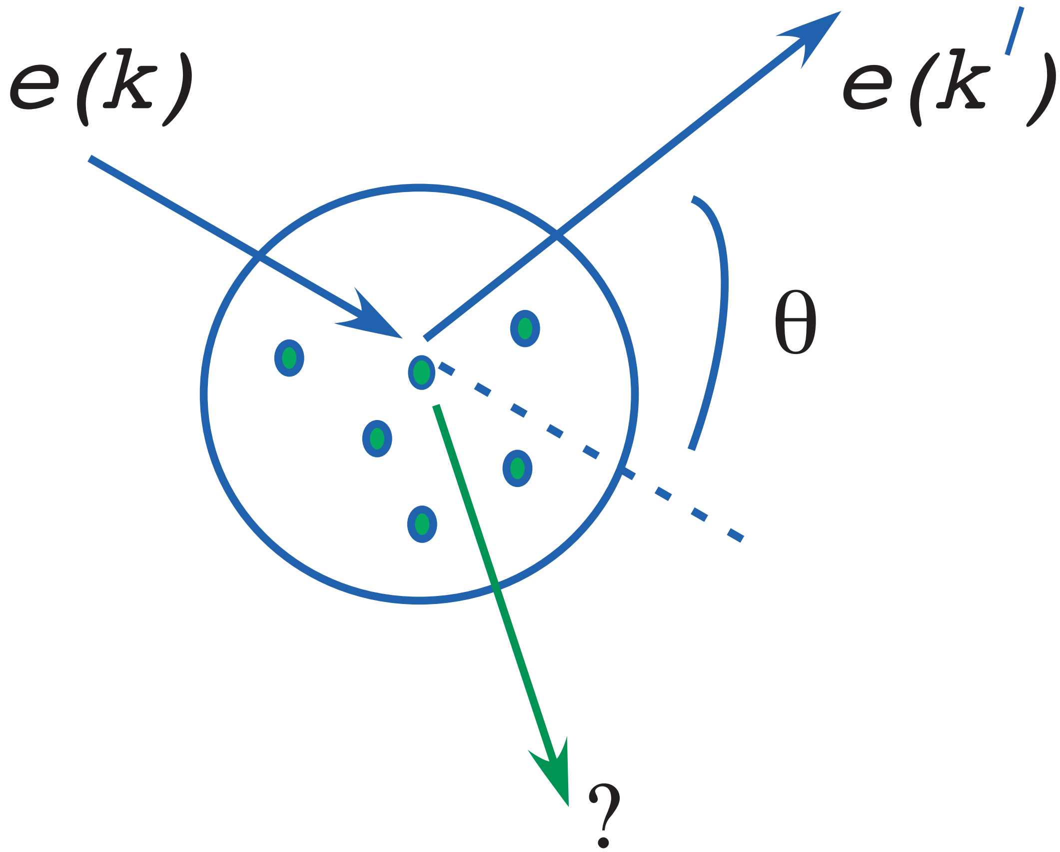
- **Quarks as Partons: "Seeing" Quarks.**

  No isolated fractional charges seen ("confinement.")

  Could such a particle be detected?

  Look closer: do high energy electrons bounce off anything hard? (SLAC 1969 – 'Rutherford-prime')

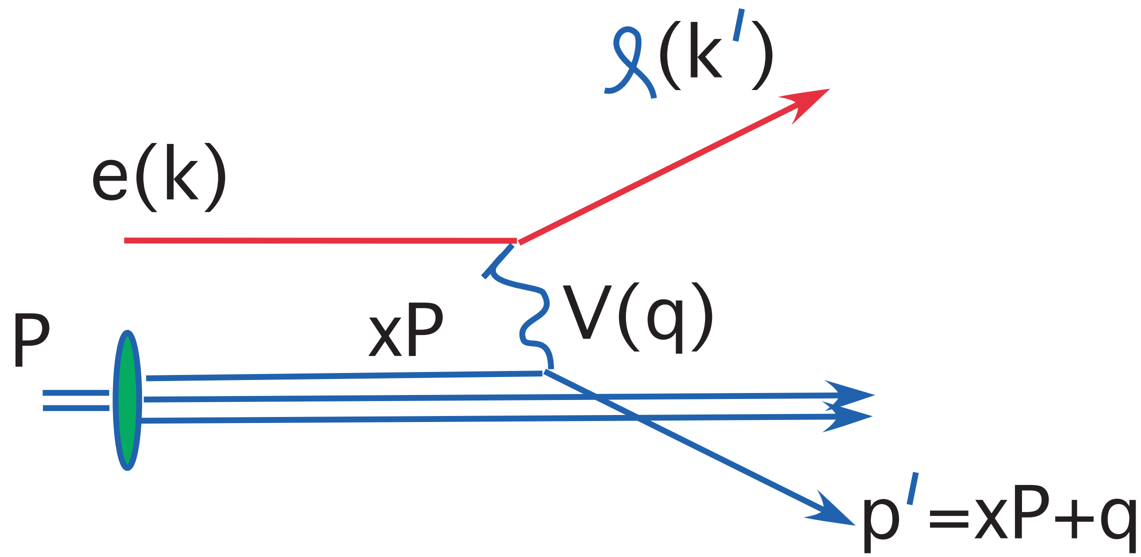
• **So look for:**



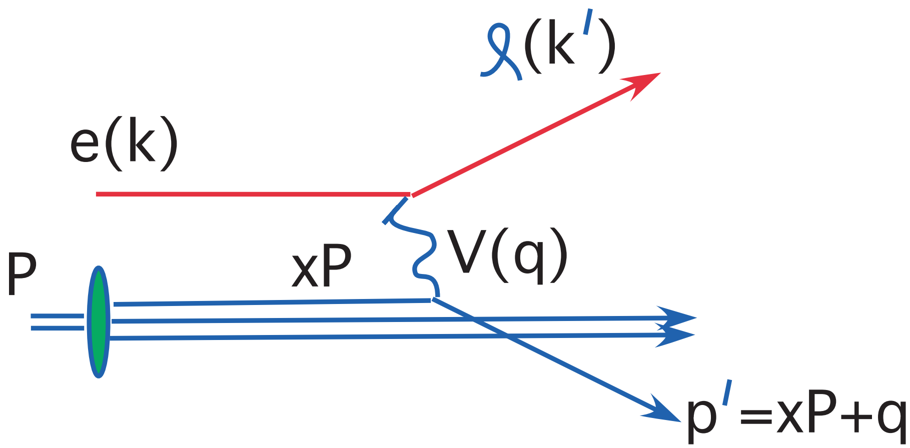
"Point-like' constituents.

The angular distribution gives information on the constituents.

Kinematics ($e + N(P) \to \ell + X$)



- $V = \gamma, Z_0 \Rightarrow \ell = e,\ \mu$, "neutral current" (NC).
- $V = W^-(e^-, \nu_e)$, $V = W^+(e^+, \bar{\nu}_e)$, also (μ, ν_μ) reactions. "charged current" (CC).
- $W^2 \equiv (p+q)^2 \gg m^2_{\text{proton}}$: Deep-inelastic scattering (DIS)



$Q^2 = -q^2 = -(k - k')^2$ **momentum transfer.**

$x \equiv \frac{Q^2}{2p\cdot q}$ **momentum fraction (from** $p'^2 = (xp + q)^2 = 0$**).**

$y = \frac{p\cdot q}{p\cdot k}$ **fractional energy transfer in** p **rest frame.**

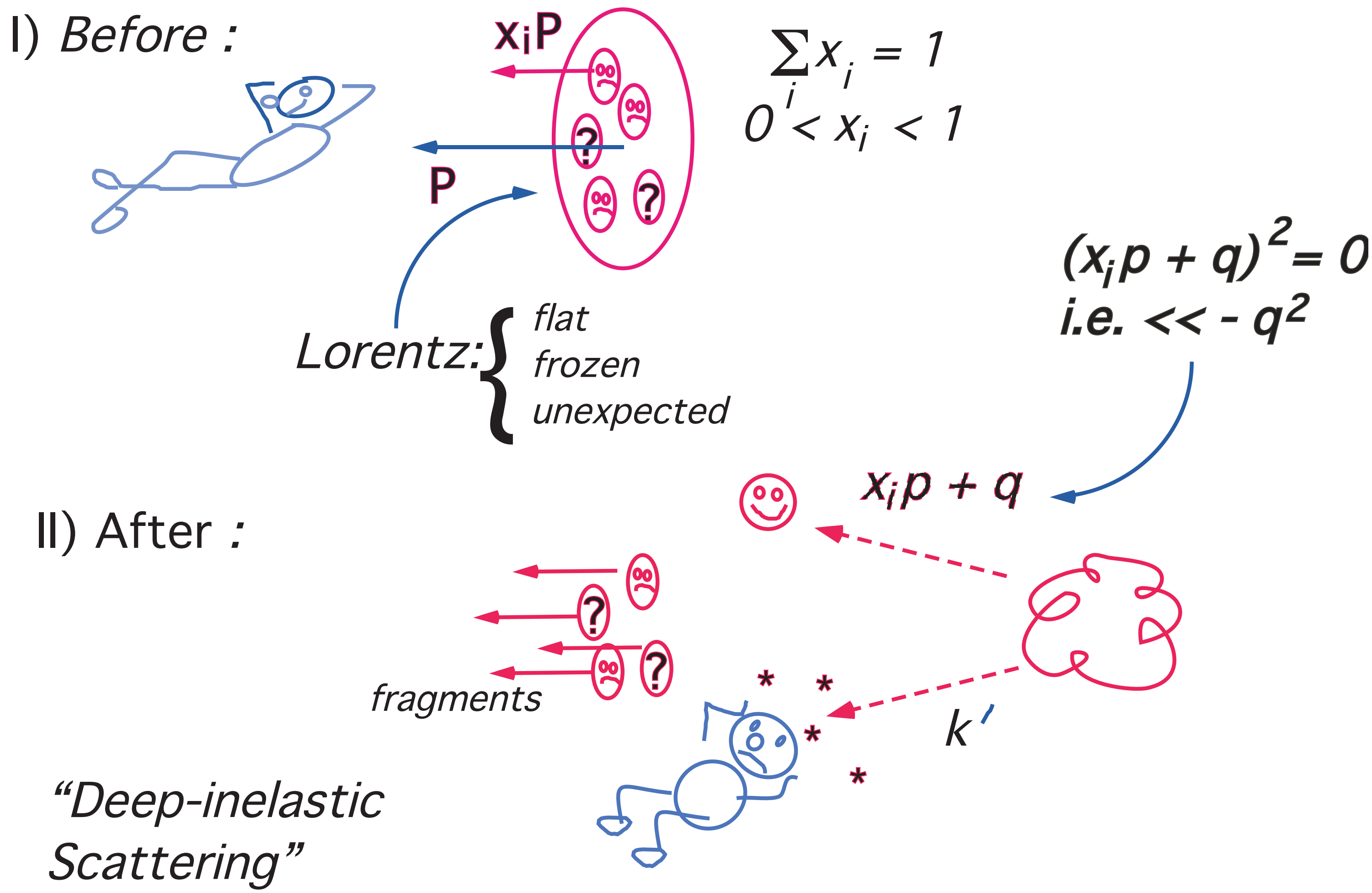
$W^2 = (p + q)^2 = \frac{Q^2}{x}(1 - x)$ **squared invariant mass of final-state hadrons.**

A useful identity:

$$xy = \frac{Q^2}{S}$$

From CTEQ Summer School 1992:

Parton Interpretation (Feynman 1969, 72)
Look in the electron's rest frame . . .



– Basic Parton Model Relation

$$\sigma_{\rm eh}(p,q) \ = \ \sum_{\text{partons } a} \int_0^1 d\xi \, \hat{\sigma}^{\rm el}_{ea}(\xi p, q) \ \phi_{a/h}(\xi)\,,$$

– where: $\sigma_{eh}(p,q)$ is the inclusive cross section $e(k)+h(p) \to e(k' = k-q) + X(p+q)$

– and: $\hat{\sigma}^{\rm el}_{ea}(\xi p, q)$ is the elastic cross section $e(k)+a(\xi p) \to e(k'-q) + a(\xi p+q)$ which sets $(\xi p+q)^2 = 0 \to \xi = -q^2/2p\cdot q \equiv x$.

– and: $\phi_{a/h}(\xi)$ is the distribution of parton a in hadron h, the "probability for a parton of type a to have momentum ξp". It is independent of the details of the hard scattering – the hallmark of factorization.

– **in words: Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron's parton distributions.**

– **The nontrivial assertion: quantum mechanical incoherence of large-q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**

– **Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. Later we'll see how this works in QCD.**

The basic elastic scattering: electron with "quark" a, fractional charge e_a, from one-photon exchange:

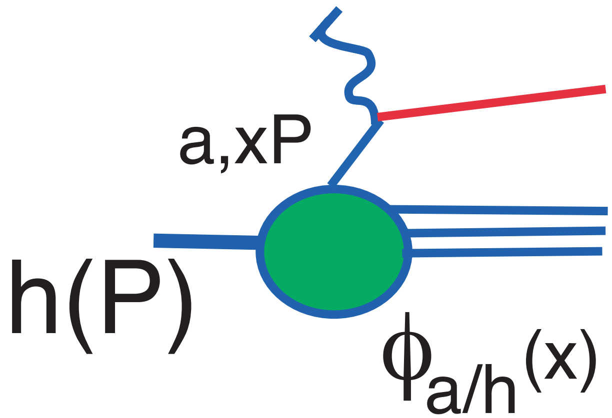
$$2\omega_{k'}(k,\xi p,q)\,\frac{d^3\sigma_{ea}^{(\mathrm{el})}}{d^3k'} \;=\; \frac{e_f^2\alpha_{EM}^2}{2s\,2p\cdot q}\left(\frac{s^2+u^2}{Q^4}\right)\delta(\xi - x)$$

where

$$s \;=\; (\xi p + k)^2\,, \quad u \;=\; (\xi p - k')^2 \quad t \;=\; -Q^2\,.$$

The "extra" delta function restricts the energy of the incoming quark, which isolates the parton distributions.
To analyze DIS in general terms, we will introduce a more general notation in terms of "structure functions" below.

– The conventional picture for distributions:



– "QM incoherence" $\Leftrightarrow$ no interactions between the outgoing scattered quark and the rest.

– Note: cross section is like a area. One parton: $1/Q^2$, area covered by $a \sim \phi_a(x)/Q^2$. For this picture to work:

$$\phi_a(x)/Q^2 \ll \pi R_p^2 .$$

Otherwise, the partons cover the proton and we can assume only a single interaction. This is called "saturation".

- **A contemporary set of parton distributions "at different scales": see "evolution" (CTEQ 2015: 1506.07443):**

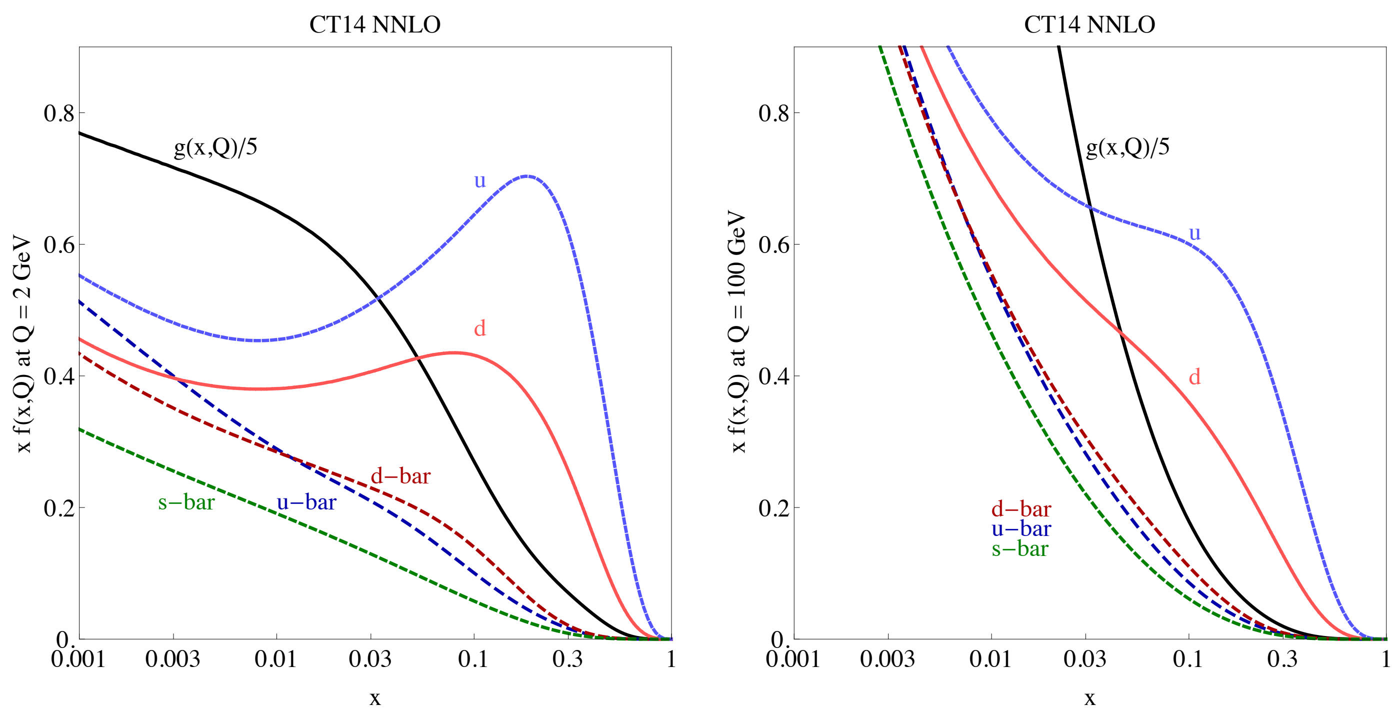
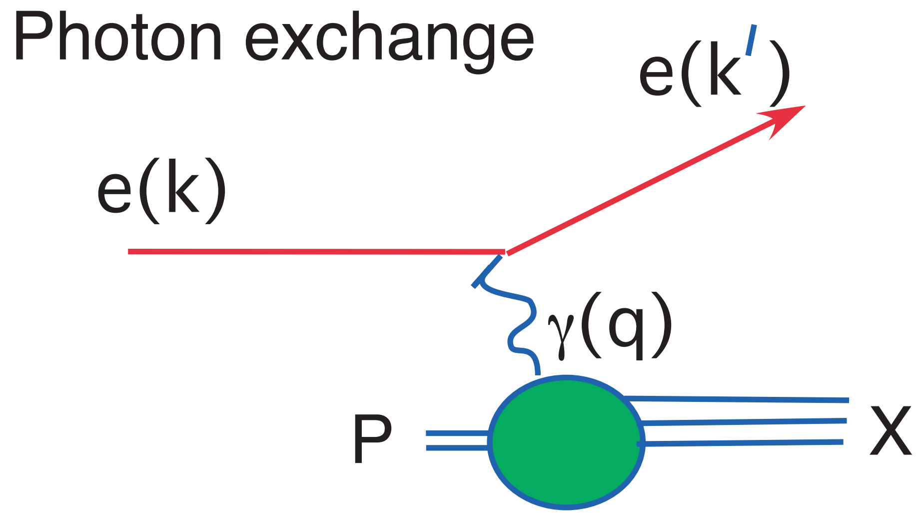


FIG. 4: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \overline{u}, d, \overline{d}, s = \overline{s}$, and g.

- **The distributions change with Q: $\phi_a(x) \to \phi_a(x, Q)$ – we'll see where this comes from.**

IB. DIS: Structure Functions and Scaling



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k) \times \frac{-ig^{\mu\mu'}}{q^2} \times \langle X|\, eJ^{\mathrm{EM}}_{\mu'}(0)\,|p,\sigma\rangle$$

- **Historically an assuption that the photon couples to hadrons by point-like current operator. Now, built into the Standard Model.**

- **The cross section:**

$$d\sigma_{\mathrm{DIS}} = \frac{1}{2^2}\frac{1}{2s}\frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda,\lambda',\sigma} |A|^2 \times (2\pi)^4 \delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the "unknown" hadronic part: $\sum |A|^2 \delta^4(\cdots) \equiv L^{\mu\nu} W_{\mu\nu}$.

- **The leptonic tensor:**

$$L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda,\lambda'} (\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k))^* \, (\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k))$$

$$= \frac{e^2}{2\pi^2} \left(k^\mu k'^{\,\nu} + k'^{\,\mu} k^\nu - g^{\mu\nu} k \cdot k' \right)$$

- **Leaves us with the "hadronic tensor":**

$$W_{\mu\nu}^{Vh} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J_\mu^{(V)}|p,\sigma\rangle^* \langle X|J_\nu^{(V)}|p,\sigma\rangle$$
$$\times\ (2\pi)^4 \delta^4(p_X - p - q)$$

where $J_\mu^{(V)}$ is the electroweak current, coupled to vector: V = photon, Z_0 or $\mathrm{W}^\pm$. It is dimensionles (an exercise.)

- **And the cross section becomes:**

$$2\omega_{k'} \frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2}\ L^{\mu\nu} W_{\mu\nu}$$

- **$W_{\mu\nu}$ has sixteen components,**
  but known properties of the strong interactions constrain $W_{\mu\nu}$ . . .

- **An example: current conservation,**

$$\partial^\mu J_\mu^{\rm EM}(x) = 0$$
$$\Rightarrow \langle X|\, \partial^\mu J_\mu^{\rm EM}(x)\, |p\rangle = 0$$
$$\Rightarrow (p_X - p)^\mu \langle X|\, J_\mu^{\rm EM}(x)\, |p\rangle = 0$$
$$\Rightarrow q^\mu W_{\mu\nu} = 0$$

- **With time-reversal, etc . . .**

$$
\begin{aligned}
W_{\mu\nu}^{Vh} = &- \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1^{Vh}(x, Q^2) \\
&+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) W_2^{Vh}(x, Q^2) \\
&- i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \; W_3^{Vh}(x, Q^2)
\end{aligned}
$$

- **Often given in terms of the dimensionless structure functions,**

$$
F_1 = W_1 \qquad F_2 = p \cdot q W_2 \qquad F_3 = p \cdot q W_3 \, .
$$

- **Note that if there is no other mass scale, the F's cannot depend on Q except indirectly through x.**

- **Structure functions in the Parton Model: The Callan-Gross Relation**

  From the "basic parton model formula":

$$\frac{d\sigma_{eh}^{\text{incl}}}{d^3k'} = \sum_{\text{quarks } f} \int d\xi \; \frac{d\sigma_{ef}^{\text{el}}(\xi p)}{d^3k'} \; \phi_{f/h}(\xi) \tag{1}$$

Can treat a quark of "flavor" f just like any hadron and get

$$\omega_{k'} \, \frac{d\sigma_{ef}^{\text{el}}(\xi p)}{d^3k'} = \frac{1}{2(\xi s)Q^4} \, L^{\mu\nu} \, W_{\mu\nu}^{ef}(k + \xi p \to k' + p')$$

Here are three exercises to try out. Charge of f is e_f.

Ex. 1: Compute $W_{\mu\nu}^{\gamma f}$ to find:

$$W_{\mu\nu}^{\gamma f} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \delta\left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{2}$$
$$+ \left(\xi p_\mu - q_\mu \frac{\xi p \cdot q}{q^2}\right) \left(\xi p_\nu - q_\nu \frac{\xi p \cdot q}{q^2}\right) \delta\left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{\xi p \cdot q}\,.$$

Ex. 2: By substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\text{quarks}\, f} e_f^2 x\, \phi_{f/p}(x) = 2x F_1(x)\,.$$

Ex. 3: Show that this relation is quite different for scalar quarks.

- The Callan-Gross relation shows the compatibility of the quark and parton models.

- In addition: parton model structure functions are independent of Q^2, a property called "scaling".

- With massless partons, there is no other massive scale. Then the F's must be Q-independent; see above.

- Approximate properties of the kinematic region explored by SLAC in late 1960's – early 1970's.

- QCD "evolution" gives corrections to this picture.

The F's, W's from DIS: (CTEQ "Handbook", Rev. Mod. Phys. 67 (1995) 157.) Note slight difference in normalization of the W's compared to these notes.

$$\frac{d\sigma^{(lh)}}{dx\,dy}=\frac{E_k y}{m}N^{(lV)}\left[2W_1^{(Vh)}(x,q^2)\sin^2(\theta/2)\right.$$
$$+W_2^{(Vh)}(x,q^2)\cos^2(\theta/2)$$
$$\left.\pm W_3^{(Vh)}(x,q^2)\frac{E+E'}{m_h}\sin^2(\theta/2)\right], \tag{3.23}$$

where the $\pm$ corresponds to $V=W^{\pm}$, and where

$$N^{(l^{\pm}\gamma)}=8\pi\alpha^2\frac{m_h E}{Q^4},$$
$$N^{(\nu W^+)}=N^{(\bar\nu W^-)}=\pi\alpha^2\frac{m_h E}{2\sin^4(\theta_W)(Q^2+M_W^2)^2}. \tag{3.24}$$

Here θ_W is the weak mixing angle, and $\pi\alpha^2/(2M_W^4\sin^4\theta_W)=G_F^2/\pi$, with G_F the Fermi constant.

Other useful expressions for this cross section are given directly in terms of y,

$$\frac{d\sigma^{(lh)}}{dx\,dy}=N^{lV}\left[\frac{y^2}{2}2xF_1^{(Vh)}+\left(1-y-\frac{m_h xy}{2E}\right)F_2^{(Vh)}\right.$$
$$\left.+\delta_V\left(y-\frac{y^2}{2}\right)xF_3^{(Vh)}\right], \tag{3.25}$$

where δ_V is $+1$ for $V=W^+$ (neutrino beam), -1 for $V=W^-$ (antineutrino beam), and zero for the photon, while m_h is the target mass.

IC. Getting at the Quark Distributions

- **Relating the parton distributions to experiment**
- **Simplifying assumptions that illustrate the general approach.**

$$\phi_{u/p} = \phi_{d/n} \qquad \phi_{d/p} = \phi_{u/n} \qquad \text{isospin}$$

$$\phi_{\bar{u}/p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} \qquad \text{symmetric sea}$$

$$\phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 \qquad \text{no heavy quarks}$$

- **Adequate to early experiments, but no longer.**

- With assumptions above, the quark-parton model gives for e, ν (W^+) and $\bar{\nu}$ (W^-) DIS (see appendix)

$$
\begin{aligned}
F_2^{(eN)}(x) &= 2xF_1^{(eN)}(x) = \sum_{f=u,d,s} e_F^2 x\phi_{f/N}(x) \\
F_2^{(\nu N)} &= 2x\left(\sum_{D=d,s,b}\phi_{D/N}(x) + \sum_{U=u,c,t}\phi_{\bar{U}/N}(x)\right) \\
F_2^{(\bar{\nu}N)} &= 2x\left(\sum_{D}\phi_{\bar{D}/N}(x) + \sum_{U}\phi_{U/N}(x)\right) \\
F_3^{(\nu N)} &= 2\left(\sum_{D}\phi_{D/N}(x) - \sum_{U}\phi_{\bar{U}/N}(x)\right) \\
F_3^{(\bar{\nu}N)} &= 2\left(-\sum_{D}\phi_{\bar{D}/N}(x) + \sum_{U}\phi_{U/N}(x)\right)
\end{aligned}
$$

- Ex: Trace the relative minus sign between quarks and anti-quarks in F_3 back to the Born diagrams.

- The distributions are actually overdetermined with these assumptions, which checks the consistency of the picture.

- Further consistency checks: Sum Rules.

$$N_{u/p} = \int_0^1 dx \left[\phi_{u/p}(x) - \phi_{\bar{u}/p}(x) \right] = 2$$

etc. for $N_{d/p} = 1$.

The most famous ones make predictions for structure functions . . .

- **The Adler Sum Rule:**

$$\int_0^1 dx\,\frac{1}{2x}\left[F_2^{(\nu n)} - F_2^{(\nu p)}\right]$$

$$= \int_0^1 dx\left[\sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x)\right]$$

$$- \int_0^1 dx\left[\sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x)\right]$$

$$= \int_0^1 dx\left[\phi_{d/n}(x) - \phi_{\bar{u}/p}(x) - \Big(\phi_{d/p}(x) - \phi_{\bar{u}/n}(x)\Big)\right]$$

$$= N_{u/p} - N_{d/p}$$

$$= 1$$

In the 2nd equality, all the extra terms from heavy quarks $D = s, b$, $U = c, t$ cancel between proton and neutron. In the 3rd, we've used isospin invariance.

- **And similarly, the Gross-Llewellyn-Smith Sum Rule:**

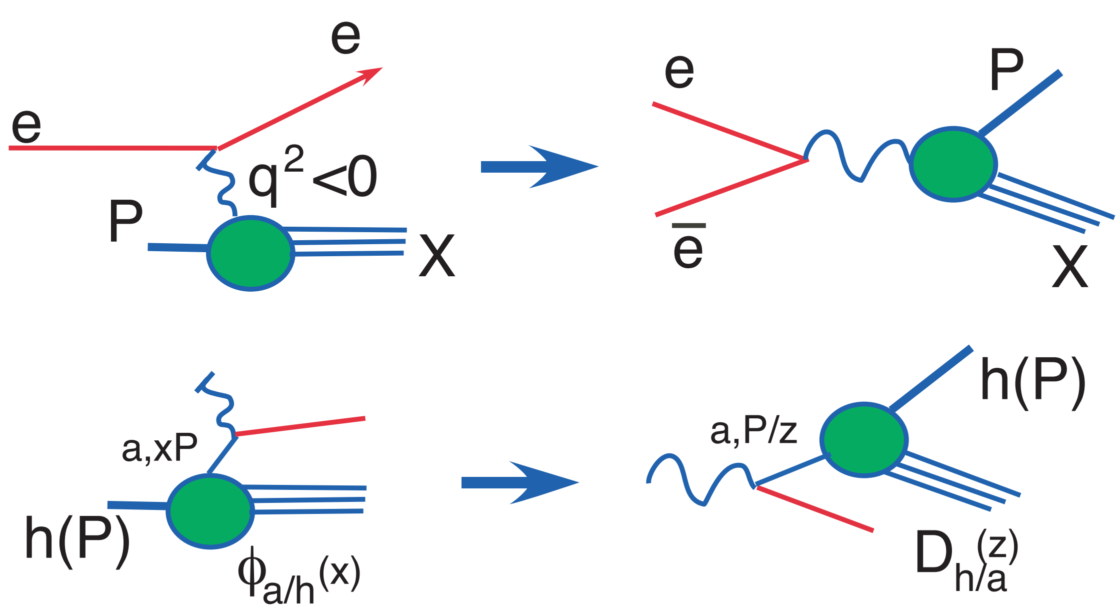
$$\begin{aligned} 3 &= N_{u/p} + N_{d/p} \\ &= \int_0^1 dx\, \frac{1}{2x} \left[xF_3^{(\nu n)} + xF_3^{(\nu p)} - \left(xF_3^{(\bar{\nu} n)} + xF_3^{(\bar{\nu} p)} \right) \right] \end{aligned}$$

Ex: work this one out from the relations of structure functions to quark and antiquark distributions.

ID. Classic Parton Model Extensions: Fragmentation and Drell Yan

- **Fragmentation functions**
- **“Crossing” applied to DIS: “Single-particle inclusive” (1PI) From scattering to pair annihilation.**

Parton distributions become “fragmentation functions”.



- **Parton model relation for 1PI: inclusive hadron from exclusive parton:**

$$\frac{d\sigma_h^{(\mathrm{incl})}(P,q)}{d^3P} = \sum_a \int_0^1 dz \; \frac{d\sigma^{(\mathrm{elas})}_{\mathrm{e^+e^-}\to a}(P/z,q)}{d^3P} \; D_{h/a}(z)$$

- **The direction of the hadron follows the direction of the parton!**

- $D_{h/a}$ **is "universal": could be in DIS (SIDIS), or hadron-hadron scattering.**

- **Heuristic justification from time dilation: Formation of hadron $h(P)$ from parton $a(P/z)$ takes a fixed time τ_0 in the rest frame of a, but much longer in the CM frame – this "fragmentation" thus decouples from $\sigma_a^{(\mathrm{elastic})}$, and is independent of mtm. transf. q (scaling).**

- For $\mathrm{e}^+(\mathrm{k}_2)\mathrm{e}^-(\mathrm{k}_1) \to \mathrm{q}(\mathrm{p}_1)\bar{\mathrm{q}}(\mathrm{p}_2)$.

Exercise: Start with the matrix element:

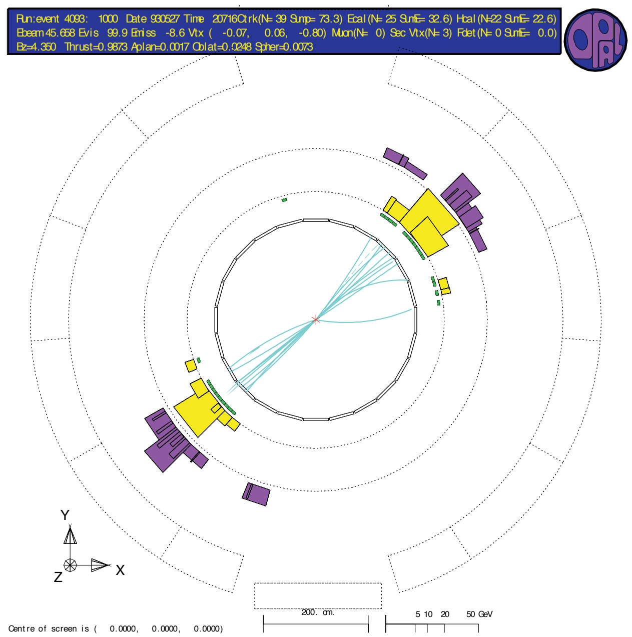
$$\mathcal{M} = e_q \frac{e^2}{Q^2} \, \overline{u}(p_1,\sigma_1)\gamma_\mu v(p_2,\sigma_2) \, \overline{v}(k_2,s_2)\gamma^\mu u(k_1,s_1) \; .$$

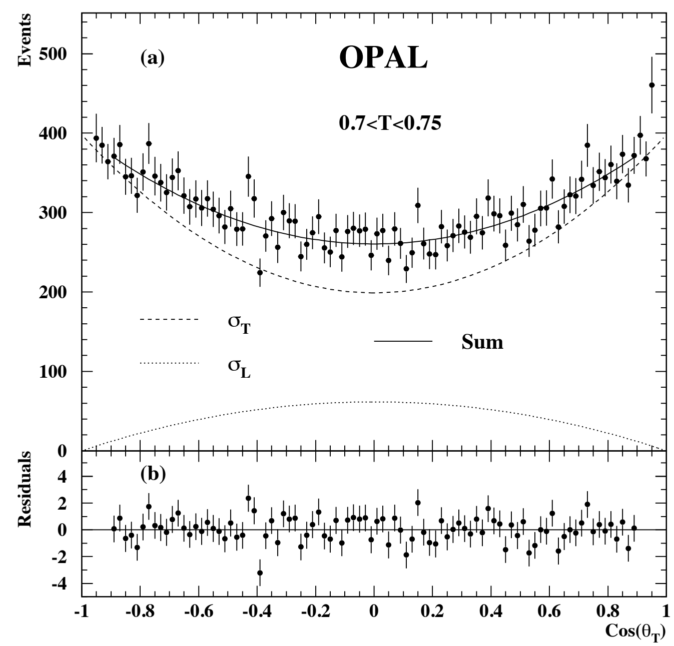
- First square and sum/average spins in $\mathcal{M}$. Then evaluate phase space at fixed angle for the "quark" p_1 in the final state to get:

$$\frac{d\sigma^{(\mathrm{elastic})}_{\mathrm{q}\bar{\mathrm{q}}\to\mu\bar{\mu}}(k_1,k_2)}{d\Omega} = \frac{1}{2Q^2}\frac{e_q^2 e^4}{32\pi^2}\, e_q^2 e^4 \left(1+\cos^2\theta\right) \; ,$$

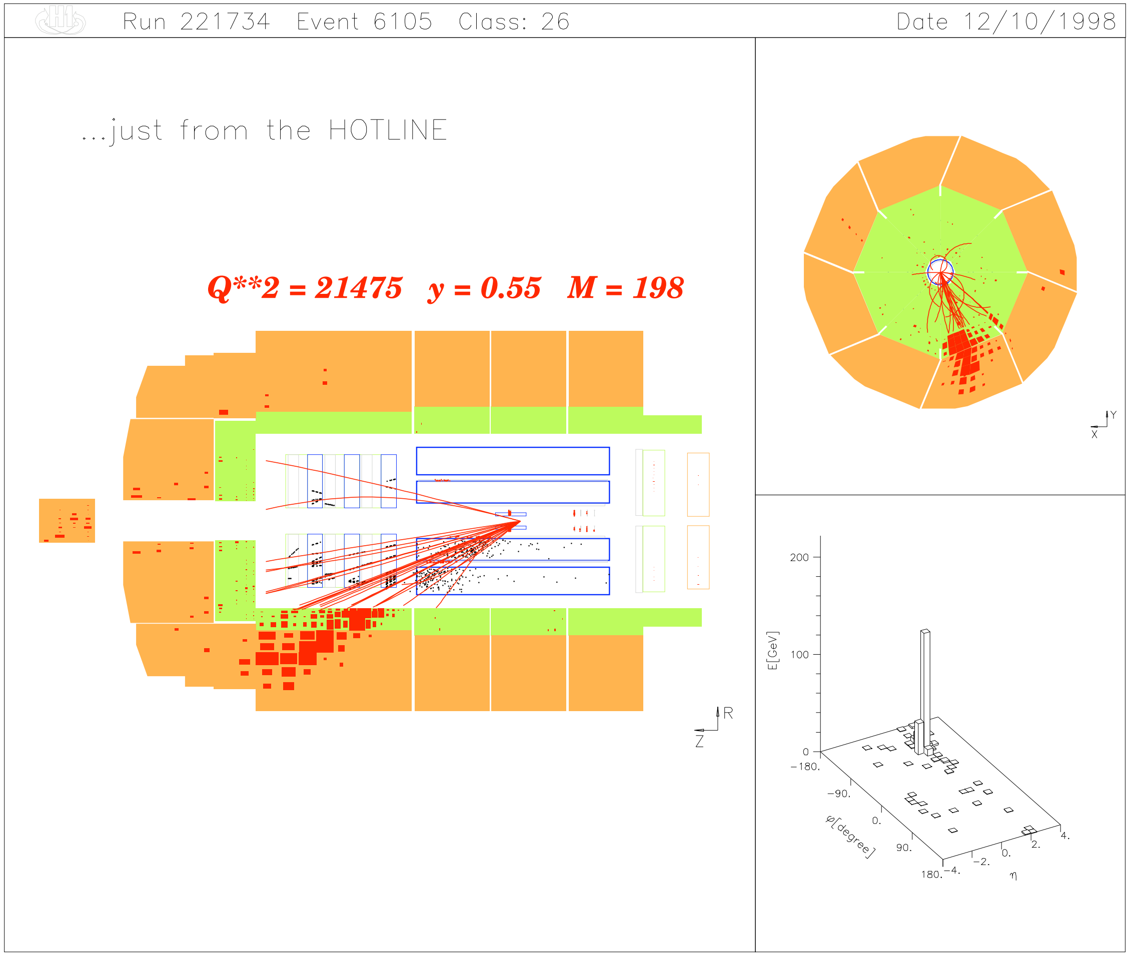
with $Q^2 = (k_1+k_2)^2$, and θ the angle between the electron and the quark.

- **The fragmentation picture suggests that almost all hadrons are aligned along parton directions $\Rightarrow$ most hadrons come out together as "jets", following the $1 + \cos^2\theta$ distribution relative to the incoming electron. And this is what happens.**
- **Hadrons should show up this way, and they do.**





- **For DIS:**



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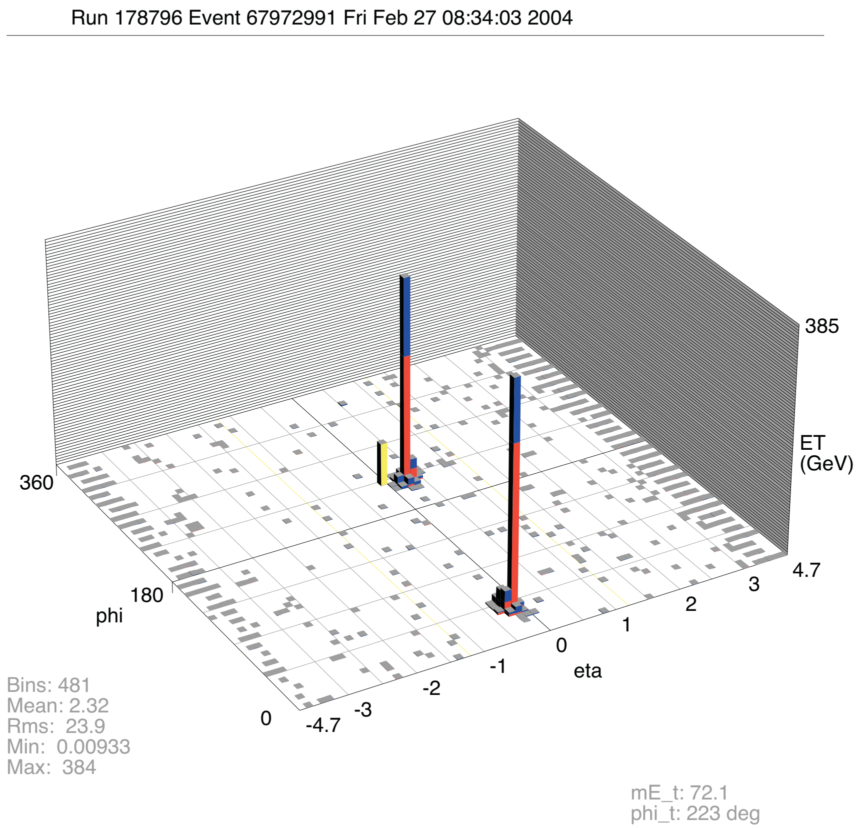
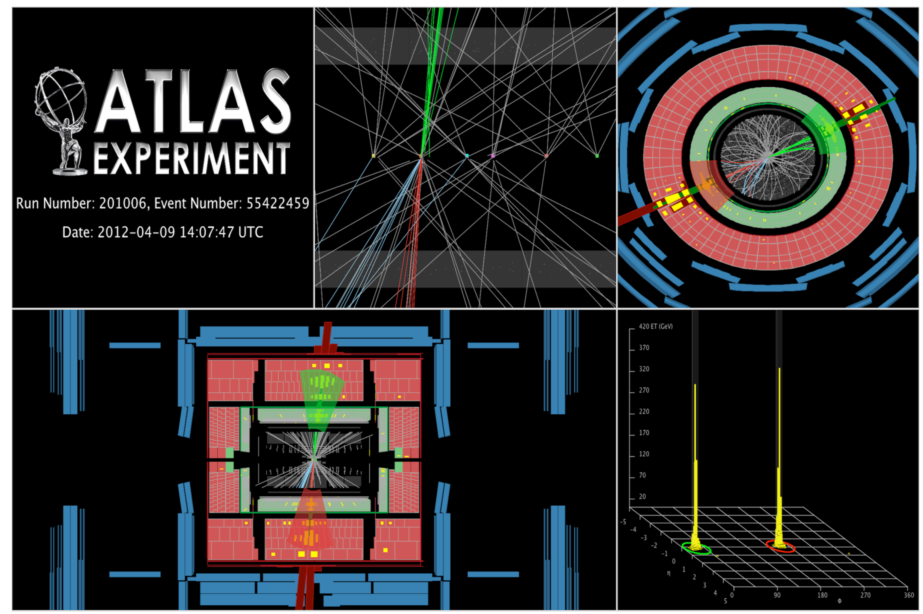
• And for nucleon-nucleon collisions:





- Finally: the Drell-Yan process

- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q ... any electroweak boson in NN scattering.

$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim$$

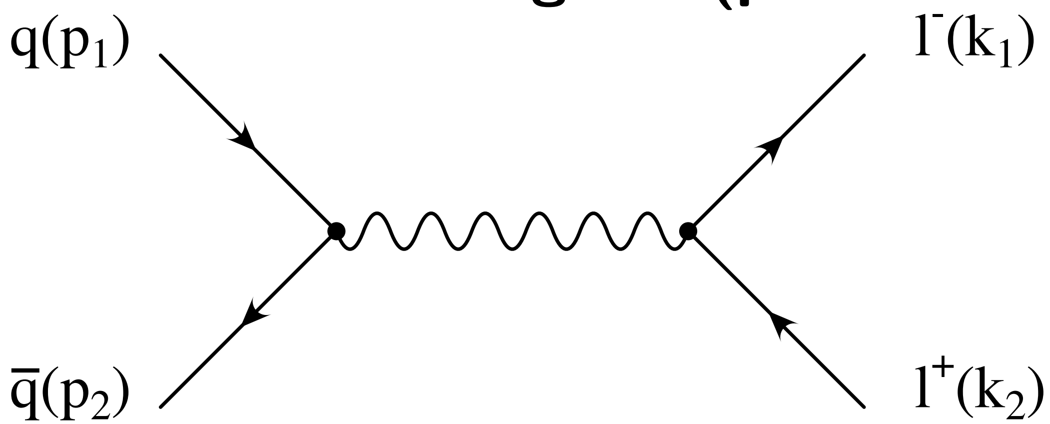
$$\int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a\bar{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$
$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS!

How it works (with colored quarks) . . .

- **The Born cross section: $\mathrm{e}^+\mathrm{e}^-$ backwards.**

$\boldsymbol{\sigma}^{\mathrm{EW,Born}}$ **is all from this diagram (parton x's set to unity):**



With this matrix element:

$$\boldsymbol{M \;=\; e_q \frac{e^2}{Q^2}\, \overline{u}(k_1,\sigma_1)\gamma_\mu v(k_2,\sigma_2)\; \overline{v}(p_2,s_2)\gamma^\mu u(p_1,s_1)}$$

- **First square and sum/average M. Then evaluate phase space.**

- Total cross section at "pair mass" $Q^2 = (x_1 p_1 + x_2 p_2)^2$

$$\sigma^{\mathrm{EW,\,elastic}}_{q\bar{q}\to\mu\bar{\mu}}(x_1 p_1, x_2 p_2) = \frac{1}{2\hat{s}} \int \frac{d\Omega}{32\pi^2} \frac{e_q^2 e^4}{3} (1 + \cos^2\theta)$$

$$= \frac{4\pi\alpha^2}{9Q^2} \sum_q e_q^2$$

With Q the pair mass and 3 for color average.

- **And measured rapidity:**

  Pair mass (Q) and rapidity

$$\eta \equiv (1/2)\ln\left(\frac{Q^+}{Q^-}\right) = (1/2)\ln\left(\frac{Q^0+Q^3}{Q^0-Q^3}\right)$$

- **ξ's are overdetermined $\rightarrow$ delta functions in the Born cross section**

$$\frac{d\sigma^{(PM)}_{NN\rightarrow\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\eta} =$$

$$\int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \sigma^{\mathrm{EW,\,Born}}_{a\bar{a}\rightarrow\mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2)$$

$$\times\delta\left(Q^2-\xi_1\xi_2 S\right)\,\delta\left(\eta-\frac{1}{2}\ln\left(\frac{\xi_1}{\xi_2}\right)\right)$$

$$\times\phi_{a/N}(\xi_1)\,\phi_{\bar{a}/N}(\xi_2)$$

- and integrating over rapidity, back to $d\sigma/dQ^2$,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha_{\mathrm{EM}}^2}{9Q^4}\right) \int_0^1 d\xi_1\, d\xi_2\, \delta\left(\xi_1\xi_2 - \tau\right) \\ \times \sum_a \lambda_a^2\, \phi_{a/N}(\xi_1)\, \phi_{\bar{a}/N}(\xi_s)$$

Found by Drell and Yan in 1970 (aside from 1/3 for color).
Analog of DIS scaling in x is DY scaling in $\tau = Q^2/S$.

- Template for all hard hadron-hadron scattering

- Ex.: fill in the results for this parton model cross section, starting from the matrix element M.

- **Appendix I: Quarks in the Standard Model**
  Electroweak interactions of quarks: SU(2)$_L$ $\times$ U(1). Their non-QCD interactions.

- **Quark and lepton fields: L(eft) and R(ight)**
  - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi$; $\psi = q, \ell$
  - **Helicity: spin along $\vec{p}$ (R=right handed) or opposite (L=left handed) in solutions to Dirac equation**
  - $\psi^{(L)}$: **expanded only in L particle solutions to Dirac eqn. R antiparticle solutions**
  - $\psi^{(R)}$: **only R particle solutions, L antiparticle**
  - **An essential feature: L and R have different interactions in general!**

– L quarks come in "weak $SU(2)$" = "weak isospin" pairs:

$$q_i^{(L)} = \begin{pmatrix} u_i \\ d_i' = V_{ij} dj \end{pmatrix} \qquad u_i^{(R)},\ d_i^{(R)}$$

$$(u, d') \qquad (c, s') \qquad (t, b')$$

$$\ell_i^{(L)} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \qquad e_i^{(R)},\ \nu_i^{(R)}$$

$$(\nu_e, e) \qquad (\nu_\mu, \mu) \qquad (\nu_\tau, \tau)$$

(We've neglected neutrino masses.)

– V_{ij} is the "CKM" matrix.

– The electroweak interactions distinguish L and R.

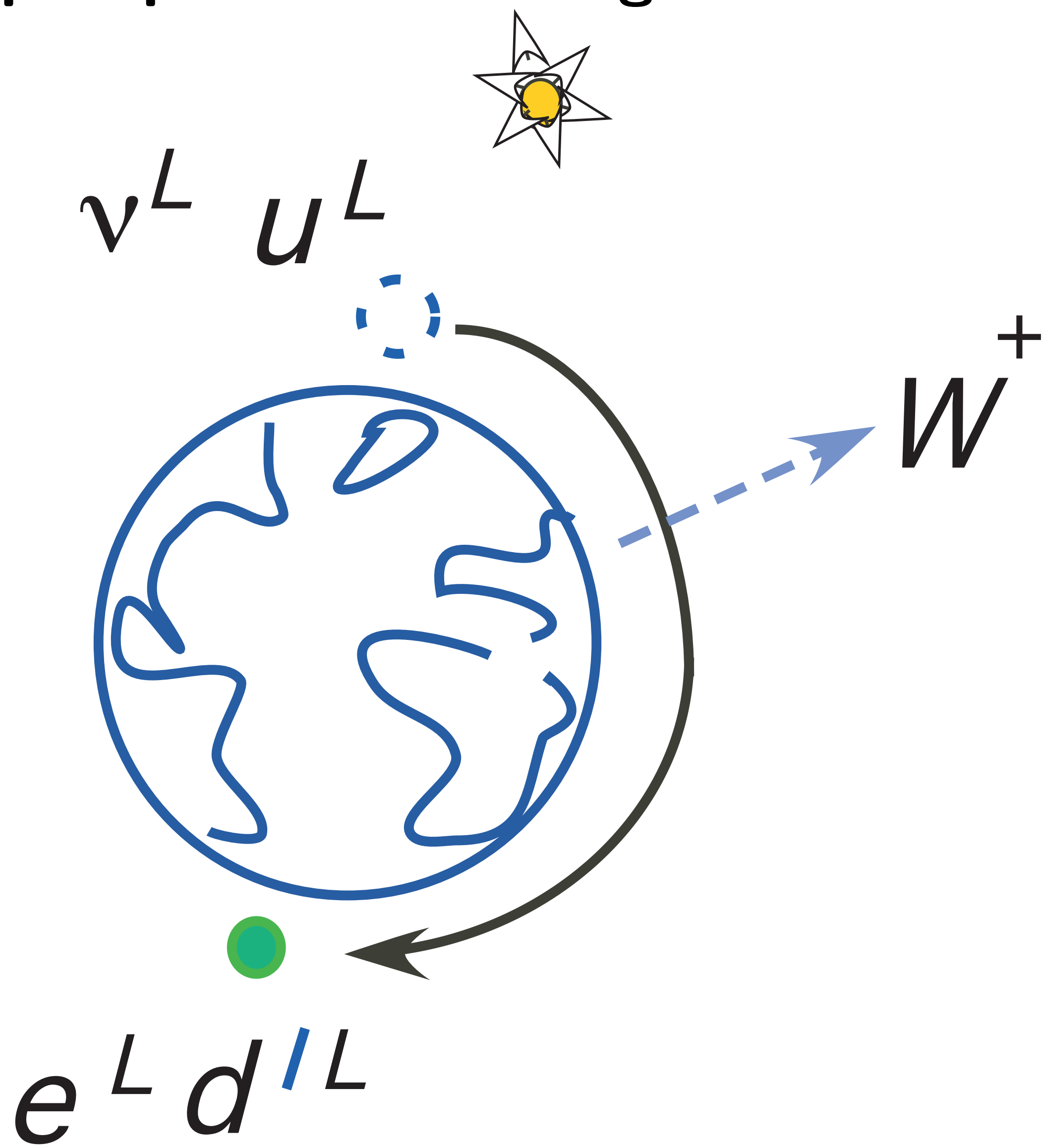
- **Weak vector bosons: electroweak gauge groups**
  - **SU(2): three vector bosons B_i with coupling g**
  - **U(1); one vector boson C with coupling g'**
  - **The physical bosons:**

$$W^{\pm} = B_1 \pm iB_2$$
$$Z = -C \sin\theta_W + B_3 \cos\theta_W$$
$$\gamma \equiv A = C \cos\theta_W + B_3 \sin\theta_W$$

$$\sin\theta_W = g'/\sqrt{g^2 + g'^2} \qquad M_W = M_Z/\cos\theta_W$$

$$e = gg'/\sqrt{g^2 + g'^2} \qquad M_W \sim g/\sqrt{G_F}$$

- **Weak isospin space: connecting u with d'**



- **Only left handed fields move around this globe.**

– The interactions of quarks and leptons with the photon, W, Z

$$\mathcal{L}_{\rm EW}^{(fermion)} = \sum_{\text{all } \psi} \bar{\psi} \left(i \not{\partial} - e \lambda_\psi \not{A} - (g m_\psi 2 M_W) h \right) \psi$$
$$- (g/\sqrt{2}) \sum_{q_i, e_i} \bar{\psi}^{(L)} \left(\sigma^+ \not{W}^+ + \sigma^- \not{W}^- \right) \psi^{(L)}$$
$$- (g/2 \cos\theta_W) \sum_{\text{all } \psi} \bar{\psi} \left(v_f - a_f \gamma_5 \right) \not{Z} \, \psi$$

– Interactions with W are through ψ_L's only.

– Neutrino Z exchange depends on $\sin^2 \theta_W$ even at low energy.

– This observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large $\rightarrow$ a need for colliders.

– Coupling to the Higgs $h \propto$ mass (special status of t).

- **Symmetry violations in the standard model:**

  - W's interact through $\psi^{(L)}$ only, $\psi = q, \ell$.

  - These are left-handed quarks & leptons; right-handed antiquarks, antileptons.

  - Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.

  - CP combination OK ($L \rightarrow_P R \rightarrow_C L$) if all else equal, but it's not (quite) . . .

  Complex phases in CKM V result in CP violation.

- **Appendix II: Structure Functions and Photon Polarizations**

In the P rest frame can take

$$q^\mu = \left(\nu; 0, 0, \sqrt{Q^2 + \nu^2}\right) \,, \qquad \nu \equiv \frac{p \cdot q}{m_p}$$

In this frame, the possible photon polarizations ($\epsilon \cdot q = 0$):

$$\epsilon_R(q) = \frac{1}{\sqrt{2}}(0; 1, -i, 0)$$
$$\epsilon_L(q) = \frac{1}{\sqrt{2}}(0; 1, i, 0)$$
$$\epsilon_{\text{long}}(q) = \frac{1}{Q}\left(\sqrt{Q^2 + \nu^2}, 0, 0, \nu\right)$$

• **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^\nu(q)\, F_\lambda(x,Q^2)$$

• **For photon exchange (Exercise 4):**

$$F_{L,R}^{\gamma e} = F_1$$
$$F_{\text{long}} = \frac{F_2}{2x} - F_1$$

• **So F_{long} vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**

- **Neutrinos: flavor of the "struck" quark is changed when a $W^{\pm}$ is exchanged. For W^{+}, a d is transformed into a linear combination of u, c, t, determined by CKM matrix (and momentum conservation).**

- **Z exchange leaves flavor unchanged but still violates parity.**

• **The Vh structure functions for $= W^+, W^-, Z$:**

$$W_{\mu\nu}^{(Vh)} - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2)$$
$$+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{m_h^2} W_2(x, Q^2)$$
$$-i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)$$

• **with dimensionless structure functions:**

$$F_1 = W_1\,, \qquad F_2 = \frac{p \cdot q}{m_h^2} W_2\,, \qquad F_3 = \frac{p \cdot q}{m_h^2} W_3$$

• $F_i^{(\nu h)}$ **gives** $W^+ h$ **scattering,** $F_i^{(\bar{\nu} h)}$ **gives** $W^- h$

• **And with spin (for the photon).**

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4z\, e^{iq\cdot z} \ \langle h(P,S) \mid J^\mu(z) J^\nu(0) \mid h(P,S)\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2)$$

$$+ \left(P^\mu - q^\mu \frac{P\cdot q}{q^2}\right)\left(P^\nu - q^\nu \frac{P\cdot q}{q^2}\right) F_2(x,Q^2)$$

$$+ i m_h\, \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q} g_1(x,Q^2) + \frac{S_\sigma (P\cdot q) - P_\sigma (S\cdot q)}{(P\cdot q)^2} g_2(x,Q^2)\right.$$

- **Parton model structure functions:**

$$F_2^{(eh)}(x) = \sum_f e_f^2\, x\, \phi_{f/h}(x)$$
$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 \left(\Delta\phi_{f/n}(x) + \Delta\bar{\phi}_{f/h}(x)\right)$$

- **Notation:** $\Delta\phi_{f/h} = \phi^{+}_{f/h} \;-\; \phi^{-}_{f/h}$ **with** $\phi^{\pm}_{f/h}(x)$ **probability for struck quark** f **to have momentum fraction** x **and helicity with (+) or against (−)** h**'s helicity.**