Electroweak Gauge Boson Production

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Electroweak interactions in the Standard Model (SM)

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \sum_{j=1}^{f} \bar{q}^j(x) i\gamma^\mu (\partial_\mu + ig_s G^a_\mu(x) \frac{\lambda^a}{2}) q^j(x)$$

$$\mathcal{L}_{EW} = \sum_f (\bar{\Psi}_f (i\gamma^\mu \partial\mu - m_f)\Psi_f - eQ_f \bar{\Psi}_f \gamma^\mu \Psi_f A_\mu) +$$

$$+ \frac{g}{2\sqrt{2}} \sum_i (\bar{a}^i_L \gamma^\mu b^i_L W^+_\mu + \bar{b}^i_L \gamma^\mu a^i_L W^-_\mu) + \frac{g}{2c_w} \sum_f \bar{\Psi}_f \gamma^\mu (I^3_f - 2s^2_w Q_f - I^3_f \gamma_5) \Psi_f Z_\mu +$$

$$-\frac{1}{4} |\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 - \frac{1}{2} |\partial_\mu W^+_\nu - \partial_\nu W^+_\mu +$$

$$-ie(W^+_\mu A_\nu - W^+_\nu A_\mu) + igc_w (W^+_\mu Z_\nu - W^+_\nu Z_\mu)|^2 +$$

$$-\frac{1}{4} |\partial_\mu Z_\nu - \partial_\nu Z_\mu + igc_w (W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 +$$

$$-\frac{1}{2} M^2_H H^2 - \frac{gM^2_H}{8M_W} H^3 - \frac{g^2 M^2_H}{32 M^2_W} H^4 + |M_W W^+_\mu + \frac{g}{2} H W^+_\mu|^2 +$$

$$+\frac{1}{2} |\partial_\mu H + iM_Z Z_\mu + \frac{ig}{2c_w} H Z_\mu|^2 - \sum_f \frac{g}{2} \frac{m_f}{M_W} \bar{\Psi}_f \Psi_f H$$

Glashow (1961); Higgs (1964,1966); Brout and Englert (1964); Guralnik, Hagen and Kibble (1964); Kibble (1967), Weinberg (1967); Salam (1968); 't Hooft, Veltman (1971)

$g = e/s_w$, $s_w \equiv \sin\theta_w$, $c_w \equiv \cos\theta_w = M_W/M_Z$, $e = \sqrt{4\pi\alpha}$; $\Psi_{L,f} = \frac{1-\gamma_5}{2}\Psi_f = (a_{L,f}, b_{L,f}) = (\nu_{e,L}, e^-_L), \ldots : SU(2)_I$ doublet for left-handed fermions

Organization of perturbative predictions for $2 \to n$ particle processes

Lagrangian + mathematical framework of perturbative QFT + a renormalization procedure $\to$ predictions for (parton-level) cross sections in terms of (measured) SM input parameters:

- Fixed order (LO, NLO, ...) ($g_s^2 = 4\pi\alpha_s, e^2 = 4\pi\alpha$):

$$d\sigma_{LO,NLO,\ldots} \propto \alpha^i \alpha_s^j |\mathcal{A}^0(e^i g_s^j)|^2 +$$

  one order higher in EW (NLO EW):

$$\alpha^{i+1}\alpha_s^j[|\mathcal{A}^1(e^{i+1}g_s^j)|^2 + 2\mathcal{R}e(\mathcal{A}^2(e^{i+2}g_s^j)\mathcal{A}^{0*}) + \mathcal{A}^1(e^{i+2}g_s^{j-1}\mathcal{A}^{1*}(e^i g_s^{j+1})] +$$

  one order higher in QCD (NLO QCD):

$$\alpha^i \alpha_s^{j+1}[|\mathcal{A}^1(e^i g_s^{j+1})|^2 + 2\mathcal{R}e(\mathcal{A}^2(e^i g_s^{j+2})\mathcal{A}^{0*}) + \mathcal{A}^1(e^{i+1}g_s^j)\mathcal{A}^{1*}(e^{i-1}g_s^{j+2})] + \ldots$$

- Resummation (LL, NLL, ...): all-order summation of classes of potentially large terms in the perturbation series, e.g., *See lectures by George Sterman.*

$$d\sigma \propto 1 + \alpha(L^2 + L + 1) + \alpha^2(L^4 + L^3 + L^2 + L + 1) + \ldots \quad L = \ln(A)$$

  reads after resummation

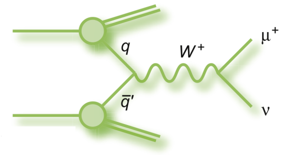
$$d\sigma_{res} \propto C(\alpha) \exp\left[Lg_1(\alpha L) + g_2(\alpha L) + \alpha g_3(\alpha L) + \ldots\right] + R(\alpha)$$

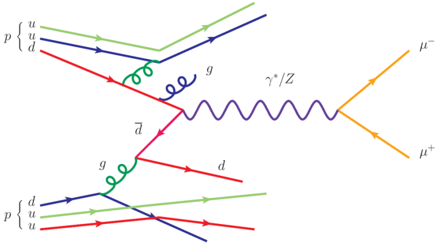
  g_1: leading logarithmic (LL) approximation
  g_2: next-to-leading logarithmic (NLL)) approximation

  Note that an increase in the logarithmic accuracy must go along with the inclusion, without double counting, of more terms in the $C(\alpha)$ series, a procedure known as matching.

Precise predictions for differential distributions over a wide kinematic range often require the combination of fixed-order and resummed predictions.

Precision physics with W and Z bosons in Drell-Yan-like processes





W and Z production processes are one of the theoretically best understood, most precise experimental probes of the Standard Model: *See lectures by Mayda Velasco.*

- Detector calibration (M_Z); tuning of multi-purpose Monte Carlo event generators.
- Search for BSM particles appearing as heavy resonances in W and Z distributions at high energies.
- Sensitive probe of proton structure, e.g., asymmetries in W^+, W^- rapidity distribution probe the $d(x, Q^2)/u(x, Q^2)$ PDF ratio.
- Precision measurement of W boson mass (M_W) and the *effective* leptonic weak mixing angle ($\sin^2 \theta^l_{eff}$): increased sensitivity to indirect signals of Beyond-the-SM (BSM) physics in EW precision observables (EWPO).

EW Precision (Pseudo-)Observables around the Z resonance at LEP/SLC

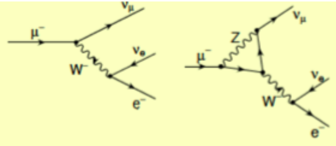
- Pseudo-observables are extracted from "real" observables (cross sections, asymmetries) by de-convoluting them of QED and QCD radiation and by neglecting terms ($\mathcal{O}(\alpha\Gamma_Z/M_Z)$) that would spoil factorization (γ, Z interference, t-dependent radiative corrections).
- The $Zf\bar{f}$ vertex is parametrized as $\gamma_\mu(G_V^f + G_A^f\gamma_5)$ with formfactors $G_{V,A}^f$, so that the partial Z width reads:

$$\Gamma_f = 4N_c^f\Gamma_0(|G_V^f|^2 R_V^f + |G_A^f|^2 R_A^f) + \Delta_{EW/QCD}$$

  $R_{V,A}^f$ describe QED, QCD radiation and Δ non-factorizable radiative corrections. Pseudo-observables are then defined as ($g_{V,A}^f = ReG_{V,A}^f$) D.Bardin *et al.*, hep-ph/9902452
  - $\sigma_h^0 = 12\pi\frac{\Gamma_e\Gamma_h}{M_Z^2\Gamma_Z^2}$, $R_{q,l} = \Gamma_{q,h}/\Gamma_{h,l}$
  - $A_{FB}^f = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \to A_{FB}^{f,0} = \frac{3}{4}A_e A_f, A_f = 2\frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}$
  - $A_{LR}(SLD) = \frac{N_L - N_R}{N_L + N_R}\frac{1}{<P_e>} \to A_{LR}^0(SLD) = A_e$
- $\sin^2\theta_{eff}^l$ is extracted from A_{FB} and A_{LR}: $4|Q_f|\sin^2\theta_{eff}^f = 1 - \frac{g_V^f}{g_A^f}$ with $g_{V,A}^f$ being *effective* couplings including radiative corrections.
  Note: At leading order, $\sin^2\theta_{eff}^f \equiv \sin^2\theta_w = 1 - \frac{M_W^2}{M_Z^2}$.

Prediction for M_W

The W boson mass can be calculated from an implicit equation relating the muon lifetime in the Fermi model to the one calculated in the SM:



$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha(0)M_Z^2}{2(M_Z^2 - M_W^2)M_W^2}[1 + \Delta r(\alpha, M_W, M_Z, m_t, M_H, \ldots)]$$

Δr describes the loop corrections to muon decay ($c_W = M_W/M_Z$):

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho(0) + 2\Delta_1 + \frac{s_w^2 - c_W^2}{s_w^2}\Delta_2 + \text{boxes, vertices, higher orders}$$

$\Delta\rho(0)$ at 1-loop is given in terms of 1-PI EW gauge boson self energies, $\Pi^T_{V_1 V_2}$:

$$\Delta\rho(0) = \frac{\Pi^T_{WW}(0)}{M_W^2} - \frac{\Pi^T_{ZZ}(0)}{M_Z^2} - 2\frac{s_W}{c_W}\frac{\Pi^T_{Z\gamma}(0)}{M_Z^2}$$

$\Delta\alpha$ describes contributions to the running of α: $\Delta\alpha = \Delta\alpha_{lep} + \Delta\alpha_{top} + \Delta\alpha^{(5)}_{had} + \ldots$.

For a review of the role of radiative corrections in EW precision physics see, e. g., A.Ferroglia, A.Sirlin (2013).

Status of Predictions for EWPOs

- To match or better exceed the experimental accuracy, some EWPOs had to be calculated even up to leading 4-loop corrections!
- Some of the most important EWPOs and their present-day and future estimated theory errors: see discussion by A.Freitas in EW WG Snowmass report, arXiv:1310.6708

Quantity	Current theory error	Leading missing terms	Est. future theory error
$\sin^2\theta^l_{\rm eff}$	4.5×10^{-5}	$\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(N_f^{\geq2}\alpha^3)$	$1...1.5\times10^{-5}$
R_b	$\sim2\times10^{-4}$	$\mathcal{O}(\alpha^2)$, $\mathcal{O}(N_f^{\geq2}\alpha^3)$	$\sim1\times10^{-4}$
Γ_Z	few MeV	$\mathcal{O}(\alpha^2)$, $\mathcal{O}(N_f^{\geq2}\alpha^3)$	<1 MeV
M_W	4 MeV	$\mathcal{O}(\alpha^2\alpha_s)$, $\mathcal{O}(N_f^{\geq2}\alpha^3)$	$<\sim1$ MeV

Since then fermionic 2-loop corrections have been completed A.Freitas, 1401.2477 [hep-ph]
New: Bosonic 2-loop correction have recently been calculated, which completes the EW 2-loop calculation: $\Delta\Gamma_Z\sim0.5MeV$ I. Dubovyk *et al*, 1804.10236[hep-ph]

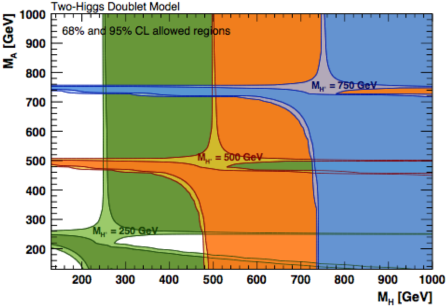
- Consider a specific BSM model, which is predictive beyond tree-level, and calculate complete BSM loop contributions to EWPOs (Z pole observables, M_W, $\sin^2\theta^l_{eff}$, ...). Example: 2HDM, MSSM
- In many new physics models, the leading BSM contributions to EWPOs are due to modifications of the gauge-boson self-energies which can be described by the *oblique* parameters S, T, U Peskin, Takeuchi (1991):

$$\Delta r \approx \Delta r^{\mathrm{SM}} + \frac{\alpha}{2s_W^2}\Delta S - \frac{\alpha c_W^2}{s_W^2}\Delta T + \frac{s_W^2 - c_W^2}{4s_W^4}\Delta U$$

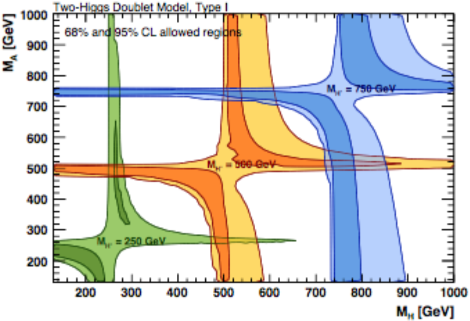
$$\sin^2\theta^l_{eff} \approx (\sin^2\theta^l_{eff})^{\mathrm{SM}} + \frac{\alpha}{4(c_W^2 - s_W^2)}\Delta S - \frac{\alpha s_W^2 c_W^2}{c_W^2 - s_W^2}\Delta T$$

Search for indirect signals of BSM physics in EWPOs

Constraints on a 2HDM model only from EWPO from GFitter: arXiv:1803.01853



Constraints on a 2HDM model from EWPO, flavor physics, and muon anomalous moment from Gfitter: arXiv:1803.01853



See also BSM constraints provided by the HEPfit collaboration: hepfit.roma1.infn.it.

Observables for the M_W measurement in W production at hadron colliders

M_W is extracted from the transverse mass $M_T(l\nu)$ and transverse momentum $p_T(l)$ distributions in W boson production:

- At LO in the W restframe ($p_T(W) = 0$ and assuming no intrinsic k_T):

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2(l)} = \frac{3}{M_W^2}\frac{1}{\left(1-\frac{4p_T(l)^2}{M_W^2}\right)^{1/2}}\left(1-\frac{2p_T(l)^2}{M_W^2}\right)$$

  $\Rightarrow$ peaks at $p_T(l) = M_W/2$ (Jacobean peak)

- Accordingly, the distribution in the transverse mass

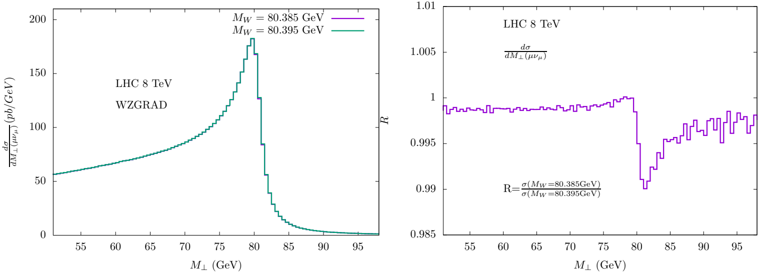
$$M_T(l\nu_l) = \sqrt{p_T(l)p_T(\nu)(1-\cos(\Phi(l)-\Phi(\nu)))}$$

  $\Delta\Phi = \pi$ and $p_T(l) = p_T(\nu) \Rightarrow M_T(l\nu) = 2p_T(l)$ has a peak at M_W. (see, e.g., *QCD and Collider Physics* by Ellis, Stirling, Webber)
  The Jacobean peak is smeared out by the finite width and non-zero transverse momentum of the W boson ($p_T(W) \neq 0$ due to parton or photon radiation).

- Due to the missing energy in $W \to l\nu_l$ events, the measurement of M_T also requires precise knowledge of $p_T(Z)$:
  $\vec{p}_T(\nu) = -\vec{p}_T(l) - \vec{u}_T$, where $\vec{u}_T = -\vec{p}_T(W)$ is the transverse momentum of hadronic recoil particles which balances $p_T(W)$.
  $p_T(W)$ can be deduced from the measured $p_T(Z)$ using predictions for the $p_T(W)/p_T(Z)$ ratio.

Sensitivity of $M_T(l\nu_l)$ to M_W

- The W mass is determined by performing template fits to the measured $M_T(l\nu_l)$ and $p_T(l)$ distributions.
- LO $M_T(\mu\nu_\mu)$ distribution in $pp \to W^+ \to \mu^+\nu_\mu$ at the 8 TeV LHC for two different values of M_W which differ by 10 MeV:

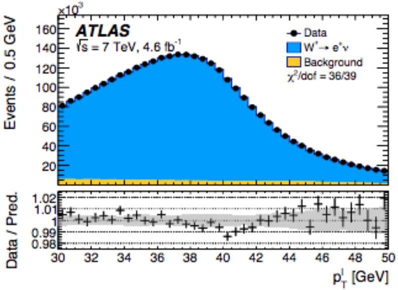


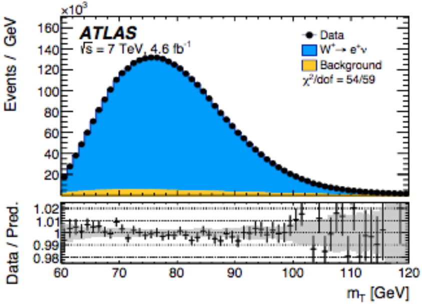
→ Predictions have to be under control at the permille level!

Repository of Drell-Yan MCs

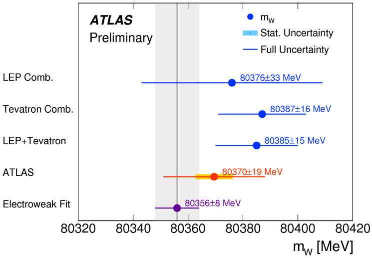
M_W measurement in $pp \to W^{\pm} \to (e^{\pm}\nu, \mu^{\pm}\nu) + X$

First *W* boson mass measurement at the LHC: The ATLAS Collaboration, arXiv:1701.07240





New world average (PDG 2017):
80.379 ± 0.012 GeV



Observable for the $\sin^2\theta^l_{eff}$ measurement in Z production at hadron colliders

- $\sin^2\theta_l^{eff}$ is extracted from the forward-backward asymmetry $A_{FB}(M_{ll}, y_{ll})$ in Z boson production with $\cos\theta$ defined in the Collins-Soper frame:

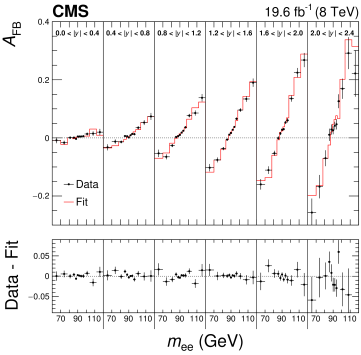
$$\cos\theta^* = \frac{|p_z(\mu^+\mu^-)|}{p_z(\mu^+\mu^-)} \frac{2[p^+(\mu^-)p^-(\mu^+) - p^-(\mu^-)p^+(\mu^+)]}{m(\mu^+\mu^-)\sqrt{m^2(\mu^+\mu^-) + p_T^2(\mu^+\mu^-)}}$$

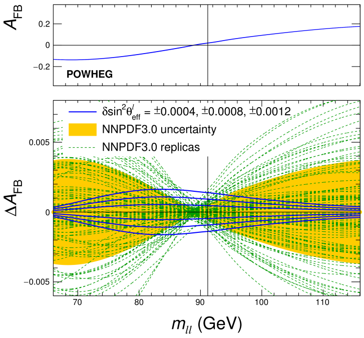
$(p^\pm = \frac{1}{\sqrt{2}}(E \pm p_z))$

- This requires a "new" parametrization of A_{FB} in terms of $\sin^2\theta_l^{eff}$ around the Z resonance in the presence of higher-order corrections and quark-couplings to the Z boson.

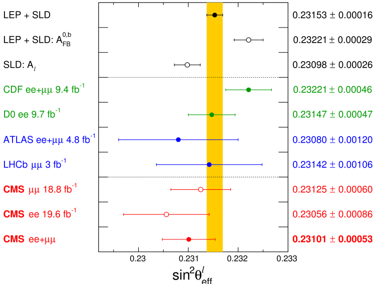
A renewed effort has just started to tackle challenges from theory (and experiment) to achieve the highest possible accuracy on M_W and $\sin^2\theta^l_{eff}$ at the LHC, and it is a good time to get involved: LHC Physics Center at CERN (LPCC) EW WG

$sin^2\theta^l_{eff}$ measurement in $pp \to \gamma, Z \to (e^+e^-, \mu^+\mu^-) + X$

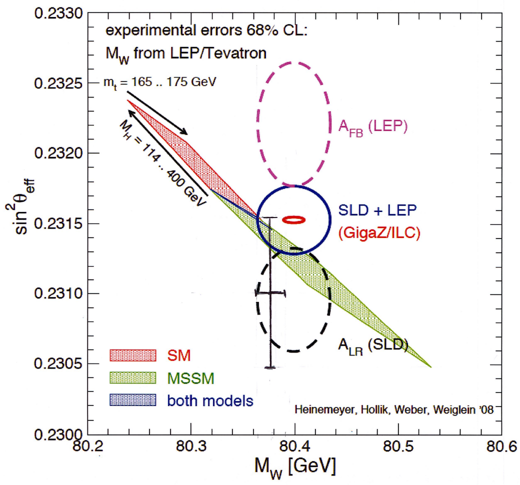




The CMS collaboration, arXiv:1806.00863



$sin^2\theta^l_{eff}$ measurement at LEP/SLD, LHC vs SM and MSSM predictions

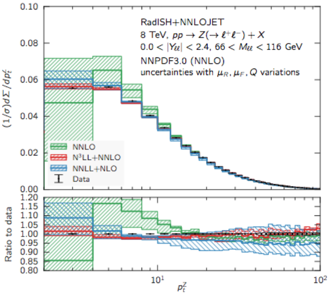


$M_W =$
80.379 ± 0.012 GeV
$\sin^2\theta^e_{eff}$ (CMS) $=$
0.23101 ± 0.00053

QCD prediction vs measurement: $p_T(Z)$ in $pp \to \gamma, Z \to l^+l^- + X$

$NNLO + N^3LL$ prediction for p_T^Z: W.Bizon *et al.*, arXiv:1805.05916

Transverse momentum $p_T(Z)$:



$Z(ll) + 1j$ production at LO ($O(\alpha^2\alpha_s)$) which is part of the NLO QCD correction to $Z(ll) + X$ production:

$$\frac{1}{\sigma}\frac{d\sigma}{dq_T^2} \propto \frac{1}{q_T^2}\alpha_s A_1 \log(\frac{M_Z^2}{q_T^2}) + \mathrm{non-log.\,terms}$$

For $q_T(Z)^2 \ll M_Z^2$ and multiple soft gluon radiation in leading approximation:

$$\frac{1}{\sigma}\frac{d\sigma}{dq_T^2} \propto \frac{1}{q_T^2}\sum_{n=1} A_n\alpha_s^n \log^{2n-1}(\frac{M^2}{q_T^2})$$

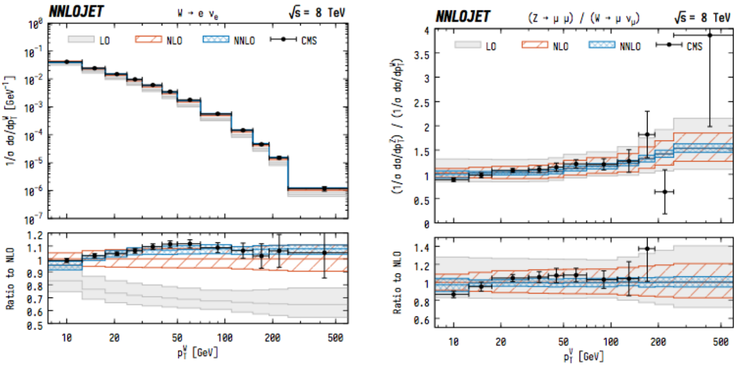
Resummation: Collins, Soper, Sterman (1985) (in b-parameter space); see also Becher, Neubert, 1007.4005 (SCET)

Table 11: Predictions for the Higgs cross sections in 13 TeV pp collisions before and after inclusion of the p_T^Z data in the global fits. The indicated errors are the PDF errors computed according to the NNPDF prescription.

	Before p_T^Z data	After p_T^Z data
$\sigma_{gg\to H}$ [pb]	48.22 ± 0.89 (1.8%)	48.61 ± 0.61 (1.3%)
σ_{VBF} [pb]	3.92 ± 0.06 (1.5%)	3.96 ± 0.04 (1.0%)

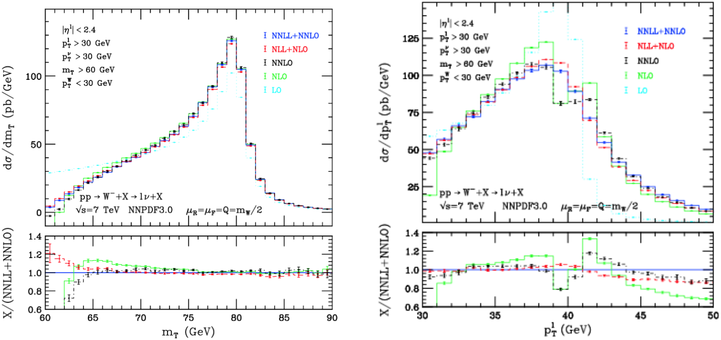
R. Boughezal *et al.*, arXiv:1705.00343

NNLO prediction for p_T^W and p_T^Z/p_T^W: Gehrmann-de Ritter *et al.*,arXiv:1712.07543



QCD prediction vs measurement: $p_T(l)$ and $M_T(l\nu)$ in $pp \to W^\pm \to (e^\pm\nu, \mu^\pm\nu) + X$

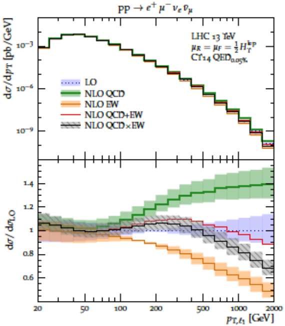
Resummation of $\alpha_s^n(m_W^2/q_T^2)\ln^m(m_W^2/q_T^2)$ at NNLL accuracy at small q_T combined with fixed order NNLO at large q_T Catani, de Florian, Ferrera, Grazzini, arXiv:1507.06937:



Electroweak radiative corrections

- EW corrections are relevant for modeling signal and background processes for searches of signals of new physics, either due to direct production, higher-dimensional operators, or the virtual presence of new particles in SM observables.
- EW corrections continue to play an especially important role in EW gauge boson production processes: V, VV, VVV (+jets) with $V = \gamma, Z, W^{\pm}$ at the LHC.
- NLO EW corrections can be numerically at least as important as NNLO QCD corrections, and for certain processes and in certain kinematic regions they may even be the dominant corrections.

$pp \to e^+\mu^-\nu_e\bar{\nu}_\mu$: S.Kallweit, J.Lindert, S. Pozzorini, M.Schönherr, arXiv:1705.00598;



Electroweak corrections: Useful resources

- Status of automation for the calculation of EW 1-loop corrections : arXiv:1605.04692
- Precision studies of observables in $pp \to W \to l_l$ and $pp \to \gamma, Z \to l^+ l$ processes at the LHC: arXiv:1606.02330
  and
  Precision Measurement of M_W: Theoretical Contributions and Uncertainties C. M. Carloni Calame,M. Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, A.Vicini arXiv:1612.02841
- Precise predictions for V+jets dark matter backgrounds J. M. Lindert, S. Pozzorini, R. Boughezal, J. M. Campbell, A. Denner, S. Dittmaier, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, S. Kallweit, P. Maierhöfer, M. L. Mangano, T.A. Morgan, A. Mück, F. Petriello, G.P. Salam, M.Schönherr, C.Williams arXiv:1705.04664
- Dictionary of electroweak (EW) corrections: S.Dittmaier in arXiv:1405.1067
  - EW Sudakov logs and mass-singular logs
  - QED corrections in PDFs and photon-induced processes
  - combination of QCD and EW corrections
  - gauge-invariant treatment of unstable W, Z bosons
  - EW input schemes
  - photon-jet separation

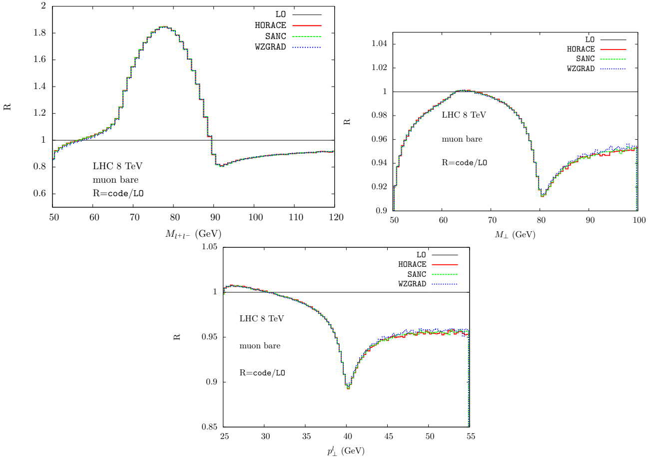
Characteristics of EW corrections

Naive estimate of relative size of EW and QCD corrections:
$\frac{\alpha(M_Z)}{\pi} \approx 0.0025$ vs. $\frac{\alpha_s(M_Z)}{\pi} \approx 0.037$ and $(\frac{\alpha_s(M_Z)}{\pi})^2 \approx 0.0014$
Possible enhancements:

- QED corrections:
  $\frac{\alpha(0)}{\pi} \log(\frac{m_f^2}{Q^2}) \approx -0.024$ for $Q = M_W, f = \mu$
  Origin: Soft/collinear FS photon radiation
  In sufficiently inclusive observables these mass singularities completely cancel. Kinoshita, Lee, Nauenberg (1962,1964)
  Depending on the experimental lepton identification cuts they can significantly affect the shape of distributions.
  IS mass singularities are factorized into PDFs which introduces a QED factorization scheme; PDFs with QED corrections and photon PDF A.Manohar, 1607.04266 (LUXqed), CT14qed, NNPDFqed, MMHT
- Weak Sudakov corrections:
  LL: $-\frac{\alpha}{\pi s_w^2} \log^2(\frac{M_V^2}{Q^2}) \approx -0.052$ for Q=2 TeV
  Origin: Remnants of UV singularities after renormalization and soft/collinear IS and FS emission of virtual and real W and Z bosons.
  Since observables are not fully inclusive in the weak isospin states these corrections do not completely cancel. M.Ciafaloni, P.Ciafaloni, D.Comelli (2000,2001) see, e.g., K.Mishra *et al*, 1308.1430; J.H.Kühn, Acta Phys.Polon.B39 (2008) for examples and a brief review

W and Z production at NLO EW at the LHC

Impact on observables usually shown as relative correction: $\delta(\%) = \frac{d\sigma_{NLO}}{d\sigma_{LO}}$



Precision studies of observables in $pp \to W \to l_l$ and $pp \to \gamma, Z \to l^+ l$ processes at the LHC arXiv:1606.02330

Mass-singular logarithms of QED origin: beyond NLO

Multiple FS photon radiation and exponentiation at LL, $L = \log(\frac{Q^2}{m_l^2})$:

- Exponentiation of YFS form factor Yennie,Frautschi,Suura (1961):

$$Y(m \ll Q) = \frac{\alpha}{\pi}\left\{2(L-1)\ln(\frac{2\Delta E_\gamma}{Q}) + \frac{1}{2}L - \frac{1}{2} - \frac{\pi^2}{6}\right\}$$

  Implemented in `WINHAC` for W production Placzek *et al* (2003), matched to NLO EW of `SANC` Bardin *et al* (2008); and in `Sherpa` M. Schönherr, F.Krauss (2008).

- QED parton shower: emission of n photons ($I_+ = \int_0^{1-\epsilon} dz P(z)$)

$$d\sigma = \exp[-\frac{\alpha}{2\pi} I_+ L] \sum_n^\infty |M_n^{LL}|^2 d\Phi_n$$

  Implemented in `HORACE` Carloni-Calame et al (2003,2004,2006), matched to full NLO EW.

- QED structure function Kuraev, Fadin (1985):
  $d\sigma = d\sigma_{LO} \int dz \Gamma(z)\theta_{cut}(zp_l)$; $\beta_l = \frac{2\alpha(0)}{\pi}(L-1)$

$$\Gamma(z, Q^2)) = \frac{\exp[-\beta_l/2\gamma_E + \frac{3}{8}\beta_l]}{\Gamma(1+\beta_l/2)} \frac{\beta_l}{2}(1-z)^{\beta_l/2-1} + \ldots + \mathcal{O}(\beta_l^4)$$

  Implemented in W production Brensing, Dittmaier, Krämer, Mück (2008) and Z production Dittmaier, Huber (2009), matched to full NLO EW.

- `POWHEG`(NLO QCD+EW)$\otimes$ (QCD+QED) PS; QED PS with `PHOTOS` (Golonka, Was (2005,2006)) or with `PYTHIA 8` for W production Carloni Calame *et al*, 1612.02841.

Initial-state photon radiation (ISR)

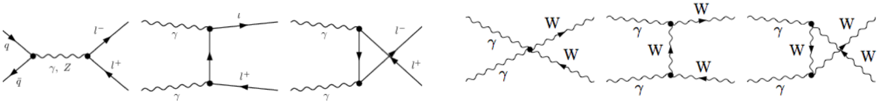
Mass singularities due to collinear radiation survive but are absorbed by universal collinear counterterms to the parton distribution functions; mass factorization done in complete analogy to QCD:

- introduces dependence on QED factorization scheme (in analogy to QCD there is a *DIS* and $\overline{MS}$ scheme) see, e.g. Baur, Keller, D.W., Phys. Rev. **D59**, 013002 (1999)

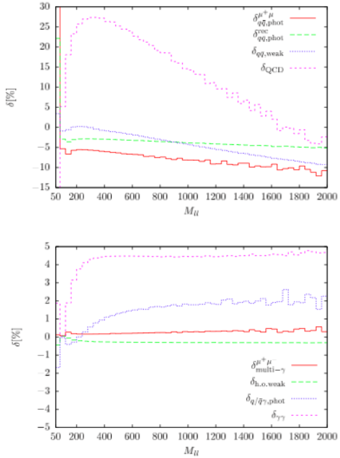
$$q_i(x,Q^2) = q_i(x)\,\left[1+\frac{\alpha}{\pi}\,Q_i^2\left\{1-\ln\delta_s-\ln^2\delta_s+\left(\ln\delta_s+\frac{3}{4}\right)\ln\left(\frac{Q^2}{m_i^2}\right)-\frac{1}{4}\lambda_{FC}\,f_{v+s}\right\}\right]$$
$$+\int_x^{1-\delta_s}\frac{dz}{z}\,q_i\left(\frac{x}{z}\right)\,\frac{\alpha}{2\pi}\,Q_i^2\left\{\frac{1+z^2}{1-z}\ln\left(\frac{Q^2}{m_i^2}\frac{1}{(1-z)^2}\right)-\frac{1+z^2}{1-z}+\lambda_{FC}\,f_c\right\}$$
$$f_{v+s}=9+\frac{2\pi^2}{3}+3\ln\delta_s-2\ln^2\delta_s$$
$$f_c=\frac{1+z^2}{1-z}\ln\left(\frac{1-z}{z}\right)-\frac{3}{2}\frac{1}{1-z}+2z+3$$

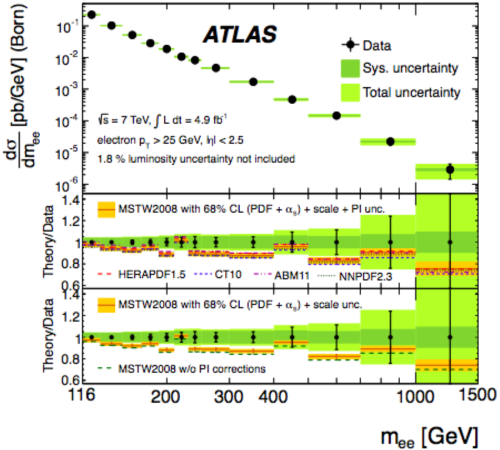
- PDFs including QED in their evolution have been made available, providing a photon PDF which allow for inclusion of photon-induced processes. See, e.g., combined LO QED$\times$ NNLO QCD DGLAP evolution with APFEL, apfel.mi.infn.it

Examples of photon-induced processes:



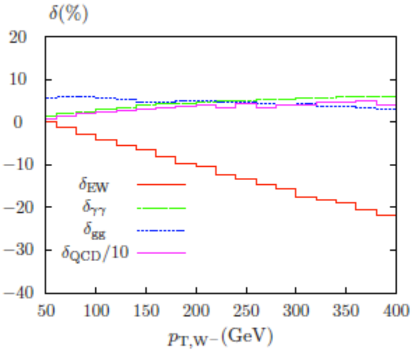
Impact of photon-induced processes in di-lepton production at the 14 TeV LHC:
The ATLAS collaboration, arXiv:1305.4192 (Theory: FEWZ NNLO+EW+W/Z rad.)
S.Dittmaier, M.Huber, arXiv:0911.2329

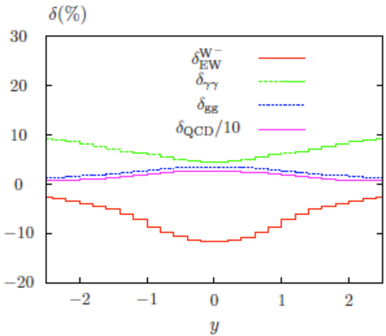




WW production at NLO EW at the 8 TeV LHC

p_T and y_w (with $M_{WW} > 500$ GeV) distributions of W^- at NLO EW at the 8 TeV LHC:

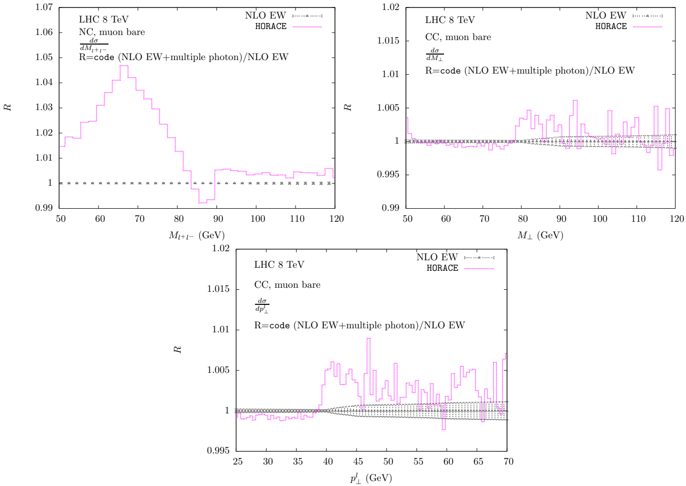




Bierweiler *et al*, arXiv:1208.3147

Interesting feature not seen in single-W production: photon-induced processes contribute considerably.

Multiple-photon radiation in Drell-Yan with HORACE



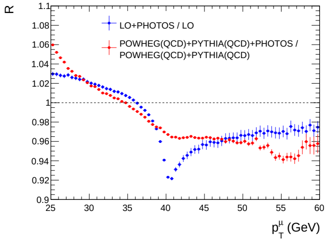
arXiv:1606.02330

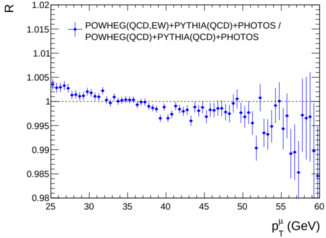
Shifts in M_W: $\delta M_W(\text{QED FSR}) \approx \mathcal{O}(100)$ MeV

$\delta M_W(mFS) \approx 2, 10$ MeV for e, μ Carloni-Calame et al (2003)

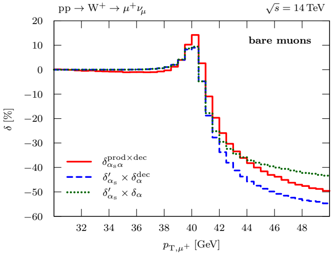
Impact of NNLO EW×QCD corrections

Implementation of NLO EW calculation in `POWHEG` (NLO QCD× parton shower): Barze *et al.* (2012); see also Bernaciak, D.W. (2012)





Comparison of initial-final factorizable $\mathcal{O}(\alpha\alpha_s)$ correction in pole approximation and a naive factorization defined as $\sigma^{LO}(1+\delta_{\alpha_s})(1+\delta_\alpha)$: S.Dittmaier, A.Huss, C.Schwinn, arXiv:1405.6897; 1403.3216

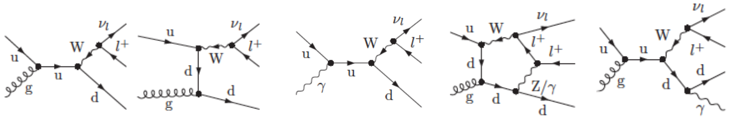


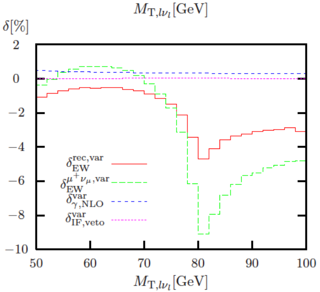
DY report, arXiv:1606.02330

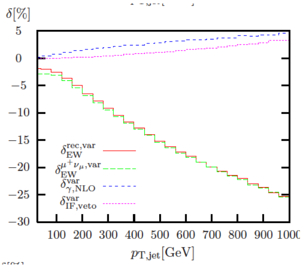
NLO Electroweak corrections to $W + j$ production

Fixed-order prediction for $pp \to W(l\nu) + 1\text{jet} + X$ up to $\mathcal{O}(\alpha_s\alpha^3)$: A.Denner, S.Dittmaier, T.Kasprzik, A. Mück, arXiv:0906.1656

$$d\sigma_{NLO} = d\sigma_{LO}(\alpha_s\alpha^2) + d\sigma_{LO,\gamma}(\alpha^3) + (d\sigma_{virtual} + d\sigma_{real})_{EW}(\alpha_s\alpha^3) + d\sigma_{NLOQCD,\gamma}(\alpha_s\alpha^3)$$







Automation of NLO EW (+QCD) calculations

"Amplitude calculators":

- Recola S.Actis *et al*, 1605.01090+ Collier A.Denner *et al* 1604.06792
- OpenLoops F.Cascioli *et al*, 1111.5206
- Gosam M.Chiesa *et al*, 1507.08579 + MadDipole T.Gehrmann *et al*, 1011.0321
- `Madgraph5_aMC@NLO` J.Alwall *et al*, 1405.0301
- `NLOX` (in preparation); Example: $pp \to Zb$ 1805.01353

Some recent results for multi-particle processes which consistently include higher-order QCD and EW corrections implemented in parton-shower MCs:

- `Recola+Sherpa` B.Biedermann *et al*, 1704.05783
  Examples: $pp \to V+$ jets, $pp \to ZZ \to 4$ leptons, $pp \to t\bar{t}H$, off-shell WWW production, 1806.00307
- `OpenLoops+Munich/Sherpa`
  Examples: $pp \to W + 1, 2, 3$ jets, S.Kallweit *et al*, 1412.5157, and $V + 1, 2$ jets with $V \to ll'$ and MEPS@NLO jet merging, S.Kallweit *et al*, 1511.08692; $pp \to 2\nu 2l$, S.Kallweit *et al*, 1705:00598
- `GOSAM+Sherpa` M.Chiesa *et al*, 1706.09022
  Examples: $pp \to \gamma\gamma + 0, 1, 2$ jets
- `Madgraph5_aMC@NLO`
  Examples: $pp \to t\bar{t} + (H, Z, W)$, S.Frixione *et al*, 1504.03446; $pp \to jj$, Frederix *et al*, 1612.06548

Appendix with more resources . . .

EW predictions for $pp \to W \to \nu l, pp \to Z, \gamma \to ll$

- Complete EW $\mathcal{O}(\alpha)$ corrections: `HORACE, RADY, SANC, W/ZGRAD2`
  U.Baur *et al*, PRD65 (2002); C.M.Carloni Calame *et al*, JHEP05 (2005)
  U.Baur, D.W., PRD70 (2004); S.Dittmaier, M.Krämer, PRD65 (2002); A.Andonov *et al*, EPJC46 (2006); Arbuzov *et al*, EPJC54 (2008); S.Dittmaier, M.Huber, JHEP60 (2010).
- Multiple final-state photon radiation: `HORACE, RADY, WINHAC, PHOTOS`
  W.Placzek *et al*, EPJC29 (2003); C.M.Carloni Calame *et al*, PRD69 (2004); S.Brensing *et al*, PRD77 (2008)
- EW Sudakov logarithms up to N^3LL Jantzen, Kühn, Penin, Smirnov (2005); brief review: J.H.Kühn, Acta Phys.Polon.B39 (2008)
- NLO EW corrections to W production implemented in POWHEG Bernaciak, W. (2012); Barze et al. (2012) $\Rightarrow$ Study of mixed QED-QCD effects
- NLO EW corrections to Z production implemented in POWHEG Barze et al. (2013) $\Rightarrow$ Study of mixed QED-QCD effects
- NLO EW corrections to Z production implemented in FEWZ (NNLO QCD) Li, Petriello (2012)
- $W+1j, Z+1j, Z+2j$(stable Z) at NLO EW, now with leptonic W, Z decays W.Hollik *et al* (2008); S.Dittmaier *et al* (2009); J.H.Kühn *et al* (2008); A.Denner *et al.* (2010); Actis *et al* (2012); weak Sudakov corr. to $Z+\leq 3$ jets in Alpgen Chiesa *et al* (2013)
- Toward W and Z production at $\mathcal{O}(\alpha\alpha_s)$ Kotikov *et al* (2008); Bonciani (2011); Kilgore, Sturm (2011); S.Dittmaier, A.Huss, C.Schwinn (2014)

QCD predictions for $pp \to W \to \nu l, pp \to Z, \gamma \to ll$

- NLO and NNLO QCD (up to $\mathcal{O}(\alpha_s^2)$): total cross sections ($\sigma_{W,Z}$) and fully differential distributions (DYNNLO, FEWZ):

  R.Hamberg *et al.*, NPB359 (1991); W.L.van Neerven *et al*, NBP382 (1992); W.T.Giele *et al*, NPB403 (1993)

  L.Dixon *et al.*, hep-ph/031226; K.Melnikov, F.Petriello, PRL96, PRD74 (2006); S.Catani *et al.*, PRL103 (2009), JHEP1005 (2010); R.Gavin *et al*, 1011.3540

- NLO QCD corrections matched to an all-order resummation of large logarithms $\ln^n(q_T/Q)$ (at NLL and NNLL accuracy) (Q: W/Z virtuality, q_T: W/Z transverse momentum).

  C.Balazs, C.-P.Yuan, PRD56 (1997) (ResBos); G.Bozzi *et al*, NPB815 (2009), arXiv:1007.2351; S.Catani *et al*, 1209.0158

- NLO QCD corrections matched to a parton shower (HERWIG, PYTHIA): MC@NLO, POWEG.

  S.Frixione, B.R.Webber, hep-ph/0612272; S.Alioli *et al*, JHEP0807 (2008)

- NNLO QCD corrections matched to a parton shower: Sherpa+BlackHat Hoeche, Li, Prestel, 1405.3607; POWHEG+MiNLO+DYNNLO Karlberg, Re, Zanderighi, 1407.2940

- $W + n$-jets ($n \leq 5$) and $Z + n$-jets ($n \leq 4$) at NLO QCD (and matched to PS).

  C.F.Berger *et al.* (2010,2009); Z.Bern *et al.* (2013); H.Ita *et al.* (2011); K.Ellis *et al.* (2009); J.Campbell *et al* (2002, 2013 (POWHEG)); B.Jaeger *et al* (2012) (POWHEG); S.Hoeche *et al* (2012)