University at Buffalo The State University of New York

# Higgs I : Fundamentals

## CTEQ Summer School 2018

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# Overview

## Lecture 1 :

- Fundamentals of the Higgs mechanism, SSB gauge invariance, masses of bosons and fermions

## Lecture 2 :

- Higgs at the LHC: Higgs production, EFT, Higgs decays. Recent Experimental results.

# More resources

- These slides can be downloaded from :
- Additionally a longer PDF writeup of this lecture:
- Also there are 4 very detailed CERN yellow reports on the Higgs boson : https://inspirehep.net/search?ln=en&ln=en&p=Handbook+of+LHC+Higgs+Cross+Sections&of=hb&action_search=Search&sf=&so=d&rm=&rg=25&sc=0

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**“Higgs boson-like particle discovery claimed at LHC”** BBC NEWS

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Scientists at the Fermilab in Batavia, Ill., on Wednesday watched the presentation about the discovery of the Higgs boson, which was shown from Geneva.

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# Global Symmetries and Gauge Theories

# U(1) Gauge Inv.

We begin by refreshing our notion of Gauge invariance.

We make the following transformation on our Dirac Spinor.

$$\psi \rightarrow e^{i\alpha(x)}\psi \qquad \overline{\psi} \rightarrow \overline{\psi}e^{-i\alpha(x)}$$

# U(1) Gauge Inv.

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$$\psi \rightarrow e^{i\alpha(x)}\psi \qquad \overline{\psi} \rightarrow \overline{\psi}e^{-i\alpha(x)}$$

Then investigate what happens to the Dirac Lagrangian

$$\mathcal{L} = i\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi$$

Under this transformation.

$$\mathcal{L} = i\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi$$

The mass term is trivially gauge invariant since

$$\overline{\psi}\psi \to \overline{\psi}e^{-i\alpha(x)}e^{i\alpha(x)}\psi = \overline{\psi}\psi$$

However the spacetime derivative is a bit tricker

$$\partial_\mu \psi \to e^{i\alpha(x)}\partial_\mu \psi + ie^{i\alpha(x)}\psi \, \partial_\mu \alpha$$

So the kinetic term is not gauge invariant on its own.

We would like to change the derivative such that it transforms as follows

$$D_\mu \psi \to e^{i\alpha(x)} D_\mu \psi$$

This “covariant derivative” will be gauge invariant.

We can achieve this transformation by defining the covariant derivative as follows

$$D_\mu = \partial_\mu - ieA_\mu$$

And we demand the following transformation of A.

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

The magic of gauge invariance is that we just mandated the inclusion of a gauge boson to our free Lagrangian. That is we recovered the interaction part of the QED Lagrangian

$$\begin{aligned}\mathcal{L} &= i\overline{\psi}\gamma^\mu D_\mu\psi - m\overline{\psi}\psi \\ &= i\overline{\psi}\gamma^\mu\partial_\mu\psi + e\overline{\psi}\gamma^\mu A_\mu\psi - m\overline{\psi}\psi\end{aligned}$$

We can get the full QED Lagrangian by including the Field Strength Tensor (which is gauge invariant)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

So

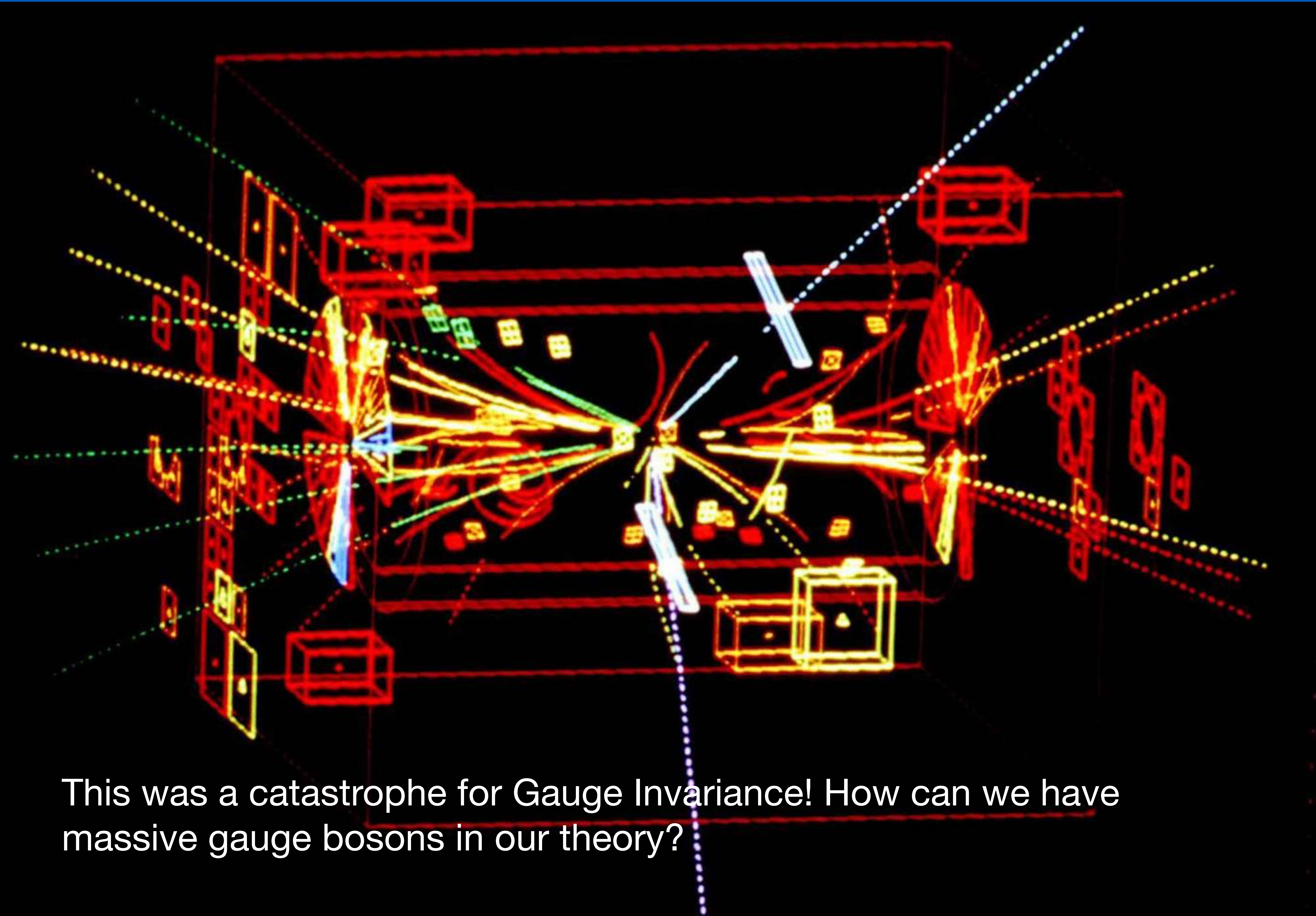
$$\mathcal{L}_{QED} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\overline{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Now, whats important for our discussion today is that its impossible to write down a mass term which respects gauge invariance since under our U(1) transformation the quadratic term

$$\mathcal{L}_m = \frac{1}{2}m^2 A^\mu A_\mu$$

behaves as follows

$$\frac{1}{2}m^2 A^\mu A_\mu \to \frac{1}{2}m^2(A^\mu + \partial^\mu \lambda)(A_\mu + \partial_\mu \lambda) \neq \frac{1}{2}m^2 A^\mu A_\mu$$



This was a catastrophe for Gauge Invariance! How can we have massive gauge bosons in our theory?

# Spontaneous Symmetry Breaking of a Gauge Theory

We start with a toy example, which actually contains nearly all of the physics we will need.

We have a Lagrangian

$$\mathcal{L} = T - V = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

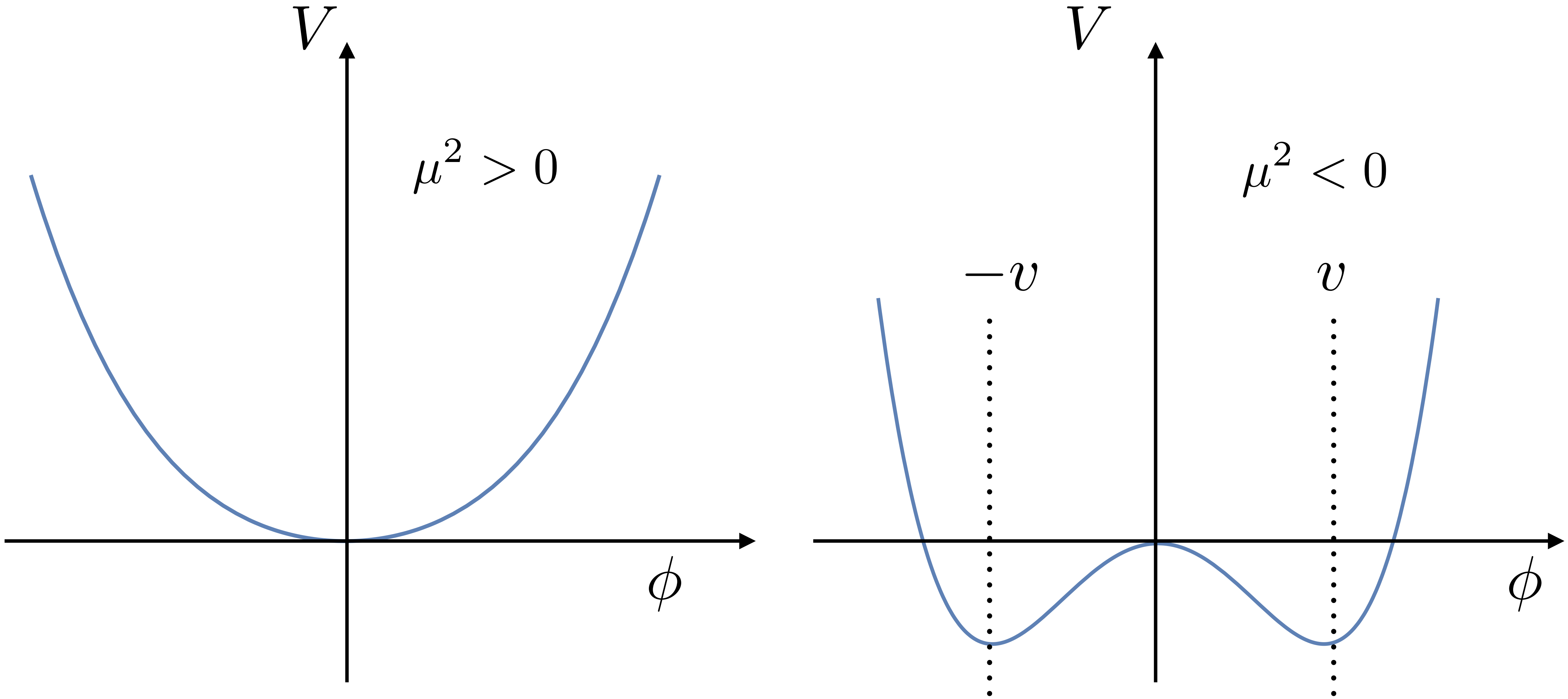
This is invariant under the following (discrete) symmetry

$$\phi \to -\phi$$

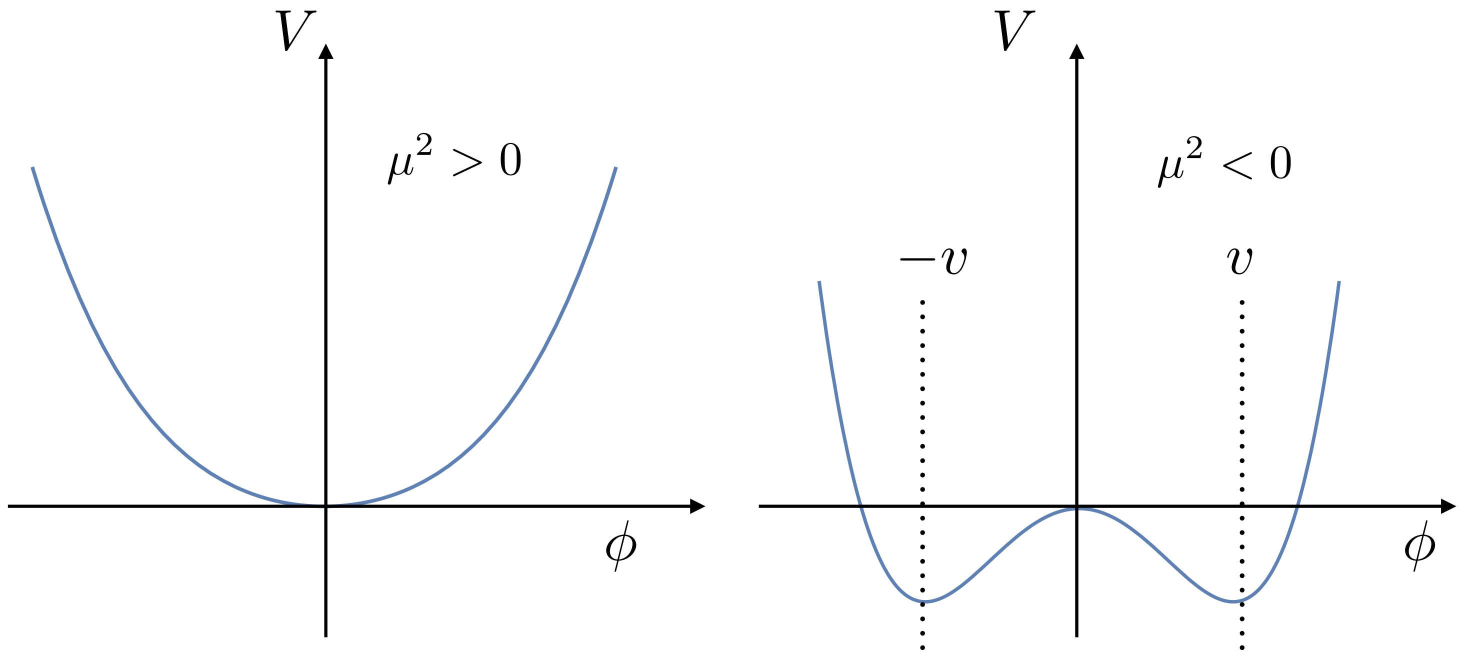
Lets look at the potential term

$$V = \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

# The potential has different types of structures depending on the sign of $\mu^2$



$$V = \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$



$$V = \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

We are interested in the minima of the potential which we find as follows

$$\frac{\partial V}{\partial \phi} = 0 \implies \phi(\mu^2 + \lambda\phi^2) = 0$$

Which we define as

$$\phi = \pm v \quad v = \sqrt{\frac{-\mu^2}{\lambda}}$$

In order to have a stable perturbation theory we want to expand our fields around the minimum. We can choose either +/- v. Lets choose +v.

$$\phi(x) = v + \eta(x)$$

Writing our Lagrangian in the new coordinates we see that

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \text{const}$$

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Comparing the Lagrangian to that of a free scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2 \phi^2$$

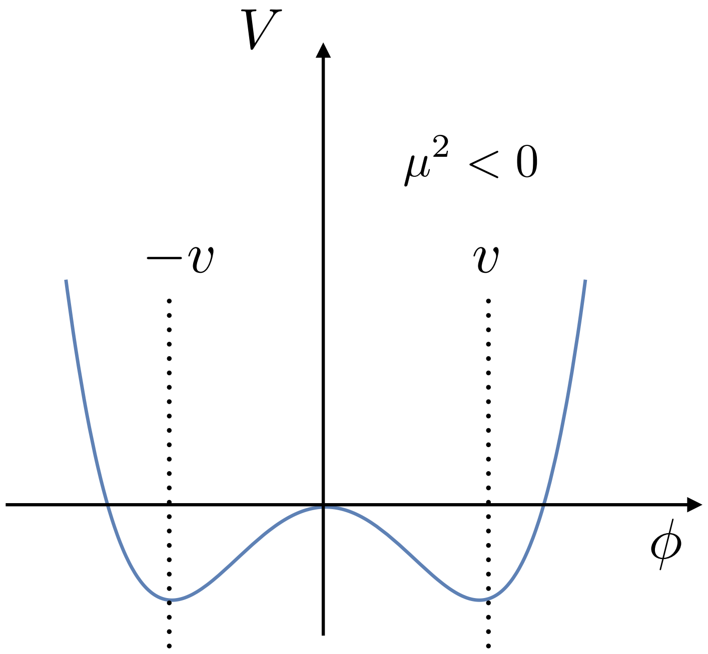
We see that the scalar field has a physical mass

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

Writing our Lagrangian in the new coordinates we see that

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \text{const}$$

$$\eta \rightarrow -\eta$$

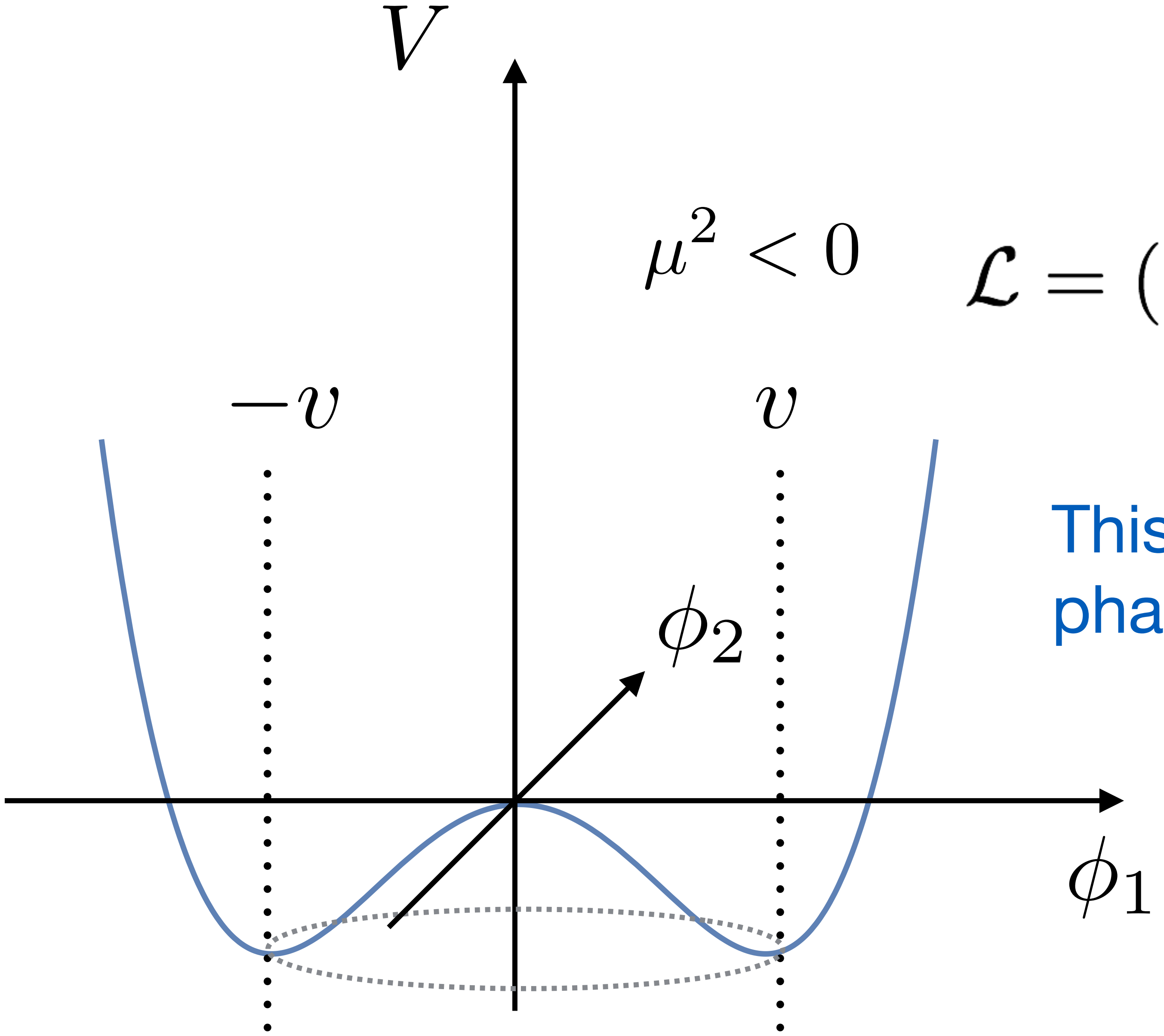


In the new coordinates the symmetry is gone

Note that we didn't change the Lagrangian, and the symmetry is still present globally. Our choice of +v versus -v breaks the symmetry.

A perturbative expansion around 0 would be inherently unstable.

# Lets consider a similar, but more complicated example. A complex scalar field with the following Lagrangian

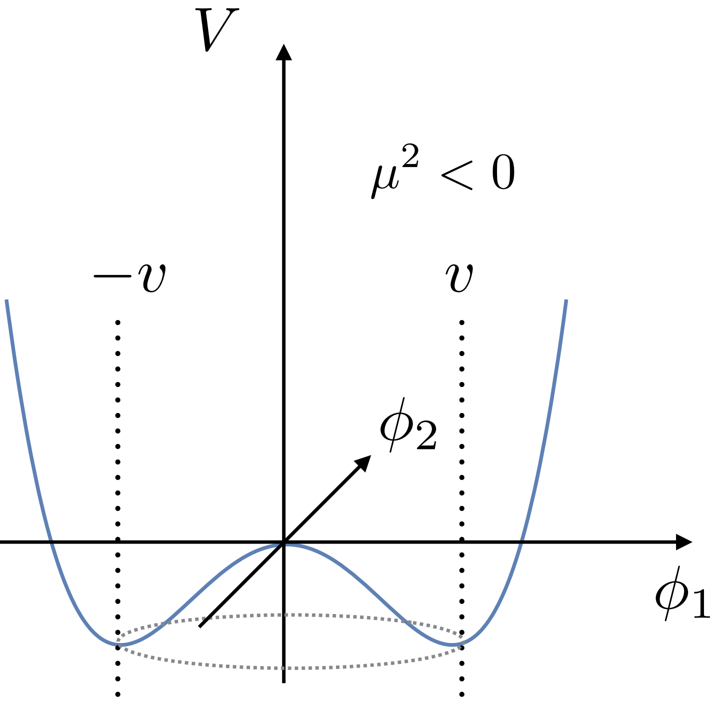


$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

This is invariant under the (global) phase rotations

$$\phi \to e^{i\alpha} \phi$$

# Writing the complex scalar field as the combination of two real fields



$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

The Lagrangian can then be written as follows

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^4 + 2\phi_1^2\phi_2^2 + \phi_2^4)$$

The minimum of the potential is a circle in the $(\phi_1, \phi_2)$ plane

$$\phi_1^2 + \phi_2^2 = v^2 \quad v^2 = -\frac{\mu^2}{\lambda}$$

We spontaneously choose to expand our fields around the point

$$\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

So that $$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

And the Lagrangian becomes

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2\eta^2 + \text{const} + \text{cubic} + \text{quartic}$$

We see one massless field and one with mass $m_\eta = \sqrt{-2\mu^2}$

The appearance of one massive and one massless boson is no accident. It is an example of **Goldstone's theorem**, which states that.

For every continuous symmetry of a physical system which is spontaneously broken there exists a massless boson

In our example we lost one symmetry (rotations in $(\phi_1, \phi_2)$ plane) so as predicted we see one massless boson

So if SSB generates masses? Where are all the Goldstone bosons?

# The Higgs mechanism

We reconsider our previous Lagrangian,

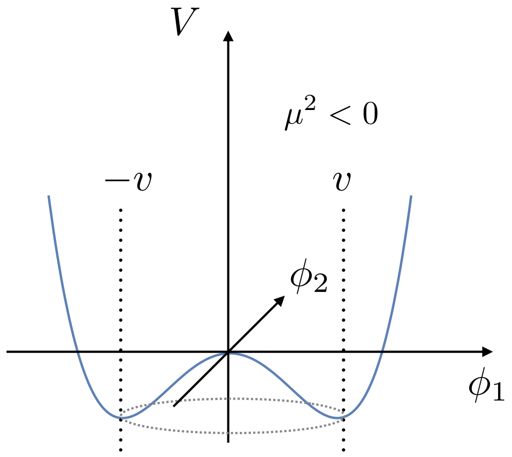
$$\mathcal{L} = (\partial^\mu \phi)^* (\partial_\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$$\phi \rightarrow e^{i\alpha(x)} \phi$$

Which means we have to introduce a gauge field to ensure gauge invariance.

Our Lagrangian is thus

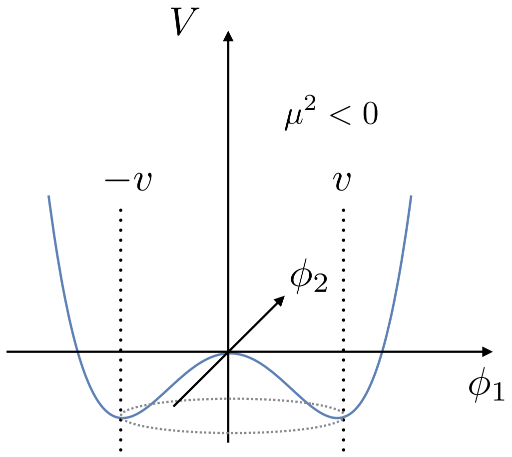
$$\mathcal{L} = (\partial^\mu + ieA^\mu) \phi^* (\partial_\mu - ieA_\mu) \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$



Expanding about the vacuum as before $\phi = \begin{pmatrix} v \\ 0 \end{pmatrix}$

And $$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

Our Lagrangian is then



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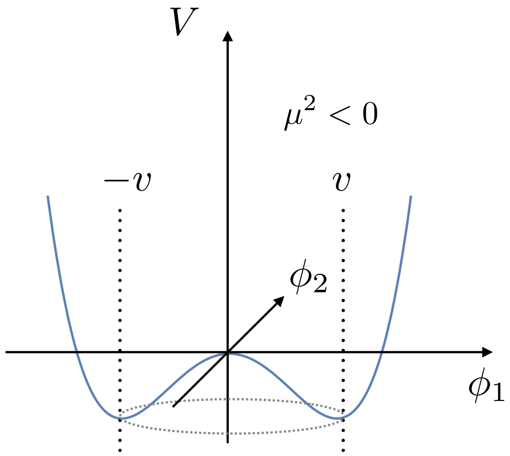
Our Lagrangian is then

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2\lambda\eta^2 + \frac{1}{2}e^2v^2A_\mu A^\mu - evA_\mu\partial^\mu\xi + \text{Interaction terms}$$

Lots to see here!



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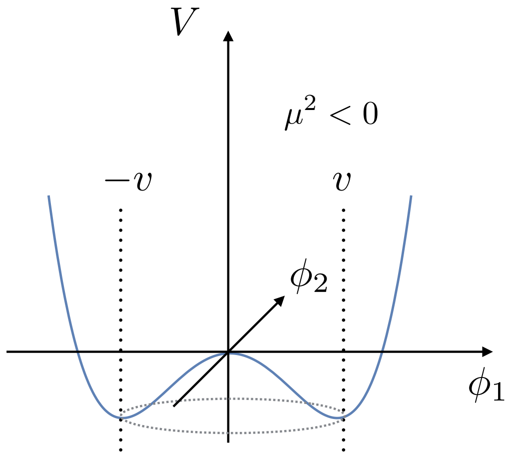
Goldstone boson



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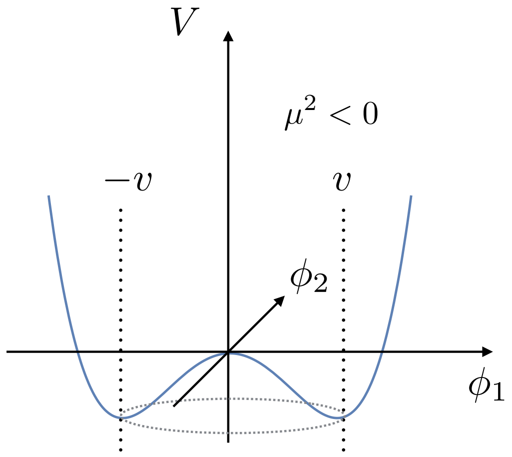
Massive scalar



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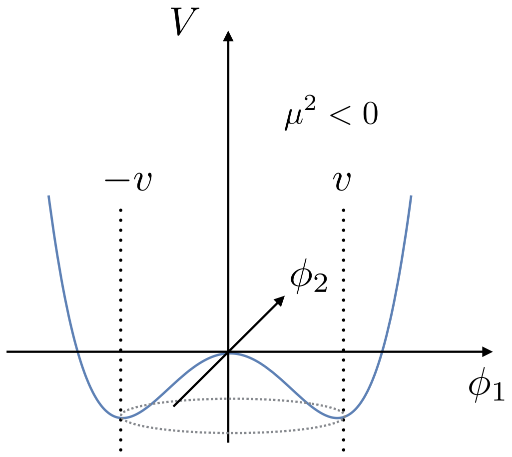
Lots to see here!

Goldstone boson

Massive scalar

Mass term for the gauge boson!



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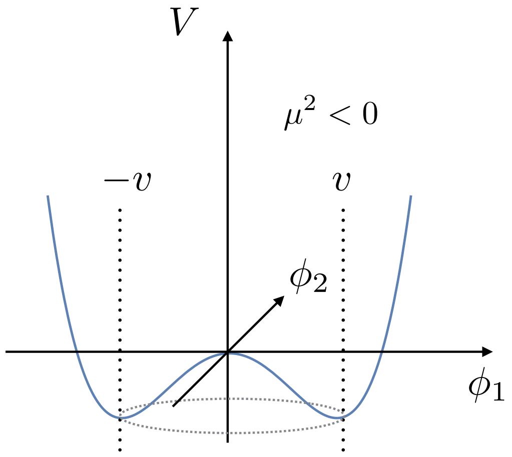
Lots to see here!

Goldstone boson

Massive scalar

Mass term for the gauge boson!

$$m_\xi = 0 \quad m_\eta = \sqrt{2\lambda v^2} \quad m_A = ev$$



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This term is nasty, and corresponds to a mixing between the Goldstone and gauge bosons.

Its appearance is related to the longitudinal degree of symmetry of the (massive) gauge boson.

Since we didn't redefine A, it still has only transverse degrees of freedom

We can write the Lagrangian in a nicer (more physical) framework by applying a gauge transformation.

Firstly we note that our choice of writing the scalar field was not unique, the following definition

$$\phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi) \implies \phi = \frac{1}{\sqrt{2}}(v + \eta)e^{i\xi/v}$$

Is equally valid, (and equivalent to lowest order in the fields)

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We therefore write out fields as follows

$$\begin{aligned} \phi &\rightarrow \frac{1}{\sqrt{2}}(v + h(x))e^{i\theta(x)/v} \\ A_\mu &\rightarrow A_\mu + \frac{1}{ev}\partial_\mu \theta \end{aligned}$$

This is equivalent to choosing a favorite gauge to work in. We call it the unitary gauge.

# In terms of these fields our Lagrangian becomes

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + \frac{1}{2}e^2 v^2 A_\mu A^\mu - \lambda v h^3 - \frac{1}{4}\lambda h^4 + \frac{1}{2}e^2 A_\mu A^\mu h^2 + v e^2 A_\mu A^\mu h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

- There are no physical Goldstone bosons in the theory (absorbed into the longitudinal degree of freedom of the gauge boson)
- One physical massive scalar (the Higgs boson)
- One massive gauge boson.

The old Lagrangian wasnt wrong, but it would have been extremely tedious to work with, we would carry around a lot of spurious ghosts in our calculation.

# SSB of a local SU(2)

We are nearly in a position to construct the SM Higgs. Our two remaining problems are 1) Generating two different masses for the W and Z bosons and 2) Generating masses for fermions.

We start by promoting our basic Lagrangian

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

To be symmetric under local SU(2) transformations of the form

$$\phi \to e^{i\alpha_a \tau_a / 2} \phi$$

Sums over the 3 (2x2 matrices) which generate SU(2)

$$\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Since there are 3 generators we introduce the following covariant derivative.

$$D_\mu \to \partial_\mu + ig\frac{\tau_a}{2}W_\mu^a$$

Which sums over 3 gauge bosons.

Under an SU(2) transformation the fields transform as follows.

$$\vec{W}_\mu \to \vec{W}_\mu - \frac{1}{g}\partial_\mu\vec{\alpha} - \vec{\alpha} \times \vec{W}_\mu$$

Putting this all together we obtain the following Lagrangian

$$\mathcal{L} = \left(\partial_\mu\phi + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu\phi\right)^\dagger\left(\partial_\mu\phi + ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu\phi\right) - V(\phi) - \frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu}$$

With our usual Higgs potential

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

Which has minima at

$$\phi^\dagger\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda}$$

We (spontaneously) choose the following vacuum state

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

And expand around it as follows (in the unitary gauge)

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

From our past experience we know that the the mass terms for the gauge bosons come from the $|D_\mu\phi|^2$ part of the Lagrangian

So for simplicity we focus on that

$$\begin{aligned}\left|ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu\phi\right|^2 &= \frac{g^2}{8}\left|\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2 \\ &= \frac{g^2v^2}{8}[(W^1_\mu)^2 + (W^2_\mu)^2 + (W^3)^2_\mu]\end{aligned}$$

So we have 3 massive gauge bosons, of equal mass.

# The Standard Model

In order to generate the mass spectrum of the SM, we need to break a more complicated structure than just a single SU(2).

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$$

Where we define hypercharge as follows

$$Q = T^3 + \frac{Y}{2}$$

Left-handed and right handed matter transforms as follows

$$\begin{aligned} \chi_L \rightarrow \chi_L' &= e^{i\alpha(\vec{x})\cdot\vec{\tau}/2+i\beta(x)Y}\chi_L \\ \psi \rightarrow \psi' &= e^{i\beta(x)Y}\psi \end{aligned}$$

Quarks and Leptons are represented by

$$\chi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \psi_R = u_R \quad \text{or} \quad d_R \qquad\qquad \chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \psi_R = e_R$$

For instance, the leptonic part of the Lagrangian

$$\mathcal{L}_1 = \overline{\chi}_L \gamma^\mu \left[ i\partial_\mu - g\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu - ig'\left(-\frac{1}{2}\right)B_\mu \right]\chi_L + \bar{e}_R\gamma^\mu\left[i\partial_\mu - g'(-1)B_\mu\right]e_R - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

Is manifestly gauge invariant.

Note the absence of any mass term in the Lagrangian, in fact the mass term

$$-m\bar{e}e = -m\bar{e}[\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5)]e$$
$$= -m(\overline{e_R}e_L + \bar{e}_L e_R)$$

Is **NOT** invariant under SU(2) rotations.

The Higgs will help us here too.

Next we need to work out the Higgs sector, The Higgs is an SU(2) doublet with hyperchage Y=1.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Q=1/2-1/2

Q=1/2+1/2

Expanding our complex scalars

$$\phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$\phi^0 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)$$

The Higgs part of the Lagrangian is then

$$\mathcal{L}_2 = \left| \left( i\partial_\mu - g\frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - g'\frac{Y}{2} B_\mu \right) \phi \right|^2 - V(\phi)$$

Covariant derivative

Higgs potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

Again we expand around the vacuum given by

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Now we get to see something really beautiful, consider the following linear combination of generators

$$Q = T^3 + \frac{Y}{2}$$

Acting on the vacuum state, i.e.

$$Q\phi_0 \propto \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

So Q annihilates the vacuum, i.e.

$$\phi_0 \rightarrow \phi_0' = e^{i\alpha(x)Q}\phi_0 = \phi_0$$

The symmetry of the vacuum wrt Q, will give us EM (and the photon)!

## Getting back to our massive bosons we see that the piece we are interested in is

$$\left|\left(-ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu - i\frac{g'}{2}B_\mu\right)\phi\right|^2 = \frac{g^2}{8}\left|\begin{pmatrix} gW^3_\mu + g'B_\mu & g(W^1_\mu - iW^2_\mu) \\ g(W^1_\mu + iW^2_\mu) & -gW^3_\mu + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

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Expanding this out yields

$$\left|\left(-ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu - i\frac{g'}{2}B_\mu\right)\phi\right|^2 = \frac{1}{8}v^2g^2\left[(W^1_\mu)^2 + (W^2_\mu)^2\right] + \frac{1}{8}v^2(g'B_\mu - gW^3_\mu)(g'B^\mu - gW^{3\mu})$$

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Or alternatively in terms of the physical W+,W- physical states

$$\left|\left(-ig\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu - i\frac{g'}{2}B_\mu\right)\phi\right|^2 = \left(\frac{1}{2}vg\right)^2 W^+_\mu W^{-\mu} + \frac{1}{8}v^2(W^3_\mu, B_\mu)\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

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We have our first big result

$$M_W = \frac{1}{2}vg$$

We can tidy up the last term by re-packing the fields into a diagonal form

$$\frac{1}{8}v^2\left[g^2(W^3_\mu)^2 - 2gg'W^3_\mu B^\mu + {g'}^2 B^2_\mu\right] = \frac{1}{8}v^2\left[gW^3_\mu - g'B_\mu\right]^2 + 0\left[gW^3_\mu + g'B_\mu\right]^2$$

Which we interpret as

$$\frac{1}{2}M_Z Z^\mu Z_\mu + \frac{1}{2}M_A A^\mu A_\mu$$

With

$$\begin{aligned} A_\mu &= \frac{gW^3_\mu + g'B_\mu}{\sqrt{g^2+g'^2}} \qquad & M_A &= 0 \\ Z_\mu &= \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2+g'^2}} \qquad & M_Z &= \frac{1}{2}v\sqrt{g^2+g'^2} \end{aligned}$$

So we predict a massless photon and a Z which is **heavier** than then W :-) Happy days.

Finally we turn our attention to the fermion mass terms, note that the following combination is gauge invariant

$$\mathcal{L}_3 = -G_e\left[\overline{\chi}_L \phi \psi_R + \text{h.c}\right]$$

Doublets

singlet

For instance for the leptons we have

$$\mathcal{L}_3 = -G_e\left[(\overline{\nu}_e, \overline{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \text{h.c}\right]$$

In our time honored tradition we expand around the new vacuum

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

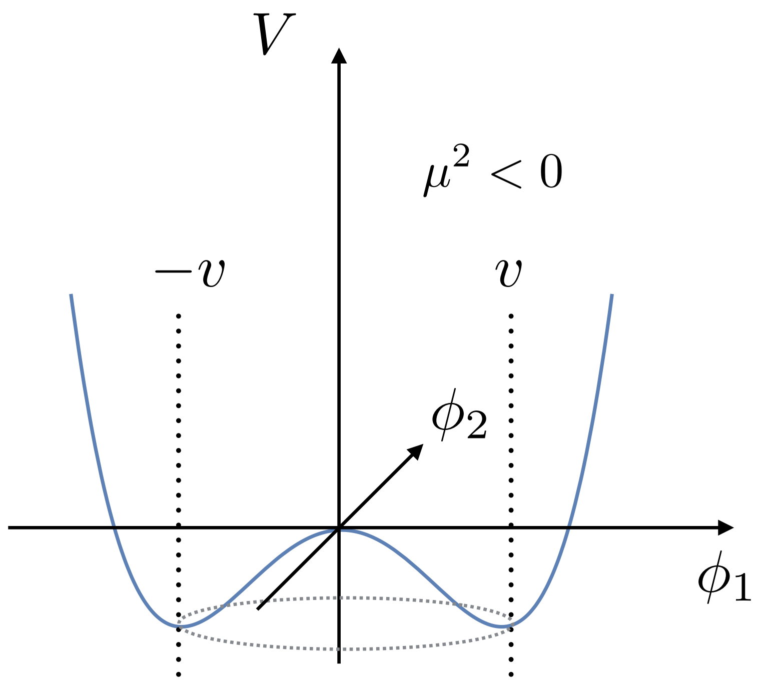
Finding

$$\mathcal{L}_3 = -\frac{G_e}{\sqrt{2}} v(\bar{e}_L e_R + \bar{e}_R e_L) - \frac{G_e}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) h$$

I.e.

$$\mathcal{L}_3 = -m_e \bar{e} e - \frac{m_e}{v} \bar{e} e h$$

# Summary



$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

- Potentials with multiple non-zero global minimum, lead to spontaneous symmetry breaking.

- By breaking SU(2)_L X U(1)_Y to U(1)_EM we are able to explain the generation of mass terms for both gauge bosons and fermions in a theoretically robust manner.