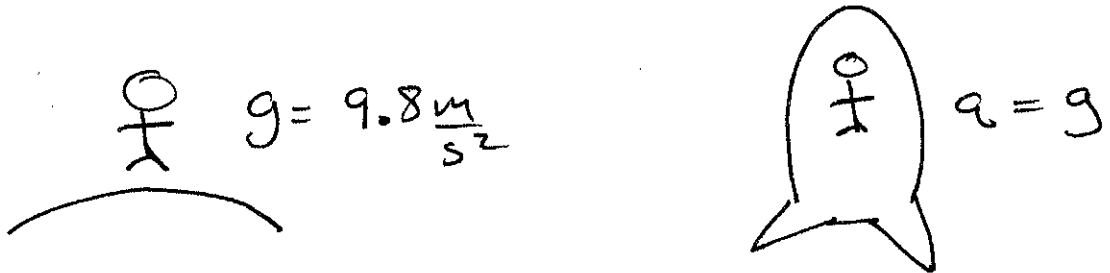


Misc. Topics: Friday 8 June 2001

F. Olness
SMU

Einstein Equivalence Principle:

Earth Ref Frame \equiv Accelerating Space Ship



Result:

① Gravitational mass \equiv inertial mass

$$F = \frac{GMm}{r^2} = mg = ma$$

② Light bends in gravitational.

③ Mass \Rightarrow grav. field

grav. field \Rightarrow curved space

∞

$$G = T - ?$$

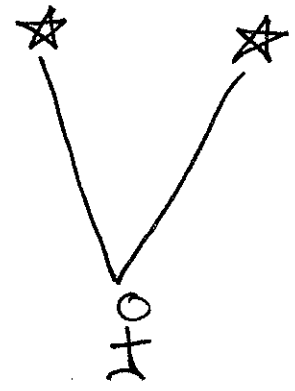
Curvature of Space Stress-Energy Tensor (Mass) Cosmological Constant and here

Test of Bending Light.

Observe 2 stars:

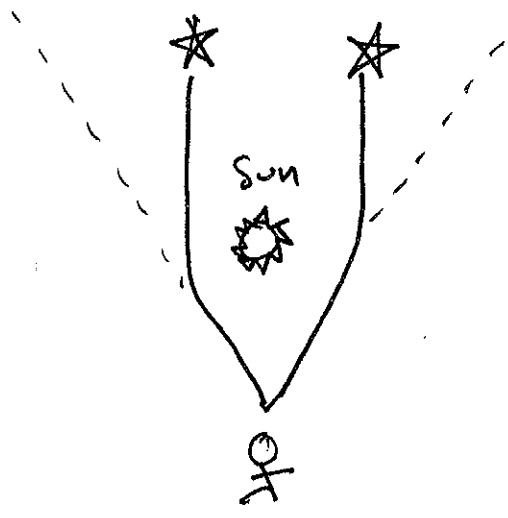
No Sun:

$$\theta = 20^\circ$$



With Sun

$$\theta = 30^\circ$$



Bending of light makes stars appear further apart.

Einstein's Equation

13

$$G = 8\pi T - \Lambda$$

Einstein Tensor
(Curvature of
Space)

Stress-Energy
Tensor
(mass)

Cosmological
Constant

IF IF $\Lambda = 0$, then if the universe has no mass ($T=0$) then the curvature = 0 ($G=0$)

But If appears $\Lambda \neq 0$, therefore empty space time is NOT Flat.

Historical Note Einstein introduced Λ to obtain "Static" Universe solution. ~ 10 years later, it was discovered that the universe was expanding.

"... my greatest blunder!"

Field Trip to Flatland

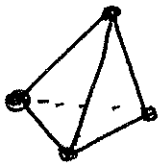
4

Q: What type of world do we live in?

A: Define Euler ~~Number~~ Number χ

$$\chi = F - E + V$$

Constant for
a given surface



$$\begin{aligned} F &= 4 \\ E &= 6 \\ V &= 4 \end{aligned}$$

$$\chi = 4 - 6 + 4 = 2$$

Sphere



$$\chi = 2$$

#holes

$$G = 0$$

Torus
(Doughnut)



$$\begin{aligned} \chi &= 0 \\ \text{Flat} \end{aligned}$$

$$G = 1$$



$$\chi = -2$$

$$G = 2$$



$$\chi = -4$$

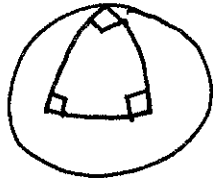
$$G = 3$$

...

$$\chi = 2 - 2G$$

On Uniform Sphere, $\Delta \neq 180^\circ$

5



$$\theta_1 + \theta_2 + \theta_3 = 270^\circ$$

In general,
$$\sum_{i=1}^3 \theta_i = \pi + 2\pi \left(\frac{A}{4\pi R^2} \right)$$

$$A = 0, \quad \sum \theta = \pi = 180^\circ$$

$$A = 4\pi R^2, \quad \sum \theta = 3\pi = 540^\circ$$

Gauss - Bonnet Theorem:

$$\int K \, dA = 2\pi \chi$$

For
Sphere:

$$\begin{array}{ccc} \swarrow & \searrow & \swarrow \\ \frac{1}{R^2} & 4\pi R^2 & 2 \end{array}$$

For

Torus:

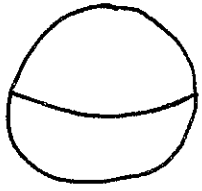
$$K = 0$$

$$\chi = 0$$

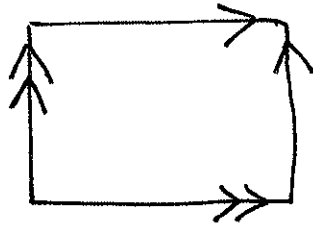
Upshot This relates an intrinsic / local quantity ($\int K dA$) to a extrinsic global (χ) quantity.

Simple 2-D Surfaces

6



Sphere

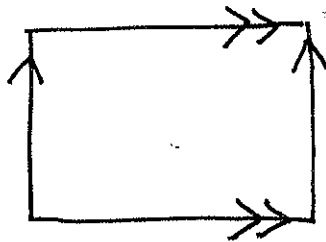


$$\chi = 2$$

$$K = \frac{1}{R^2}$$

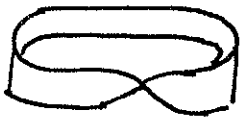


Torus

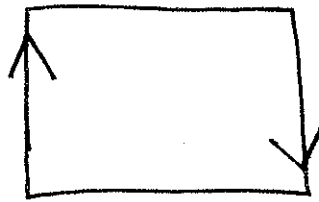


$$\chi = 0$$

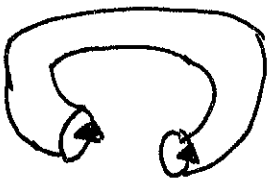
$$K = 0$$



Möbius
Strip

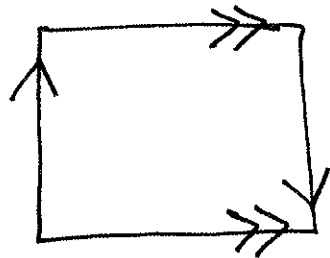


Non-orientable



invent

Klein
Bottle



Exercise: Play Tic-Tac-Toe on the above