Resonant Pipes & Harmonic Series:

INTRODUCTION:

We are going to study different resonant systems, and try to model them as either **open** or **closed** resonant pipes. Of course, real systems are not quite so simple, so we will be studying how closely such a real system matches the theoretical ideal.

EXPERIMENT:

Step 1: Practice:
You will be given a resonant pipe. Spend a few minutes experimenting with it to discover how to efficiently generate as many resonant frequencies as possible. When you have mastered your instrument and are ready for performance, proceed to a tuner and we will measure the frequencies of these resonances.

Step 2: Performance:
(You may find it useful to examine the example table shown below to understand how to fill in the data.)
Measure the frequencies of as many resonances as possible. Have one person play the instrument and center the tone without looking at the meter, and have a second person take a reading. Fill in the notes in the following table starting from the highest resonance you can play, and working downward.

Most of you will be using musical chromatic tuners to determine your frequency. The tuner will tell you the note (e.g., F#), and the cents above/below this note (e.g., +40).
Record the notes as follows: F_5# +40, would be 40 cents an F_5-sharp. You must figure out the octave. There will be a piano available to assist you.

Step 3: Frequencies:
Once you have determined the note (e.g., F_5# +40), use the table to compute your frequency. 3 digits accuracy should be plenty. Then compute the difference between the two frequencies. *(Note, if you made an error, or are missing a resonance, it will show up here in the difference. If you find an anomaly, go back and cross check for errors.)*

Step 4: Fundamental Frequency:
You may or may not have measured the fundamental (lowest) frequency. Assuming you did not, you can estimate what this should be by stepping down in frequency. First, compute the average difference, and then use this to step down to obtain the fundamental frequency.

In the example table, I measured down to the 700Hz resonance. Using the average difference of 200Hz, I was able to determine the fundamental frequency was 100Hz. I indicate that the values below 700Hz are theory not measurement using the last column.

Step 5: Harmonic Series:
Do the resonant notes from your instrument appear to be all multiples of the fundamental frequency (e.g., $1f_0$, $2f_0$, $3f_0$, $4f_0$, $5f_0$, ...) or only the odd multiples (e.g., $1f_0$, $3f_0$, $5f_0$,...). Does you instrument appear to be harmonic, or odd-harmonic?

Step 6: Length of Instrument:
Measure the length of your instrument in meters. Compute the expected fundamental if this were both a open and closed pipe. How does this compare with what you found in Step 5?
Be sure to use the real value for the speed of sound (c=331±0.6/C), not my fake 400m/s.

Repeat:

Repeat the above steps so that you perform this experiment for a total of 3 resonant systems. I prefer that you try and use at least one closed, and one open system.

QUESTIONS:

How closely did real instruments match ideal theoretical models? What elements would make the real instruments differ from the models?

Other general conclusions and observations?

EXAMPLE CALCULATION:

Note:	Frequency	Difference	Measured Y/N
F_6# +20	1500		Y
E_6 -20	1300	200	Y
C_6# -10	1100	200	Y
A_5 +40	900	200	Y
F_5 +00	700	200	Y
	500		N
	300		N
	100		N
	-100		N

$$f_{closed} = \frac{c}{4\,L} = 100\text{Hz}$$

$$f_{open} = \frac{c}{2\,L} = 200\text{Hz}$$

CALCULATION WORKSHEET:

Note:	Frequency	Difference	Measured Y/N

$f_{closed} = \frac{c}{4\,L} =$

$f_{open} = \frac{c}{2\,L} =$

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A	440	-50	-40	-30	-20	-10	0	10	20	30	40	50
0	C	15.886	15.978	16.071	16.164	16.257	**16.352**	16.446	16.542	16.637	16.734	16.831
0	C#	16.831	16.928	17.026	17.125	17.224	**17.324**	17.424	17.525	17.627	17.729	17.832
0	D	17.832	17.935	18.039	18.143	18.248	**18.354**	18.460	18.567	18.675	18.783	18.892
0	D#	18.892	19.001	19.111	19.222	19.333	**19.445**	19.558	19.671	19.785	19.900	20.015
0	E	20.015	20.131	20.248	20.365	20.483	**20.602**	20.721	20.841	20.962	21.083	21.205
0	F	21.205	21.328	21.452	21.576	21.701	**21.827**	21.953	22.080	22.208	22.337	22.466
0	F#	22.466	22.596	22.727	22.859	22.991	**23.125**	23.259	23.393	23.529	23.665	23.802
0	G	23.802	23.940	24.079	24.218	24.359	**24.500**	24.642	24.784	24.928	25.072	25.218
0	G#	25.218	25.364	25.511	25.658	25.807	**25.957**	26.107	26.258	26.410	26.563	26.717
0	A	26.717	26.872	27.028	27.184	27.342	**27.500**	27.659	27.820	27.981	28.143	28.306
0	A#	28.306	28.470	28.635	28.801	28.967	**29.135**	29.304	29.474	29.645	29.816	29.989
0	B	29.989	30.163	30.337	30.513	30.690	**30.868**	31.047	31.226	31.407	31.589	31.772
1	C	31.772	31.956	32.141	32.328	32.515	**32.703**	32.893	33.083	33.275	33.468	33.661
1	C#	33.661	33.856	34.053	34.250	34.448	**34.648**	34.849	35.050	35.253	35.458	35.663
1	D	35.663	35.870	36.077	36.286	36.497	**36.708**	36.921	37.135	37.350	37.566	37.784
1	D#	37.784	38.003	38.223	38.444	38.667	**38.891**	39.116	39.343	39.571	39.800	40.030
1	E	40.030	40.262	40.496	40.730	40.966	**41.203**	41.442	41.682	41.924	42.167	42.411
1	F	42.411	42.656	42.904	43.152	43.402	**43.654**	43.906	44.161	44.417	44.674	44.933
1	F#	44.933	45.193	45.455	45.718	45.983	**46.249**	46.517	46.787	47.058	47.330	47.605
1	G	47.605	47.880	48.158	48.437	48.717	**48.999**	49.283	49.569	49.856	50.145	50.435
1	G#	50.435	50.727	51.021	51.317	51.614	**51.913**	52.214	52.516	52.821	53.126	53.434
1	A	53.434	53.744	54.055	54.368	54.683	**55.000**	55.319	55.639	55.961	56.286	56.612
1	A#	56.612	56.940	57.269	57.601	57.935	**58.270**	58.608	58.948	59.289	59.632	59.978
1	B	59.978	60.325	60.675	61.026	61.380	**61.735**	62.093	62.453	62.815	63.178	63.544
2	C	63.544	63.913	64.283	64.655	65.030	**65.406**	65.785	66.166	66.550	66.935	67.323
2	C#	67.323	67.713	68.105	68.500	68.897	**69.296**	69.697	70.101	70.507	70.915	71.326
2	D	71.326	71.739	72.155	72.573	72.993	**73.416**	73.841	74.269	74.699	75.132	75.567
2	D#	75.567	76.005	76.446	76.888	77.334	**77.782**	78.232	78.686	79.141	79.600	80.061
2	E	80.061	80.525	80.991	81.460	81.932	**82.407**	82.884	83.364	83.847	84.333	84.822
2	F	84.822	85.313	85.807	86.304	86.804	**87.307**	87.813	88.322	88.833	89.348	89.865
2	F#	89.865	90.386	90.910	91.436	91.966	**92.499**	93.034	93.573	94.115	94.661	95.209
2	G	95.209	95.761	96.315	96.873	97.434	**97.999**	98.567	99.138	99.712	100.289	100.870
2	G#	100.870	101.455	102.043	102.634	103.228	**103.826**	104.428	105.033	105.641	106.253	106.869
2	A	106.869	107.488	108.110	108.737	109.366	**110.000**	110.637	111.278	111.923	112.571	113.223
2	A#	113.223	113.879	114.539	115.202	115.870	**116.541**	117.216	117.895	118.578	119.265	119.956
2	B	119.956	120.651	121.350	122.053	122.760	**123.471**	124.186	124.905	125.629	126.357	127.089
3	C	127.089	127.825	128.565	129.310	130.059	**130.813**	131.571	132.333	133.099	133.870	134.646
3	C#	134.646	135.426	136.210	136.999	137.793	**138.591**	139.394	140.202	141.014	141.831	142.652
3	D	142.652	143.479	144.310	145.146	145.987	**146.832**	147.683	148.538	149.399	150.264	151.135
3	D#	151.135	152.010	152.891	153.777	154.668	**155.563**	156.465	157.371	158.283	159.200	160.122
3	E	160.122	161.049	161.982	162.921	163.865	**164.814**	165.769	166.729	167.695	168.666	169.643
3	F	169.643	170.626	171.614	172.609	173.608	**174.614**	175.626	176.643	177.666	178.696	179.731
3	F#	179.731	180.772	181.819	182.872	183.932	**184.997**	186.069	187.147	188.231	189.321	190.418
3	G	190.418	191.521	192.631	193.746	194.869	**195.998**	197.133	198.275	199.424	200.579	201.741
3	G#	201.741	202.910	204.085	205.267	206.456	**207.652**	208.855	210.065	211.282	212.506	213.737
3	A	213.737	214.975	216.221	217.473	218.733	**220.000**	221.274	222.556	223.846	225.142	226.446
3	A#	226.446	227.758	229.078	230.405	231.739	**233.082**	234.432	235.790	237.156	238.530	239.912
3	B	239.912	241.301	242.699	244.105	245.519	**246.942**	248.372	249.811	251.258	252.714	254.178
4	C	254.178	255.650	257.131	258.621	260.119	**261.626**	263.141	264.666	266.199	267.741	269.292
4	C#	269.292	270.852	272.421	273.999	275.586	**277.183**	278.788	280.403	282.028	283.661	285.305
4	D	285.305	286.957	288.620	290.292	291.973	**293.665**	295.366	297.077	298.798	300.529	302.270
4	D#	302.270	304.021	305.782	307.553	309.335	**311.127**	312.929	314.742	316.565	318.399	320.244
4	E	320.244	322.099	323.965	325.841	327.729	**329.628**	331.537	333.458	335.389	337.332	339.286
4	F	339.286	341.252	343.229	345.217	347.217	**349.228**	351.251	353.286	355.333	357.391	359.461
4	F#	359.461	361.544	363.638	365.745	367.863	**369.994**	372.138	374.294	376.462	378.643	380.836
4	G	380.836	383.042	385.261	387.493	389.738	**391.995**	394.266	396.550	398.847	401.158	403.482
4	G#	403.482	405.819	408.170	410.535	412.913	**415.305**	417.711	420.130	422.564	425.012	427.474
4	A	427.474	429.950	432.441	434.946	437.466	**440.000**	442.549	445.113	447.691	450.285	452.893
4	A#	452.893	455.517	458.155	460.809	463.479	**466.164**	468.864	471.580	474.312	477.060	479.823
4	B	479.823	482.603	485.399	488.211	491.039	**493.883**	496.744	499.622	502.516	505.427	508.355
5	C	508.355	511.300	514.262	517.241	520.237	**523.251**	526.282	529.331	532.397	535.482	538.584
5	C#	538.584	541.704	544.842	547.998	551.172	**554.365**	557.577	560.807	564.055	567.323	570.609
5	D	570.609	573.915	577.240	580.583	583.947	**587.330**	590.732	594.154	597.596	601.058	604.540
5	D#	604.540	608.042	611.564	615.107	618.670	**622.254**	625.859	629.484	633.131	636.798	640.487
5	E	640.487	644.198	647.930	651.683	655.458	**659.255**	663.074	666.915	670.779	674.664	678.573
5	F	678.573	682.504	686.457	690.434	694.434	**698.456**	702.503	706.572	710.665	714.782	718.923
5	F#	718.923	723.087	727.276	731.489	735.727	**739.989**	744.276	748.587	752.924	757.285	761.672
5	G	761.672	766.084	770.522	774.986	779.475	**783.991**	788.532	793.100	797.695	802.316	806.964
5	G#	806.964	811.638	816.340	821.069	825.825	**830.609**	835.421	840.261	845.128	850.024	854.948
5	A	854.948	859.901	864.882	869.892	874.932	**880.000**	885.098	890.225	895.382	900.569	905.786
5	A#	905.786	911.033	916.311	921.619	926.958	**932.328**	937.728	943.161	948.624	954.120	959.647
5	B	959.647	965.206	970.797	976.421	982.077	**987.767**	993.489	999.244	1005.032	1010.855	1016.710

A	440	-50	-40	-30	-20	-10	0	10	20	30	40	50
6	C	1016.7	1022.6	1028.5	1034.5	1040.5	**1046.5**	1052.6	1058.7	1064.8	1071.0	1077.2
6	C#	1077.2	1083.4	1089.7	1096.0	1102.3	**1108.7**	1115.2	1121.6	1128.1	1134.6	1141.2
6	D	1141.2	1147.8	1154.5	1161.2	1167.9	**1174.7**	1181.5	1188.3	1195.2	1202.1	1209.1
6	D#	1209.1	1216.1	1223.1	1230.2	1237.3	**1244.5**	1251.7	1259.0	1266.3	1273.6	1281.0
6	E	1281.0	1288.4	1295.9	1303.4	1310.9	**1318.5**	1326.1	1333.8	1341.6	1349.3	1357.1
6	F	1357.1	1365.0	1372.9	1380.9	1388.9	**1396.9**	1405.0	1413.1	1421.3	1429.6	1437.8
6	F#	1437.8	1446.2	1454.6	1463.0	1471.5	**1480.0**	1488.6	1497.2	1505.8	1514.6	1523.3
6	G	1523.3	1532.2	1541.0	1550.0	1559.0	**1568.0**	1577.1	1586.2	1595.4	1604.6	1613.9
6	G#	1613.9	1623.3	1632.7	1642.1	1651.7	**1661.2**	1670.8	1680.5	1690.3	1700.0	1709.9
6	A	1709.9	1719.8	1729.8	1739.8	1749.9	**1760.0**	1770.2	1780.5	1790.8	1801.1	1811.6
6	A#	1811.6	1822.1	1832.6	1843.2	1853.9	**1864.7**	1875.5	1886.3	1897.2	1908.2	1919.3
6	B	1919.3	1930.4	1941.6	1952.8	1964.2	**1975.5**	1987.0	1998.5	2010.1	2021.7	2033.4
7	C	2033.4	2045.2	2057.0	2069.0	2080.9	**2093.0**	2105.1	2117.3	2129.6	2141.9	2154.3
7	C#	2154.3	2166.8	2179.4	2192.0	2204.7	**2217.5**	2230.3	2243.2	2256.2	2269.3	2282.4
7	D	2282.4	2295.7	2309.0	2322.3	2335.8	**2349.3**	2362.9	2376.6	2390.4	2404.2	2418.2
7	D#	2418.2	2432.2	2446.3	2460.4	2474.7	**2489.0**	2503.4	2517.9	2532.5	2547.2	2561.9
7	E	2561.9	2576.8	2591.7	2606.7	2621.8	**2637.0**	2652.3	2667.7	2683.1	2698.7	2714.3
7	F	2714.3	2730.0	2745.8	2761.7	2777.7	**2793.8**	2810.0	2826.3	2842.7	2859.1	2875.7
7	F#	2875.7	2892.3	2909.1	2926.0	2942.9	**2960.0**	2977.1	2994.3	3011.7	3029.1	3046.7
7	G	3046.7	3064.3	3082.1	3099.9	3117.9	**3136.0**	3154.1	3172.4	3190.8	3209.3	3227.9
7	G#	3227.9	3246.6	3265.4	3284.3	3303.3	**3322.4**	3341.7	3361.0	3380.5	3400.1	3419.8
7	A	3419.8	3439.6	3459.5	3479.6	3499.7	**3520.0**	3540.4	3560.9	3581.5	3602.3	3623.1
7	A#	3623.1	3644.1	3665.2	3686.5	3707.8	**3729.3**	3750.9	3772.6	3794.5	3816.5	3838.6
7	B	3838.6	3860.8	3883.2	3905.7	3928.3	**3951.1**	3974.0	3997.0	4020.1	4043.4	4066.8
8	C	4066.8	4090.4	4114.1	4137.9	4161.9	**4186.0**	4210.3	4234.6	4259.2	4283.9	4308.7
8	C#	4308.7	4333.6	4358.7	4384.0	4409.4	**4434.9**	4460.6	4486.5	4512.4	4538.6	4564.9
8	D	4564.9	4591.3	4617.9	4644.7	4671.6	**4698.6**	4725.9	4753.2	4780.8	4808.5	4836.3
8	D#	4836.3	4864.3	4892.5	4920.9	4949.4	**4978.0**	5006.9	5035.9	5065.0	5094.4	5123.9
8	E	5123.9	5153.6	5183.4	5213.5	5243.7	**5274.0**	5304.6	5335.3	5366.2	5397.3	5428.6
8	F	5428.6	5460.0	5491.7	5523.5	5555.5	**5587.7**	5620.0	5652.6	5685.3	5718.3	5751.4
8	F#	5751.4	5784.7	5818.2	5851.9	5885.8	**5919.9**	5954.2	5988.7	6023.4	6058.3	6093.4
8	G	6093.4	6128.7	6164.2	6199.9	6235.8	**6271.9**	6308.3	6344.8	6381.6	6418.5	6455.7
8	G#	6455.7	6493.1	6530.7	6568.6	6606.6	**6644.9**	6683.4	6722.1	6761.0	6800.2	6839.6
8	A	6839.6	6879.2	6919.1	6959.1	6999.5	**7040.0**	7080.8	7121.8	7163.1	7204.6	7246.3
8	A#	7246.3	7288.3	7330.5	7373.0	7415.7	**7458.6**	7501.8	7545.3	7589.0	7633.0	7677.2
8	B	7677.2	7721.6	7766.4	7811.4	7856.6	**7902.1**	7947.9	7994.0	8040.3	8086.8	8133.7
9	C	8133.7	8180.8	8228.2	8275.9	8323.8	**8372.0**	8420.5	8469.3	8518.4	8567.7	8617.3
9	C#	8617.3	8667.3	8717.5	8768.0	8818.8	**8869.8**	8921.2	8972.9	9024.9	9077.2	9129.8
9	D	9129.8	9182.6	9235.8	9289.3	9343.1	**9397.3**	9451.7	9506.5	9561.5	9616.9	9672.6
9	D#	9672.6	9728.7	9785.0	9841.7	9898.7	**9956.1**	10013.7	10071.7	10130.1	10188.8	10247.8
9	E	10248	10307	10367	10427	10487	**10548**	10609	10671	10732	10795	10857
9	F	10857	10920	10983	11047	11111	**11175**	11240	11305	11371	11437	11503
9	F#	11503	11569	11636	11704	11772	**11840**	11908	11977	12047	12117	12187
9	G	12187	12257	12328	12400	12472	**12544**	12617	12690	12763	12837	12911
9	G#	12911	12986	13061	13137	13213	**13290**	13367	13444	13522	13600	13679
9	A	13679	13758	13838	13918	13999	**14080**	14162	14244	14326	14409	14493
9	A#	14493	14577	14661	14746	14831	**14917**	15004	15091	15178	15266	15354
9	B	15354	15443	15533	15623	15713	**15804**	15896	15988	16081	16174	16267
10	C	16267	16362	16456	16552	16648	**16744**	16841	16939	17037	17135	17235