

## Homework #2: Phys 3320: Prof. Olness Fall 2015

*Due 15 September 2015*

Consider the equation:

$$x'' + 2\gamma x' + \omega_0^2 x = Q_0 \cos(\omega_D t)$$

The term  $2\gamma x'$  is the friction (dissipative) term.

The term  $Q_0 \cos(\omega_D t)$  is the driving term.

- 1) Case:  $x'' + 0 + \omega_0^2 x = 0$ .
- 2) Case:  $x'' + 2\gamma x' + \omega_0^2 x = 0$
- 3) Case:  $x'' + 0 + \omega_0^2 x = Q_0 \cos(\omega_D t)$
- 4) Case:  $x'' + 2\gamma x' + \omega_0^2 x = Q_0 \cos(\omega_D t)$

Note, you may find it convenient to define:  $\omega_1^2 = \omega_0^2 - \gamma^2$ .

Note, also find it easier to replace  $Q_0 \cos(\omega_D t)$  by  $Q_0 \text{Exp}(i \omega_D t)$ .

Boundary Conditions:  $x[t]=x_0$ ,  $x'[t]=v_0$ .

For each of the 4 cases above, use Mathematica to obtain the solution including boundary conditions. Simplify the solution to roughly match the form you derived on paper. Plot the solution for a selection of variables that display the key features of the solution.

Here are a few observations I expect you to make. This is an example—I expect you to come up with more.

- For case 2, display a) under-damped, b) critically damped, and c) over-damped, and plot these curves on the same plot for a selection of BC. Comment.
- For case 3, vary  $\gamma$  and  $\omega_D$ , and comments. What happens when  $\gamma$  is small? When  $\omega_D$  is close to  $\omega_0$ ?