

# Prelab 1:

## Measurement and Measurement Error

PHYS 1320  
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### 1 Introduction

Error in this context does not mean mistake but rather refers to the uncertainty in a measurement. All measurements in practice and even in principle have some error associated with them; no measured quantity can be determined with infinite precision.

#### 1.1 Statistical Errors (also known as Random Errors)

Most measurements involve reading a scale. The fineness of the scale markings (how close together they are) is limited and the width of the scale lines is nonzero. In every case, the final reading must be estimated and is therefore uncertain. This kind of scale-reading error is random since we expect that half of the time the estimate will be too small, and the other half of the time the estimate will be too large. We expect that random errors should cancel on average, that is, many measurements of the same quantity should produce a more reliable estimate. Statistical errors can be controlled by performing a sufficiently large number of measurements. The error estimate on a single scale reading can be taken as half of the scale width. For example, if you were measuring length with a scale marked in millimeters, you might quote the reading as  $17.0 \text{ mm} \pm 0.5 \text{ mm}$ . Here  $17.0 \text{ mm}$  is the measured value and  $\pm 0.5 \text{ mm}$  is the error on the measurement. If you measured the same length many times, you would expect the error on the measurement to decrease. This is indeed the case.

The best estimate of the measured quantity is the “**Mean**” or “**Average**” of all the measurements. Simply add all the individual measurements together and divide by the number of measurements. We denote the average with a little bar over the top of the measured quantity. So an average value for  $x$  would be written

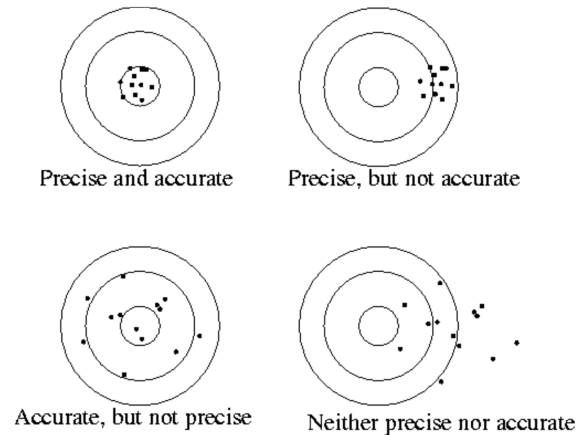
$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i = \frac{1}{N} (x_1 + x_2 + x_3 + \dots + x_N)$$

where  $\bar{x}$  is the average value of  $x$ ,  $x_i$  is the  $i^{\text{th}}$  measurement of  $x$ , and  $N$  is the total number of measurements taken. The  $\sum_{i=1}^N$  tells us that we need add up all the measurements of  $x$ .

The best estimate of the error associated with the mean value is called the “**Error on the Mean**” and is given by the error on a single measurement divided by the square root of the number of measurements taken. Obviously, this will decrease as the number of measurements increases. The final reading for a quantity should always be quoted as: (Mean)  $\pm$  (Error on the Mean). The Mean and the Error on the Mean must always have the same units.

## 1.2 Systematic Errors

These errors are more insidious than statistical errors. Systematic errors are difficult to detect, and the sizes of systematic errors are difficult to estimate. Increasing the number of measurements has no effect on systematic errors because the error is always in the same direction (all measurements too high, or all measurements too low). Careful instrument calibration and understanding of the measurement being made are part of prevention. For example, suppose that you are using a stopwatch to time runners in the 100-meter dash. You are quite adept at making the measurement, but – unknown to you – the watch runs 5 % fast. All times will be 5 % too high, but there will be no immediately obvious indication of a problem.



If you happen to be familiar with the runners' normal times, you might notice that everyone seems to be having a slow day. To prevent such problems, one should calibrate the stopwatch with a known standard such as the National Institute of Standards and Technology's standard time service on short wave radio. The rules are: 1) the error should have one significant figure; 2) the number of decimal places in the measurement should be the same as the number of decimal places in the error. Always remember: **There is no such thing as "human error"**. Try to find the deeper cause for any uncertainty or variation.

