Chapter 22

Electric Fields

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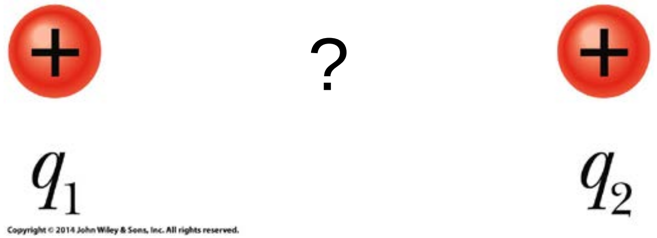
22-1 The Electric Field

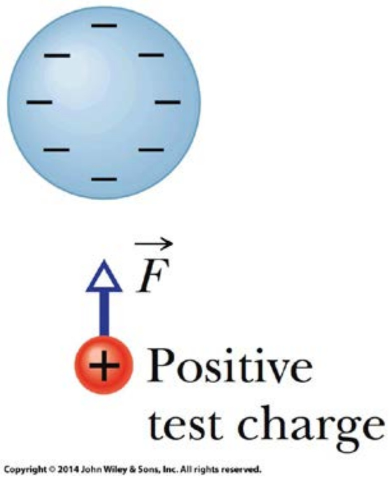


How does particle 1 “know” of the presence of particle 2? That is, since the particles do not touch, how can particle 2 push on particle 1—how can there be such an action at a distance?

Electric Field

The explanation that we shall examine here is this: Particle 2 sets up an electric field at all points in the surrounding space, even if the space is a vacuum. If we place particle 1 at any point in that space, particle 1 knows of the presence of particle 2 because it is affected by the electric field particle 2 has already set up at that point. Thus, particle 2 pushes on particle 1 not by touching it as you would push on a coffee mug by making contact. Instead, particle 2 pushes by means of the electric field it has set up.

Electric Field

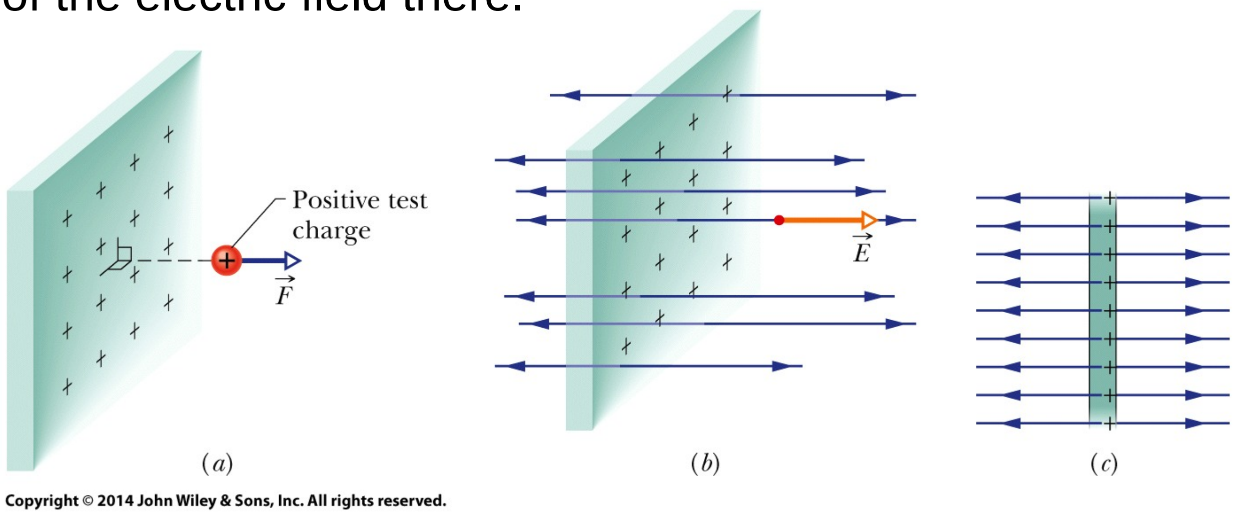


The electric field ***E*** at any point is defined in terms of the electrostatic force ***F*** that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

Electric Field Lines

Electric field lines help us visualize the direction and magnitude of electric fields. The electric field vector at any point is tangent to the field line through that point. The density of field lines in that region is proportional to the magnitude of the electric field there.



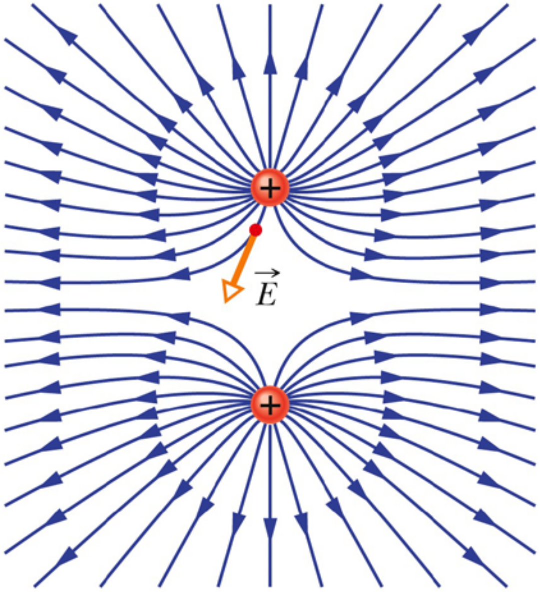
(a) The force on a positive test charge near a very large, non-conducting sheet with uniform positive charge on one side. (b) The electric field vector ***E*** at the test charge's location, and the nearby electric field lines, extending away from the sheet. (c) Side view.

Electric Field Lines

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

(1) The electric field vector at any given point must be tangent to the field line at that point and in the same direction, as shown for one vector.

(2) A closer spacing means a larger field magnitude.



Field lines for two particles with equal positive charge. Doesn’t the pattern itself suggest that the particles repel each other?

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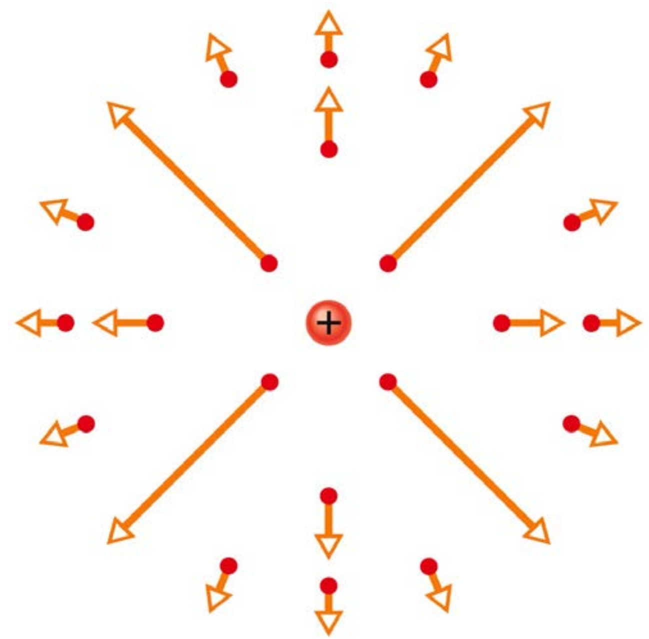
22-2 The Electric Field Due to a Charged Particle

The magnitude of the electric field ***E*** set up by a particle with charge *q* at distance *r* from the particle is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

The **electric field vectors** set up by a positively charged particle all point directly away from the particle. Those set up by a negatively charged particle all point directly toward the particle.

If more than one charged particle sets up an electric field at a point, the net electric field is the vector **sum** of the individual electric fields—**electric fields obey the superposition principle**.



The electric field vectors at various points around a positive point charge.

22-3 The Electric Field Due to a Dipole

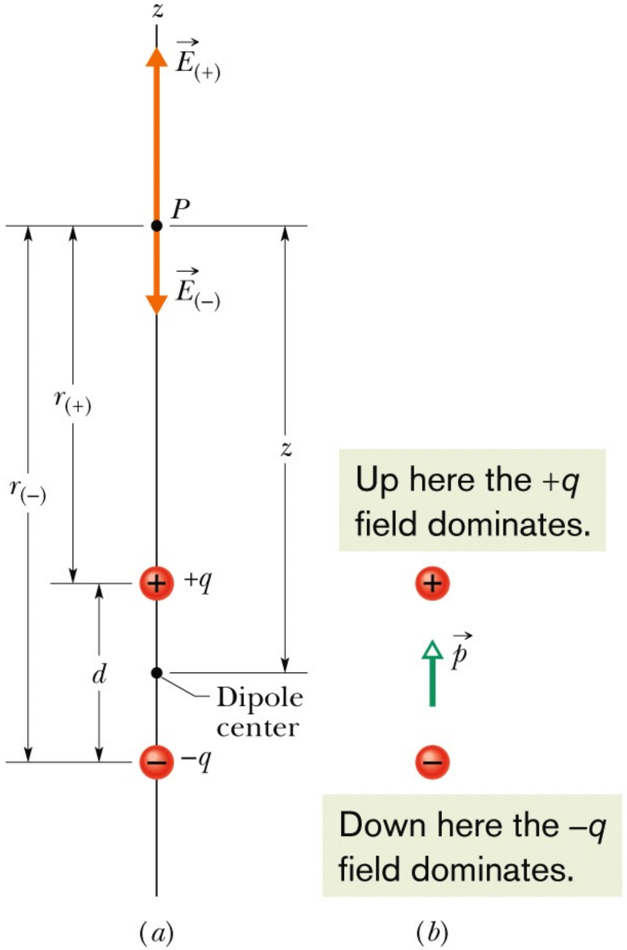
Electric Dipole

An electric dipole consists of two particles with charges of equal magnitude *q* but opposite signs, separated by a small distance *d*.

The magnitude of the electric field set up by an electric dipole at a distant point on the dipole axis (which runs through both particles) can be written in terms of either the product *qd* or the magnitude *p* of the dipole moment:

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3},$$

where z is the distance between the point and the center of the dipole.



22-4 The Electric Field Due to a Line of Charge

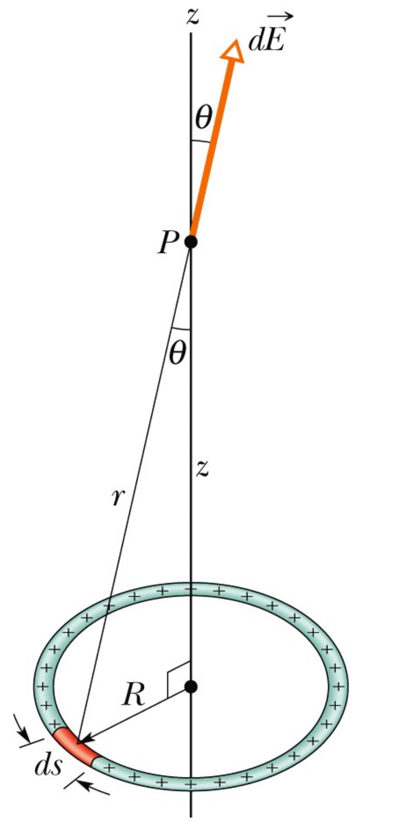
Key Concepts

- The equation for the electric field set up by a particle does not apply to an extended object with charge (said to have a **continuous charge distribution**).
- To find the electric field of an extended object at a point, we first consider the electric field set up by a charge element dq in the object, where the element is small enough for us to apply the equation for a particle. Then we sum, via integration, components of the electric fields ***dE*** from all the charge elements.
- Because the individual electric fields ***dE*** have different magnitudes and point in different directions, we first see if symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

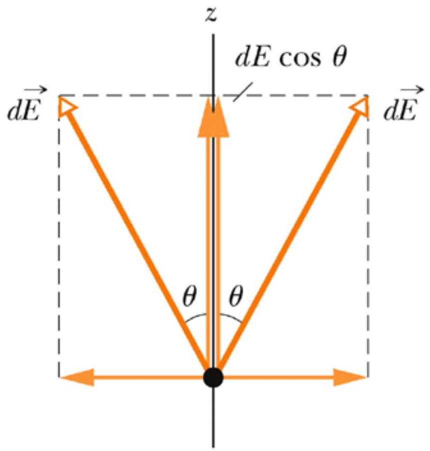
22-4 The Electric Field Due to a Line of Charge

Charged Ring

Canceling Components - Point P is on the axis: In the Figure (right), consider the charge element on the opposite side of the ring. It too contributes the field magnitude ***dE*** but the field vector leans at angle θ in the opposite direction from the vector from our first charge element, as indicated in the side view of Figure (bottom). Thus the two perpendicular components cancel. All around the ring, this cancelation occurs for every charge element and its symmetric partner on the opposite side of the ring. So we can neglect all the perpendicular components.



A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field ***dE*** at point *P*.



The components perpendicular to the z axis cancel; the parallel components add.

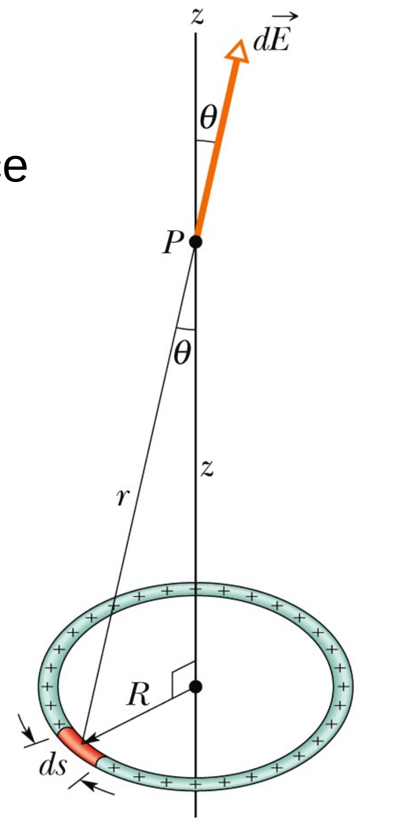
22-4 The Electric Field Due to a Line of Charge

Charged Ring

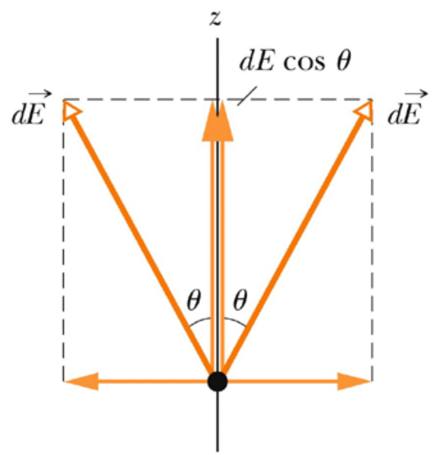
Adding Components. From the figure (bottom), we see that the parallel components each have magnitude ***dE*** cos*θ*. We can replace cos*θ* by using the right triangle in the Figure (right) to write

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}$$

And, $$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{z\lambda}{(z^2+R^2)^{3/2}}\,ds.$$ gives us the parallel field component from each charge element



The components perpendicular to the z axis cancel; the parallel components add.



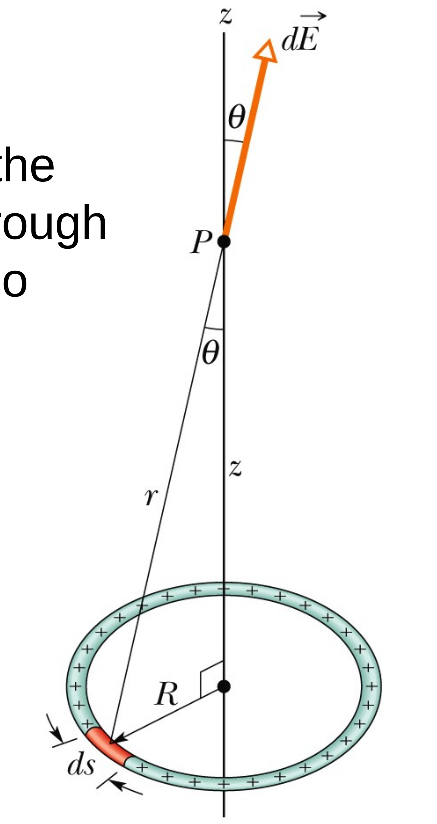
A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field ***dE*** at point *P*.

22-4 The Electric Field Due to a Line of Charge

Charged Ring

Integrating. Because we must sum a huge number of these components, each small, we set up an integral that moves along the ring, from element to element, from a starting point (call it *s=0*) through the full circumference (*s=2πR*). Only the quantity *s* varies as we go through the elements. We find

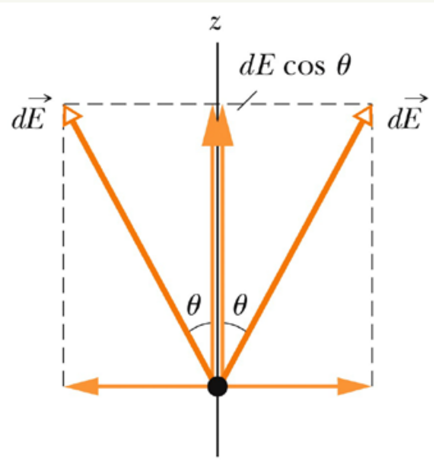
$$E = \int dE \cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} \int_0^{2\pi R} ds$$



Finally,

$$E = \frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} \quad \text{(charged ring).}$$

The components perpendicular to the z axis cancel; the parallel components add.



A ring of uniform positive charge. A differential element of charge occupies a length *ds* (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point *P*.

22-5 The Electric Field Due to a Charged Disk

We superimpose a ring on the disk as shown in the Figure, at an arbitrary radius $r \le R$. The ring is so thin that we can treat the charge on it as a charge element dq. To find its small contribution dE to the electric field at point P, on the axis, in terms of the ring's charge dq and radius r we can write

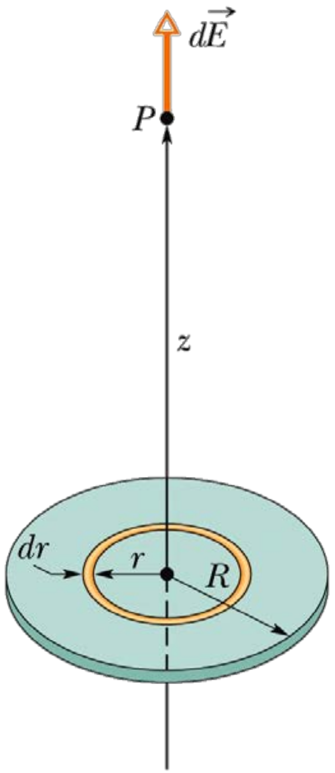
$$dE = \frac{dq\, z}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}}.$$

Then, we can sum all the ***dE*** contributions with

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr.$$

We find

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)}$$



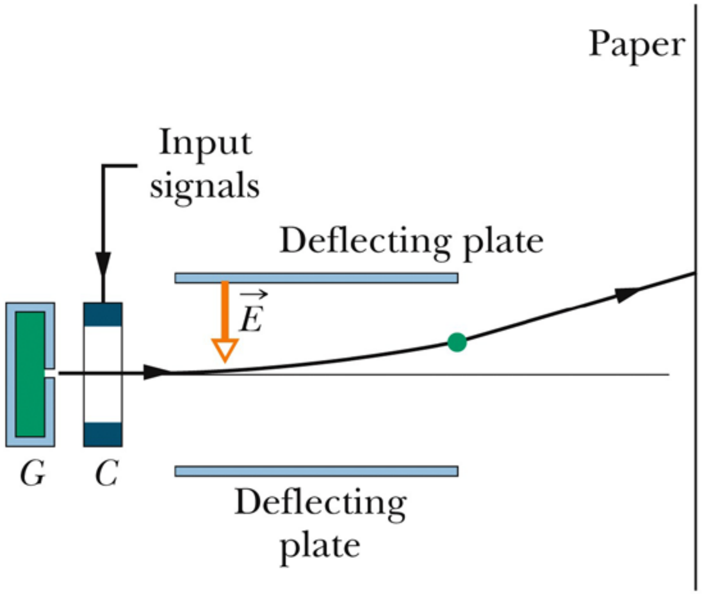
A disk of radius R and uniform positive charge. The ring shown has radius r and radial width dr. It sets up a differential electric field ***dE*** at point P on its central axis.

22-6 A Point Charge in an Electric Field

If a particle with charge *q* is placed in an external electric field ***E***, an electrostatic force ***F*** acts on the particle:

$$\vec{F} = q\vec{E}.$$

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.



Ink-jet printer. Drops shot from generator *G* receive a charge in charging unit *C*. An input signal from a computer controls the charge and thus the effect of field ***E*** on where the drop lands on the paper.

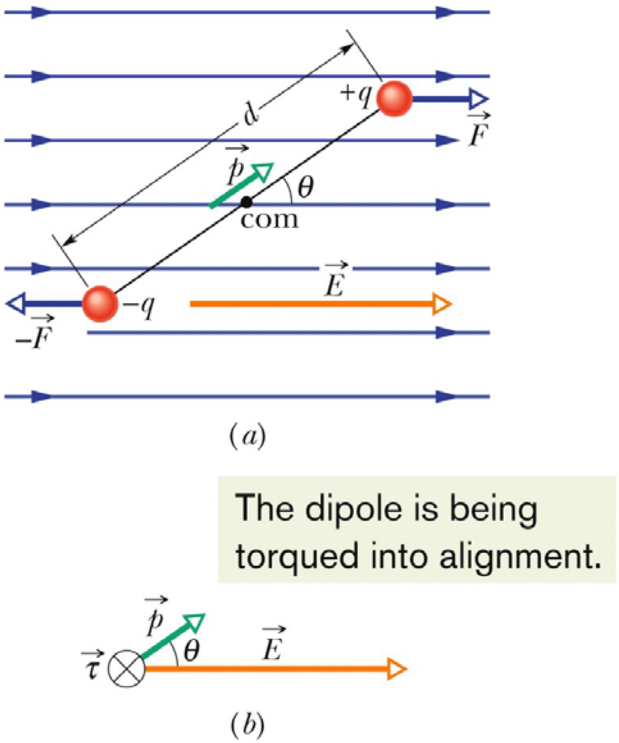
22-7 A Dipole in an Electric Field

The torque on an electric dipole of dipole moment ***p*** when placed in an external electric field ***E*** is given by a cross product:

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

A potential energy *U* is associated with the orientation of the dipole moment in the field, as given by a dot product:

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$



(a) An electric dipole in a uniform external electric field ***E***. Two centers of equal but opposite charge are separated by distance *d*. The line between them represents their rigid connection.

(b) Field E: causes a torque ***τ*** on the dipole. The direction of ***τ*** is into the page, as represented by the symbol (x-in a circle) .

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22 Summary

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}. \qquad \textbf{Eq. 22-1}$$

Electric Field Lines

- provide a means for visualizing the directions and the magnitudes of electric fields

Field due to a Point Charge

- The magnitude of the electric field ***E*** set up by a point charge *q* at a distance *r* from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}. \qquad \textbf{Eq. 22-3}$$

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \textbf{Eq. 22-9}$$

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \qquad \textbf{Eq. 22-26}$$

22 Summary

Force on a Point Charge in an Electric Field

- When a point charge *q* is placed in an external electric field ***E***

$$\vec{F} = q\vec{E}.$$

Eq. 22-28

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\vec{\tau} = \vec{p} \times \vec{E}.$$

Eq. 22-34

- The dipole has a potential energy *U* associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}.$$

Eq. 22-38