Chapter 24

Electric Potential

24-1 Electric Potential

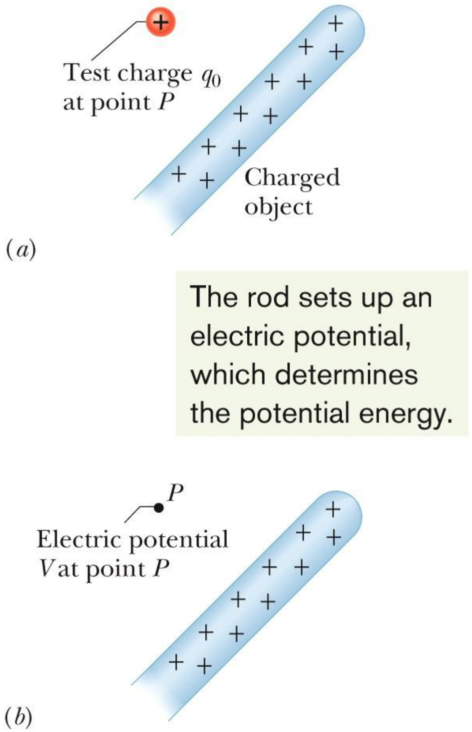
The electric potential V at a point P in the electric field of a charged object is

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}$$

where W_∞ is the work that would be done by the electric force on a positive test charge q_0 were it brought from an infinite distance to *P*, and *U* is the electric potential energy that would then be stored in the test charge–object system.

If a particle with charge *q* is placed at a point where the electric potential of a charged object is *V*, the electric potential energy *U* of the particle–object system is

$$U = qV.$$



(a) A test charge has been brought in from infinity to point P in the electric field of the rod.
(b) We define an electric potential V at P based on the potential energy of the configuration in (a).

WILEY

24-1 Electric Potential

Change in Electric Potential. If the particle moves through a potential difference *ΔV*, the change in the electric potential energy is

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$

Work by the Field. The work W done by the electric force as the particle moves from *i* to *f:*

$$W = -\Delta U = -q\,\Delta V = -q(V_f - V_i).$$

Conservation of Energy. If a particle moves through a change *ΔV* in electric potential without an applied force acting on it, applying the conservation of mechanical energy gives the change in kinetic energy as

$$\Delta K = -q\,\Delta V = -q(V_f - V_i).$$

Work by an Applied Force. If some force in addition to the electric force acts on the particle, we account for that work

$$\Delta K = -\Delta U + W_{app} = -q\,\Delta V + W_{app}.$$

24-2 Equipotential Surfaces and the Electric Field

Adjacent points that have the same electric potential form an **equipotential surface**, which can be either an imaginary surface or a real, physical surface.

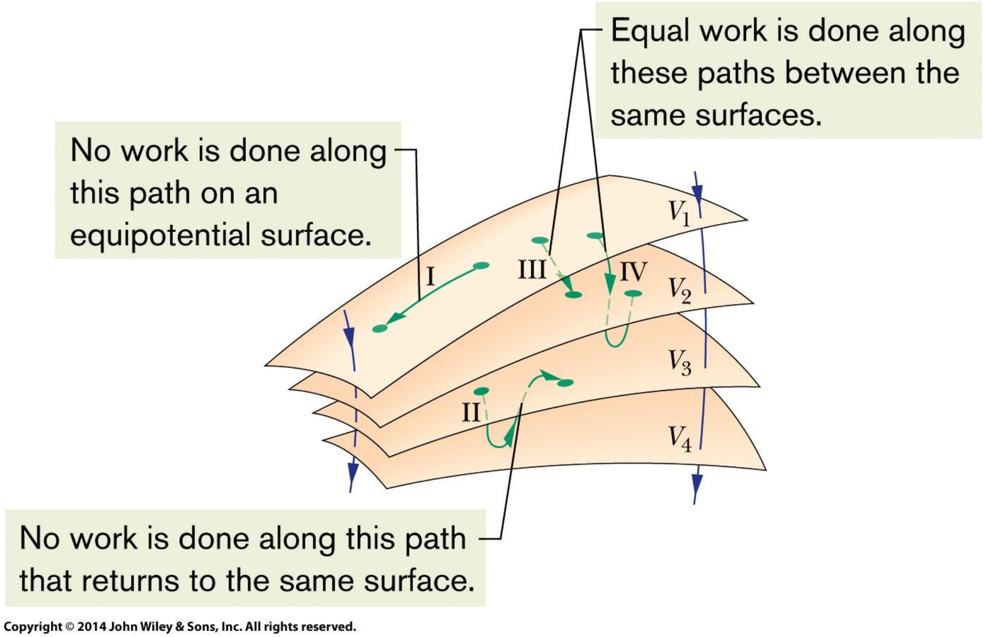


Figure shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. The work done by the electric field on a charged particle as the particle moves from one end to the other of paths **I** and **II** is zero because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential. The work done as the charged particle moves from one end to the other of paths **III** and **IV** is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths **III** and **IV** connect the same pair of equipotential surfaces.

24-2 Equipotential Surfaces and the Electric Field

The electric potential difference between two points *i* and *f* is

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

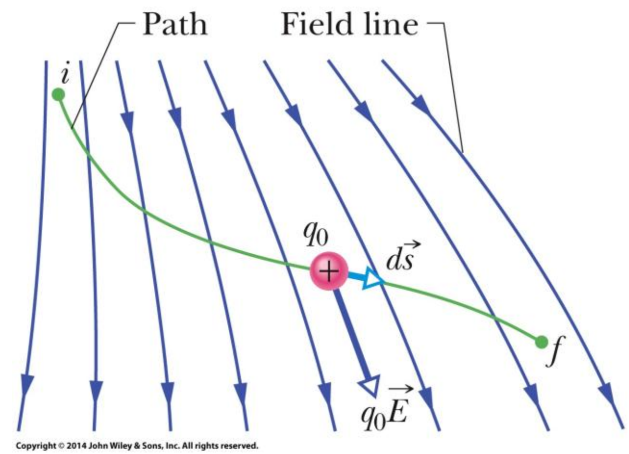
where the integral is taken over any path connecting the points. If the integration is difficult along any particular path, we can choose a different path along which the integration might be easier.

If we choose $V_i = 0$, we have, for the potential at a particular point,

$$V = -\int_i^f \vec{E} \cdot d\vec{s}.$$

In a uniform field of magnitude E, the change in potential from a higher equipotential surface to a lower one, separated by distance Δx, is

$$\Delta V = -E\,\Delta x.$$



A test charge q_0 moves from point *i* to point *f* along the path shown in a non-uniform electric field. During a displacement $d\mathbf{s}$, an electric force $q_0\mathbf{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

24-3 Potential due to a Charged Particle

We know that the electric potential difference between two points *i* and *f* is

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

For radial path

$$V_f - V_i = -\int_R^\infty E\, dr.$$

The magnitude of the electric field at the site of the test charge

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

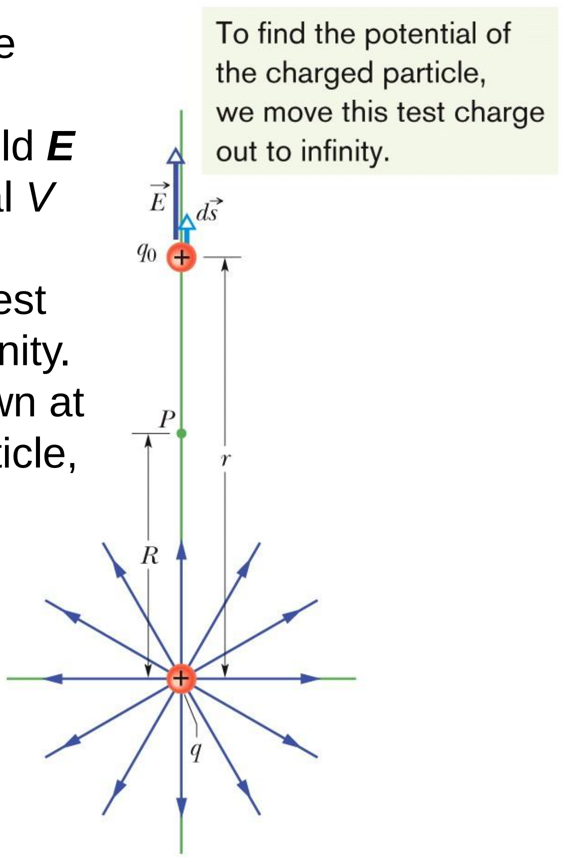
We set $V_f = 0$ (at ∞) and $V_i = V$ (at R)

$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\, dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

Solving for V and switching R to r, we get

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

In this figure the particle with positive charge q produces an electric field $\boldsymbol{E}$ and an electric potential V at point P. We find the potential by moving a test charge q_0 from P to infinity. The test charge is shown at distance r from the particle, during differential displacement $d\boldsymbol{s}$.



as the electric potential V due to a particle of charge q at any radial distance r from the particle.

Potential due to a group of Charged Particles

The potential due to a collection of charged particles is

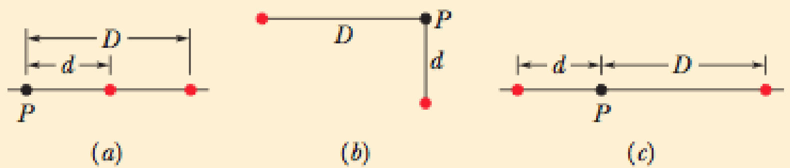
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \quad (n \text{ charged particles}).$$

Thus, the potential is the algebraic sum of the individual potentials, with no consideration of directions.

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Checkpoint 3

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



Answer: Same net potential (a)=(b)=(c)

24-4 Potential due to a Electric Dipole

The net potential at P is given by

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

We can approximate the two lines to P as being parallel and their length difference as being the leg of a right triangle with hypotenuse d (Fig. b). Also, that difference is so small that the product of the lengths is approximately r^2.

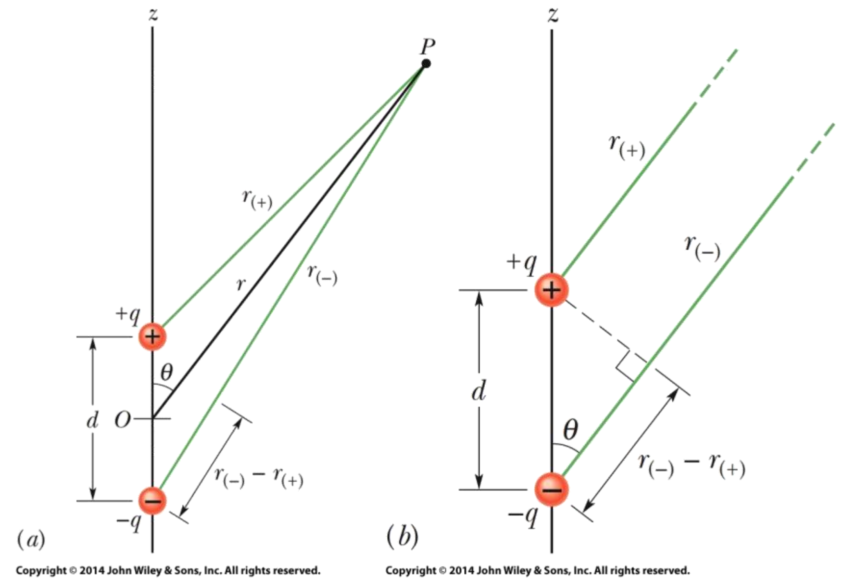
$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

We can approximate *V* to be

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2},$$

where θ is measured from the dipole axis as shown in Fig. a. And since *p=qd,* we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \quad \text{(electric dipole)}$$



(a) Point P is a distance *r* from the midpoint O of a dipole. The line OP makes an angle θ with the dipole axis.

(b) If P is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length *r*, and the dashed black line is approximately perpendicular to the line of length $r_{(-)}$.

24-5 Potential due to a Continuous Charge Distribution

For a continuous distribution of charge (over an extended object), the potential is found by
(1) dividing the distribution into charge elements dq that can be treated as particles and then
(2) summing the potential due to each element by integrating over the full distribution:

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}.$$

We now examine two continuous charge distributions, a line and a disk.

24-5 Potential due to a Continuous Charge Distribution

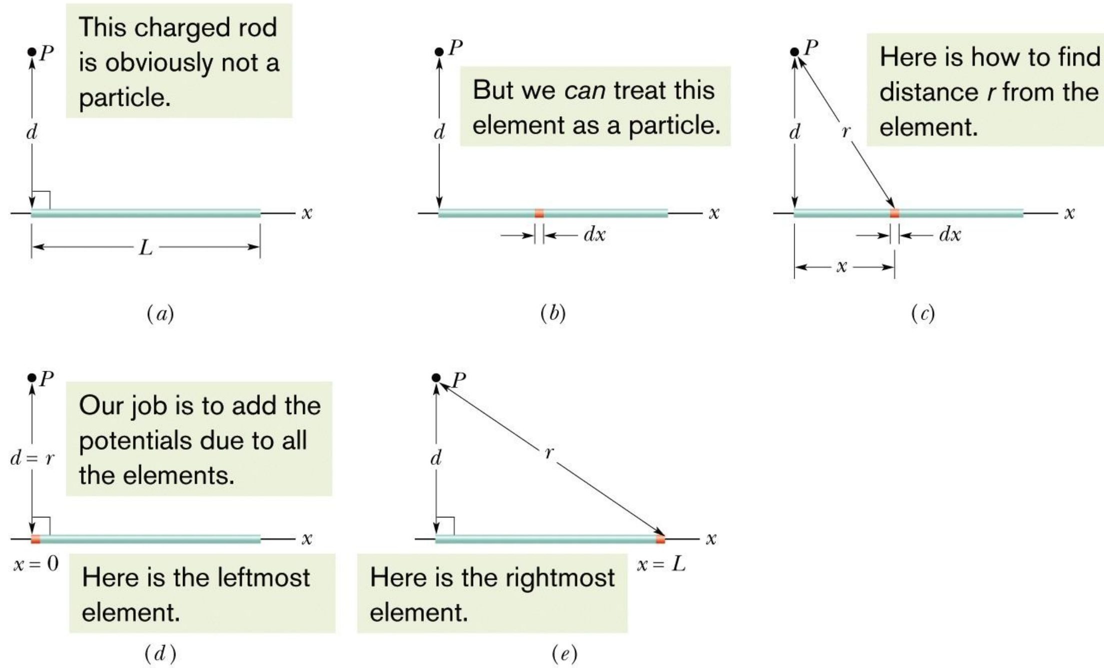
Line of Charge

Fig. a has a thin conducting rod of length L. As shown in fig. b the element of the rod has a differential charge of

$$dq = \lambda\, dx.$$

This element produces an electric potential dV at point P (fig c) given by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(x^2 + d^2)^{1/2}}.$$



We now find the total potential V produced by the rod at point P by integrating dV along the length of the rod, from $x = 0$ to $x = L$ (Figs.d and e)

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx$$

Simplified to,

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{L + (L^2 + d^2)^{1/2}}{d}\right].$$

24-5 Potential due to a Continuous Charge Distribution

Charged Disk

In figure, consider a differential element consisting of a flat ring of radius *R'* and radial width *dR'*. Its charge has magnitude

$$dq = \sigma(2\pi R')(dR'),$$

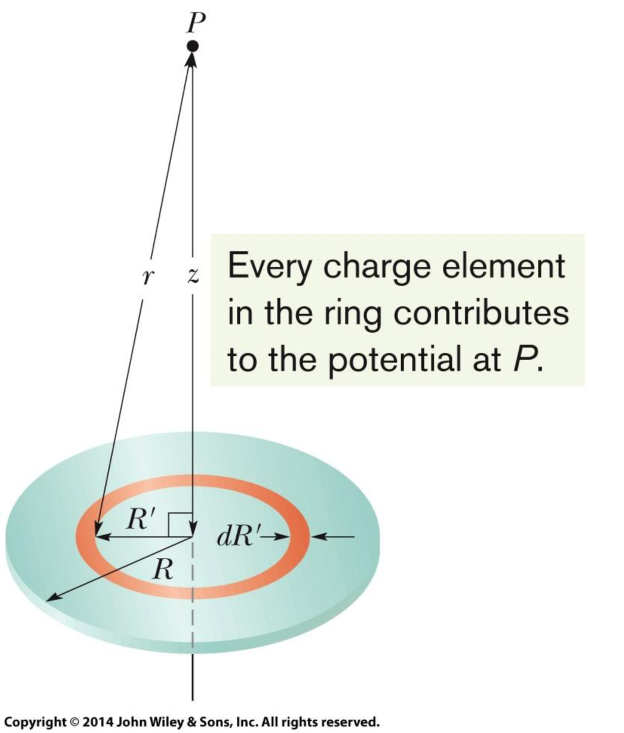
in which $(2\pi R')(dR')$ is the upper surface area of the ring. The contribution of this ring to the electric potential at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}.$$

We find the net potential at P by adding (via integration) the contributions of all the rings from $R'=0$ to $R'=R$:

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'\,dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right).$$

Note that the variable in the second integral is *R'* and not *z*



A plastic disk of radius *R*, charged on its top surface to a uniform surface charge density *σ*. We wish to find the potential V at point P on the central axis of the disk.

24-6 Calculating the Field from the Potential

Suppose that a positive test charge q_0 moves through a displacement $d\mathbf{s}$ from one equipotential surface to the adjacent surface. The work the electric field does on the test charge during the move is $-q_0dV$. On the other hand the work done by the electric field may also be written as the scalar product $(q_0\mathbf{E})\bullet d\mathbf{s}$. Equating these two expressions for the work yields

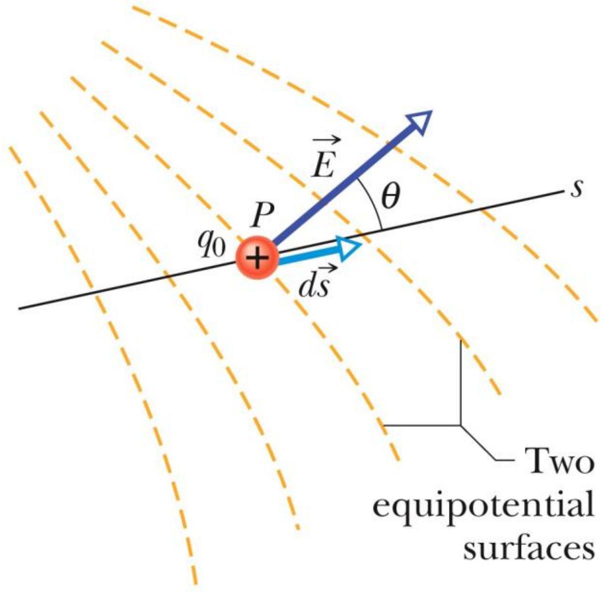
$$-q_0\,dV = q_0E(\cos\theta)\,ds,$$

or,

$$E\cos\theta = -\frac{dV}{ds}.$$

Since $E\cos\theta$ is the component of $\mathbf{E}$ in the direction of $d\mathbf{s}$, we get,

$$E_s = -\frac{\partial V}{\partial s}.$$



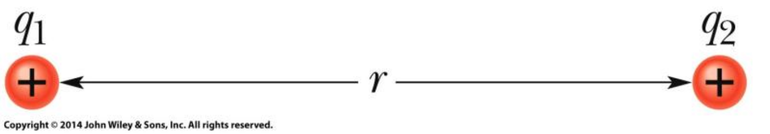
A test charge q_0 moves a distance $d\mathbf{s}$ from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement $d\mathbf{s}$ makes an angle θ with the direction of the electric field $\mathbf{E}$.

WILEY

24-7 Electric Potential Energy of a System of Charged Particles

The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other. For two particles at separation *r*,



Two charges held a fixed distance *r* apart.

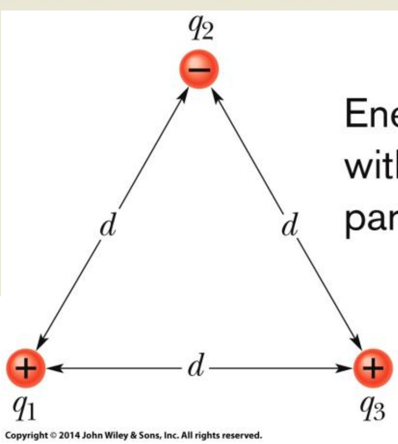
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \quad \text{(two-particle system).}$$

Sample Problem 24.06 Potential energy of a system of three charged particles

Figure 24-19 shows three charged particles held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

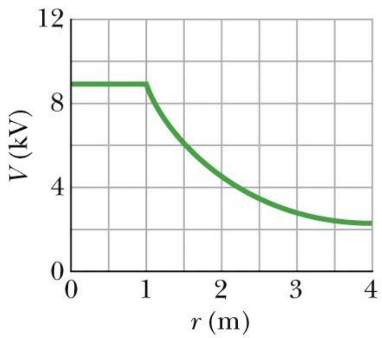


Energy is associated with each pair of particles.

Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

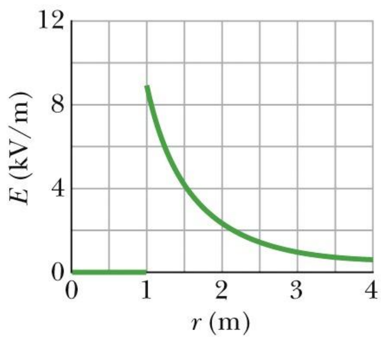
24-8 Potential of a Charged Isolated Conductor

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.



(a)

(a) A plot of *V(r)* both inside and outside a charged spherical shell of radius *1.0 m*.



(b)

(a) A plot of *E(r)* for the same shell.

Courtesy Westinghouse Electric Corporation

It is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal.

24 Summary

Electric Potential

- The electric potential *V* at point *P* in the electric field of a charged object:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}, \qquad \textbf{Eq. 24-2}$$

Electric Potential Energy

- Electric potential energy *U* of the particle-object system:

$$U = qV. \qquad \textbf{Eq. 24-3}$$

- If the particle moves through potential *ΔV*:

$$\Delta U = q\,\Delta V = q(V_f - V_i). \qquad \textbf{Eq. 24-4}$$

Mechanical Energy

- Applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q\,\Delta V. \qquad \textbf{Eq. 24-9}$$

- In case of an applied force in a particle

$$\Delta K = -q\,\Delta V + W_{app}. \qquad \textbf{Eq. 24-11}$$

- In a special case when *ΔK=0:*

$$W_{app} = q\,\Delta V \quad (\text{for } K_i = K_f). \qquad \textbf{Eq. 24-12}$$

Finding V from *E*

- The electric potential difference between two point *I* and *f* is:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}, \qquad \textbf{Eq. 24-18}$$

24 Summary

Potential due to a Charged Particle

- due to a single charged particle at a distance r from that particle :

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$ **Eq. 24-26**

- due to a collection of charged particles

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}.$$ **Eq. 24-27**

Potential due to an Electric Dipole

- The electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$ **Eq. 24-30**

Potential due to a Continuous Charge Distribution

- For a continuous distribution of charge:

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$ **Eq. 24-32**

Calculating *E* from V

- The component of ***E*** in any direction is:

$$E_s = -\frac{\partial V}{\partial s}.$$ **Eq. 24-40**

Electric Potential Energy of a System of Charged Particle

- For two particles at separation *r*:

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$ **Eq. 24-46**