Chapter 26

Current and Resistance

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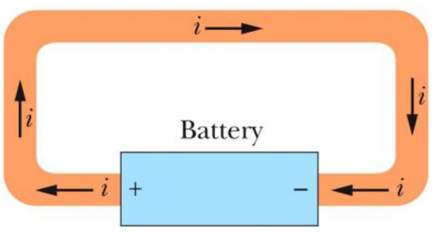
26-1 Electric Current

As Fig. (a) reminds us, any isolated conducting loop—regardless of whether it has an excess charge — is all at the same potential. No electric field can exist within it or along its surface.

(a)

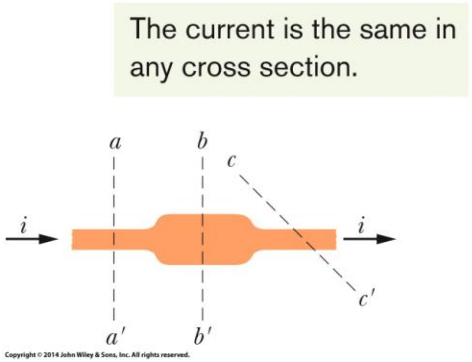
If we insert a battery in the loop, as in Fig. (b), the conducting loop is no longer at a single potential. Electric fields act inside the material making up the loop, exerting forces on internal charges, causing them to move and thus establishing a **current**. (The diagram assumes the motion of positive charges moving clockwise.)



(b)

Figure c shows a section of a conductor, part of a conducting loop in which current has been established. If charge *dq* passes through a hypothetical plane (such as aa’) in time *dt*, then the current *i* through that plane is defined as

$$i = \frac{dq}{dt} \quad \text{(definition of current).}$$



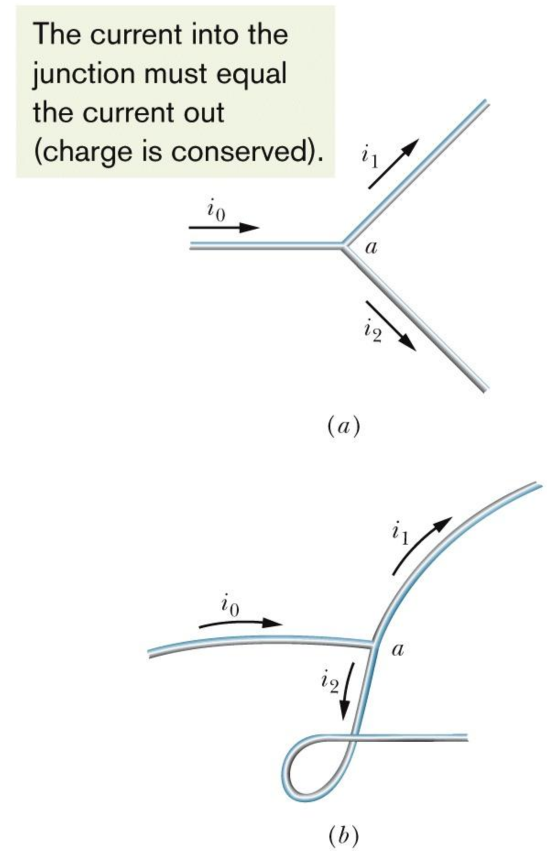
(c)

26-1 Electric Current

Figure (a) shows a conductor with current i_0 splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$i_0 = i_1 + i_2.$$

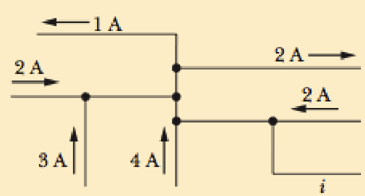
Figure (b) suggests, bending or reorienting the wires in space does not change the validity of the above equation Current arrows show only a direction (or sense) of flow along a conductor, not a direction in space.



A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

Checkpoint 1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current i in the lower right-hand wire?



Answer: 8A with arrow pointing right

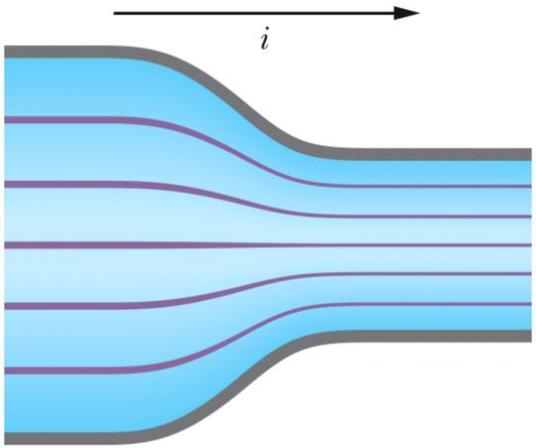
26-2 Current Density

Current *i* (a scalar quantity) is related to **current density *J*** (a vector quantity) by

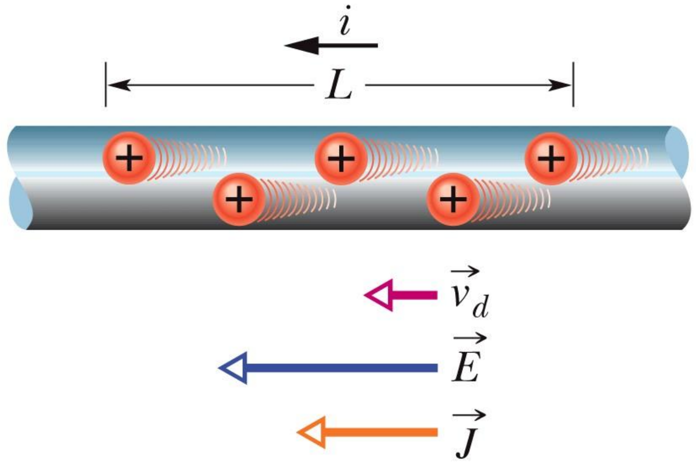
$$i = \int \vec{J} \cdot d\vec{A}.$$

where *d**A*** is a vector perpendicular to a surface element of area *d**A*** and the integral is taken over any surface cutting across the conductor. The current density ***J*** has the same direction as the velocity of the moving charges if they are positive charges and the opposite direction if the moving charges are negative.

Streamlines representing current density in the flow of charge through a constricted conductor.



26-2 Current Density



Conduction electrons are actually moving to the right but the conventional current *i* is said to move to the left.

Current is said to be due to positive charges that are propelled by the electric field. In the figure, positive charge carriers drift at speed v_d in the direction of the applied electric field ***E*** which here is applied to the left. By convention, the direction of the current density ***J*** and the sense of the current arrow are drawn in that same direction, as is the drift speed v_d.

The drift velocity v_d is related to the current density by

$$\vec{J} = (ne)\vec{v}_d.$$

Here the product *ne*, whose SI unit is the coulomb per cubic meter (C/m^3), is the **carrier charge density**.

26-3 Resistance and Resistivity

If we apply the same potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its **electrical resistance**. The resistance *R* of a conductor is defined as

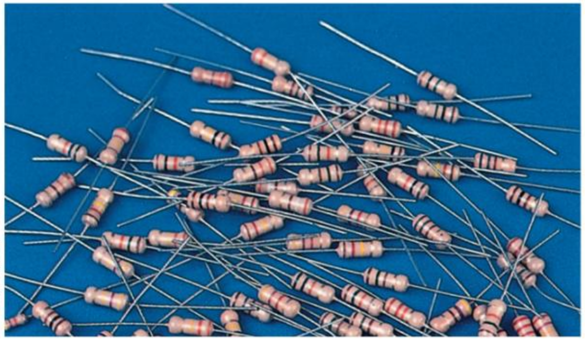
$$R = \frac{V}{i} \quad \text{(definition of } R\text{)}.$$

where *V* is the potential difference across the conductor and *i* is the current through the conductor. Instead of the resistance R of an object, we may deal with the **resistivity** *ρ* of the material:

$$\rho = \frac{E}{J} \quad \text{(definition of } \rho\text{)}.$$

The reciprocal of resistivity is **conductivity** *σ* of the material:

$$\sigma = \frac{1}{\rho} \quad \text{(definition of } \sigma\text{)}.$$



The Image Works

Assortment of Resistors

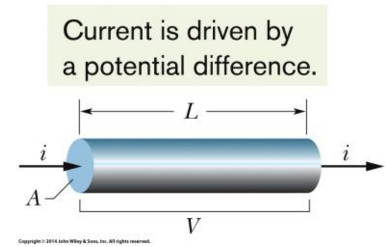
26-3 Resistance and Resistivity

Resistance is a property of an object. Resistivity is a property of a material.

The resistance R of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A}.$$

Here A is the cross-sectional area.

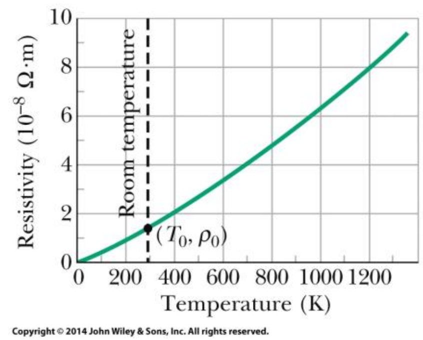


A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

The resistivity ρ for most materials changes with temperature. For many materials, including metals, the relation between ρ and temperature T is approximated by the equation

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

Here T_0 is a reference temperature, ρ_0 is the resistivity at T_0, and α is the temperature coefficient of resistivity for the material.



The resistivity of copper as a function of temperature.

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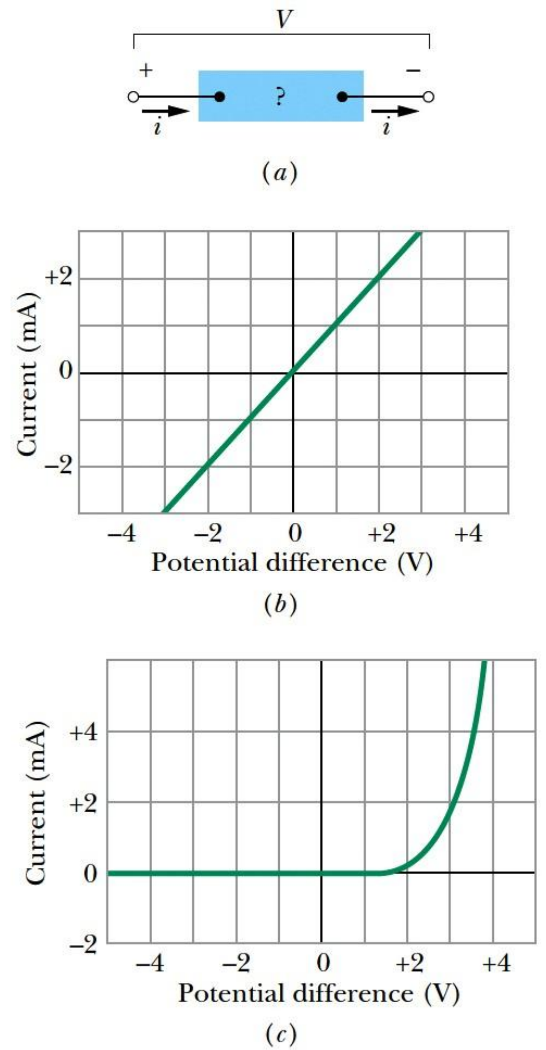
26-4 Ohm's Law

Figure (a) shows how to distinguish among devices. A potential difference *V* is applied across the device being tested, and the resulting current *i* through the device is measured as *V* is varied in both magnitude and polarity.

Figure (b) is a plot of *i* versus *V* for one device. This plot is a straight line passing through the origin, so the ratio *i/V* (which is the slope of the straight line) is the same for all values of V. This means that the resistance $R = V/i$ of the device is independent of the magnitude and polarity of the applied potential difference *V*.

Figure (c) is a plot for another conducting device. Current can exist in this device only when the polarity of *V* is positive and the applied potential difference is more than about 1.5 *V*. When current does exist, the relation between *i* and *V* is not linear; it depends on the value of the applied potential difference *V*.



26-4 Ohm's Law

Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

$$I \sim V \quad \text{or} \quad I = V/R$$

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

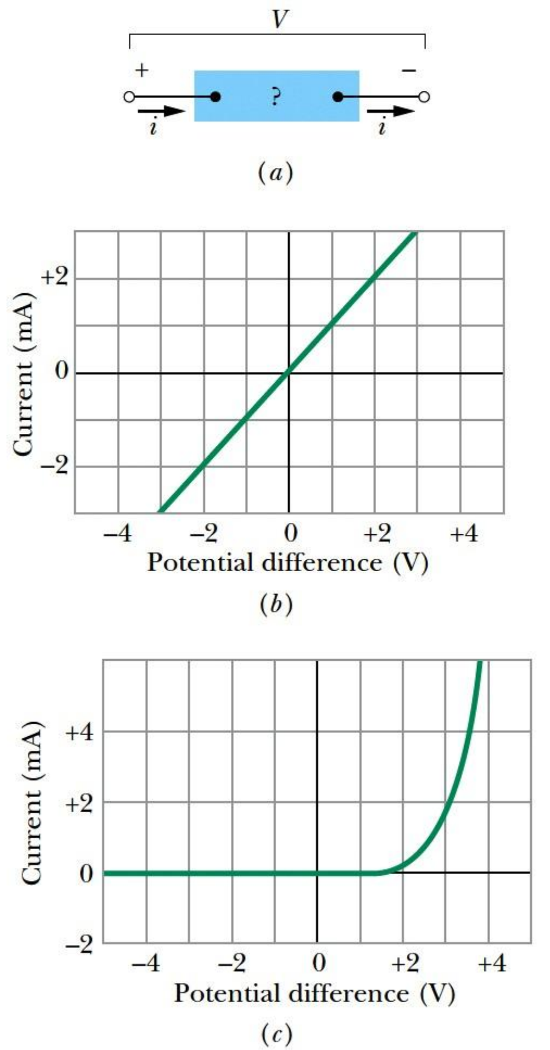
A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

Checkpoint 4

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). From these data, determine which device does not obey Ohm's law.

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

Answer: Device 2 does not follow ohm's law.



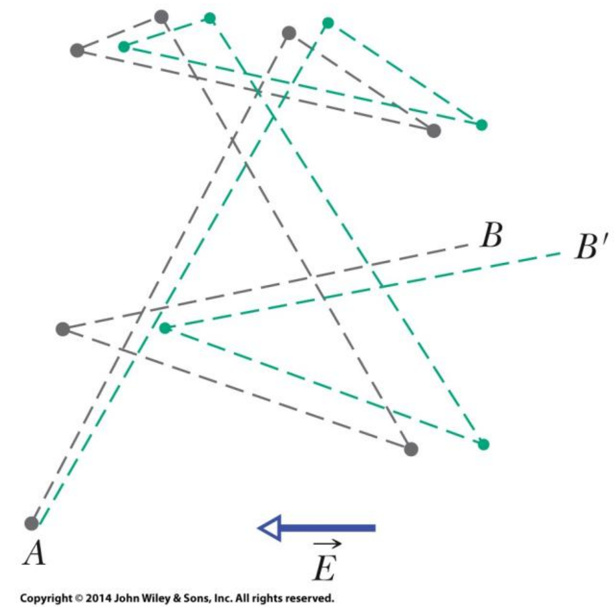
A Microscopic View

The assumption that the conduction electrons in a metal are free to move like the molecules in a gas leads to an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Here *n* is the number of free electrons per unit volume and *τ* is the mean time between the collisions of an electron with the atoms of the metal.

Metals obey Ohm's law because the mean free time *τ* is approximately independent of the magnitude *E* of any electric field applied to a metal.



The gray lines show an electron moving from A to B, making six collisions en route. The green lines show what the electron's path might be in the presence of an applied electric field ***E***. Note the steady drift in the direction of -***E***.

26-5 Power, Semiconductors, Superconductors

The battery at the left supplies energy to the conduction electrons that form the current.

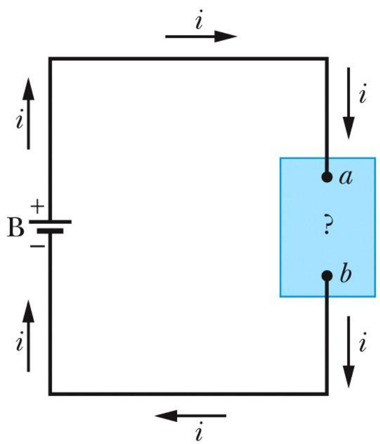


Figure shows a circuit consisting of a battery B that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude V across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal b.

The power *P*, or rate of energy transfer, in an electrical device across which a potential difference *V* is maintained is

$$P = iV \quad \text{(rate of electrical energy transfer).}$$

If the device is a resistor, the power can also be written as

$$P = i^2R \quad \text{(resistive dissipation)}$$

or,

$$P = \frac{V^2}{R} \quad \text{(resistive dissipation).}$$

26-5 Power, Semiconductors, Superconductors

Semiconductors are materials that have few conduction electrons but can become conductors when they are doped with other atoms that contribute charge carriers.

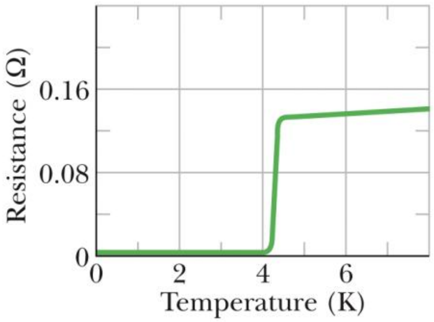
In a semiconductor, *n* (number of free electrons) is small (unlike conductor) but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a decrease of resistivity with increasing temperature, as indicated by the negative temperature coefficient of resistivity for silicon in Table 26-2.

$$\rho = \frac{m}{e^2 n \tau}.$$

Table 26-2 Some Electrical Properties of Copper and Silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	8.49×10^{28}	1×10^{16}
Resistivity, $\Omega \cdot m$	1.69×10^{-8}	2.5×10^{3}
Temperature coefficient of resistivity, K^{-1}	$+4.3 \times 10^{-3}$	-70×10^{-3}

Superconductors are materials that lose all electrical resistance below some critical temperature. Most such materials require very low temperatures, but some become superconducting at temperatures as high as room temperature.



The resistance of mercury drops to zero at a temperature of about 4 K.

26 Summary

Current

- The electric current i in a conductor is defined by

$$i = \frac{dq}{dt}.$$ **Eq. 26-1**

Current Density

- Current is related to current density by

$$i = \int \vec{J} \cdot d\vec{A},$$ **Eq. 26-4**

Drift Speed of the Charge Carriers

- Drift speed of the charge carriers in an applied electric field is related to current density by

$$\vec{J} = (ne)\vec{v}_d,$$ **Eq. 26-7**

Resistance of a Conductor

- Resistance R of a conductor is defined by

$$R = \frac{V}{i}$$ **Eq. 26-8**

- Similarly the resistivity and conductivity of a material is defined by

$$\rho = \frac{1}{\sigma} = \frac{E}{J}$$ **Eq. 26-10&12**

- Resistance of a conducting wire of length L and uniform cross section is

$$R = \rho\frac{L}{A}$$ **Eq. 26-16**

Change of *ρ* with Temperature

- The resistivity of most material changes with temperature and is given as

$$\rho - \rho_0 = \rho_0\alpha(T - T_0).$$ **Eq. 26-17**

26 Summary

Ohm's Law

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance R (defined by **Eq. 26-8** as *V/i*) is independent of the applied potential difference *V.*

Resistivity of a Metal

- By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}. \qquad \textbf{Eq. 26-22}$$

Power

- The power P, or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV \qquad \textbf{Eq. 26-26}$$

- If the device is a resistor, we can write

$$P = i^2R = \frac{V^2}{R} \qquad \textbf{Eq. 26-27\&28}$$