Chapter 27

Circuits

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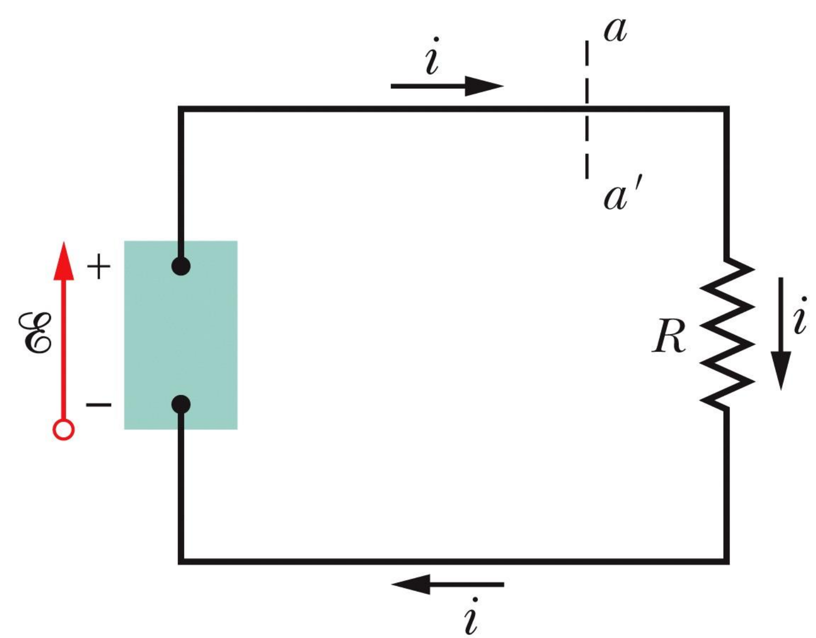
27-1 Single-Loop Circuits

To produce a steady flow of charge, you need a "charge pump," a device that—by doing work on the charge carriers—maintains a potential difference between a pair of terminals. We call such a device an ***emf* device**, and the device is said to provide an *emf* $\mathscr{E}$, which means that it does work on charge carriers.

Figure shows an *emf* device (consider it to be a battery) that is part of a simple circuit containing a single resistance *R*. The *emf* device keeps one of its terminals (called the positive terminal and often labeled +) at a higher electric potential than the other terminal (called the negative terminal and labeled -). We can represent the *emf* of the device with an arrow that points from the negative terminal toward the positive terminal as in Figure. A small circle on the tail of the *emf* arrow distinguishes it from the arrows that indicate current direction.



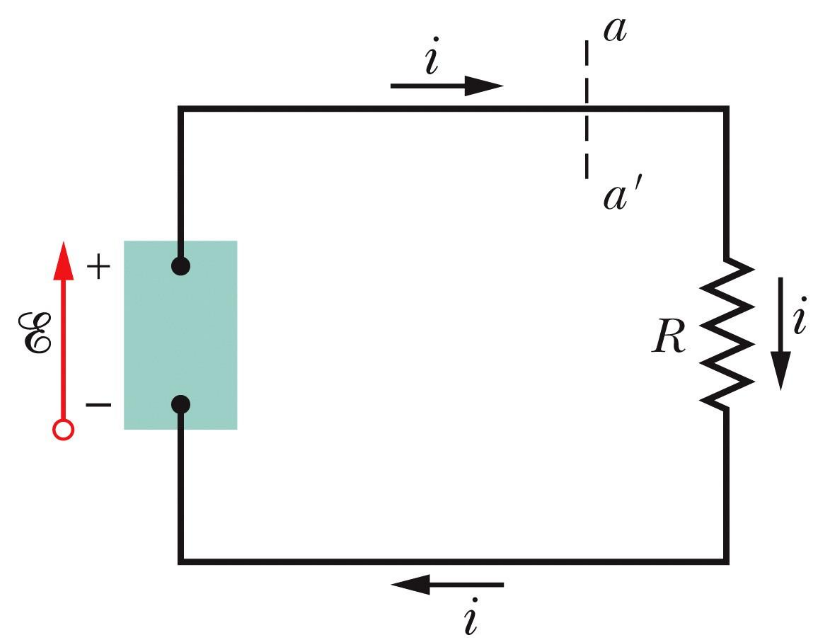
27-1 Single-Loop Circuits

An emf device does work on charges to maintain a potential difference between its output terminals. If *dW* is the work the device does to force positive charge *dq* from the negative to the positive terminal, then the *emf* (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{).}$$

An **ideal *emf* device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the *emf*.

A **real *emf* device** has internal resistance. The potential difference between its terminals is equal to the *emf* only if there is no current through the device.



Calculating Current in a Single-Loop Circuits

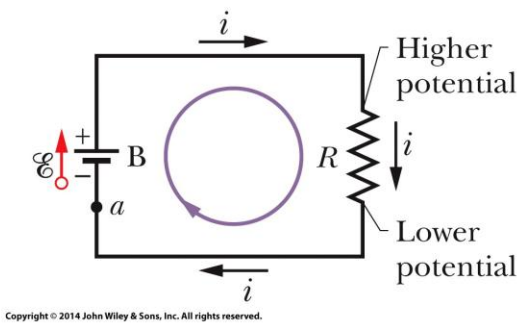
Energy Method

Equation, $P=i^2R$, tells us that in a time interval dt an amount of energy given by $i^2R\,dt$ will appear in the resistor (shown in the figure) as thermal energy. This energy is said to be **dissipated**. (Because we assume the wires to have negligible resistance, no thermal energy will appear in them.)

During the same interval, a charge $dq = i\,dt$ will have moved through battery B, and the work that the battery will have done on this charge is

$$dW = \mathscr{E}\,dq = \mathscr{E}i\,dt.$$

The battery drives current through the resistor, from high potential to low potential.



From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

$$\mathscr{E}i\,dt = i^2R\,dt.$$

Which gives us

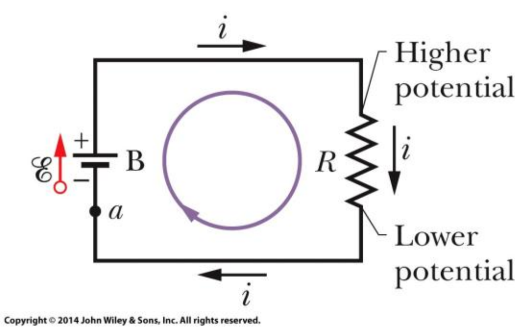
$$i = \frac{\mathscr{E}}{R}.$$

Calculating Current in a Single-Loop Circuits

Potential Method

In the figure, let us start at point a, whose potential is V_a, and mentally walk clockwise around the circuit until we are back at point a, keeping track of potential changes as we move. Our starting point is at the low-potential terminal of the battery. Because the battery is ideal, the potential difference between its terminals is equal to $\mathscr{E}$. When we pass through the battery to the high-potential terminal, the change in potential is $+\mathscr{E}$.

The battery drives current through the resistor, from high potential to low potential.



After making a complete loop, our initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \mathscr{E} - iR = V_a.$$

The value of V_a cancels from this equation, which becomes

$$\mathscr{E} - iR = 0.$$

Which gives us

$$i = \frac{\mathscr{E}}{R}.$$

Calculating Current in a Single-Loop Circuits

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

Internal Resistance

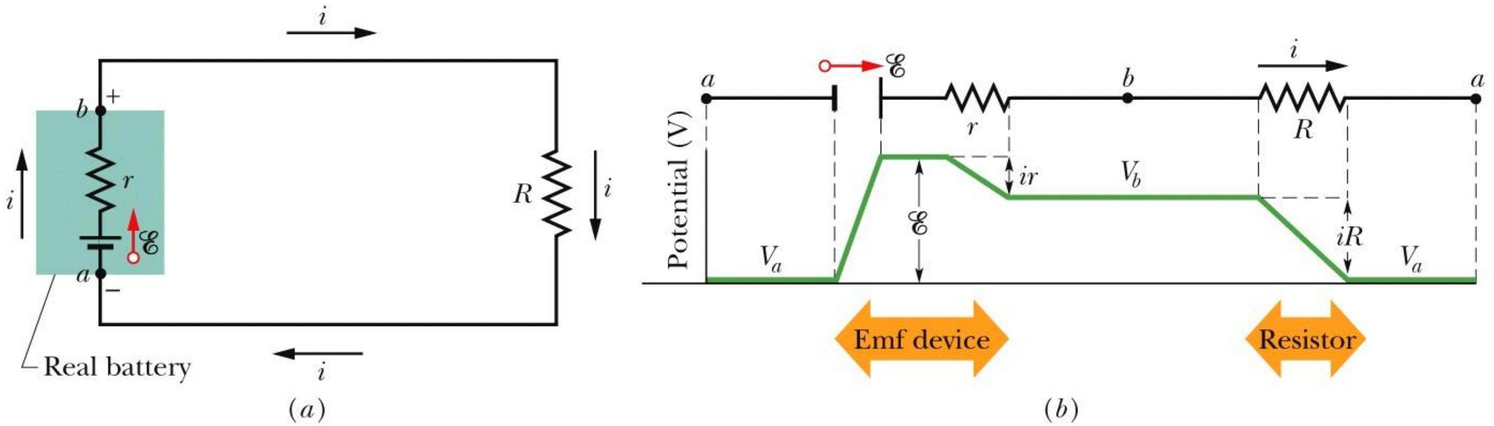


Figure (a) shows a real battery, with internal resistance *r*, wired to an external resistor of resistance *R*. The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an un removable feature of the battery. Figure (b) shows graphically the changes in electric potential around the circuit. Now if we apply the loop rule clockwise beginning at point a, the changes in potential give us $\mathscr{E} - ir - iR = 0.$

Solving for the current we find, $i = \dfrac{\mathscr{E}}{R + r}.$

Resistance in Series

Figure (a) shows three resistances connected in series to an ideal battery with *emf* $\mathscr{E}$. The resistances are connected one after another between a and b, and a potential difference is maintained across a and b by the battery. The potential differences that then exist across the resistances in the series produce identical currents *i* in them. To find total resistance R_{eq} in Fig. (b), we apply the loop rule to both circuits. For Fig. (a), starting at a and going clockwise around the circuit, we find

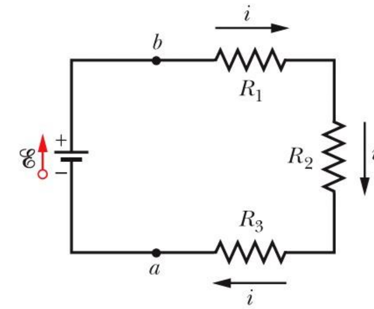
$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0, \quad \text{or} \quad i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}.$$

For Fig. (b), with the three resistances replaced with a single equivalent resistance R_{eq}, we find

$$\mathscr{E} - iR_{eq} = 0, \quad \text{or} \quad i = \frac{\mathscr{E}}{R_{eq}}.$$

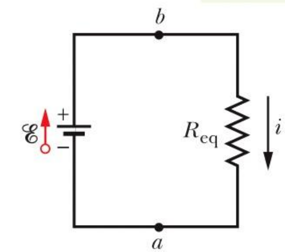
Equating them, we get,

$$R_{eq} = R_1 + R_2 + R_3. \quad \Rightarrow \quad R_{eq} = \sum_{j=1}^{n} R_j \quad (n \text{ resistances in series}).$$



(a)

Series resistors and their equivalent have the same current ("ser-i").

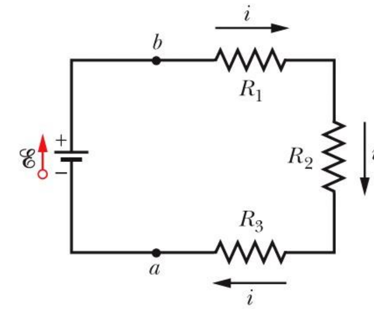


(b)

Resistance in Series

When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same *total* potential difference V as the actual resistances.



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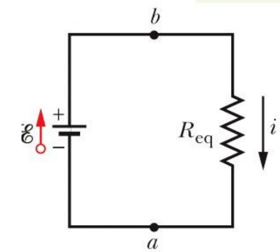


(b)

Checkpoint 2

In Fig. *a*, if $R_1 > R_2 > R_3$, rank the three resistances according to (a) the current through them and (b) the potential difference across them, greatest first.

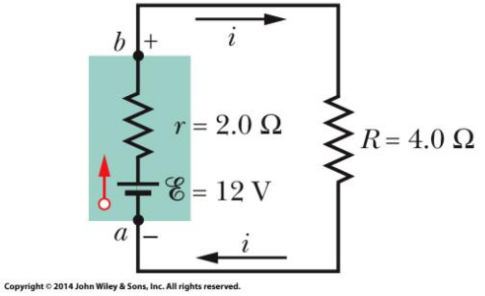
Answer: (a) current is same for all resistors in series.

(b) V_1, V_2, and V_3

Potential Difference

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

The internal resistance reduces the potential difference between the terminals.



Potential Difference across a real battery: In the Figure, points a and b are located at the terminals of the battery. Thus, the potential difference V_b - V_a is the terminal-to-terminal potential difference V across the batter and is given by:

$$V = \mathscr{E} - ir.$$

Grounding a Circuit: Grounding a circuit usually means connecting one point in the circuit to a conducting path to Earth's surface (actually to the electrically conducting moist dirt and rock below ground)

Power of *emf* Device: The rate P_{emf} at which the *emf* device transfers energy both to the charge carriers and to internal thermal energy is

$$P_{\text{emf}} = i\mathscr{E} \quad \text{(power of emf device).}$$

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27-2 Multiloop Circuits

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

The current into the junction must equal the current out (charge is conserved).

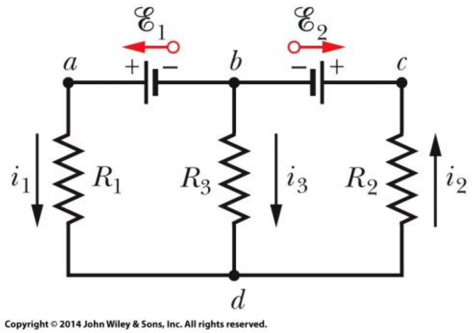


Figure shows a circuit containing more than one loop. If we traverse the left-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$\mathscr{E}_1 - i_1R_1 + i_3R_3 = 0.$$

If we traverse the right-hand loop in a counterclockwise direction from point b, the loop rule gives us

$$-i_3R_3 - i_2R_2 - \mathscr{E}_2 = 0.$$

If we had applied the loop rule to the big loop, we would have obtained (moving counterclockwise from b) the equation

$$\mathscr{E}_1 - i_1R_1 - i_2R_2 - \mathscr{E}_2 = 0.$$

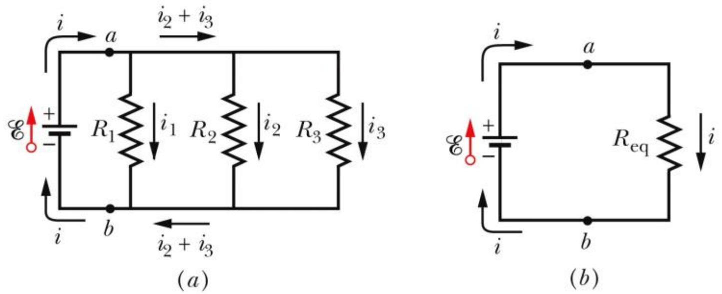
which is the sum of two small loops equations.

27-2 Multi-Loop Circuits

Resistances in Parallel

Figure (a) shows three resistances connected in parallel to an ideal battery of *emf* $\mathscr{E}$. The applied potential difference V is maintained by the battery. Fig. b, the three parallel resistances have been replaced with an equivalent resistance R_{eq}.

Parallel resistors and their equivalent have the same potential difference ("par-V").



To derive an expression for R_{eq} in Fig. (b), we first write the current in each actual resistance in Fig. (a) as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

where V is the potential difference between a and b. If we apply the junction rule at point a in Fig. (a) and then substitute these values, we find

$$i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

If we replaced the parallel combination with the equivalent resistance R_{eq} (Fig. b), we would have $i = \frac{V}{R_{eq}}.$ and thus substituting the value of *i* from above equation we get,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

27-2 Multi-Loop Circuits

Resistance and capacitors

Table 27-1 **Series and Parallel Resistors and Capacitors**

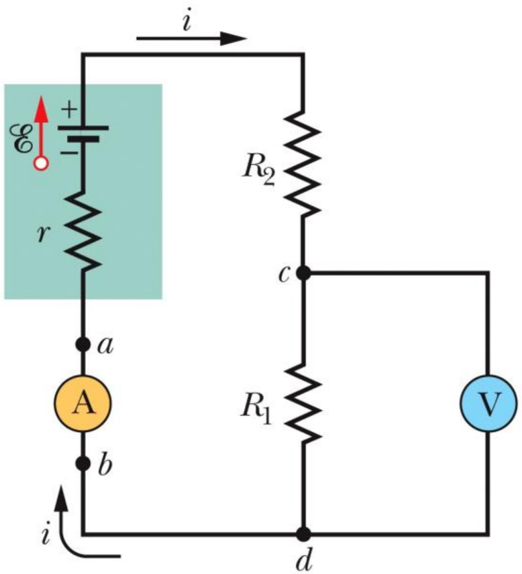
Series	Parallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$ Eq. 27-7	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$ Eq. 27-24	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ Eq. 25-20	$C_{eq} = \sum_{j=1}^{n} C_j$ Eq. 25-19
Same current through all resistors	Same potential difference across all resistors	Same charge on all capacitors	Same potential difference across all capacitors

Checkpoint 4

A battery, with potential V across it, is connected to a combination of two identical resistors and then has current i through it. What are the potential difference across and the current through either resistor if the resistors are (a) in series and (b) in parallel?

Answer: (a) Potential difference across each resistor: *V/2*
Current through each resistor: *i*
(b) Potential difference across each resistor: *V*
Current through each resistor: *i/2*

27-3 The Ammeter and The Voltmeter



An instrument used to measure currents is called an **ammeter**. To measure the current in a wire, you usually have to break or cut the wire and insert the ammeter so that the current to be measured passes through the meter. In the figure, ammeter A is set up to measure current *i*. It is essential that the resistance R_A of the ammeter be very much smaller than other resistances in the circuit. Otherwise, the very presence of the meter will change the current to be measured.

A meter used to measure potential differences is called a **voltmeter**. To find the potential difference between any two points in the circuit, the voltmeter terminals are connected between those points without breaking or cutting the wire. In the Figure, voltmeter V is set up to measure the voltage across R_1. It is essential that the resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected. This is to insure that only a negligible current passes through the voltmeter, otherwise, the meter alters the potential difference that is to be measured.

27-4 *RC* Circuits

Charging a capacitor: The capacitor of capacitance *C* in the figure is initially uncharged. To charge it, we close switch S on point a. This completes an *RC* series circuit consisting of the capacitor, an ideal battery of *emf* $\mathscr{E}$, and a resistance *R*.

The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

in which $C\mathscr{E} = q_0$ is the equilibrium (final) charge and $RC = \tau$ is the capacitive time constant of the circuit. During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC} \quad \text{(charging a capacitor).}$$

And the voltage is:

$$V_C = \frac{q}{C} = \mathscr{E}(1 - e^{-t/RC}) \quad \text{(charging a capacitor).}$$

The product *RC* is called the **capacitive time constant** of the circuit and is represented with the symbol τ.

$$\tau = RC \quad \text{(time constant).}$$

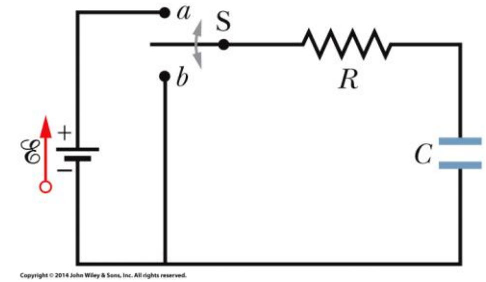
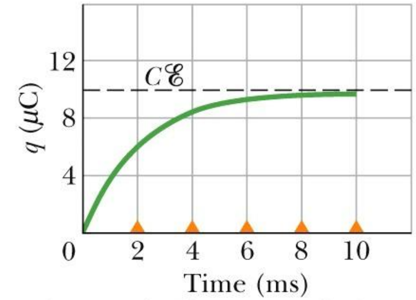


Figure: RC circuit



The plot shows the buildup of charge on the capacitor of the above figure.

27-4 *RC* Circuits

Discharging a capacitor: Assume now that the capacitor of the figure is fully charged to a potential V_0 equal to the emf $\mathscr{E}$ of the battery. At a new time $t=0$, switch S is thrown from a to b so that the capacitor can discharge through resistance R.

When a capacitor discharges through a resistance R, the charge on the capacitor decays according to

$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor),}$$

where q_0 ($=CV_0$) is the initial charge on the capacitor.

During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC} \quad \text{(discharging a capacitor).}$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

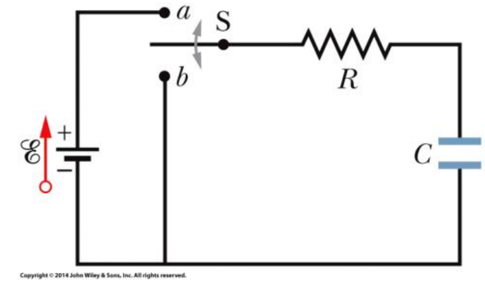
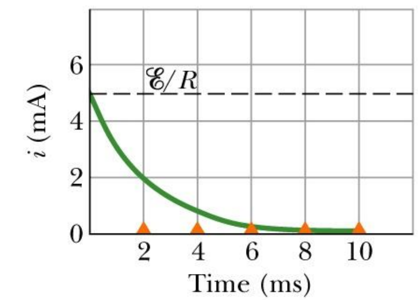


Figure: RC circuit



A plot shows the decline of the charging current in the circuit of the above figure.

26 Summary

Emf

- The **emf** (work per unit charge) of the device is

$$\mathscr{E} = \frac{dW}{dq} \quad \text{(definition of } \mathscr{E}\text{)}.$$ **Eq. 27-1**

Single-Loop Circuits

- Current in a single-loop circuit:

$$i = \frac{\mathscr{E}}{R + r},$$ **Eq. 27-4**

Power

- The rate P of energy transfer to the charge carriers is

$$P = iV,$$ **Eq. 27-14**

- The rate P_r at which energy is dissipated as thermal energy in the battery is $$P_r = i^2 r.$$ **Eq. 27-16**

- The rate P_{emf} at which the chemical energy in the battery changes is

$$P_{\text{emf}} = i\mathscr{E}.$$ **Eq. 27-17**

Series Resistance

- When resistances are in series

$$R_{\text{eq}} = \sum_{j=1}^{n} R_j$$ **Eq. 27-7**

Parallel Resistance

- When resistances are in parallel

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^{n} \frac{1}{R_j}$$ **Eq. 27-24**

RC Circuits

- The charge on the capacitor increases according to

$$q = C\mathscr{E}(1 - e^{-t/RC})$$ **Eq. 27-33**

- During the charging, the current is

$$i = \frac{dq}{dt} = \left(\frac{\mathscr{E}}{R}\right)e^{-t/RC}$$ **Eq. 27-34**

- During the discharging, the current is

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$ **Eq. 27-40**