Chapter 32

Maxwell Equations; Magnetism of Matter

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32-1 Gauss' Law for Magnetic Fields

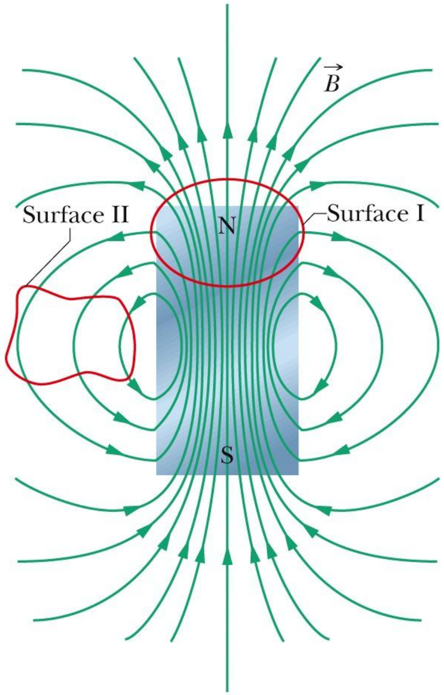
Learning Objectives

32.01 Identify that the simplest magnetic structure is a magnetic dipole.

32.02 Calculate the magnetic flux ϕ through a surface by integrating the dot product of the magnetic field vector ***B*** and the area vector *d**A*** (for patch elements) over the surface.

32.03 Identify that the net magnetic flux through a Gaussian surface (which is a closed surface) is zero.

32-1 Gauss' Law for Magnetic Fields



The field lines for the magnetic field ***B*** of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

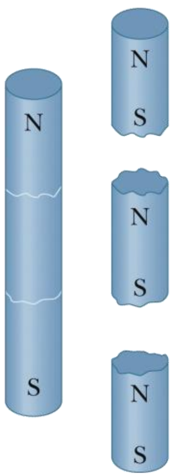
Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux Φ_B through any closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Contrast this with Gauss' law for electric fields,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

Gauss' law for magnetic fields says that there can be no net magnetic flux through the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface.



If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

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32-2 Induced Magnetic Fields

Learning Objectives

32.04 Identify that a changing electric flux induces a magnetic field.

32.05 Apply Maxwell's law of induction to relate the magnetic field induced around a closed loop to the rate of change of electric flux encircled by the loop.

32.06 Draw the field lines for an induced magnetic field inside a capacitor with parallel circular plates that are being charged, indicating the orientations of the vectors for the electric field and the magnetic field.

32.07 For the general situation in which magnetic fields can be induced, apply the Ampere–Maxwell (combined) law.

32-2 Induced Magnetic Fields

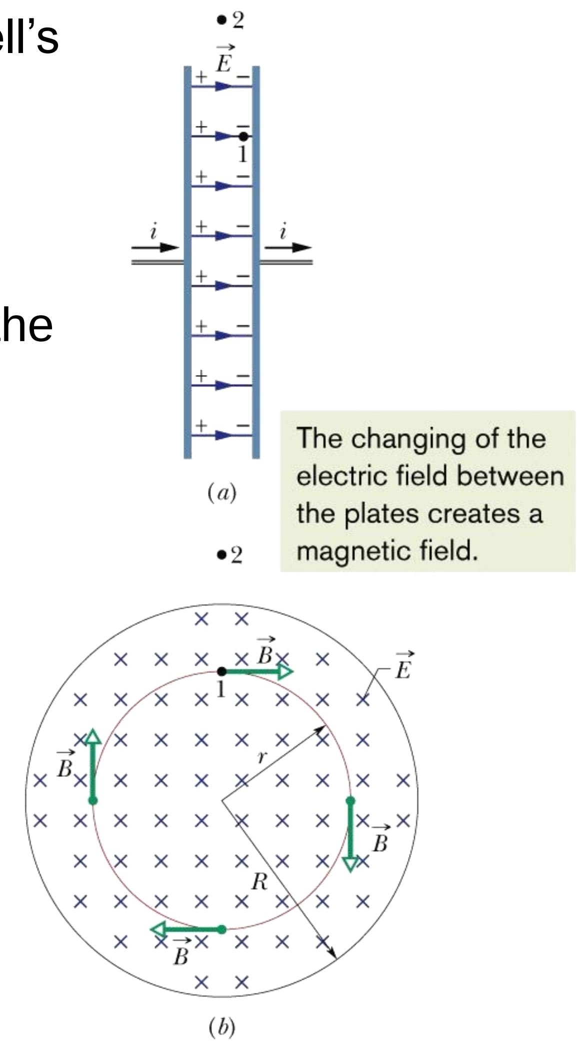
A changing electric flux induces a magnetic field ***B***. Maxwell's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Relates the magnetic field induced along a closed loop to the changing electric flux ϕ_E through the loop.

Charging a Capacitor.

As an example of this sort of induction, we consider the charging of a parallel-plate capacitor with circular plates. The charge on our capacitor is being increased at a steady rate by a constant current *i* in the connecting wires. Then the electric field magnitude between the plates must also be increasing at a steady rate.



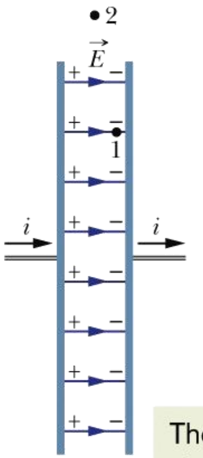
32-2 Induced Magnetic Fields

A changing electric flux induces a magnetic field ***B***. Maxwell's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

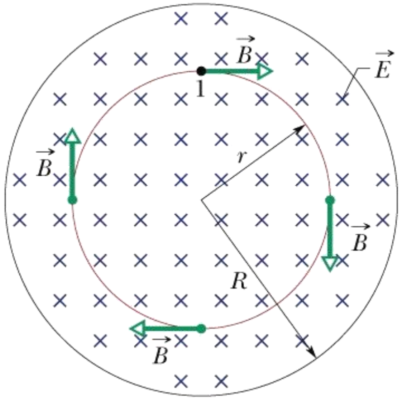
Relates the magnetic field induced along a closed loop to the changing electric flux ϕ_E through the loop.

Charging a Capacitor (continued)
Figure (b) is a view of the right-hand plate of Fig. (a) from between the plates. The electric field is directed into the page. Let us consider a circular loop through point 1 in Figs.(a) and (b), a loop that is concentric with the capacitor plates and has a radius smaller than that of the plates. Because the electric field through the loop is changing, the electric flux through the loop must also be changing. According to the above equation, *this changing electric flux induces a magnetic field around the loop.*



(a)

The changing of the electric field between the plates creates a magnetic field.



(b)

32-2 Induced Magnetic Fields

Ampere-Maxwell Law

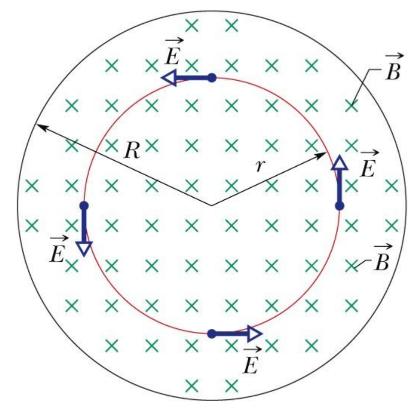
Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

gives the magnetic field generated by a current i_{enc} encircled by a closed loop.

Thus, the two equations (the other being Maxwell's Law) that specify the magnetic field ***B*** produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$



The induced $\vec{E}$ direction here is opposite the induced $\vec{B}$ direction in the preceding figure.

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of Eq. is zero, and so the Eq. reduces to Ampere's law. When there is a change in electric flux but no current (such as inside or outside the gap of a charging capacitor), the second term on the right side of Eq. is zero, and so Eq. reduces to Maxwell's law of induction.

32-3 Displacement Current

Learning Objectives

32.08 Identify that in the Ampere–Maxwell law, the contribution to the induced magnetic field by the changing electric flux can be attributed to a fictitious current ("displacement current") to simplify the expression.

32.09 Identify that in a capacitor that is being charged or discharged, a displacement current is said to be spread uniformly over the plate area, from one plate to the other.

32.10 Apply the relationship between the rate of change of an electric flux and the associated displacement current.

32.11 For a charging or discharging capacitor, relate the amount of displacement current to the amount of actual current and identify that the displacement current exists only when the electric field within the capacitor is changing.

32.12 Mimic the equations for the magnetic field inside and outside a wire with real current to write (and apply) the equations for the magnetic field inside and outside a region of displacement current.

Learning Objectives (Contd.)

32.13 Apply the Ampere–Maxwell law to calculate the magnetic field of a real current and a displacement current.

32.14 For a charging or discharging capacitor with parallel circular plates, draw the magnetic field lines due to the displacement current.

32.15 List Maxwell's equations and the purpose of each.

32-3 Displacement Current

If you compare the two terms on the right side of Eq. (Ampere-Maxwell Law), you will see that the product $\varepsilon_0(d\phi_E/dt)$ must have the dimension of a current. In fact, that product has been treated as being a fictitious current called the **displacement current** i_d:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Ampere-Maxwell Law then becomes,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$

where $i_{d,enc}$ is the displacement current encircled by the integration loop.

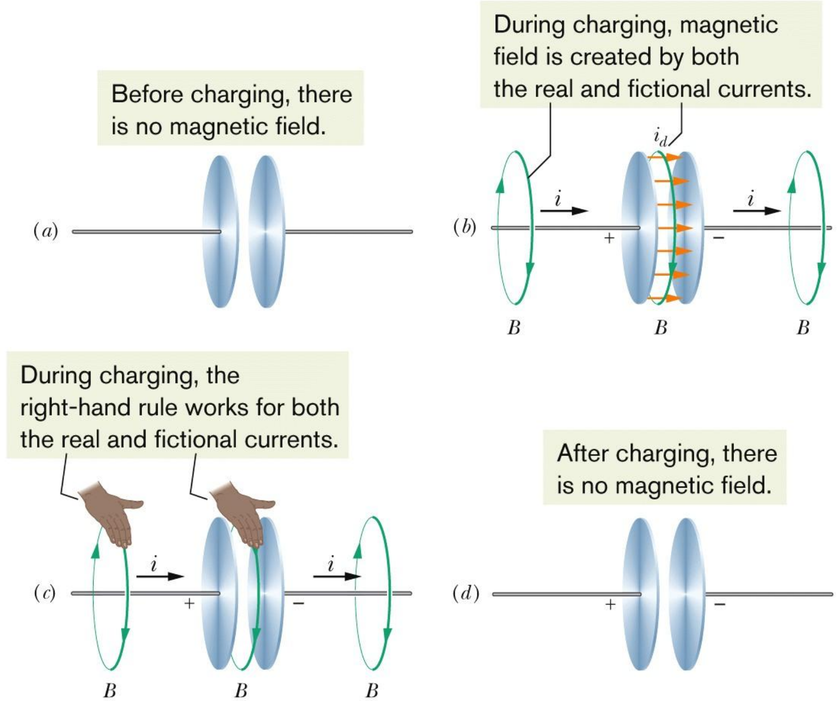
32-3 Displacement Current

Finding the Induced Magnetic Field:
In Chapter 29 we found the direction of the magnetic field produced by a real current *i* by using the right-hand rule. We can apply the same rule to find the direction of an induced magnetic field produced by a fictitious displacement current i_d, as is shown in the center of Fig. (c) for a capacitor.
Then, as done previously, the magnitude of the magnetic field at a point inside the capacitor at radius *r* from the center is

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r$$

the magnitude of the magnetic field at a point outside the capacitor at radius *r* is

$$B = \frac{\mu_0 i_d}{2\pi r}$$



(a) Before and (d) after the plates are charged, there is no magnetic field. (b) During the charging, magnetic field is created by both the real current and the (fictional) (c) displacement current. (c) The same right-hand rule works for both currents to give the direction of the magnetic field.

32-3 Displacement Current

The four fundamental equations of electromagnetism, called Maxwell's equations and are displayed in Table 32-1.

Table 32-1 Maxwell's Equations[a]

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$	Relates induced magnetic field to changing electric flux and to current

[a]Written on the assumption that no dielectric or magnetic materials are present.

These four equations explain a diverse range of phenomena, from why a compass needle points north to why a car starts when you turn the ignition key. They are the basis for the functioning of such electromagnetic devices as electric motors, television transmitters and receivers, telephones, scanners, radar, and microwave ovens.

32-4 Magnets

Learning Objectives

32.16 Identify lodestones.

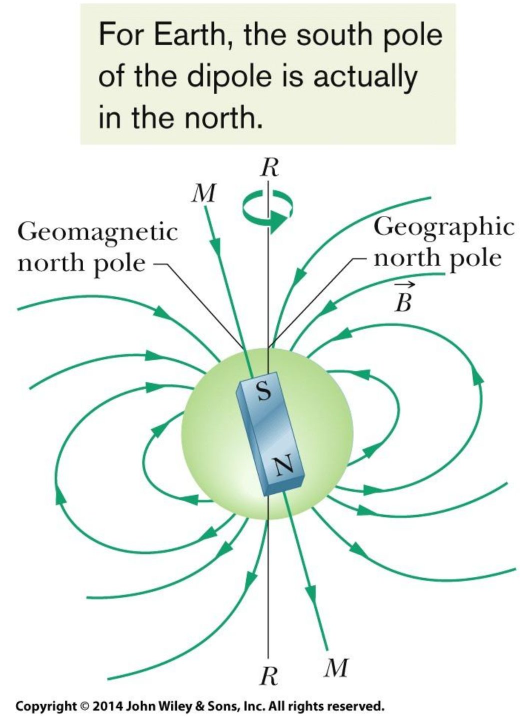
32.17 In Earth's magnetic field, identify that the field is approximately that of a dipole and also identify in which hemisphere the north geomagnetic pole is located.

32.18 Identify field declination and field inclination.

The Magnetism of Earth

Earth is a huge magnet; for points near Earth's surface, its magnetic field can be approximated as the field of a huge bar magnet — a magnetic dipole — that straddles the center of the planet. Figure shown here is an idealized symmetric depiction of the dipole field, without the distortion caused by passing charged particles from the Sun.

The direction of the magnetic field at any location on Earth's surface is commonly specified in terms of two angles. The **field declination** is the angle (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field. The **field inclination** is the angle (up or down) between a horizontal plane and the field's direction.



Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

32-5 Magnetism and Electrons

Learning Objectives

32.19 Identify that a spin angular momentum $\boldsymbol{S}$ (usually simply called spin) and a spin magnetic dipole moment $\boldsymbol{\mu}_s$ are intrinsic properties of electrons (and also protons and neutrons).

32.20 Apply the relationship between the spin vector $\boldsymbol{S}$ and the spin magnetic dipole moment vector $\boldsymbol{\mu}_s$.

32.21 Identify that $\boldsymbol{S}$ and $\boldsymbol{\mu}_s$ cannot be observed (measured); only their components on an axis of measurement (usually called the z axis) can be observed.

32.22 Identify that the observed components $\boldsymbol{S}_z$ and $\boldsymbol{\mu}_{s,z}$ are quantized and explain what that means.

32.23 Apply the relationship between the component $\boldsymbol{S}_z$ and the spin magnetic quantum number m_s, specifying the allowed values of m_s.

32.24 Distinguish spin up from spin down for the spin orientation of an electron.

32.25 Determine the z components $\boldsymbol{\mu}_{s,z}$ of the spin magnetic dipole moment, both as a value and in terms of the Bohr magneton $\boldsymbol{\mu}_B$.

Learning Objectives (Contd.)

32.26 If an electron is in an external magnetic field, determine the orientation energy U of its spin magnetic dipole moment $\boldsymbol{\mu}_s$.

32.27 Identify that an electron in an atom has an orbital angular momentum $\mathbf{L}$ and an orbital magnetic dipole moment $\boldsymbol{\mu}_{\text{orb}}$.

32.28 Apply the relationship between the orbital angular momentum $\mathbf{L}$ and the orbital magnetic dipole moment $\boldsymbol{\mu}_{\text{orb}}$.

32.29 Identity that $\mathbf{L}$ and $\boldsymbol{\mu}_{\text{orb}}$ cannot be observed but their components $L_{\text{orb,z}}$ and $\mu_{\text{orb,z}}$ on a z (measurement) axis can.

32.30 Apply the relationship between the component $L_{\text{orb,z}}$ of the orbital angular momentum and the orbital magnetic quantum number m_l, specifying the allowed values of m_l.

32.31 Determine the z components $\mu_{\text{orb,z}}$ of the orbital magnetic dipole moment, both as a value and in terms of the Bohr magneton μ_B.

Learning Objectives (Contd.)

32.32 If an atom is in an external magnetic field, determine the orientation energy *U* of the orbital magnetic dipole moment $\boldsymbol{\mu}_{orb}$.

32.33 Calculate the magnitude of the magnetic moment of a charged particle moving in a circle or a ring of uniform charge rotating like a merry-go-round at a constant angular speed around a central axis.

32.34 Explain the classical loop model for an orbiting electron and the forces on such a loop in a non-uniform magnetic field.

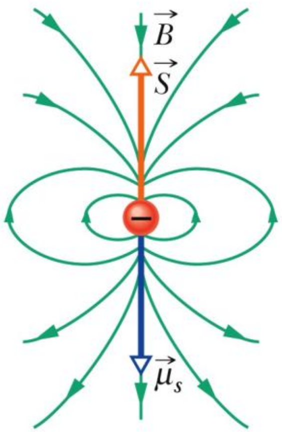
32.35 Distinguish among diamagnetism, paramagnetism, and ferromagnetism.

32-5 Magnetism and Electrons

Spin Magnetic Dipole Moment. An electron has an intrinsic angular momentum called its spin angular momentum (or just spin) ***S***; associated with this spin is an intrinsic spin magnetic dipole moment $\boldsymbol{\mu}_s$. (By intrinsic, we mean that ***S*** and $\boldsymbol{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.) Vectors ***S*** and $\boldsymbol{\mu}_s$ are related by

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

For an electron, the spin is opposite the magnetic dipole moment.



in which *e* is the elementary charge (1.60×10^{-19} C) and *m* is the mass of an electron (9.11×10^{-31} kg). The minus sign means that $\boldsymbol{\mu}_s$ and ***S*** are oppositely directed.

For a measurement along a z axis, the component $\boldsymbol{S}_z$ can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \quad \text{for } m_s = \pm½$$

Similarly, $$\mu_{s,z} = \pm\frac{eh}{4\pi m} = \pm\mu_B,$$ where μ_B is the Bohr magneton:

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T}.$$

32-5 Magnetism and Electrons

Spin Magnetic Dipole Moment. An electron has an intrinsic angular momentum called its spin angular momentum (or just spin) ***S***; associated with this spin is an intrinsic spin magnetic dipole moment $\boldsymbol{\mu}_s$. (By intrinsic, we mean that ***S*** and $\boldsymbol{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.) Vectors ***S*** and $\boldsymbol{\mu}_s$ are related by

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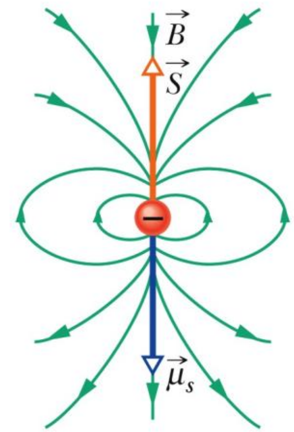

Energy. When an electron is placed in an external magnetic field $\boldsymbol{B}_{ext}$, an energy *U* can be associated with the orientation of the electron's spin magnetic dipole moment $\boldsymbol{\mu}_s$ just as an energy can be associated with the orientation of the magnetic dipole moment $\boldsymbol{\mu}$ of a current loop placed in ***B***. The orientation energy for the electron is

$$U = -\vec{\mu}_s \cdot \vec{B}_{\text{ext}} = -\mu_{s,z} B_{\text{ext}},$$

where the z axis is taken to be in the direction of $\boldsymbol{B}_{ext}$.

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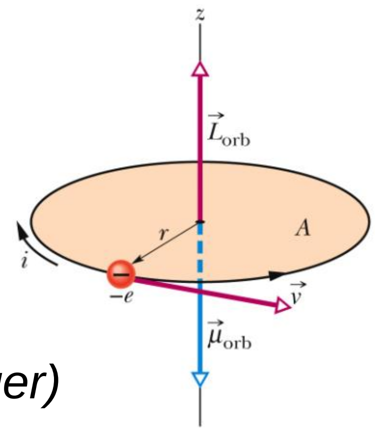
32-5 Magnetism and Electrons

Orbital Magnetic Dipole Moment. When it is in an atom, an electron has an additional angular momentum called its orbital angular momentum $\boldsymbol{L}_{orb}$. Associated with $\boldsymbol{L}_{orb}$ is an orbital magnetic dipole moment $\boldsymbol{\mu}_{orb}$ the two are related by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$



The minus sign means that $\boldsymbol{\mu}_{orb}$ and $\boldsymbol{L}_{orb}$ have opposite directions. Orbital angular momentum is quantized and can have only measured values given by

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \qquad \text{for } m_l = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit integer})$$

An electron moving at constant speed *v* in a circular path of radius *r* that encloses an area *A*.

The associated magnetic dipole moment is given by

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_B.$$

The energy U associated with the orientation of the orbital magnetic dipole moment in an external magnetic field B_{ext} is

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}}.$$

32-6 Diamagnetism

Learning Objectives

32.36 For a diamagnetic sample placed in an external magnetic field, identify that the field produces a magnetic dipole moment in the sample, and identify the relative orientations of that moment and the field.

32.37 For a diamagnetic sample in a non-uniform magnetic field, describe the force on the sample and the resulting motion.

32-6 Diamagnetism

A diamagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment directed opposite $\vec{B}_{ext}$. If the field is nonuniform, the diamagnetic material is repelled *from* a region of greater magnetic field *toward* a region of lesser field.

Levitating Frog: The frog in the figure is diamagnetic (as is any other animal). When the frog was placed in the diverging magnetic field near the top end of a vertical current- carrying solenoid, every atom in the frog was repelled upward, away from the region of stronger magnetic field at that end of the solenoid. The frog moved upward into weaker and weaker magnetic field until the upward magnetic force balanced the gravitational force on it, and there it hung in midair. The frog is not in discomfort because every atom is subject to the same forces and thus there is no force variation within the frog.

Courtesy A.K. Geim, University of Manchester, UK

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32-7 Paramagnetism

Learning Objectives

32.38 For a paramagnetic sample placed in an external magnetic field, identify the relative orientations of the field and the sample's magnetic dipole moment.

32.39 For a paramagnetic sample in a non-uniform magnetic field, describe the force on the sample and the resulting motion.

32.40 Apply the relationship between a sample's magnetization *M*, its measured magnetic moment, and its volume.

32.41 Apply Curie's law to relate a sample's magnetization *M* to its temperature *T*, its Curie constant *C*, and the magnitude B of the external field.

32.42 Given a magnetization curve for a paramagnetic sample, relate the extent of the magnetization for a given magnetic field and temperature.

32.43 For a paramagnetic sample at a given temperature and in a given magnetic field, compare the energy associated with the dipole orientations and the thermal motion.

32-7 Paramagnetism

A paramagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the paramagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

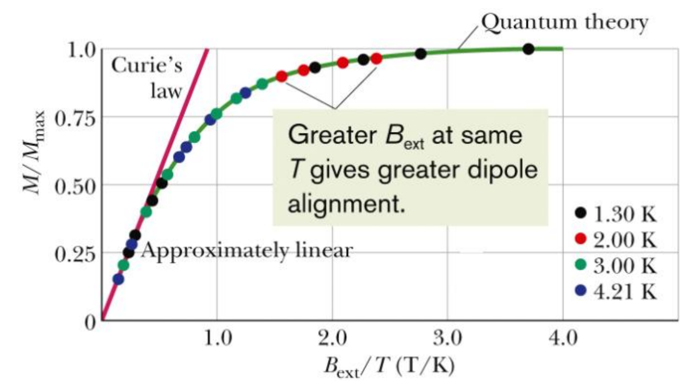
Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented, with no net moment, unless the material is in an external magnetic field $\boldsymbol{B}_{ext}$, where the dipoles tend to align with that field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}.$$

Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{max} = N\mu/V$.

At low values of the ratio $\boldsymbol{B}_{ext}/T$,

$$M = C\frac{B_{ext}}{T}$$



where T is the temperature (in kelvins) and C is a material's Curie constant.

In a nonuniform external field, a paramagnetic material is attracted to the region of greater magnetic field.

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32-8 Ferromagnetism

Learning Objectives

32.44 Identify that ferromagnetism is due to a quantum mechanical interaction called exchange coupling.

32.45 Explain why ferromagnetism disappears when the temperature exceeds the material's Curie temperature.

32.46 Apply the relationship between the magnetization of a ferromagnetic sample and the magnetic moment of its atoms.

32.47 For a ferromagnetic sample at a given temperature and in a given magnetic field, compare the energy associated with the dipole orientations and the thermal motion.

32.48 Describe and sketch a Rowland ring.

32.49 Identify magnetic domains.

32.50 For a ferromagnetic sample placed in an external magnetic field, identify the relative orientations of the field and the magnetic dipole moment.

32-8 Ferromagnetism

Learning Objectives (Contd.)

32.51 Identify the motion of a ferromagnetic sample in a non-uniform field.

32.52 For a ferromagnetic object placed in a uniform magnetic field, calculate the torque and orientation energy.

32.53 Explain hysteresis and a hysteresis loop.

32.54 Identify the origin of lodestones.

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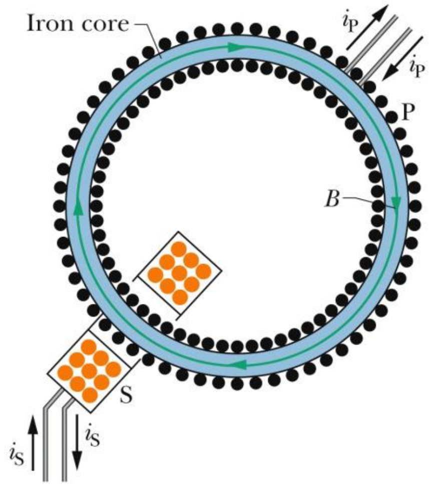
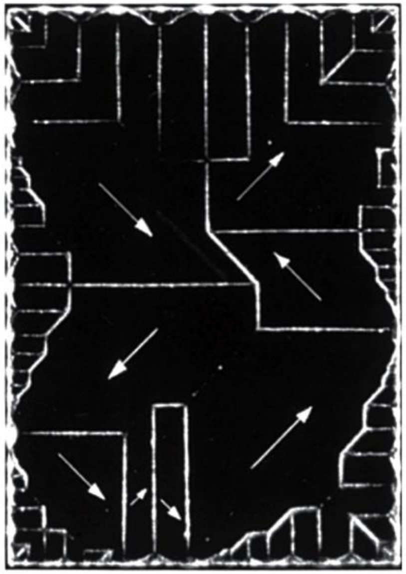
32-8 Ferromagnetism



A Rowland ring. A primary coil P has a core made of the ferromagnetic material to be studied (here iron). The core is magnetized by a current i_P sent through coil P. (The turns of the coil are represented by dots.) The extent to which the core is magnetized determines the total magnetic field ***B*** within coil P. Field ***B*** can be measured by means of a secondary coil S.

A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

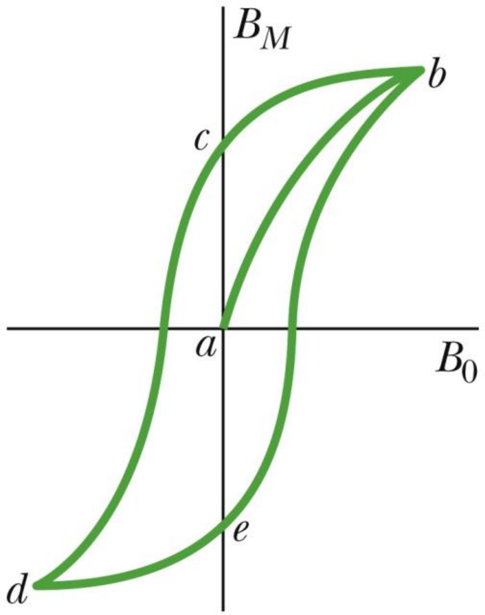
The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions know as **magnetic domains**.



Courtesy Ralph W. DeBlois

A photograph of domain patterns within a single crystal of nickel; white lines reveal the boundaries of the domains. The white arrows superimposed on the photograph show the orientations of the magnetic dipoles within the domains and thus the orientations of the net magnetic dipoles of the domains. The crystal as a whole is unmagnetized if the net magnetic field (the vector sum over all the domains) is zero.

32-8 Ferromagnetism



A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

The lack of retraceability shown in the Figure is called **hysteresis**, and the curve *bcdeb* is called a **hysteresis loop**. Note that at points *c* and *e* the iron core is magnetized, even though there is no current in the toroid windings; this is the familiar phenomenon of permanent magnetism.

Hysteresis can be understood through the concept of magnetic domains. Evidently the motions of the domain boundaries and the reorientations of the domain directions are not totally reversible. When the applied magnetic field B0 is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some "memory" of their alignment after the initial increase. This memory of magnetic materials is essential for the magnetic storage of information.

32 Summary

Gauss' Law for Magnetic Fields

- Gauss' law for magnetic fields,

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0, \qquad \textbf{Eq. 32-1}$$

Maxwell's Extension of Ampere's Law

- A changing electric field induces a magnetic field given by,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \textbf{Eq. 32-3}$$

- Maxwell's law and Ampere's law can be written as the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \qquad \textbf{Eq. 32-5}$$

Displacement Current

- We define the fictitious displacement current due to a changing electric field as

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}. \qquad \textbf{Eq. 32-10}$$

- Equation 32-5 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc} \qquad \textbf{Eq. 32-11}$$

Maxwell's Equations

- Four equations are as follows:

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

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32 Summary

Spin Magnetic Dipole Moment

- Spin angular momentum of electron is associated with spin magnetic dipole momentum through,

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}.$$ **Eq. 32-22**

- For a measurement along a *z* axis, the component S_z can have only the values given by

$$S_z = m_s\frac{h}{2\pi}, \quad \text{for } m_s = \pm\tfrac{1}{2},$$ **Eq. 32-23**

- Similarly,

$$\mu_{s,z} = \pm\frac{eh}{4\pi m} = \pm\mu_B,$$ **Eq. 32-24 &26**

- Where the Bohr magneton is

$$\mu_B = \frac{eh}{4\pi m} = 9.27\times10^{-24}\ \text{J/T}.$$ **Eq. 32-25**

- The energy *U*

$$U = -\vec{\mu}_s\cdot\vec{B}_{ext} = -\mu_{s,z}B_{ext}.$$ **Eq. 32-27**

Orbital Magnetic Dipole Momentum

- Angular momentum of an electron is associated with orbital magnetic dipole momentum as

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}.$$ **Eq. 32-28**

- Orbital angular momentum is quantized,

$$L_{orb,z} = m_\ell\frac{h}{2\pi},$$
$$\text{for } m_\ell = 0, \pm1, \pm2, \ldots, \pm(\text{limit}).$$ **Eq. 32-29**

- The associated magnetic dipole moment is given by

$$\mu_{orb,z} = -m_\ell\frac{eh}{4\pi m} = -m_\ell\mu_B.$$ **Eq. 32-30&31**

- The energy *U*

$$U = -\vec{\mu}_{orb}\cdot\vec{B}_{ext} = -\mu_{orb,z}B_{ext}.$$ **Eq. 32-32**

32 Summary

Diamagnetism

- Diamagnetic materials exhibit magnetism only when placed in an external magnetic field; there they form magnetic dipoles directed opposite the external field. In a nonuniform field, they are repelled from the region of greater magnetic field.

Paramagnetism

- Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented unless the material is in an external magnetic field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}. \qquad \textbf{Eq. 32-28}$$

- Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{max} = N\mu/V$. At low values of the ratio $\boldsymbol{B}_{ext}/T$,

$$M = C\frac{B_{ext}}{T} \qquad \textbf{Eq. 32-39}$$

Ferromagnetism

- The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.