Capacitance

Goal

- To determine the capacitance of a parallel-plate capacitor.

Equipment

Parallel-plate capacitor, ohmmeter, capacitance meter, ruler.

The Big Picture

A **capacitor** is a device that can store energy in the electric field that develops between its oppositely charged conducting **plates**. A plate is a conductor of any size or shape. Two plates form a capacitor.

When a voltage V (from a battery, for example) is applied across a capacitor with capacitance C, positive charge $+Q$ accumulates on one plate while negative charge $-Q$ accumulates on the other plate. These quantities are related by the formula

$$Q = CV$$

so that if the voltage is doubled, then the charge on each plate is also doubled. It takes work (supplied by the battery) to separate the positive and negative charges. That work is converted into the energy in the electric field between the plates and this energy can be recovered later.

Theory

The capacitance of a parallel-plate capacitor can be derived theoretically using Gauss' Law and the relation between electric field and electric potential, all of which you have already studied.

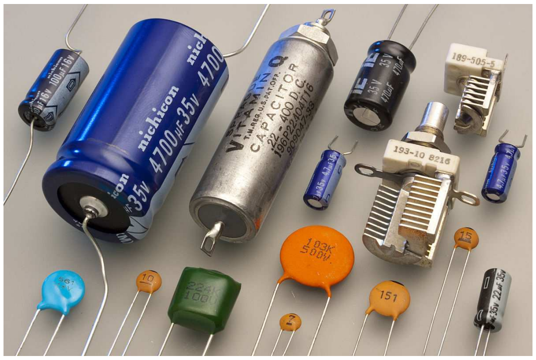

Figure 1: Capacitors are manufactured with many different shapes and capacities. From Ref 3.

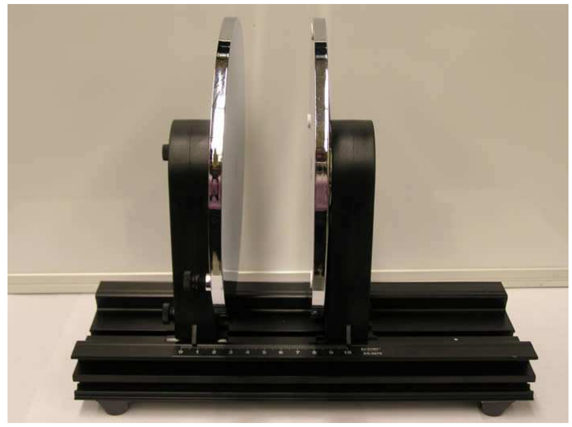

Figure 2: A parallel-plate capacitor in which the plate spacing is variable. From Ref 4.

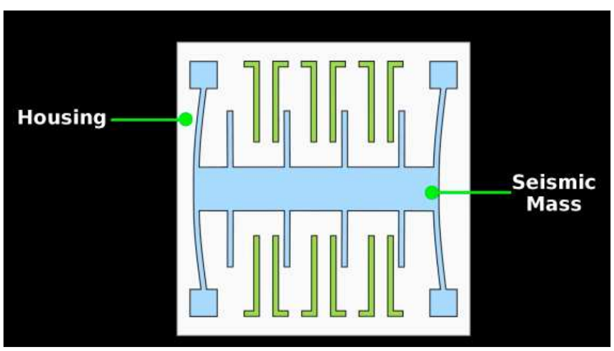

Figure 3: The accelerometer in a mobile phone uses the capacitance measurement to infer a change in position of the seismic mass. The two plates are colored blue and green. From Ref 5.

Figure 4: A defibrillator is a large capacitor that is used to deliver an electric shock to a heart muscle undergoing dysrhythmias. From Ref 6.

We will assume that the separation d between the plates is much smaller than the size of the plates. This will allow us to ignore the **fringing field** that bulges out from the edges of a real capacitor, so the only electric field is between the plates.

Surround one plate (say the positively charged one) with a Gaussian surface that is parallel to the plates between the plates and closes outside the plates. The Gaussian surface is chosen so that $d\vec{A}$ is parallel to the electric field between the plates. Gauss' Law is

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

The left-hand side is just EA, where A is the area of the plate and the right-hand side is $\frac{Q}{\epsilon_0}$. Solving for the electric field, we get

$$E = \frac{Q}{A\epsilon_0}$$

Next, we will integrate the electric field from the negative plate to the positive plate to calculate the voltage difference between the plates.

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

The integral can be done along any path, but a straight line path along one of the electric field lines is the most convenient. The voltage is simply

$$V = Ed$$

Now we are ready to derive the capacitance of the parallel-plate capacitor. Substitute the electric field from Gauss' Law into the voltage equation above and we see

$$V = \frac{Q}{A\epsilon_0} d$$

which we can solve for the charge

$$Q = \left(\frac{\epsilon_0 A}{d} \right) V$$

Compare this formula to $Q = CV$ and the capacitance must be

$$C = \frac{\epsilon_0 A}{d}$$

This formula is valid for a parallel-plate capacitor with vacuum between the plates. If an insulating material (air, glass, plastic, etc.) called a **dielectric** sits between the plates, then the permittivity of free space ϵ_0 is multiplied by the **dielectric constant** κ (Greek letter kappa).

Procedure

1. Slide the bottom plate until the two metal plates completely overlap, then anchor the stage on the guide rail by fastening the two screws. The screws are spring loaded so you will need to push gently while tightening.

2. Connect the digital multi-meter to the two plates and measure the resistance between them. As the plates do not touch each other, the multi-meter reads infinite resistance.

3. Now raise the bottom plate until the multi-meter just starts to read 0 ohm. Record the reading of micrometer head and set that to be $d = 0$ mm.

4. Replace the ohmmeter with the capacitance meter.

5. Measure the plate separation d using the micrometer built into the apparatus while measuring the capacitance C using the meter. Record many pairs of d and C values with error estimates as you move the plates further apart.

6. Measure the length and width of the plate with error estimates using the ruler to determine the plate area A.

7. Set the plate separation d to some small value, say 1 mm.

8. Loosen the two screws that used to anchor the stage. The area A that the two plates overlap can be changed by sliding the bottom plate. Measure C and A pairs as you slide the bottom plate to many different positions.

Analysis

1. For fixed (maximal) plate area A_{max}, plot capacitance C versus $1/d$. Why do we plot $1/d$ instead of d?

2. For each value of plate separation d, compare the experimentally measured capacitance to the theoretical capacitance from the formula derived above.

3. For fixed (small) plate separation d, plot capacitance C versus variable plate area A.

4. For each value of effective plate area A, compare the experimentally measured capacitance to the theoretical capacitance from the formula derived above.

5. Discuss the validity of ignoring the fringing fields, especially for large plate separation d and small effective plate area A.

Literature

1. Halliday, Resnick, & Walker, *Fundamentals of Physics* Vol. 1 (10th ed.), Ch 25: Capacitance, Wiley, 2014.

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