Error Analysis & Significant Figures in the Lab

Anatomy of a Measurement

There are three parts to a measurement:

1. Numeric value (magnitude)
2. Uncertainty
3. Unit

Uncertainties

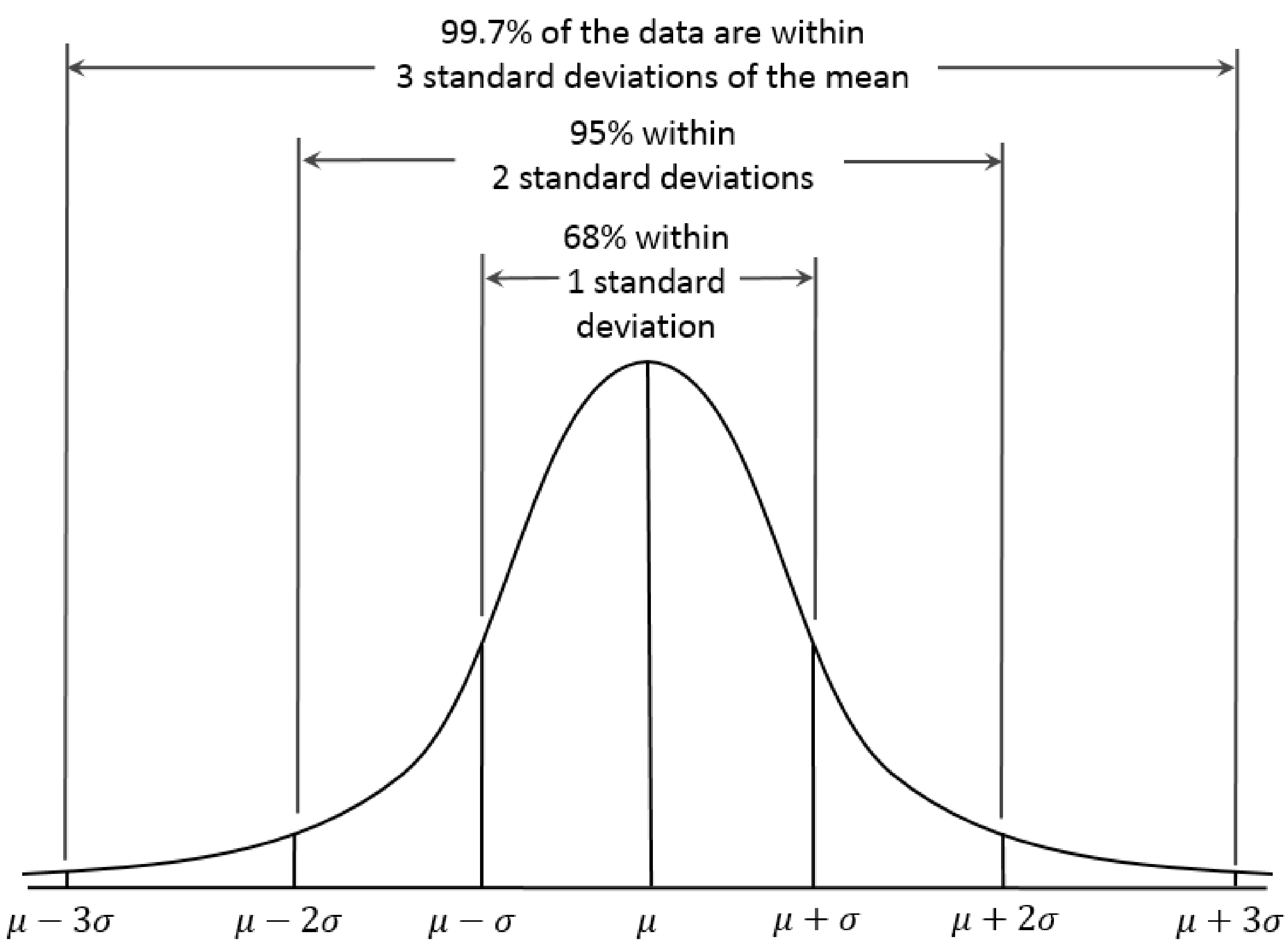
RoT: If its an uncertainty in the lab it has ***exactly one significant figure***, period, end of story.

Device related

- **RoT:** Analog Device: ½ the smallest scale division.
- **RoT:** Digital Device: One of the least displayed digit.

Statistical alternative

Experience shows, if a single measurement is repeated multiple times and the resulting measurements are displayed in a histogram (frequency plot), the resulting plot is a Gauss curve.



The width of the curve at half maximum (the standard deviation) is representative of the ***average uncertainty*** in a ***single*** measurement.

RoT: Because the mean +/- 2 sigma includes 95% of the measurements, a quick and crude estimate of sigma is (max – min) /4

Average value of multiple measurements of a single quantity

Device related

The best value of all the measurements is represented by the average which is uncertain by the EOM (Error of the Mean).

Statistical alternative

The most probable value, the mean value (average) best represents all of the measurements and is uncertain by the SDOM (Standard Deviation of The Mean).

Guass curve and measurements (under construction)

Principal under-girding all error propagation

If you know $x \pm \Delta x$ and $y = f(x)$, what is Δy?

We use the best linear approximation to the function $y = f(x)$ at x to estimate the uncertainty in y. The slope of the tangent line to the function at x is used to approximate the value of Δy. That is, $\Delta y = |(slope)| * \Delta x$. As long as $x - \Delta x < x < x + \Delta x$, then $y - \Delta y < y < y + \Delta y$. We use absolute value because uncertainties are intrinsically positive quantities and the slope can be negative.

Precision

RoT: Measured/Calculated value cannot be more precise than its uncertainty. Q must have the same number of decimals as its uncertainty delta q.

Trick: To ***increase the precision*** in a measured item/quantity, ***include MULTIPLE*** **items*/quantities in a SINGLE*** measurement. Then you can distribute the uncertainty of the single measurement equally among the multiple items included in the single measurement.

Trick: To ***increase the precision*** in a measured item/quantity, ***take multiple SINGLE measurements (n trials) and use the average value***. Then you can use EOM or SDOM which scales as 1/sqrt(n). for example if you have ten trials and EOM has 1 decimal, take an additional 90 trials and EOM will have 2 decimals. To get 3 decimals you need to take an additional 9,900 more trials. Clearly this trick has its limits.