PHYS 1105 SMU Physics Dept.

Gravitation and Dark Matter

Goal: To calculate the amount of Dark Matter in galaxy NGC 134

The (Very) Big Picture

The force of gravity acts upon any object with mass and is caused by the mass of another object. This is summarized by Newton's formula for the magnitude of the force F of attraction of mass m_1 on mass m_2 when they are separated by a distance r between their centers

$$F = G\, m_1\, m_2\, r^{-2}$$

where G = 6.67 x 10^{-11} N m^2 kg^{-2} is a universal constant of nature. 300 years ago, Newton was the first to realize that this formula could explain the motion of celestial objects like planets and moons as well as the trajectories of projectiles on Earth. When m_1 is the mass and r is the radius of the Earth, F is the force that causes free-fall acceleration of a mass m_2 near the surface of the Earth (9.80 m/s^2). If you were to go far from the Earth, however, this force and therefore acceleration would reduce as the distance r between you and the center of the Earth gets larger, according to the formula above. Nowadays, the same formula is used to describe the motion of satellites and spacecraft that visit other planets.

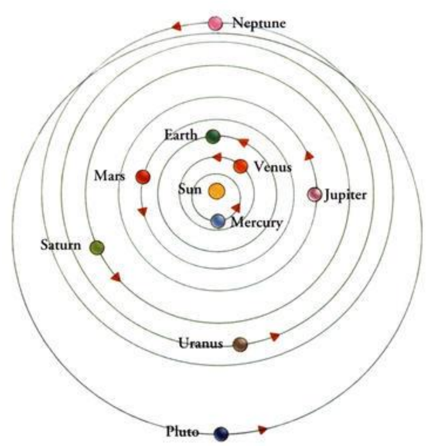


Illustration 1: A schematic diagram of the planets orbiting the Sun in our solar system. Their orbits due to the Sun are actually elliptical in shape, not exactly circular, and receive additional small corrections from the gravitational pull of other planets.

Newton's formula for the force of gravity, and the changes in motion that force causes, has often been used by astronomers to discover things that have mass, but cannot been seen with telescopes. For example, the planet Neptune was first discovered when unexpected changes in the orbit of Uranus led Alexis Bouvard to deduce that its orbit was subject to gravitational perturbation by an unknown planet. Neptune was subsequently observed with a telescope on 23 September 1846, confirming its existence.

Over billions of years, stars have been drawn by the force of gravity into rotating galaxies, like our own Milky Way, which typically consist of a spherical central bulge surrounded by a flat disc or spiral shape. Astronomers can calculate the mass in a galaxy by observing the speed *v* of stars orbiting in circular motion at radius *r* about the center of the galaxy. The centripetal acceleration v^2 / r of any given star in this circular motion is due to the net force of gravity from all the mass of the galaxy within the given star's orbital radius (assuming the mass is distributed in a spherically-symmetric way), in the same way that the force of gravity from the Sun provides the main contribution to the centripetal acceleration keeping the planets in orbit. By measuring how far from and how fast a given star is rotating around the center of a galaxy, you can deduce how strong a gravitational force must be acting on the star from Newton's formula and hence the mass producing that force.

Illustration 2: Galaxy M31, the Andromeda Galaxy, is a rotating spiral galaxy about 2.5 million light years from Earth. It contains about one trillion stars. This recent image was taken by NASA's Galaxy Evolution Explorer [Ref 1]

How do astronomers measure the rotation speed of stars in another galaxy? The galaxy may be many millions of light years away and take hundreds of thousands of years to rotate, too slow for us to see it turning directly. In general the rotating spiral galaxy will be tilted relative to our line of sight, so stars on one side are moving towards us and on the other side of the galaxy are moving away from us. By the Doppler effect – the same phenomenon the police use with radio waves to catch you speeding - the visible light emitted by stars becomes either blue-shifted (i.e. becomes bluer) or red-shifted, depending on whether the stars are moving toward or away from us. There is a precise relationship between speed and this color shift.

Measuring the rotation speeds of stars in galaxies in this way and applying Newton's formula, the results for the mass in galaxies differs wildly from what you would expect just by counting stars. Illustration 3 shows how the planets in our solar system correctly follow a pattern predicted by Newton's law, that the rotation speed diminishes rapidly with increasing orbital radius. However, stars orbiting in spiral galaxies, including our own Milky Way shown in Illustration 4, do not exhibit this rapid fall-off of orbital speed at all. It's as if there is some extra force contributing to their enhanced centripetal acceleration, which cannot be accounted for just from the gravity of other stars.

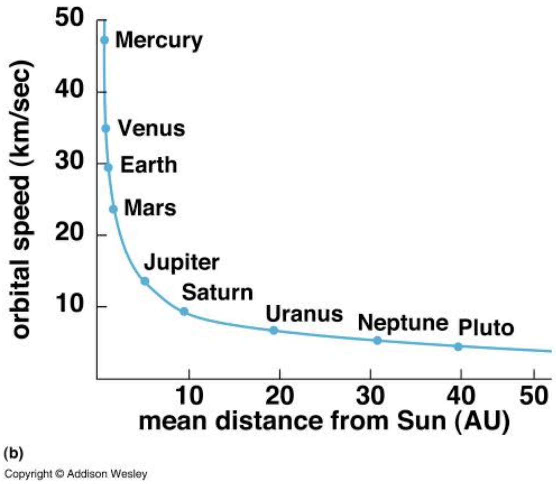


Illustration 3: Rotation curve of planets in the solar system, from Ref 2.

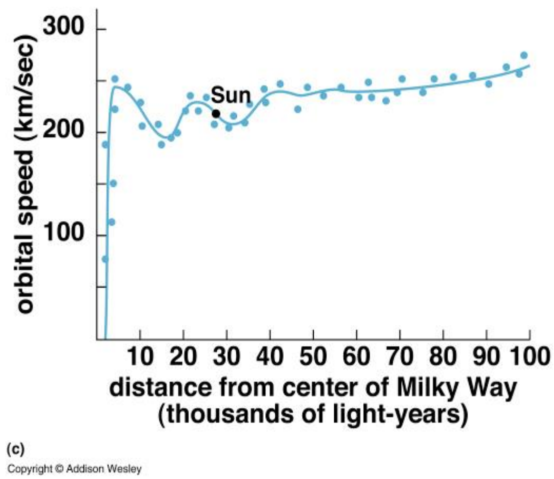


Illustration 4: Rotation curve of stars in the Milky Way galaxy, from Ref 2*.*

In recent years, as one explanation after another has been gradually discounted, the existence of so-called "Dark Matter" has emerged as the most likely explanation for the strange observed rotation speeds of stars in galaxies. Dark Matter has mass, so exerts a gravitational force. But it does not emit or reflect light, so cannot therefore be detected by telescopes. While we are able to deduce the mass of Dark Matter in galaxies – it turns out that most of the galaxy's mass is dark – no-one yet knows what Dark Matter really is. It is not regular matter like atoms or nuclei; it must be a new kind of matter.

References

1. The Galaxy Next Door, https://www.nasa.gov/mission_pages/galex/pia15416.html, accessed June 20 2017

2. The Essential Cosmic Perspective, J. O. Bennet *et al.*, Pearson (2015), quoted in http://ircamera.as.arizona.edu/NatSci102/NatSci102/lectures/darkmatter.htm, accessed June 20 2017.

Procedure

Total Mass

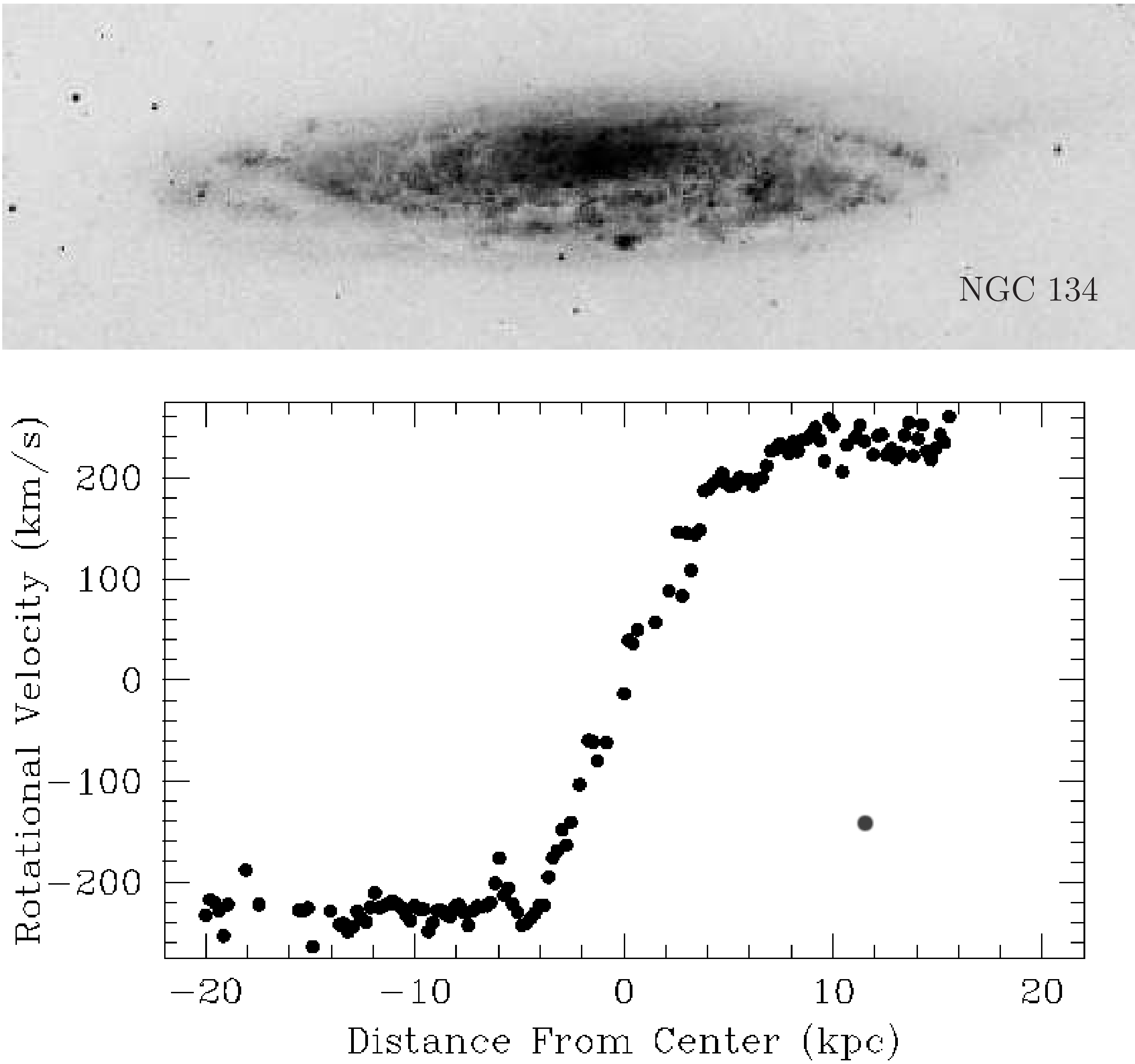


Figure 1: The top panel shows a real telescope image of galaxy NGC 134. The bottom panel shows the galaxy's "rotation curve", which is a plot of the galaxy's rotational velocity - the tangential velocity of circular motion - as a function of the distance from the center of the galaxy. The image and the plot have been scaled so that the positions where the velocities are plotted line up with the positions on the image where the velocities were measured.

The units of distance kpc (kiloparsecs) are used by astronomers to handle the enormous distances they deal with; they are related to the SI units by 1 kpc = 3.086×10^{19} m.

Radius (kpc)	Rotational Velocity (km/s)	Total Gravitational Mass Within R ($M_\odot$)
-15		
$+15$		
-3		
$+3$		

Rotation curves can be used to measure the total gravitational mass in stars, gas, dark matter, and anything that exerts a gravitational force in a galaxy.

- Use the plot of NGC 134's rotation curve to measure the average rotation speed of the galaxy at the four radii listed in the table.
- Assuming the mass *M* of the part of the galaxy enclosed by a circular orbit at radius *R* can be treated as a point mass, find a formula for the mass *M* in terms of *R,* rotational velocity *V*, and the gravitational constant G.
- Hence, use your values for radius and velocity to determine the total mass of the galaxy within each radius in units of solar masses*. Include these values in the table above.

*A solar mass is the mass of our Sun, denoted $M_\odot = 1.989 \times 10^{30}$ kg

Baryonic Mass

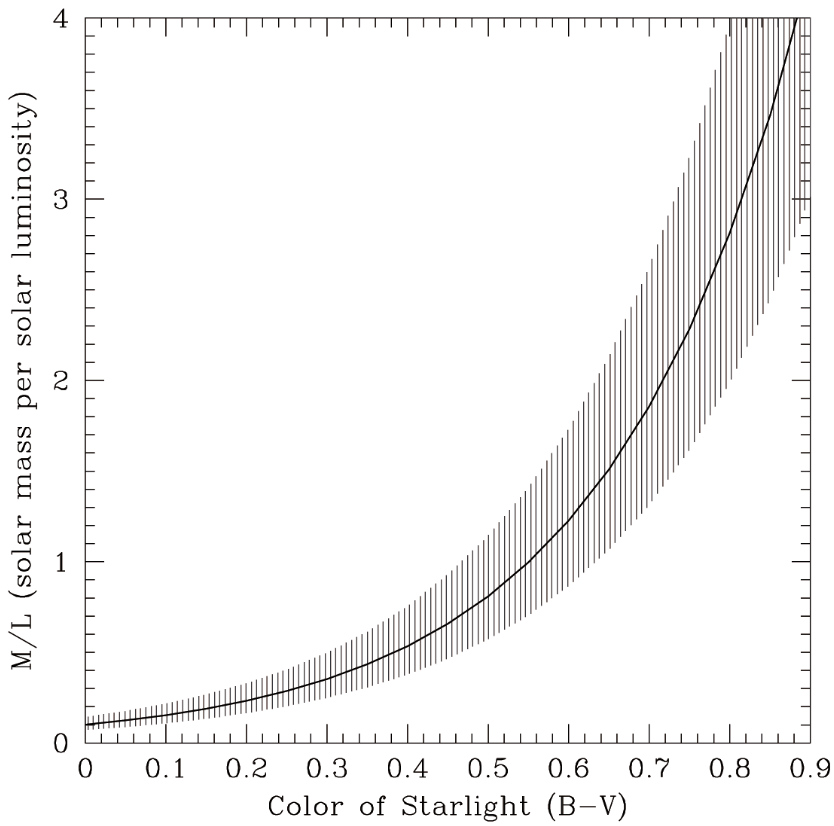
To measure what fraction (if any) of the total mass is dark matter, one needs to subtract off the mass due to normal matter, like stars and gas, that is made of atoms and nuclei.

To measure the mass in stars, one begins by measuring how much light power (luminosity) is emitted by the stars. To convert the total luminosity of all stars to the mass of all stars, astronomers build models that help them determine what mass in stars is needed to produce the luminosity of the galaxy. This relationship is quantified by the stellar "mass-to-light ratio" (*M* / *L*), defined as

$$\text{Mass}_{stars} = (M/L) \times \text{Luminosity}_{stars}.$$

Thus if a galaxy had a mass-to-light ratio of $M/L = 2\ M_\odot /\ L_\odot$, and a luminosity of 10^{10} solar luminosities, then the galaxy probably has about 2×10^{10} solar masses of stars.

The plot below shows a current “best estimate” of how the mass-to-light ratio of a galaxy varies with the galaxy’s color given by the so-called B - V index used by astronomers, along with a shaded region indicating the range of uncertainty. The galaxy's color is governed by the temperature of its stars, with smaller index values meaning bluer (hotter) stars.



Aside from the stars, some of the galaxies' mass will come from gas. Astronomers can measure the power of the light emitted from the gas to get an idea of how much there is. In this lab, we will assume this has already been done.

Radius (kpc)	Stellar Luminosity Within R ($L_\odot$)	Average $B-V$ Color of Stars Within R	Stellar Mass-to-Light Ratio ($M_\odot / L_\odot$)	Stellar Mass Within R ($M_\odot$)	Gas Mass Within R ($M_\odot$)	Normal Matter (Stars+Gas) Within R ($M_\odot$)
$-/+$ 15	$2.8\times10^{10}\,L_\odot$	0.35			$1.1\times10^{10}\,M_\odot$	
$-/+$ 3	$0.5\times10^{10}\,L_\odot$	0.55			$0.1\times10^{10}\,M_\odot$	

- At each of the radii in the table above, you are provided with the total luminosity and average B − V color of the stars inside that radius. Using the plot showing how the stellar mass-to-light ratio changes with color, estimate the appropriate stellar mass-to-light ratio for the inner parts of the galaxy and the galaxy as a whole.
- Measure the average stellar mass-to-light ratio within each radius and add it to the table.
- Using the luminosity and average stellar mass-to-light ratios, calculate the stellar mass contained within each radius and add it to the table.
- Including the gas mass (provided in the table), calculate the total amount of normal matter in the galaxy at each radius and add it to the table.
- Copy your answers for the total "gravitational" mass and the mass in normal matter within each radius into the next table.
- Hence calculate the percentage of the mass that must be due to dark matter within each radius. Use the difference in the results at +/-3 and +/-15 kpc to estimate the uncertainty in the % at these radii and record this with your %.

Radius (kpc)	Normal Matter (Stars +Gas) Within R ($M_\odot$)	Total "Gravitational" Mass Within R ($M_\odot$)	% of Total Mass Due to Dark Matter
-15			
$+15$			
-3			
$+3$			

Questions

1. Based on your results in the final table, explain whether or not there is evidence for the existence of dark matter.

2. Based on your results in the final table above, what can you deduce about how normal matter is distributed in the galaxy by comparison with the dark matter?

3. In calculating the total mass M within radius R, the assumption that it can be replaced by a point mass when calculating the gravitational force using Newton's law is only true if its actual distribution is spherically symmetric. If Dark Matter is spherically symmetric in galaxies, explain whether this assumption is approximately self-consistent at small and and large radii, based on the observed shape of galaxies and your results for mass distributions.

4. Because of dust obscuring the view, the luminosity of NGC 134 may be somewhat brighter than you assumed. If the actual luminosity is brighter than you assumed above, explain how the percentage of dark matter in your galaxy would be changed.

5. Recent studies suggest that only $\sim$15% of the mass of the Universe is in normal matter, on average. Are your results consistent with this within errors? How might you explain any differences between the fraction of dark matter in NGC 134 and the fraction of dark matter in the Universe as a whole?