Linear Momentum and Collisions

Goal

- To determine the muzzle velocity of a projectile.

Equipment

Ballistic pendulum, projectile, C-clamp, ruler, triple beam balance.

The Big Picture

Some quantities in physics do not change in time, which makes them especially useful in calculations. We say that such quantities are **conserved**. Some examples of conserved quantities are total energy E_{total}, total electric charge Q_{total}, and total linear momentum $\vec{p}_{\text{total}}$. In fact each individual vector component of the linear momentum is conserved separately ($p_{x,\text{total}}$ is conserved, $p_{y,\text{total}}$ is conserved, etc.)

For particles moving with speeds far smaller than the speed of light c, the linear momentum is simply the product of the particle's mass and its velocity.

$$\vec{p} = m\vec{v}$$

(In Albert Einstein's Special Theory of Relativity, this nonrelativistic formula is altered, but the newly defined linear momentum is still conserved.)

Why is total linear momentum conserved? It turns out that there is a deep and beautiful connection between conserved quantities and symmetries of the Universe proven by the mathematician Amalie Emmy Noether in 1915. While the proof will need to wait until graduate school, the conceptual version is that linear momentum is conserved because the laws of physics are translationally invariant; this means that you can perform an experiment in the lab, or a mile from the lab, or wait until the Earth has moved 100,000 miles in its orbit, and the results will be the same. In the same way, Noether's theorem says that total energy

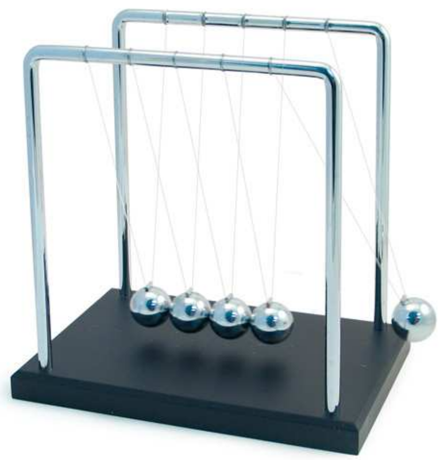

Figure 1: In Newton's cradle, the collisions between the suspended steel balls are approximately elastic, conserving both kinetic energy and momentum.

Figure 2: A ballistic pendulum is used to measure the muzzle velocity of a BB. The collision between the BB and the wood block is completely inelastic. Kinetic energy is not conserved, but momentum is conserved. From Ref 3.

Figure 3: The Astro Blaster toy uses approximately elastic collisions between different mass balls to achieve surprising heights, much larger than the height from which it is dropped. From Ref 4.

is conserved because the laws of physics are time invariant; you can perform an experiment today, or tomorrow, or in 50 years, and the result will not change.

Theory

Conservation of linear momentum is related to Newton's third law of equal action and reaction forces. To see this, imagine that there are only two particles which exert forces on each other: particle 1 exerts a force $\vec{F}_{12}$ on particle 2, and particle 2 exerts a force $\vec{F}_{21}$ on particle 1. If the forces respect Newton's third law, then

$$\vec{F}_{12} = -\vec{F}_{21}.$$

Now we can use Newton's second law $\vec{F} = \frac{d\vec{p}}{dt}$, where $\vec{p}$ is the linear momentum. (The more familiar form of Newton's second law, $\vec{F} = m\vec{a}$, is only true if the mass m does not change. The form we have written is more general.)

$$\frac{d\vec{p}_2}{dt} = -\frac{d\vec{p}_1}{dt} \Longrightarrow \frac{d}{dt}(\vec{p}_2 + \vec{p}_1) = 0 \Longrightarrow \vec{p}_2 + \vec{p}_1 = \text{constant}.$$

This means that total linear momentum of the two-particle system does not change with time, in other words it is conserved. The individual momenta $\vec{p}_1$ or $\vec{p}_2$ may change but the sum of all the momenta $\vec{p}_{\text{total}}$ will remain constant.

In any mechanical collision, the forces involved will obey Newton's third law. As a consequence, the total linear momentum will be conserved in all collisions. Whether or not kinetic energy is also conserved in the collision is a different question. Collisions in which the total kinetic energy is the same before and after the collision are called "elastic". If kinetic energy is not conserved, then the collision is called "inelastic". If the two particles stick together after the collision and move off with a common velocity, then the collision is called "completely inelastic" or "perfectly inelastic". In all of these cases, though, linear momentum is still conserved.

It is important to realize that while in inelastic collisions although kinetic energy is not conserved, the total energy is conserved. Kinetic energy can be converted into other forms of energy such as thermal energy (if the colliding particles heat up), sound energy (if the collision is audible), etc.

Procedure

1. **Carefully** unscrew the hinge pin of the pendulum.
2. Measure the mass of the one ball which you will use throughout this experiment, and separately measure the mass of the (ball plus the pendulum without the hinge pin), using the triple beam balance at the instructor's desk.
3. **Carefully** reattach the pendulum to the base with the hinge pin.
4. Level the ballistic pendulum using the built-in bubble level. Adjust the height of the legs by turning the feet.

5. Use the C-clamp to attach the gun base rigidly to the lab bench. This will prevent recoil of the gun. Ensure that the base is still level even after the C-clamp is attached.
6. Measure the length of the pendulum from the center of the hinge pin to the center of mass of the pendulum, labeled C/M on the pendulum shaft.
7. Place the metal ball whose mass you recorded on the end of the gun, and push back until the spring locks into position.
8. Make sure that the pendulum is hanging freely, but is not swinging. **Do not** change the spring tension using the knob at the end of the gun during the experiment. Fire the gun.
9. After the pendulum comes to rest, read the angle to which the pendulum has risen. If the red indicator falls on a black line, the number of degrees is even; if the red indicator falls between adjacent black lines, the number of degrees is odd.
10. Repeat the last three steps several times.

Analysis

1. Use the prelab assignment to derive a formula for the muzzle velocity v_m using only the following five variables:
   (a) the small projectile mass, m
   (b) the larger (ball+pendulum) mass, M
   (c) the length of the pendulum, L
   (d) the angle to which the pendulum rises, θ
   (e) the acceleration due to gravity in Dallas, $g = 9.80$ m/s$^2 \pm$ 0.01 m/s^2
2. Calculate the average muzzle velocity with an error estimate. Take the error in the angle to be zero. You will not be able to propagate the error in the muzzle velocity until you have a formula with vm on one side of the equals sign and the five measured quantities on the other side.
3. How does this speed compare to the muzzle velocity measured in the Acceleration and Freefall Lab? Which result is more reliable and why?
4. Why is it incorrect to equate the kinetic energy lost by the projectile to the potential energy gained by the projectile/pendulum combination? In other words, why is mechanical energy not conserved in this experiment? Where does it go?
5. Calculate the kinetic energy of the bullet before the collision. No error estimate is required.
6. Calculate the change in potential energy of the bullet and pendulum after they have both come to rest. No error estimate is required.
7. Identify at least two sources of statistical error.

8. Identify at least two sources of systematic error.

Literature

1. Halliday, Resnick, & Walker, *Fundamentals of Physics* Vol. 1 (10th ed.), Ch 8: Potential Energy and Conservation of Energy, Wiley, 2014.

2. Halliday, Resnick, & Walker, *Fundamentals of Physics* Vol. 1 (10th ed.), Ch 9: Center of Mass and Linear Momentum, Wiley, 2014.

3. Experiment Mi-03: Bullet Velocity with Ballistic Pendulum, `http://teacher.pas.rochester.edu/PhysicsDemos/Mechanics/Mi_ConsLinearMomentum/Mi-03/Mi-03.html`, accessed 26 May 2017.

4. Astro Blaster `http://www.arborsci.com/astro-blaster`, Arbor Scientific, accessed 26 May 2017.