2. The initial speed of the car is

$$v = (80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 22.2 \text{ m/s}.$$

The tire radius is $R = 0.750/2 = 0.375$ m.

(a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 11-2 leads to

$$\omega_0 = \frac{v_{\text{com}0}}{R} = \frac{22.2 \text{ m/s}}{0.375 \text{ m}} = 59.3 \text{ rad/s}.$$

(b) With $\theta = (30.0)(2\pi) = 188$ rad and $\omega = 0$, Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow |\alpha| = \frac{(59.3 \text{ rad/s})^2}{2(188 \text{ rad})} = 9.31 \text{ rad/s}^2.$$

(c) Equation 11-1 gives $R\theta = 70.7$ m for the distance traveled.

7. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30° = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves the roof) is (Eq. 11-5)

$$K_f = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} I\omega^2 .$$

Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \tfrac{1}{2} MR^2$ (Table 10-2(c)), conservation of energy leads to

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2 .$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10 \text{ m}}\sqrt{\frac{4}{3}(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 63 \text{ rad/s}.$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$v_{0x} = v_0 \cos 30° = 5.4 \text{ m/s}$$
$$v_{0y} = v_0 \sin 30° = 3.1 \text{ m/s}.$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = H = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74 \text{ s}.$$

Then we substitute this into the x equation and obtain $x = (5.4 \text{ m/s})(0.74 \text{ s}) = 4.0$ m.

21. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\times\vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = 0$, $y = -4.0$, $z = 3.0$, $F_x = 2.0$, $F_y = 0$, and $F_z = 0$. Then we obtain

$$\vec{\tau} = \vec{r}\times\vec{F} = \left(6.0\hat{\text{j}} + 8.0\hat{\text{k}}\right)\text{N}\cdot\text{m}.$$

This has magnitude $\sqrt{(6.0\ \text{N}\cdot\text{m})^2 + (8.0\ \text{N}\cdot\text{m})^2} = 10\ \text{N}\cdot\text{m}$ and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the $+y$ direction) is $\tan^{-1}(8/6) = 53°$.

(b) In the above expression, we set $x = 0$, $y = -4.0$, $z = 3.0$, $F_x = 0$, $F_y = 2.0$, and $F_z = 4.0$. Then we obtain $\vec{\tau} = \vec{r}\times\vec{F} = (-22\,\text{N}\cdot\text{m})\hat{\text{i}}$. This has magnitude $22\ \text{N}\cdot\text{m}$ and points in the $-x$ direction.

51. No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved.

Let I_1 be the rotational inertia of the wheel that is originally spinning (at ω_i) and I_2 be the rotational inertia of the wheel that is initially at rest. Then by angular momentum conservation, $L_i = L_f$, or $I_1\,\omega_i = (I_1 + I_2)\,\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

where ω_f is the common final angular velocity of the wheels.

(a) Substituting $I_2 = 2I_1$ and $\omega_i = 800$ rev/min, we obtain

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i = \frac{I_1}{I_1 + 2(I_1)}(800\ \text{rev/min}) = \frac{1}{3}(800\ \text{rev/min}) = 267\ \text{rev/min}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2.$$

Therefore, the fraction lost is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{I\omega_i^2/6}{I\omega_i^2/2} = \frac{2}{3} = 0.667.$$

90. Information relevant to this calculation can be found in Appendix C or on the inside front cover of the textbook. The angular speed is constant so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.3 \times 10^{-5} \text{ rad/s}.$$

Thus, with $m = 84$ kg and $R = 6.37 \times 10^{6}$ m, we find

$$\ell = mR^2\omega = 2.5 \times 10^{11} \text{ kg} \cdot \text{m}^2/\text{s}.$$