CHAPTER 3

22. Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\begin{aligned} \vec{E} &= 3.73\ \hat{\mathrm{i}} + 4.70\ \hat{\mathrm{j}} \\ \vec{F} &= 1.29\ \hat{\mathrm{i}} - 4.83\ \hat{\mathrm{j}} \\ \vec{G} &= 1.45\ \hat{\mathrm{i}} + 3.73\ \hat{\mathrm{j}} \\ \vec{H} &= -5.20\ \hat{\mathrm{i}} + 3.00\ \hat{\mathrm{j}} \\ \vec{E} + \vec{F} + \vec{G} + \vec{H} &= 1.28\ \hat{\mathrm{i}}\ + 6.60\ \hat{\mathrm{j}}. \end{aligned}$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28\text{ m})^2 + (6.60\text{ m})^2} = 6.72\text{ m}$.

(c) Its angle measured counterclockwise from the $+x$ axis is $\tan^{-1}(6.60/1.28) = 79.0°$.

(d) Using the conversion factor π rad $= 180°$, $79.0° = 1.38$ rad.

28. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\ \hat{\mathrm{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50° north of east). For their respective second parts: $\vec{B}$ is 0.80 m at 30° north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A} + \vec{B} = (0.5\text{ m})\hat{\mathrm{i}} + (0.8\text{ m})(\cos 30°\ \hat{\mathrm{i}} + \sin 30°\ \hat{\mathrm{j}}) = (1.19\text{ m})\ \hat{\mathrm{i}} + (0.40\text{ m})\ \hat{\mathrm{j}}.$$

The equation relating these is $\vec{A} + \vec{B} = \vec{C} + \vec{D}$, where

$$\vec{C} = (1.60\text{ m})(\cos 50.0°\hat{\mathrm{i}} + \sin 50.0°\hat{\mathrm{j}}) = (1.03\text{ m})\hat{\mathrm{i}} + (1.23\text{ m})\hat{\mathrm{j}}$$

(a) We find $\vec{D} = \vec{A} + \vec{B} - \vec{C} = (0.16\text{ m})\hat{\mathrm{i}} + (-0.83\text{ m})\hat{\mathrm{j}}$, and the magnitude is $D = 0.84$ m.

(b) The angle is $\tan^{-1}(-0.83/0.16) = -79°$, which is interpreted to mean 79° south of east (or 11° east of south).

33. Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.

(b) Using the Right-Hand Rule, the vector $\vec{a}\times\vec{b}$ points in the $\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}$, or the $+z$ direction.

(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12$.

(d) The vector $-\vec{a}\times\vec{b}$ points in the $-\hat{\mathrm{i}}\times\hat{\mathrm{j}}=-\hat{\mathrm{k}}$, or the $-z$ direction.

(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12$.

(f) The vector points in the $+z$ direction, as in part (a).

37. We apply Eq. 3-30 and Eq.3-23. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\hat{\mathrm{i}}+5.0\hat{\mathrm{j}}+6.0\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b} \times \vec{c}) = (3.0)(-8.0) + (3.0)(5.0) + (-2.0)(6.0) = -21.$$

(b) We note that $\vec{b} + \vec{c} = 1.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}} + 3.0\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)(1.0) + (3.0)(-2.0) + (-2.0)(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0)-(-2.0)(-2.0)]\,\hat{\mathrm{i}} + [(-2.0)(1.0)-(3.0)(3.0)]\,\hat{\mathrm{j}} \\ &\quad + [(3.0)(-2.0)-(3.0)(1.0)]\,\hat{\mathrm{k}} \\ &= 5\hat{\mathrm{i}} - 11\hat{\mathrm{j}} - 9\hat{\mathrm{k}}\end{aligned}$$

38. Using the fact that

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}},\quad \hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}},\quad \hat{\mathrm{k}}\times\hat{\mathrm{i}}=\hat{\mathrm{j}}$$

we obtain

$$2\vec{A} \times \vec{B} = 2\left(2.00\hat{\mathrm{i}}+3.00\hat{\mathrm{j}}-4.00\hat{\mathrm{k}}\right)\times\left(-3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}+2.00\hat{\mathrm{k}}\right) = 44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}.$$

Next, making use of

$$\begin{aligned}\hat{\mathrm{i}}\cdot\hat{\mathrm{i}} &= \hat{\mathrm{j}}\cdot\hat{\mathrm{j}} = \hat{\mathrm{k}}\cdot\hat{\mathrm{k}} = 1 \\ \hat{\mathrm{i}}\cdot\hat{\mathrm{j}} &= \hat{\mathrm{j}}\cdot\hat{\mathrm{k}} = \hat{\mathrm{k}}\cdot\hat{\mathrm{i}} = 0\end{aligned}$$

we have

$$\begin{aligned}3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right) &= 3\left(7.00\hat{\mathrm{i}}-8.00\hat{\mathrm{j}}\right)\cdot\left(44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}\right) \\ &= 3[(7.00)(44.0)+(-8.00)(16.0)+(0)(34.0)] = 540.\end{aligned}$$