Chapter 37

1. From the time dilation equation $\Delta t = \gamma \Delta t_0$ (where Δt_0 is the proper time interval, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$), we obtain

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}.$$

The proper time interval is measured by a clock at rest relative to the muon. Specifically, $\Delta t_0 = 2.2000\ \mu s$. We are also told that Earth observers (measuring the decays of moving muons) find $\Delta t = 16.000\ \mu s$. Therefore,

$$\beta = \sqrt{1 - \left(\frac{2.2000\ \mu s}{16.000\ \mu s}\right)^2} = 0.99050.$$

3. (a) The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval Δt_0) and 1000 years according to the clocks on Earth, which measure Δt. We solve Eq. 37-7 for β:

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1 - \left(\frac{1\text{y}}{1000\text{y}}\right)^2} = 0.99999950.$$

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question that has occasionally precipitated debates among professional physicists.

7. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

where $\Delta t_0 = 120$ y. This yields $\Delta t = 2684\text{ y} \approx 2.68 \times 10^3$ y.

11. The length L of the rod, as measured in a frame in which it is moving with speed v parallel to its length, is related to its rest length L_0 by $L = L_0/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Since γ must be greater than 1, L is less than L_0. For this problem, $L_0 = 1.70$ m and $\beta = 0.630$, so

$$L = L_0\sqrt{1-\beta^2} = (1.70\,\text{m})\sqrt{1-(0.630)^2} = 1.32\,\text{m}.$$

72. Using Eq. 37-10, we obtain $\beta = \dfrac{v}{c} = \dfrac{d/c}{t} = \dfrac{6.0\text{ y}}{2.0\,\text{y} + 6.0\,\text{y}} = 0.75.$