PHYS 5380: The Weak Interaction and Neutrino Mixing

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October 11 and 13, 2017

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Outline

Once More: Quark Mixing

A Review of Quark Mixing

- The suppression of decay rates when strangeness quantum number changes in a reaction was explained by proposing that the weak interaction doesn't "see" quarks in as distinct in the way we might think of them (e.g. via having different masses)
  - The weak interaction sees the up quark as we think of it, but it doesn't see the down and strange quarks in the same way. Instead, it sees a "down type" quark states described as a mixture of actual down and strange

$$|d'\rangle = \cos\theta_c\,|d\rangle + \sin\theta_c\,|s\rangle \tag{1}$$

- The interaction of the W boson is universal with regard to the u and d' states, but since physical reality is changed depending on whether the d' manifests as a d or s quark in a meson, this leads to suppression of strangeness-changing reactions in hadrons.

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The Jelly Bean Analogy

- Represent color axes mathematically:

$$\hat{r}, \hat{g} \longrightarrow |r\rangle, |g\rangle \tag{2}$$

(in linear algebra, $|i\rangle$ represents a column vector, $\langle i|$ a row vector, with the property $|\langle i|i\rangle| = 1$)

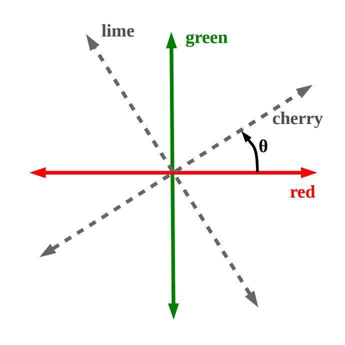
- Represent flavor axes:

$$|c\rangle, |l\rangle \tag{3}$$

- Write one set in terms of the other:

$$|c\rangle = \cos\theta\,|r\rangle + \sin\theta\,|g\rangle \tag{4}$$

$$|l\rangle = -\sin\theta\,|r\rangle + \cos\theta\,|g\rangle \tag{5}$$



Predicting a fourth quark: charm

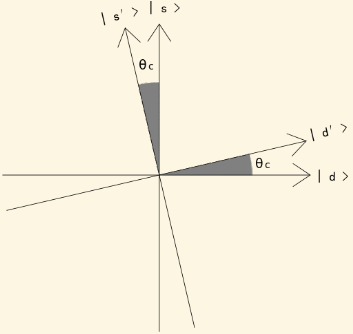
By the early 1970s, there were reasons to believe that there might be a fourth quark. In the theory, this modifies our quark picture from a single doublet to two doublets as follows:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \longrightarrow \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \tag{6}$$

such that

$$|d'\rangle = \cos\theta_c\,|d\rangle + \sin\theta_c\,|s\rangle \tag{7}$$

$$|s'\rangle = -\sin\theta_c\,|d\rangle + \cos\theta_c\,|s\rangle \tag{8}$$



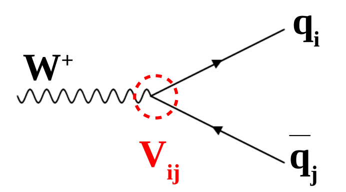
A likely candidate for charm matter was found in 1974, and charm mesons (pairings of a charm quark with either a u, d, or s) themselves were discovered not long after and fit this picture. It is remarkable that this simple idea worked even on a quark no one had seen before.

The Cabibbo Mixing Matrix

$$\begin{pmatrix} |d'\rangle \\ |s'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \end{pmatrix} \quad (10)$$

This is the Cabibbo Mixing Matrix. Its elements, V_{ij}, encode the degree with which a transition will occur between quarks i, j in a natural process, as mediated by the weak interaction.



$$P(W^+ \to q_i\bar{q}_j) \propto |V_{ij}|^2 \quad (11)$$

The matrix elements can be complex numbers, but with just four quarks they are real numbers.

The Full Cabibbo-Kabayashi-Maskawa (CKM) Quark Mixing Matrix

$$\begin{pmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{pmatrix} \tag{12}$$

Why extend to at least 6 quarks? Such a matrix is the minimum size needed to add a complex component to the otherwise real number components of the matrix. That complex component leads to CP violation, but that is a discussion for another time.

$$\theta_{12} = 13.00 \pm 0.03^\circ; \theta_{13} = 0.202 \pm 0.009^\circ; \theta_{23} = 2.39 \pm 0.05^\circ; \delta_{13} = 1.19 \pm 0.03\mathrm{rad}. \tag{14}$$

The first angle is the Cabibbo Angle. Note that it is about what we estimated. δ_{13} is the complex phase. These are not predicted; they must be measured. Values computed using data in Ref. [1].

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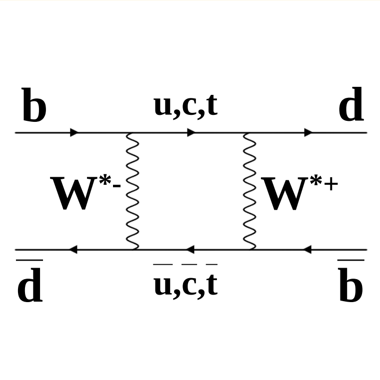
Quark Meson Mixing — Example: B-Mixing

Because of quark mixing in the weak interaction, diagrams like the one below for the neutral B meson ($B^0 = \overline{b}d$) are possible. What do you observe about the effect of these two sequential flavor-changing weak interactions?



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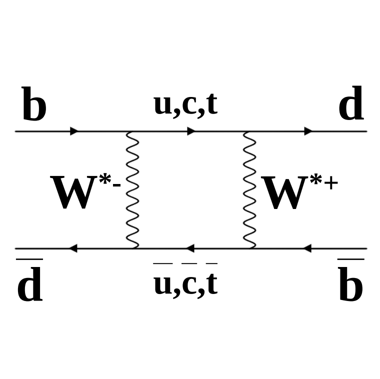

It is possible to produce a population of B^0 mesons, wait a little bit (Δt), then count the B^0 and $\overline{B}^0$ mesons and find $N_{\overline{B}^0} \neq 0$! Do we see this in data?

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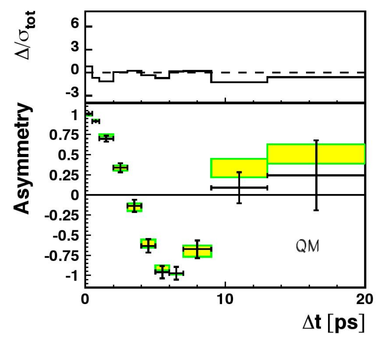
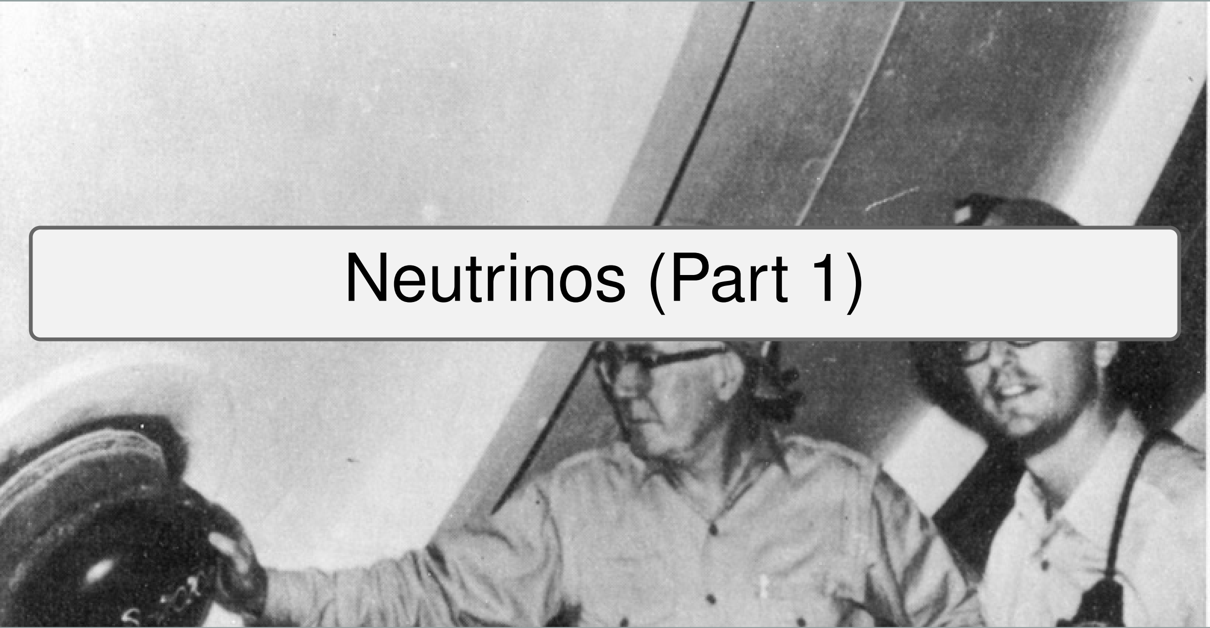


The figure at the left is compiled from data from the so-called "B-Factory Experiments" (BaBar and Belle) [2] and shows how the "flavor asymmetry",

$$\mathcal{A} \equiv \frac{P_{\text{not mixing}}(\Delta t) - P_{\text{mixing}}(\Delta t)}{P_{\text{not mixing}}(\Delta t) + P_{\text{mixing}}(\Delta t)} \tag{15}$$

changes with the time after production of a population of B mesons, in picoseconds. Note the impressive agreement of data (black points with error bars) with the predictions of quantum mechanics (QM, colored boxes).



Neutrinos (Part 1)

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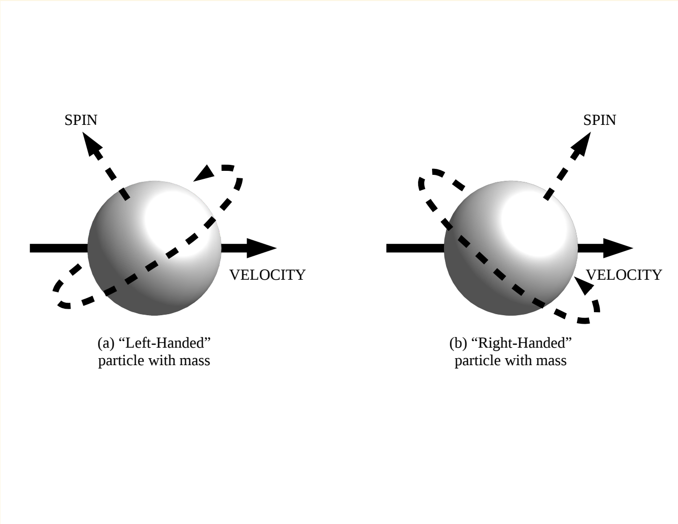

Visualization of "handedness" - how spin angular momentum projects onto linear momentum. Illustration by Stephen Sekula for the book "Reality in the Shadows (or, What the Heck's the Higgs?)" (forthcoming from YBK Publishing Synthesis)

All observed W mediated processes indicate left-handed spin polarization of $W^{\pm}$. So far there is no evidence for right-handed W current. That leads to the uncomfortable list of quarks and leptons from the perspective of their spins: left-handed doublets, right-handed singlets and no info whether right-handed neutrino exist.

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$u_R \quad c_R \quad t_R$$

$$d_R \quad s_R \quad b_R \qquad e_R \quad \mu_R \quad \tau_R$$

d,s and b quarks can be treated as superposition of all three charge -1/3 quarks - mismatch of quantum states of quarks when they are involved in weak interactions. They are eigenstates of flavor in the production, but oscillate when propagate freely.

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Trouble with neutrinos

Three flavors of neutrinos, each associated with charged lepton

- ν_e - electron neutrino
- ν_μ – muon neutrino
- ν_τ – tau neutrino

Lepton number conservation →
interactions/decays conserve "electronic", "muonic" and "tauonic" characteristics

Experiments measuring electron energy spectrum from radioactive decays to electron and neutrino have not been able to establish any deviation of the shape of the spectrum from that expected for massless neutrino
→ conclusion valid until 1990ties: "neutrino have mass = 0" and move with the speed of light

Theoretical arguments about neutrino mass come from cosmology. Big Bang model predicts a ratio of number of photons (cosmic microwave background) to the total energy of the neutrinos providing an upper limit on the sum of the three neutrinos masses of 50 MeV. More recent stringent analysis of cosmological data set upper limit at 0.3 eV. Do they have a mass?

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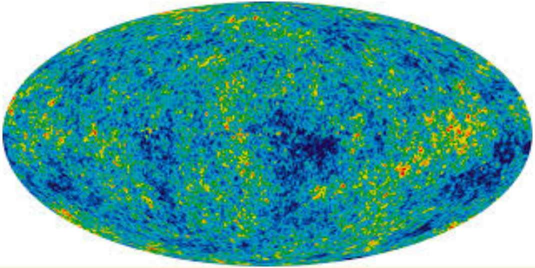
Sources

- **Big Bang** – there is a neutrino cosmic background like there is a microwave background – but they are too low energy to be detected



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Sources

- Big Bang ~300/cm^3 in the universe
- Sun and stars (~ 65 billions/cm^2 per second)
- Secondary products of cosmic rays interactions
- Reactors
- Neutrinos from Hell (heavy elements in earth core, magma and mantle)

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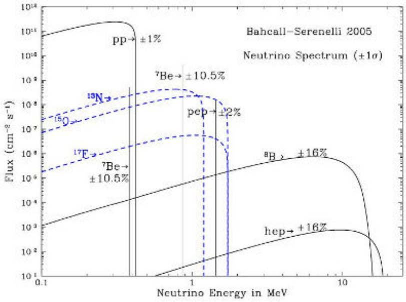
Neutrino flux on Earth predicted by Standard Solar Model



Neutrino flux at Earth predicted by the standard solar model of 2005.
The neutrinos produced in the pp chain are shown in black, neutrinos produced by the CNO cycle are shown in blue. The solar neutrino spectrum predicted by the BS05(OP) standard solar model. The neutrino fluxes from continuum sources are given in units of number $cm^{-2}\ s^{-1}\ MeV^{-1}$ at one astronomical unit, and the line fluxes are given in number $cm^{-2}\ s^{-1}$.

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Aside - How does our SUN work ?
→ nuclear physics

Three main sequences of proton-proton fusion with relative rates depending on the temperature and pressure at the core
→ copious production of photons who's outwards pressure counteracts gravitation forces
→ most of the energy from the star dominated by type-1 pp reaction is carried out by neutrinos

Star is stable as long as the there is sufficient amount of hydrogen to sustain pp fusion.
Our Sun formed ~4.6 years ago and will remain stable for another ~5 billion years.
The future of a star after pp fuel is exhausted depends on its mass. Our Sun will become red giant with radius expanding perhaps to the Earth orbit.
The nuclear processes inside all stars are assumed to be the same. In all scenario the cosmos is full of neutrinos.

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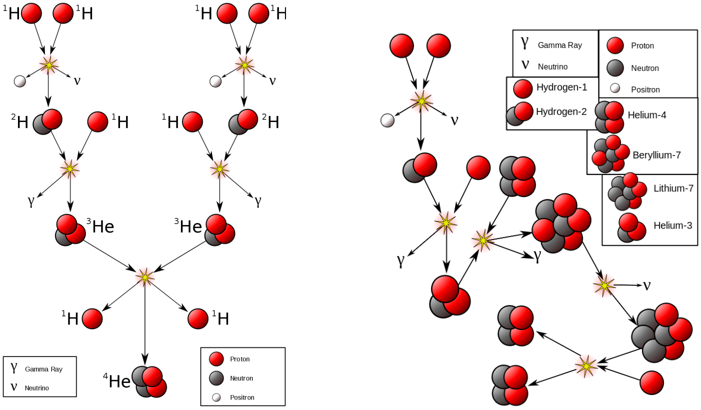
Three pp reaction chains



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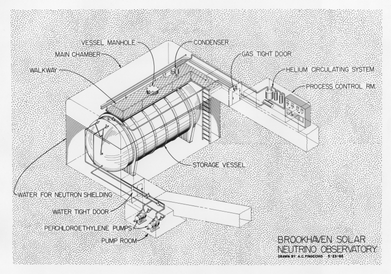
First indications of troubles: Davis experiment in Homestake mine
Davis searched for neutrinos coming from the fusion processes in the SUN using a radiochemical method where charged current neutrino interaction with a nucleus of ^{37}CL would result in a creation of ^{37}Ar atom. He used a tank filled with a cleaning fluid named tetrachloroethylene. The threshold neutrino energy for this reaction is 0.814 MeV – well in the middle of the expected solar neutrino spectrum for pp fusion reactions. The fluid was periodically purged with helium gas that removed the argon. The argon was then separated out by cooling. (Liquid helium ~4K, liquid argon ~70K). The argon atoms were counted via electron capture radioactive decays (nuclear physics effect of proton rich nucleus capturing an electron from an inner shell crating neutron and electron neutrino.

$p + e^- \rightarrow n + \nu_e$



Water tank shielding from external neutrons

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Davis results during first 15 years of operations varied between 8 and 15 events per year and were largely ignored. Eventually, it became a problem as the Solar Model became established and its predictions for luminous flux, and heavy nuclei production (metalization) that can be detected n the Sun's surface became confirmed by spectral measurements.
Davis experiment has indicated deficit of ~50% of the expected electron neutrinos.
→ Solar neutrino problem

There have been several later experiments confirming this deficit. Wikipedia always emphasizes the latest experiment with largest data samples but these just repeat original ideas at larger scales.

The explanation of this deficit is that neutrino does not remain fixed to one flavor. It changes flavor with time (or equivalently a distance of travel from its origin). This explanation assigns a non-zero mass to the neutrino, but them most likely mass range today is of the order of 10^{-5} eV, i.e, about 10 orders of magnitude smaller that the mass of electron.

Consequence of neutrino flavor oscillation:
Neutrino in the Sun start as ν_e During the passage through the Sun and to the Earth a fraction of these neutrinos oscillates to become ν_μ .
The electron neutrino has typical energy (<10 MeV) sufficient to produce an electron in flavor conserving interaction can with a nucleus of a detector on Earth. However an ν_μ with energy 10 MeV cannot produce a muon since m_μ = 105.6 MeV
→ it cannot be detected

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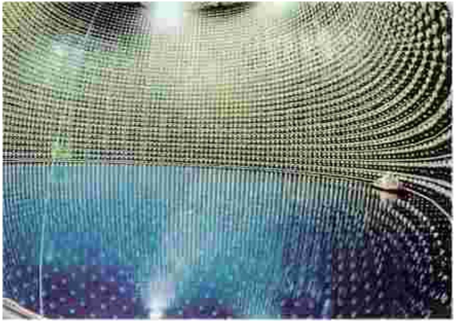
Super-Kamiokande
cylinder ~40m diameter, 40 m tall with 50,000 tons of purified water
>11,000 phototubes 20” diameter
muon neutrino beam from KEK Laboratory ($\pi^+ \rightarrow \mu^+ \nu_\mu$) $\rightarrow$ appearance of electron neutrinos



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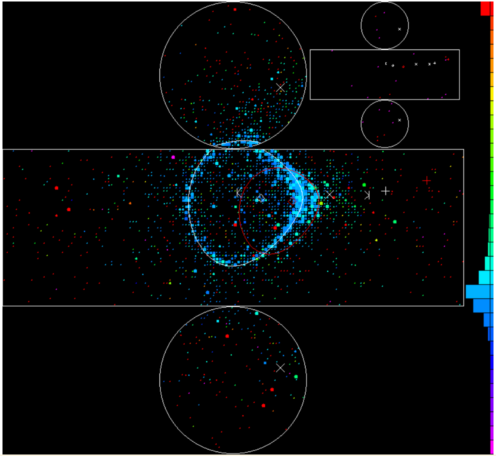
Super-Kamiokande

K2K neutrino event at Super-Kamiokande reconstructed as a pi-zero. Pi-zero decays to two gammas which make two electron-like rings.



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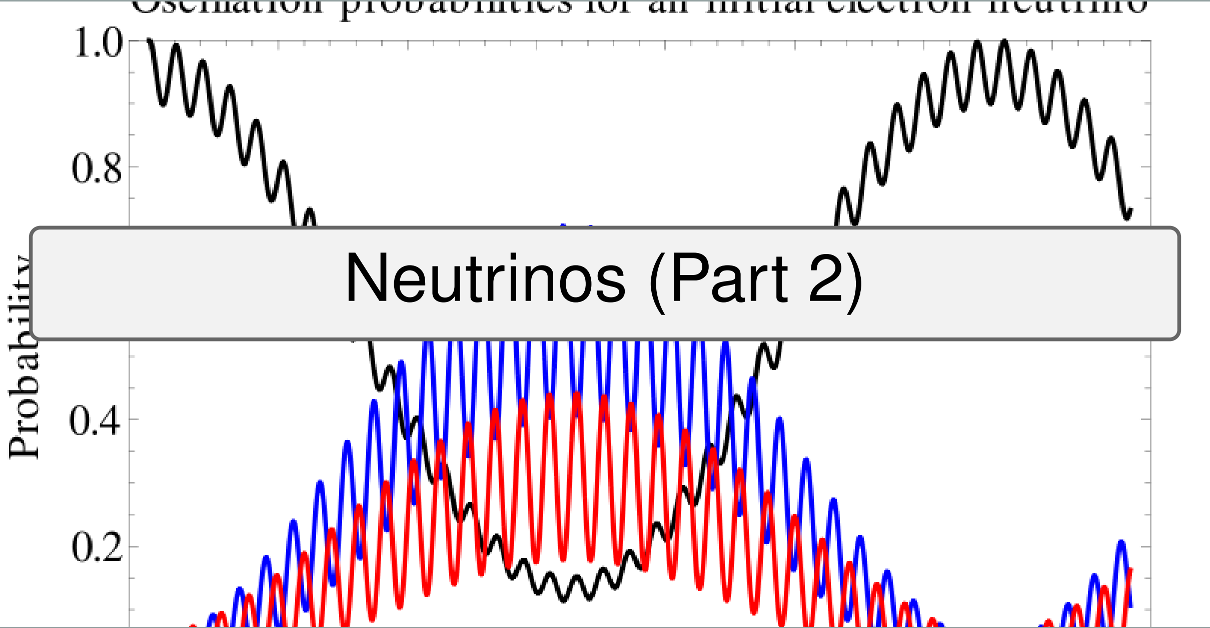

Neutrinos (Part 2)

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Neutrino oscillations: each neutrino is a superposition of 3 neutrino states with definite mass.
Neutrinos are produced in weak process in flavor eigenstates – definite flavor
As neutrino propagates in space, the quantum mechanical phase advances differently due to different masses. Neutrino produced as an electron neutrino can become after some distance a muon neutrino.
Since mass differences are small, the effect becomes visible only at large distances.

$$|\nu_\alpha> \;=\; \sum_i U^*_{\alpha i}\,|\nu_i>$$ neutrino with definite flavor: α = e, μ, τ

$$|\nu_i> \;=\; \sum_\alpha U_{\alpha i}\,|\nu_\alpha>$$ neutrino with definite mass: i=1, 2, 3

Unitary transformation

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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General Derivation

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

- Where $|\nu_\alpha\rangle$ are the flavor eigenstates , $|\nu_i\rangle$ are the mass eigenstates with mass m_i and $U_{\alpha i}$ is the neutrino analogue of the CKM matrix, i.e. the mixing matrix.

$$|\nu(t)\rangle = \sum_i U_{\alpha i} e^{-iE_i t} e^{i\vec{p}\cdot\vec{x}} |\nu_i\rangle$$

- Gives the eigenstates at a later time t and position $\vec{x}$.

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Intensity

- Let us assume the neutrino interacts weakly at time t, and we tag it as a flavor eigenstate $\left|\nu_{\beta}\right\rangle$. Then we have an intensity:

$$I_{\beta\alpha} = \left|\left\langle \nu_{\beta} \middle| \nu(t) \right\rangle\right|^{2} = \left| \sum_{i} U_{\alpha i} U^{*}_{\beta i} e^{-iE_{i}t} \right|^{2}$$

- We use the ultra-relativistic limit so that:

$$E_{i} = p + \frac{m_{i}^{2}}{2p}$$

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Mixing Equation

- So that:

$$I_{\beta\alpha} = \left| \sum_i U_{\alpha i} U^*_{\beta i} e^{-i\frac{m_i^2}{2p}t} \right|^2$$

- Which will serve as the standard mixing equation.

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Two Generation Mixing

- If only two generations (say, electron and muon) participate, then:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Setting $\alpha = 1$ for the initial state, there are two intensities, one for each value of β related by:

$$I_{11} = 1 - I_{21}$$

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Two Generation Mixing

- And,

$$I_{21} = 4\cos^2\theta \sin^2\theta \sin^2\left[\frac{(m_1^2 - m_2^2)t}{4p}\right] = \sin^2 2\theta \sin^2\left[\frac{(m_1^2 - m_2^2)t}{4p}\right]$$

- Which has three important limits:
- 1) $\frac{\left|m_1^2 - m_2^2\right| t}{4p} \ll 1$

When we are close to the source (small *t*) , no oscillations are noticeable.

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Other Cases

- 2) $\frac{\left|m_1^2 - m_2^2\right| t}{4p} \sim 1$
- A pattern is noticeable as t varies, so the precise calculation of $\left|m_1^2 - m_2^2\right|$ is possible.
- 3) $\frac{\left|m_1^2 - m_2^2\right| t}{4p} \gg 1$ The experiment will average over the rapid oscillations, resulting in

$$\overline{I_{21}} = \frac{1}{2}\sin^2 2\theta, \overline{I_{11}} = \frac{1}{2}(1+\cos^2 2\theta)$$

We have oscillations, but cannot measure the mass difference.

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Three Generation Mixing

- For three generation mixing, oscillations can be described in terms of four angles: one CP-violating phase and three differences of masses squared, only two of which are independent.
- Experimental evidence suggests that two of the mass eigenstates are more degenerate with each other than they are with the third:

$$\left|\Delta m_{13}^2\right| \simeq \left|\Delta m_{23}^2\right| \gg \left|\Delta m_{12}^2\right|$$

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Three Generation Mixing

- This simplifies the mixing equation so that:

$$I_{\beta\alpha} = 4\left|U_{\alpha 3}U^{*}_{\beta 3}\right|^{2} \sin^{2}\left(\frac{\Delta m^{2}_{23} t}{4p}\right)$$

- Which can be rewritten as:

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The Grand Finale

$$I(\nu_e \rightarrow \nu_\mu) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2(\frac{\Delta m_{23}^2 t}{4p})$$

$$I(\nu_e \rightarrow \nu_\tau) = \cos^2\theta_{23}\sin^2 2\theta_{13}\sin^2(\frac{\Delta m_{23}^2 t}{4p})$$

$$I(\nu_\mu \rightarrow \nu_\tau) = \cos^4\theta_{13}\sin^2 2\theta_{23}\sin^2(\frac{\Delta m_{23}^2 t}{4p})$$

- Where $\left|\Delta m_{23}^2\right| \simeq 2.5\times10^{-3} eV^2, \theta_{23} \simeq 45^\circ$ from experiment evidence so far.

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The dependence of the oscillation phase on the distance travel and neutrino energy can be written as

$$\frac{\Delta m^2\, c^3\, L}{4\hbar E} = \frac{\mathrm{GeV\,fm}}{4\hbar c} \times \frac{\Delta m^2}{\mathrm{eV}^2}\,\frac{L}{\mathrm{km}}\,\frac{\mathrm{GeV}}{E} \approx 1.27 \times \frac{\Delta m^2}{\mathrm{eV}^2}\,\frac{L}{\mathrm{km}}\,\frac{\mathrm{GeV}}{E}$$

The probability of a neutrino changing its flavor is (2-neutrino oscillations):

$$P_{\alpha\to\beta,\alpha\neq\beta} = \sin^2(2\theta)\,\sin^2\left(1.27\frac{\Delta m^2 L}{E}\,\frac{[\mathrm{eV}^2]\,[\mathrm{km}]}{[\mathrm{GeV}]}\right)$$

General case

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\,\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right) + 2\sum_{i>j}\mathrm{Im}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\,\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right),$$

$\Delta m^2 \sim 10^{-4}$ eV

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Phase δ is non-zero only if neutrino oscillation violates CP symmetry
-> different rates of oscillations
Neutrino is called a Majorana neutrino if it is identical to its antiparticle.
Otherwise it is called a Dirac neutrino.
Oscillation becomes complicated for thee neutrinos

We do not know

- what are the neutrino masses or even an ordering of their values
- whether the oscillations are two-ways or three-ways
- whether neutrinos are Dirac or Majorana

We know from precision measurements of Z boson decays that there are only 3 light neutrinos associated with 3 families of quarks and leptons.
We cannot exclude existence of heavy neutrinos with masses above m(Z/2) ~ 45 GeV/c^2

→ Dune

Major US national program centered at Fermilab (produce intense neutrino beams) with near detectors at Fermilab and Homestake mine in Lead, South Dakota 810 miles from Fermilab.

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Sterile neutrinos - another attempt to explain the problem of chirality i.e., spin of the neutrino

Neutrino oscillations indicate that neutrinos have non-zero mass and therefore move with velocities less than speed of light c. Their helicity is therefore non-invariant because now it is possible to move faster than they and observe opposite helicity. But right handed neutrinos have not been observed.

Postulate – the right handed neutrinos interact only via gravity which is very weak. They may have any mass and if they are sufficiently heavy they may be an explanation for the "dark mass" problem.

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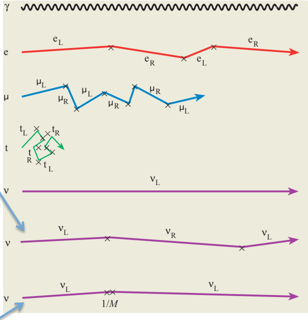
Higgs and neutrino mass

In general, the Standard Model provides a mechanism for particles to acquire mass through interactions with Higgs boson. Higgs boson has spin zero thus it is neither left-handed nor right-handed. Quantum Field Theory plus Lorenz invariance shows that a particle interaction with Higgs boson results in a left-handed particle becoming right-handed and then left-handed over the second interaction with Higgs and so on. The frequency of such interactions is proportional to paticle mass.

Extensions od the Standard Model with Dirac right handed neutrinos → right-handed W is heavy and has weak coupling, 10^{26} weaker then with ordinary neutrino
→Arkani-Hamed, Dimopoulos,Dvali – superstring theory: – right-handed neutrinos may move in extra dimensions while all other particles do not

Majorana neutrinos – are their own antiparticles. The left-handed neutrino interacting with Higgs produces a very heavy right-handed allowed by Heisenberg uncertainty principle. That one interacts right away with Higgs producing another left-handed state. → left-handed neutrinos are very heavy



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