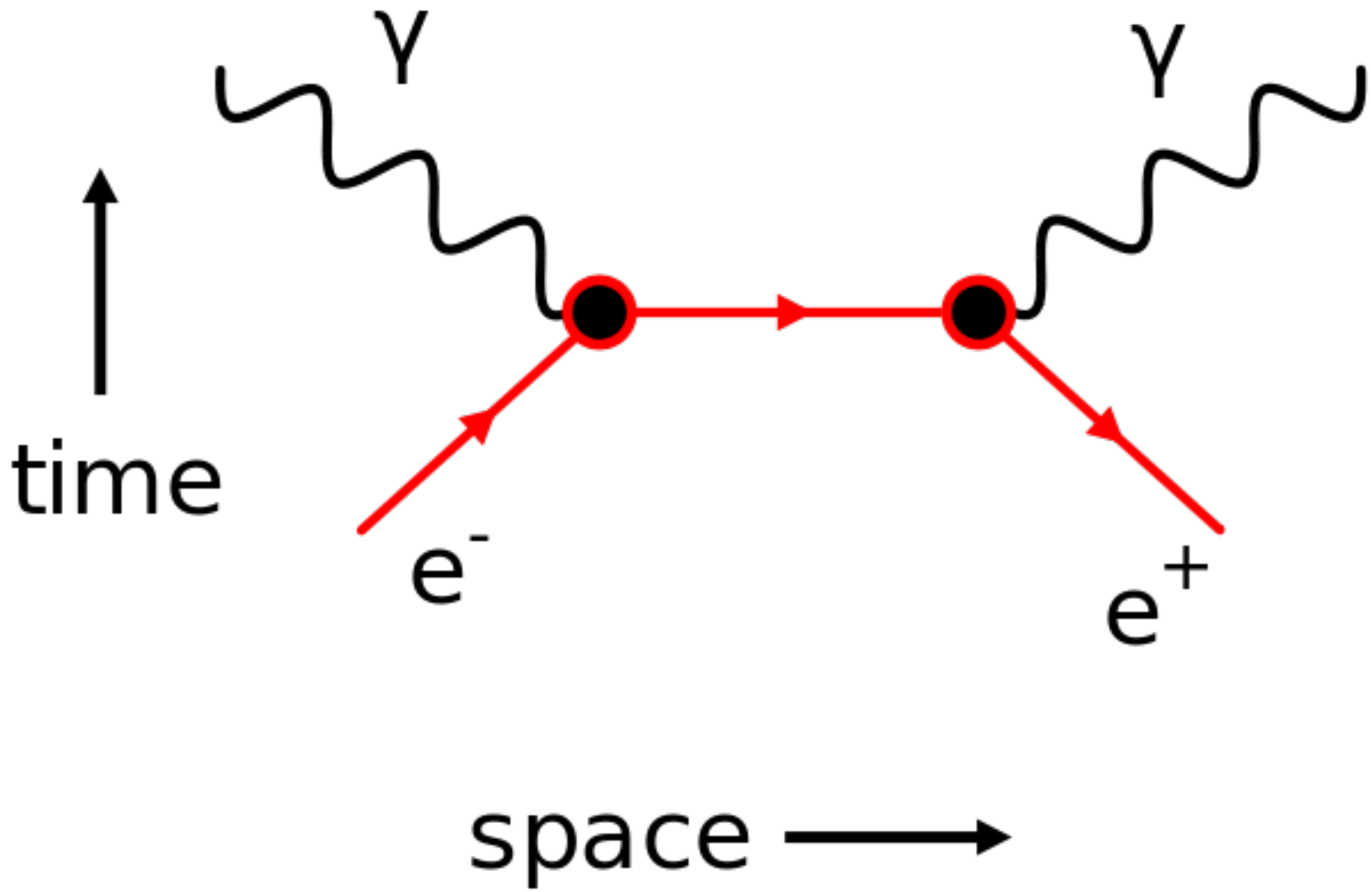
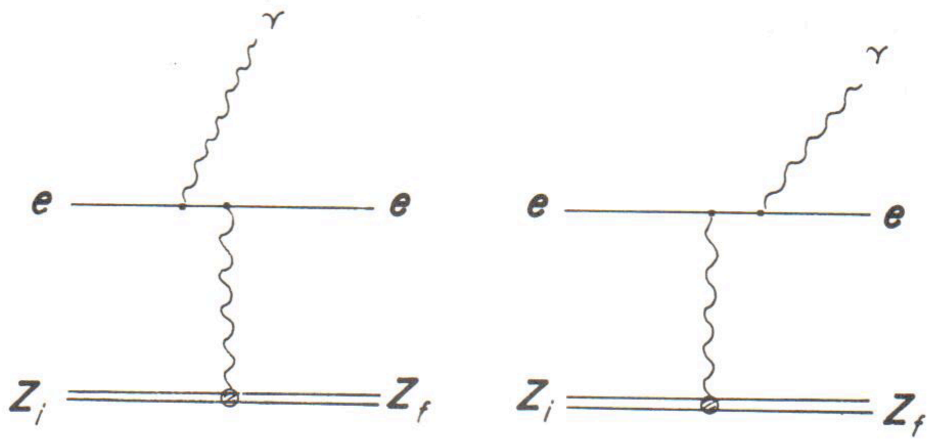
Electron-positron annihilation in classical electrodynamics



Satisfies conservation of linear momentum and total energy.
Annihilation into single photon is forbidden
(angular momentum/polarization problem).

During passage through matter electron loses energy via electromagnetic radiation. This is called synchrotron radiation in circular accelerators and bremsstrahlung in passage through matter.



Cross-section per atom – Bethe-Heitler

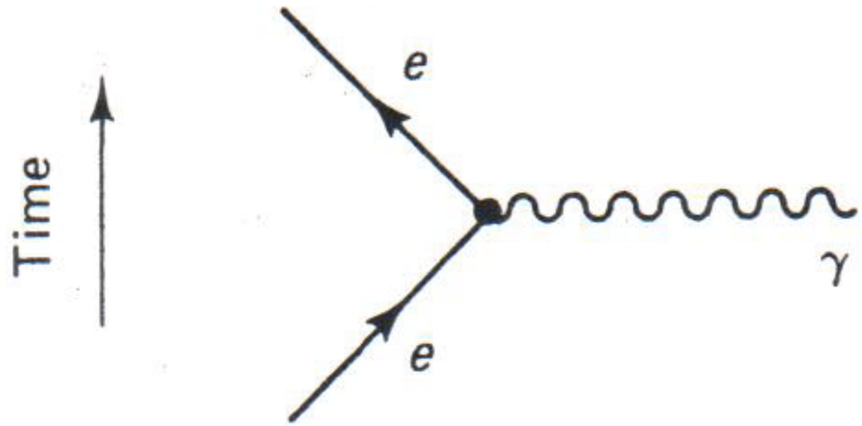
$$\sigma(Z,E) = \frac{Z\pi r_0^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

E = total energy of the incident positron

$\gamma = E/m_e c^2$

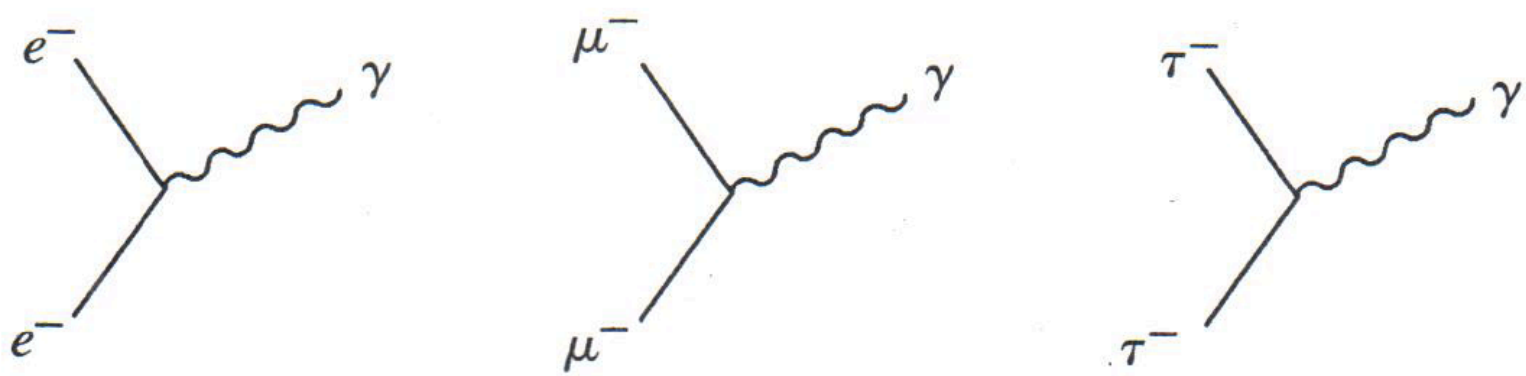
r_0 = classical electron radius

Electromagnetic interactions in quantum electrodynamics

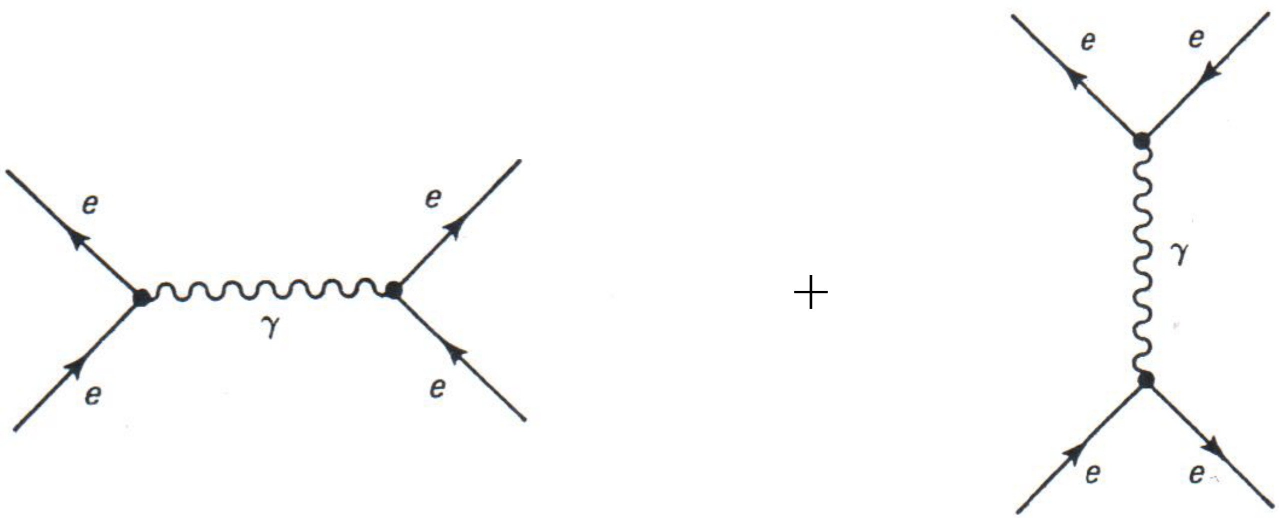


Here photon is virtual, i.e., it has an energy equivalent to mass

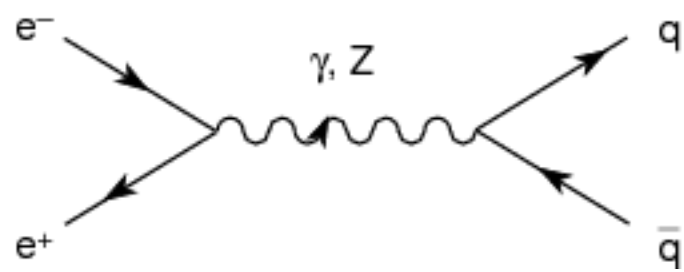
True for any charged particle, e.g., charged leptons



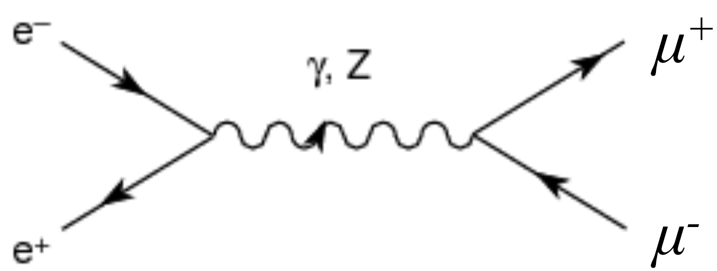
Special case: Bhabha (elastic) scattering $e^+e^- \rightarrow e^+ e^-$



In electromagnetic interaction photon couples to the charge.
The e+e- annihilation results in a virtual photon. This photon then can couple to any particle-antiparticle pair allowed by the available energy of the photon.



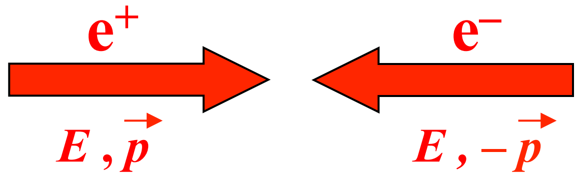
Left side of this diagram is the same for annihilation into pairs of muons



The ratio of cross sections is called R

PHYSICS WITH e^+e^- COLLIDERS

Two beams circulating in opposite directions in the same magnetic ring and colliding head-on



A two-step process: $e^+ + e^- \rightarrow$ virtual photon $\rightarrow f + \bar{f}$

f **: any electrically charged elementary spin ½ particle (μ, quark)**
(excluding e^+e^- elastic scattering)

Virtual photon energy – momentum : $E_\gamma = 2E\gamma,\ p_\gamma = 0 \rightarrow Q^2 = E_Y{}^2 - p_\gamma{}^2 c^2 = 4E^2$

Cross - section for $e^+e^- \rightarrow f\bar{f}$:

$$\sigma = \frac{2\pi\alpha^2\hbar^2c^2}{3Q^2} e_f{}^2 \beta(3-\beta)$$

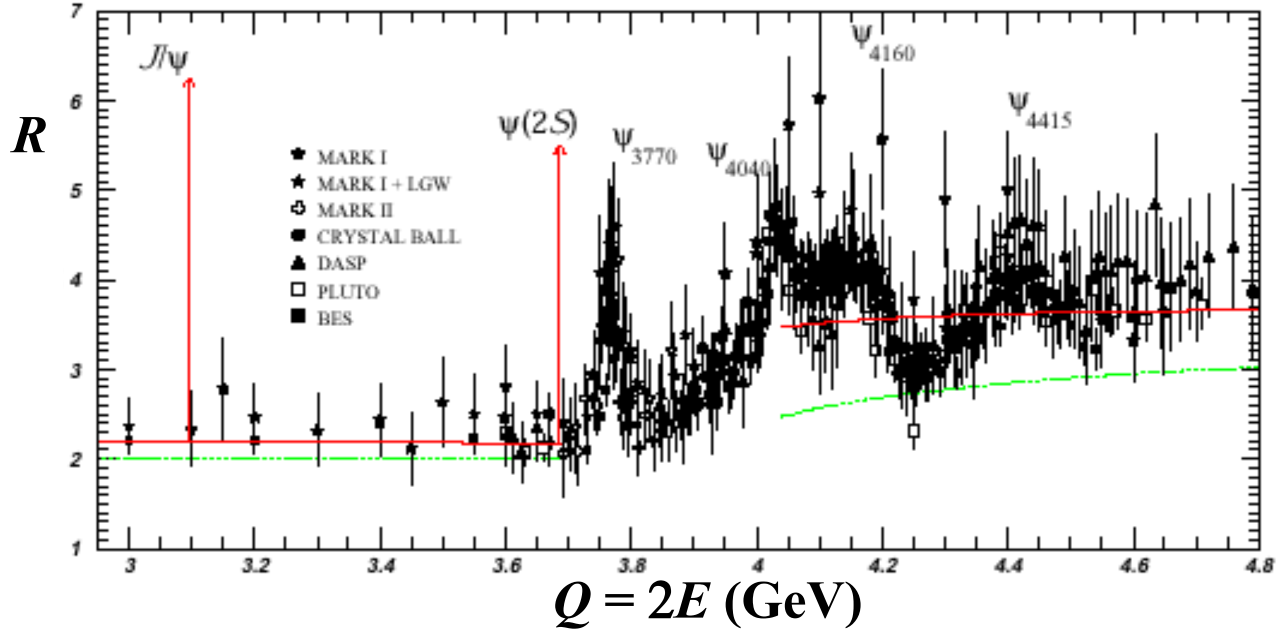
a = $e^2/(\hbar c) \approx 1/137$
e_f: electric charge of particle f (units |e|)
β= v/c of outgoing particle f

(formula precisely verified for $e^+e^- \rightarrow \mu^+\mu^-$)

Assumption: $e^+e^- \rightarrow$ quark (q) + antiquark ($\bar{q}$) $\rightarrow$ hadrons
➔ at energies $E >> m_q c^2$ (for q = u , d , s) $\beta \approx 1$:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = e_u{}^2 + e_d{}^2 + e_s{}^2 = \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$$

Experimental results from the Stanford e^+e^- collider SPEAR (1974 –75):



- For Q <3. 6 GeV $R \approx$ 2. If each quark exists in three different states, $R \approx 2$ is consistent with 3 x (2/3). This would also solve the Ω^- problem of 3 s quarks in the same state
- Between 3 and 4.5 GeV, the peaks and structures are due to the production of quark-antiquark bound states and resonances of a fourth quark ("charm", *c*) of electric charge q = +2/3

- Above 4.6 GeV $R \approx$4.3. Expect $R \approx$2 (from *u*, *d*, *s*) + 3×(4/9) = 3.3 from the addition of the *c* quark alone. So the data suggest pair production of an additional elementary spin ½ particle with electric charge = 1 (later identified as the τ – lepton (no strong interaction) with mass ≈ 1777 MeV/c^2).

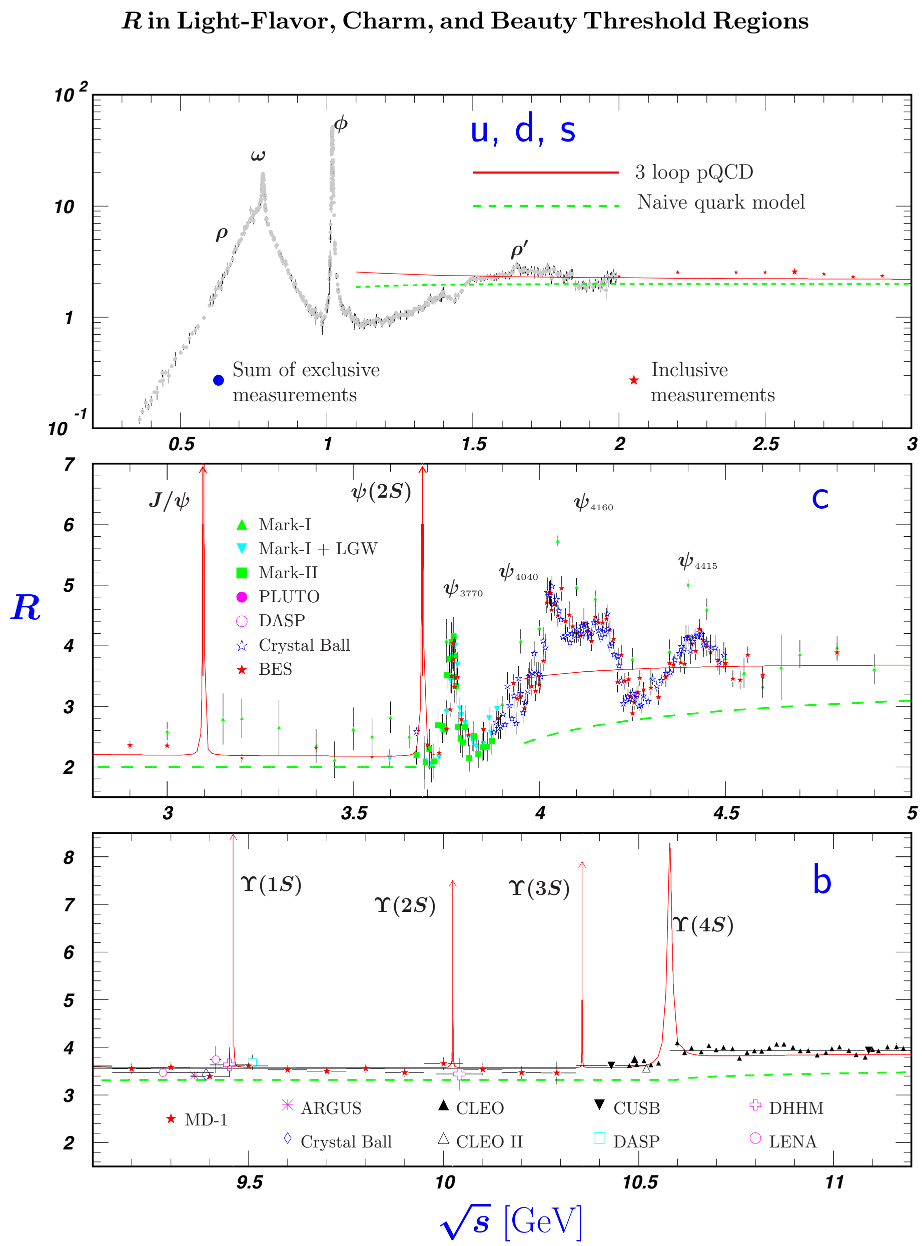


Figure 49.6: R in the light-flavor, charm, and beauty threshold regions. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are the same as in Fig. 49.5. **Note:** CLEO data above $\Upsilon(4S)$ were not fully corrected for radiative effects, and we retain them on the plot only for illustrative purposes with a normalization factor of 0.8. The full list of references to the original data and the details of the R ratio extraction from them can be found in [arXiv:hep-ph/0312114]. The computer-readable data are available at

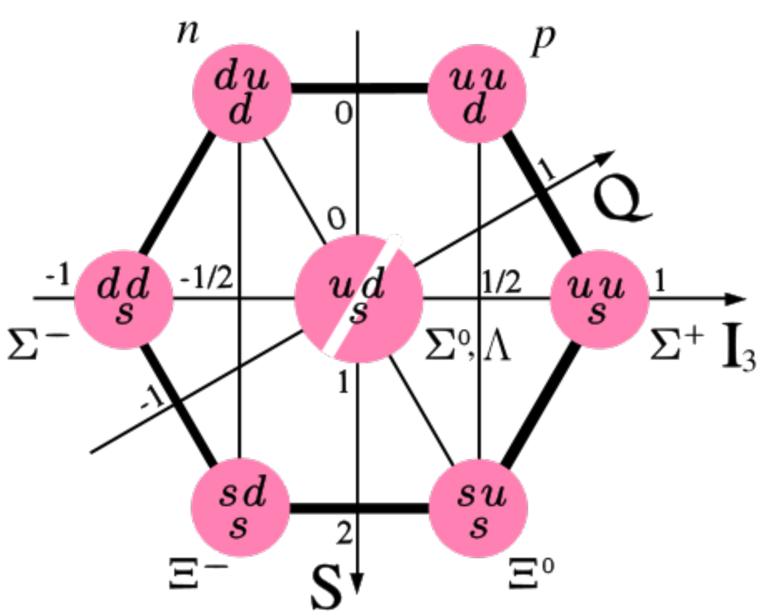
Newly discovered charm was easily incorporated into the quark Model. The multiplets became more complicated and 3-dimensional. The quarks and leptons became organized into two groups:

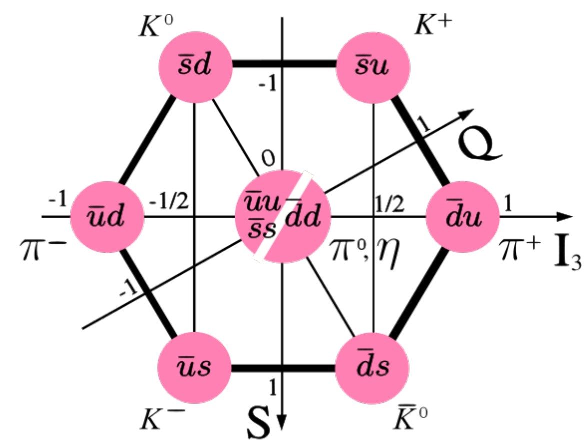
- *up + down quarks with electron and ν_e*
- *charm and strange quarks with muon and ν_μ*

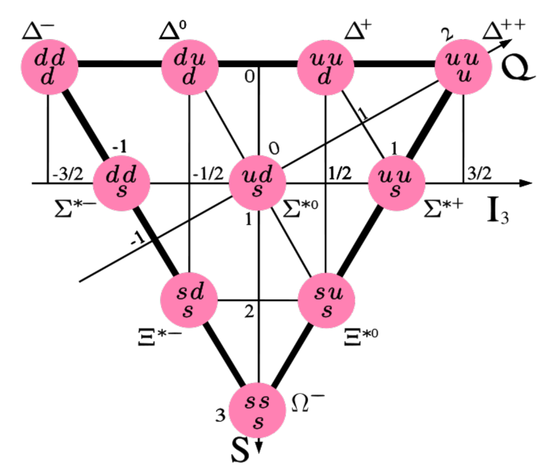
1950 - 1974

100's of elementary particles: μ, π, ρ, Σ, Ξ, Λ, Ω, ψ, ω, η and many others

→ Quark Model

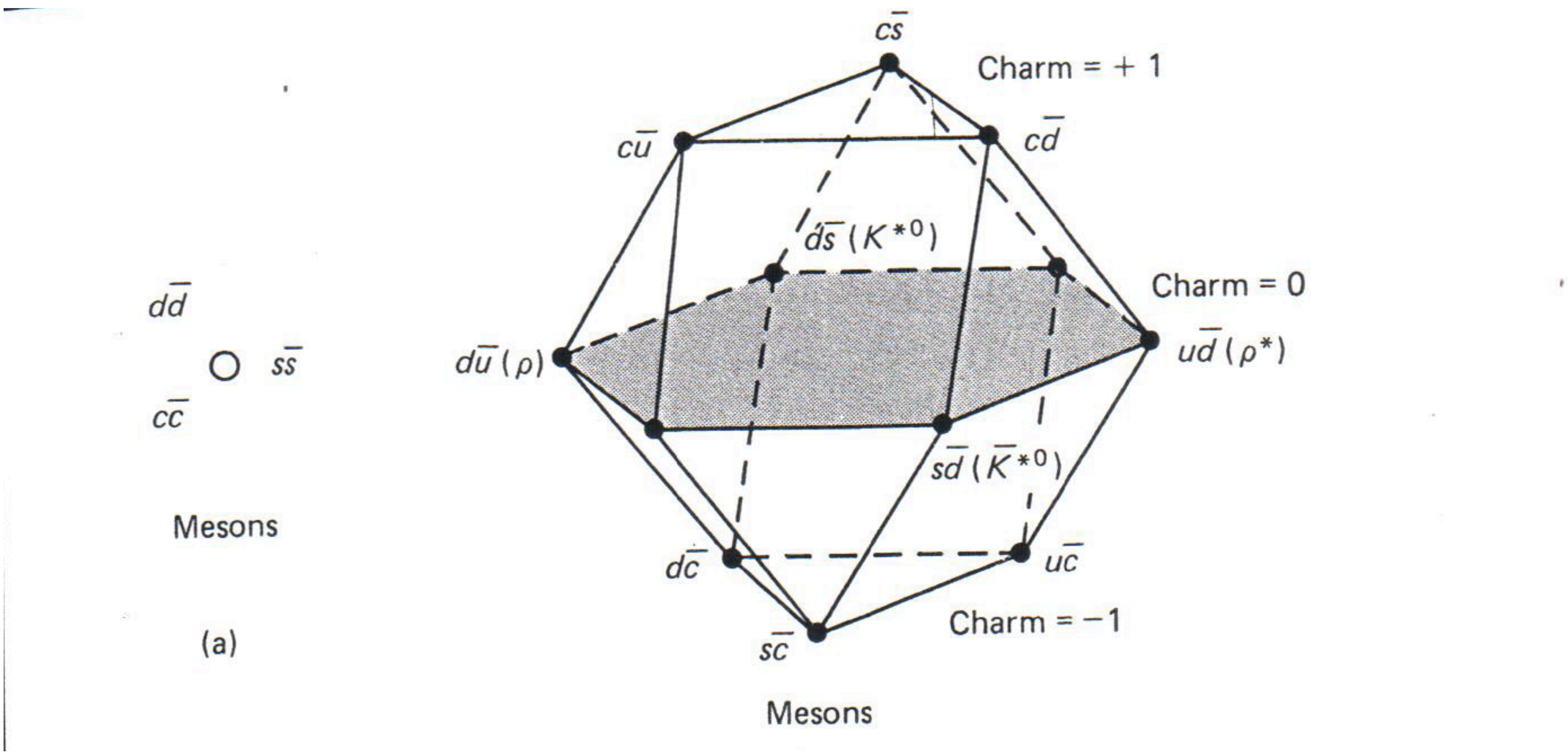




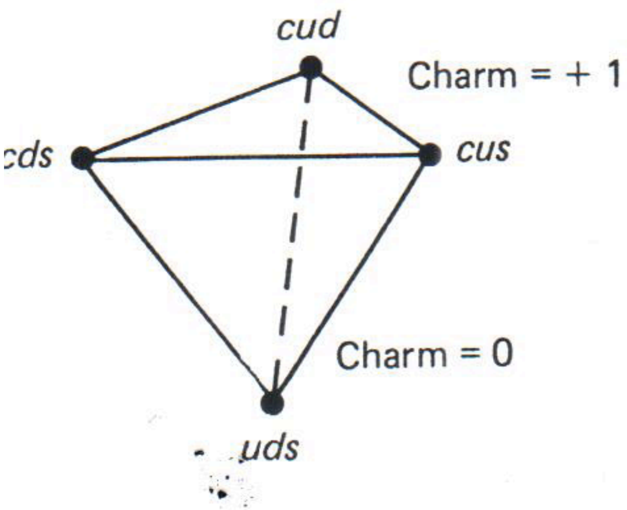


Supermultiplets with four quarks

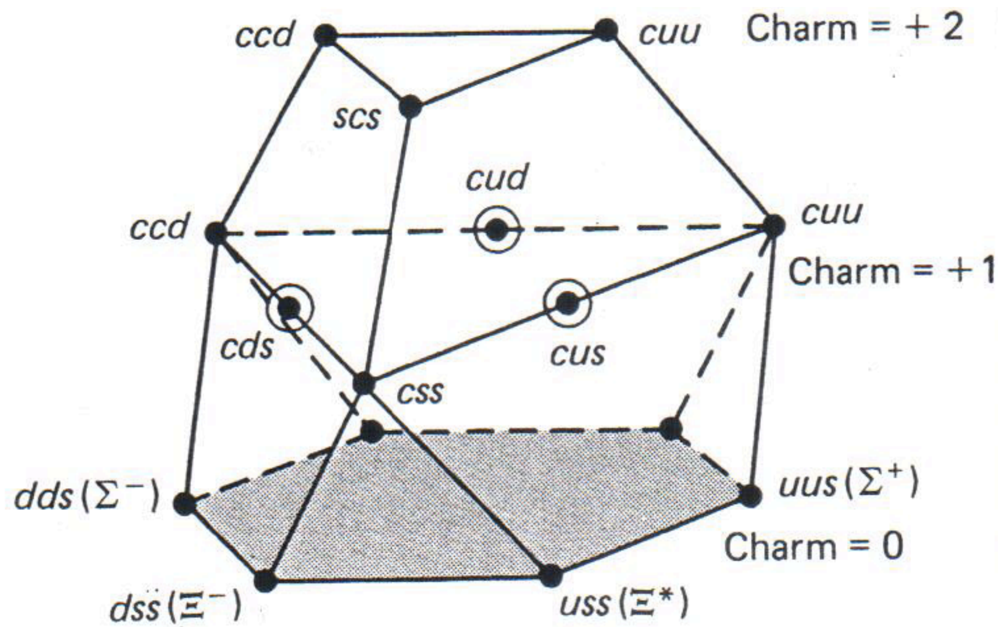
Mesons



Baryons

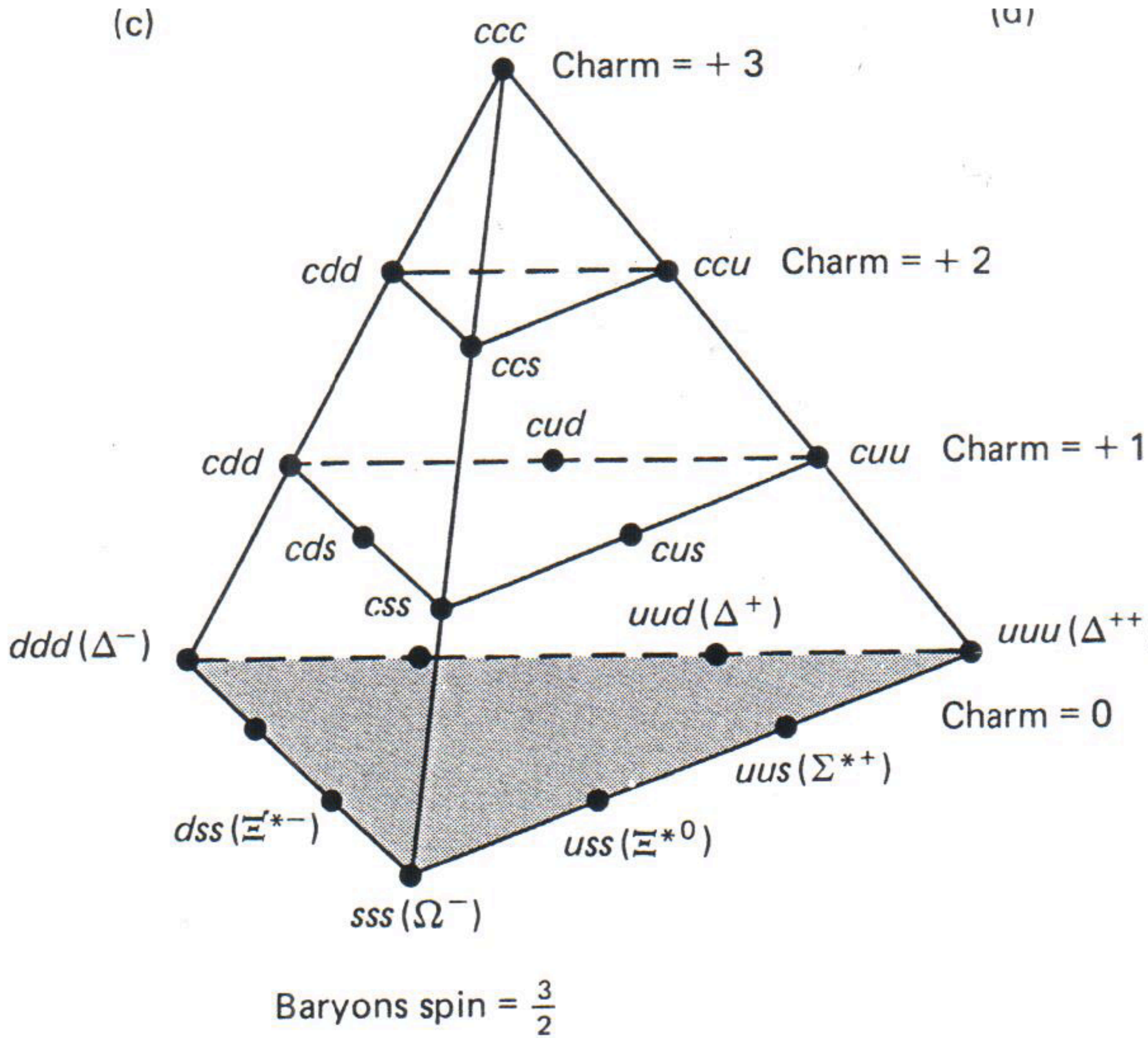


Baryons



Baryons spin = $\frac{1}{2}$

Baryons (u,d,s,c) spin 3/2



Two generations of quarks and leptons

- *up + down quarks with electron and* ν_e
- *charm and strange quarks with muon and* ν_μ

The value of R above cm energy of 4 GeV was still too large, and then τ lepton was discovered. It did not fit anywhere.

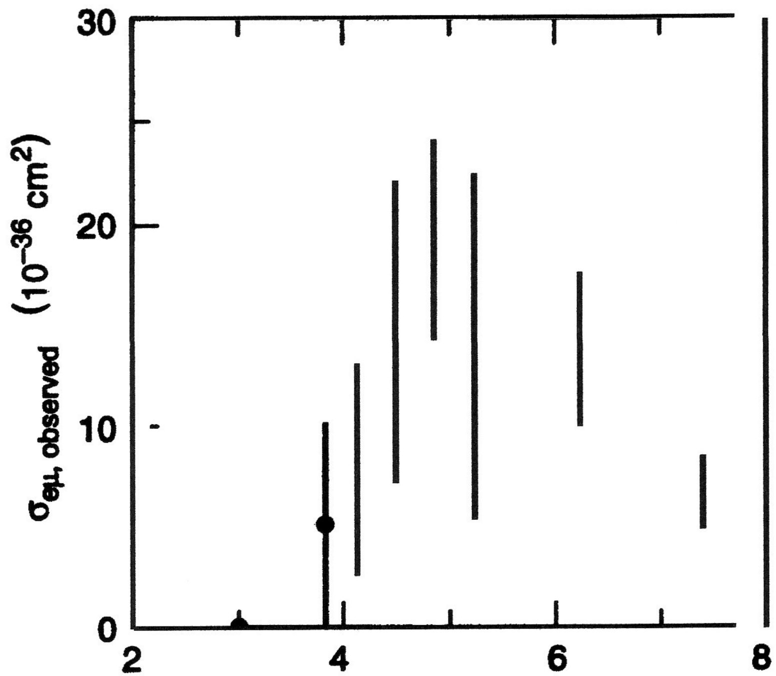
Need one more group – third generation of quarks and leptons with τ – charged lepton - being the first element of this group observed

Expect two more quarks and another neutrino bottom and top quarks, tau neutrino.

$$e^+ + e^- \rightarrow \tau^\pm + \tau^\mp$$

$\tau^\pm \rightarrow e^\pm \nu\bar{\nu}$, $\tau^\mp \rightarrow \mu^\mp \nu\bar{\nu}$

Final state : an electron – muon pair + missing energy



Q = 2*E* (GeV)

em production
weak decay

Evidence for production of pairs of heavy leptons $\tau^\pm$

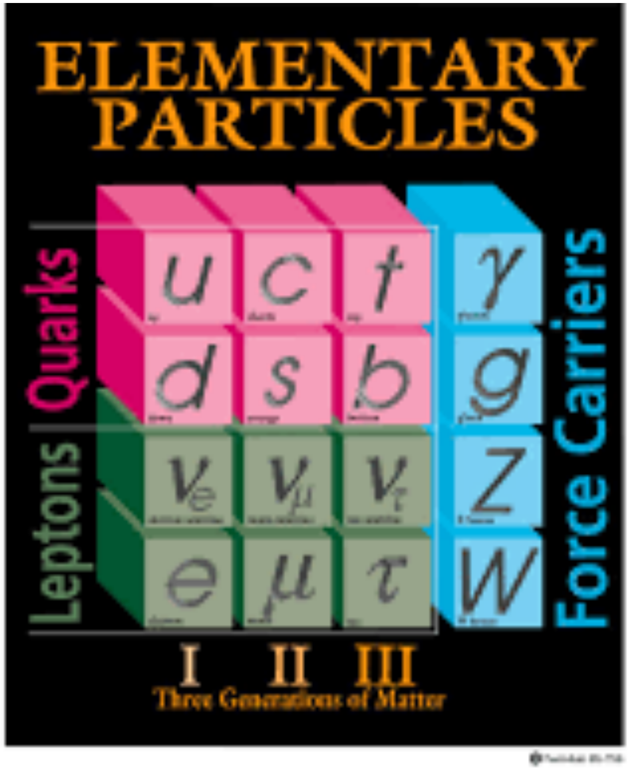
Completely unexpected – required establishment of a third family of quarks and leptons.

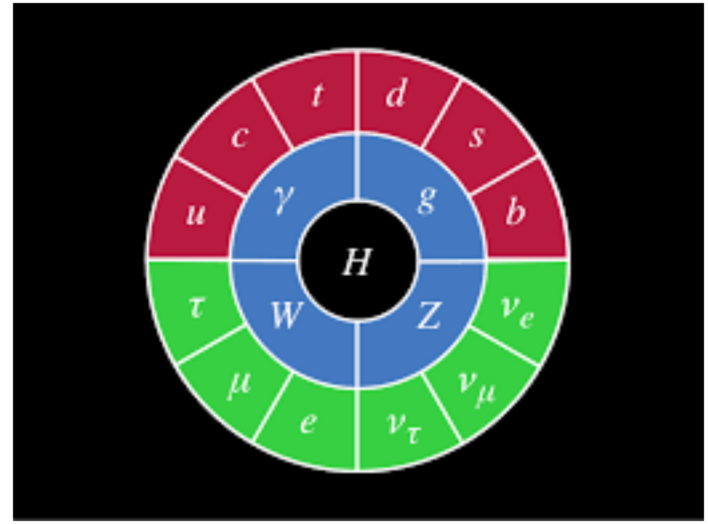
The parton model - Feynman, Yang, Bjorken

- *Quark content of the proton and neutron led to a question how the quarks are distributed inside. Feynman noticed that uncertainty principle allows for appearance of virtual quark-antiquark pairs: "wee" or "sea" quarks. The proton would then have three "valence" quarks providing the global characteristics and a sea of the virtual "wee" quarks.*
- *The proton-proton or meson-proton collisions would consist of a sum of valence-valence , valence-sea and sea-sea quarks collisions.*
- *As a consequence – we should observe "limiting fragmentation" where collisions between valence quarks carry most of the parents energies and the resulting distributions of secondaries depends solely on the distributions of energy carried by valence quarks. Sea quarks collisions should result in low cm energy secondaries and should be produced centrally in the cm rest frame. Since the number of virtual states depends on the energy, this central production of secondaries should increase proportionally to the energy of the parent particles collision.*

•*There is no hint in the theory to explain the origin of a distribution of energy carried by individual types of quarks inside the proton or neutron. Instead the pdf (parton distribution functions) are derived from a series of measurements using e± and neutrino and antineutrino beams. These are then used in Monte Carlo simulation of various physics processes. Fred Olness and Pavel Nadolsky are among leaders of CTEQ Collaboration doing that work.*

•*Aside: Even though we are able to simulate various processes with very high precision there is no understanding of the origin of pdf's and even more disturbing there is no understanding of the origin of the proton spin. Now that Higgs particle has been discovered there is an additional problem – we do not understand how proton gets its mass.*





Problem with number (3 generations with increasing masses)

6 quarks + 6 antiquarks (each in 3 colors)
3 charged leptons + 3 charged antileptons
3 neutrinos + 3 antineutrinos (only left-handed)
} *fermions spin = 1/2*

γ, Z, W^+, W^-, 8 gluons carrying color + anticolor } *bosons, spin = 1*
Higgs *spin = 0*

59 elementary particles

Symmetries

Conservation laws for strong and electromagnetic interactions (typically production and fast decays in high energy collisions)

Energy
Momentum
Angular momentum
Parity
Charge
Time reversal
Isospin
Baryon number

New quantum numbers:
flavor
color

Strong and weak interactions

THE MODERN THEORY OF STRONG INTERACTIONS:

the interactions between quarks based on "Colour Symmetry" Quantum ChromoDynamics (QCD) formulated in the early 1970's

- **Each quark exists in three states of a new quantum number named "color"**
- **Particles with color interact strongly through the exchange of spin 1 particles named "gluons", in analogy with electrically charged particles interacting electromagnetically through the exchange of spin 1 photons**

A MAJOR DIFFERENCE WITH THE ELECTROMAGNETIC INTERACTION

Electric charge: positive or negative

Photons have no electric charge and there is no direct photon-photon interaction

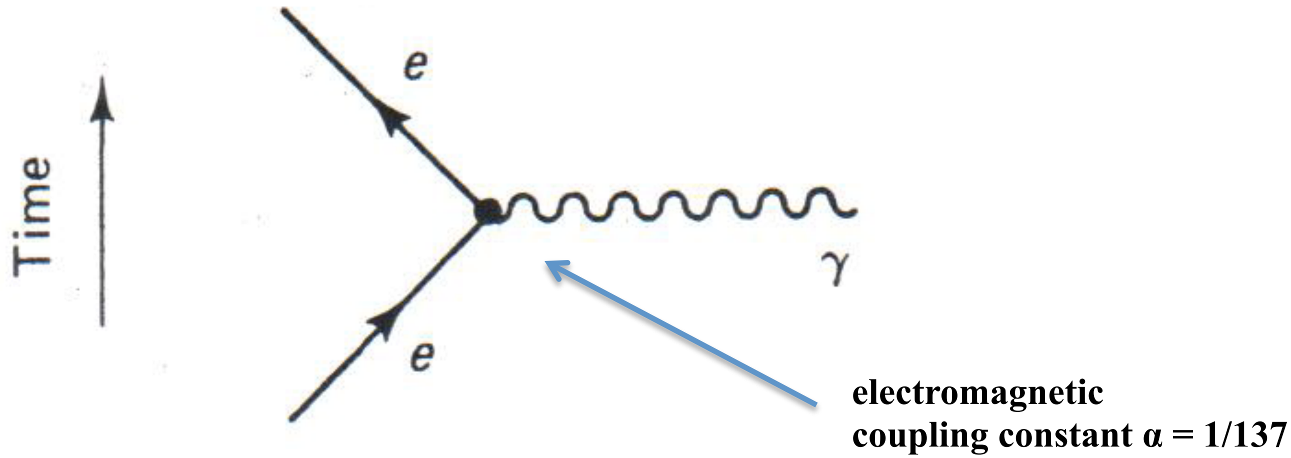
Colour: three varieties

Mathematical consequence of color symmetry: the existence of eight gluons with eight variety of colors, with direct gluon – gluon interaction

- **The observed hadrons (baryons, mesons) are colorless combinations of colored quarks and gluons**

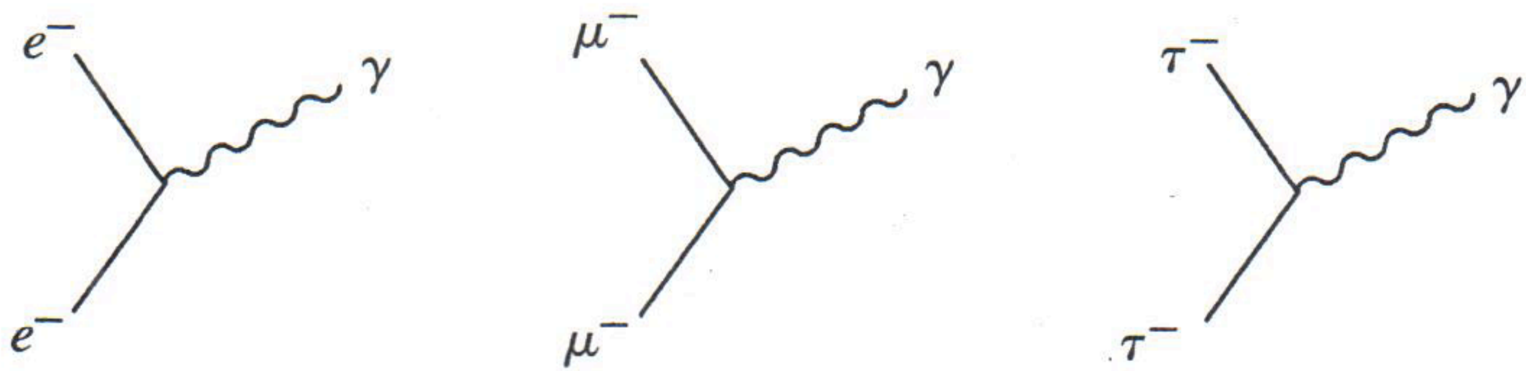
- **The strong interactions between baryons, mesons is an "apparent" interaction between colorless objects, in analogy with the apparent electromagnetic interaction between electrically neutral atoms**

Electromagnetic interactions in quantum electrodynamics



Here photon is virtual, i.e., it has an energy equivalent to mass

True for any charged particle, e.g., charged leptons, quarks,⋯



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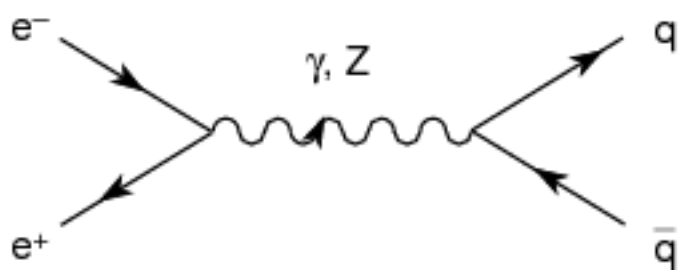
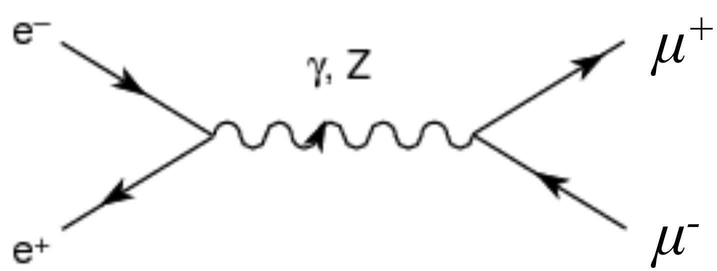


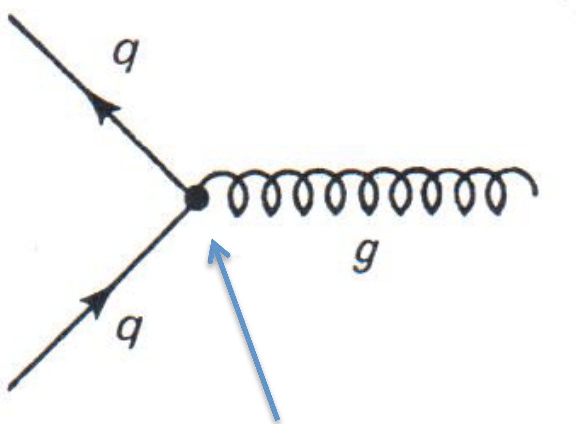
Weak interactions can proceed via
exchange of weak boson Z.
Similarity allowed for unification
of mathematical description of weak and
electromagnetic interactions
→ electroweak interactions

Left side of this diagram is the same for annihilation into pairs of muons

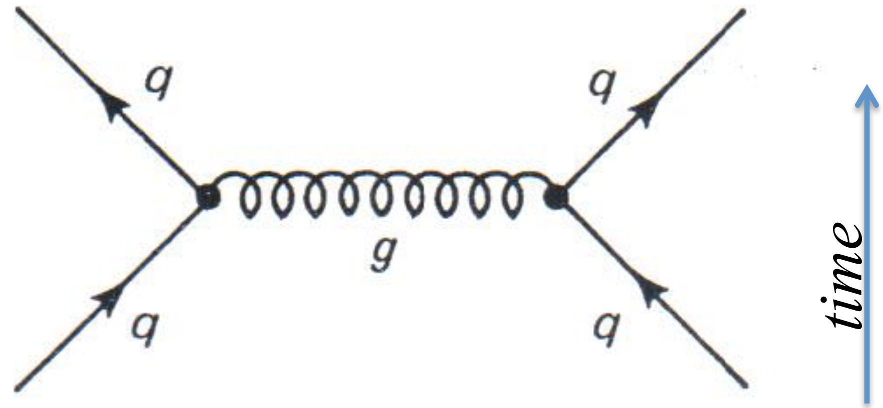


The ratio of cross sections is called R

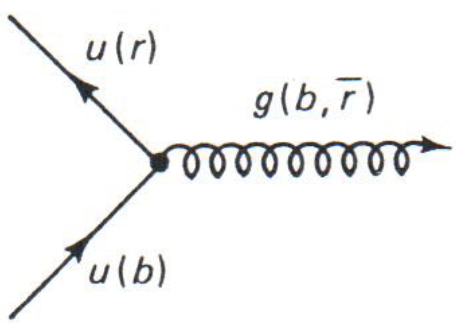
*Carrier of the **strong** force - gluon*



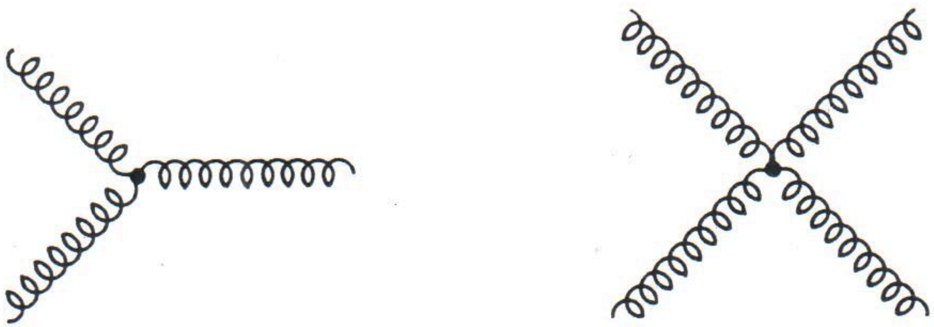
strong coupling constant $\alpha_s \sim 0.1$



Gluon carries color-anti-color combination



Since gluons carry color, they can also couple to themselves
Recall, photon does not carry the charge and cannot couple to itself.



QCD postulates quark confinement – all hadrons (baryons and mesons) are formed as color singlets (no color) out of quarks and gluons.
The process of "hadronization" i.e., formation of individual hadrons or jets of hadrons out of a single quark is not addressed by the theory.
Phenomenological models are based on data obtained in e+e- collisions, electron – proton interactions and neutrino – proton/neutron collisions.
These are usually based on a picture of breaking of a color string where a new hadron is produced out of vacuum at each break. This picture is based on a QCD postulate that the strong force increases with distance.
F ~ -kx, i.e., quarks cannot get out but are asymptotically free at very short distances.

Free quarks, gluons have never been observed experimentally; only indirect evidence from the study of hadrons – WHY?

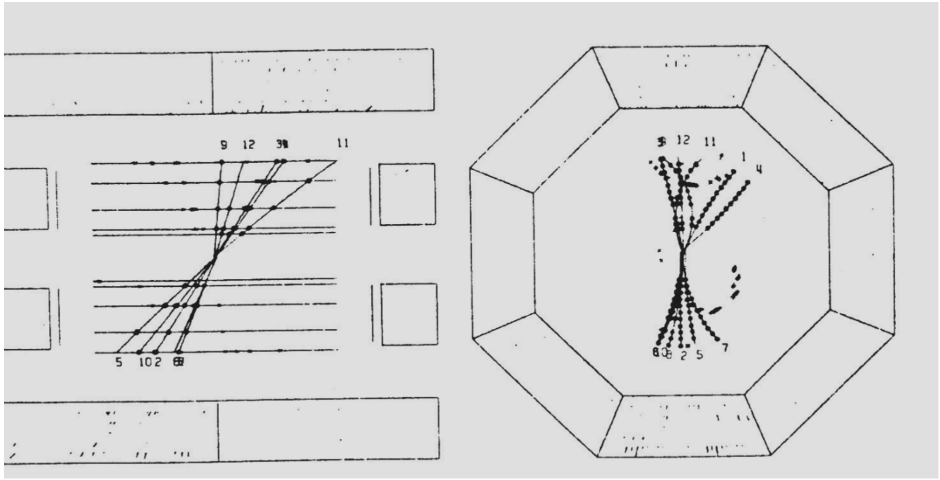
CONFINEMENT: colored particles are confined within colorless hadrons because of the behavior of the color forces at large distances

The attractive force between coloured particles increases with distance → increase of potential energy → production of quark – antiquark pairs which neutralize color → formation of colorless hadrons (hadronization)

At high energies (e.g., in $e^+e^- \rightarrow q + \bar{q}$) expect the hadrons to be produced along the initial direction of the $q - \bar{q}$ pair → production of hadronic "jets"

CONFINEMENT, HADRONIZATION: properties deduced from observation. So far, the properties of color forces at large distance have no precise mathematical formulation in QCD.

$e^+ + e^- \rightarrow$ hadrons
A typical event at
$Q = 2E = 35$ GeV:
reconstructed
charged particle tracks



A typical proton-antiproton collision at the CERN $\bar{p}$ p collider (630 GeV) producing high-energy hadrons at large angles to the beam axis (UA2 experiment, 1985)