Accelerators Ideal Case

Goal of an accelerator: increase energy of CHARGED particles

- Increase energy

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q \int_{\vec{r}_1}^{\vec{r}_2} (\vec{E} + \vec{v} \times \vec{B})\, d\vec{r}$$

- The particle trajectory direction d**r** parallel to **v**

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q \int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{r} = qU$$

- **…**increase of energy with electric fields

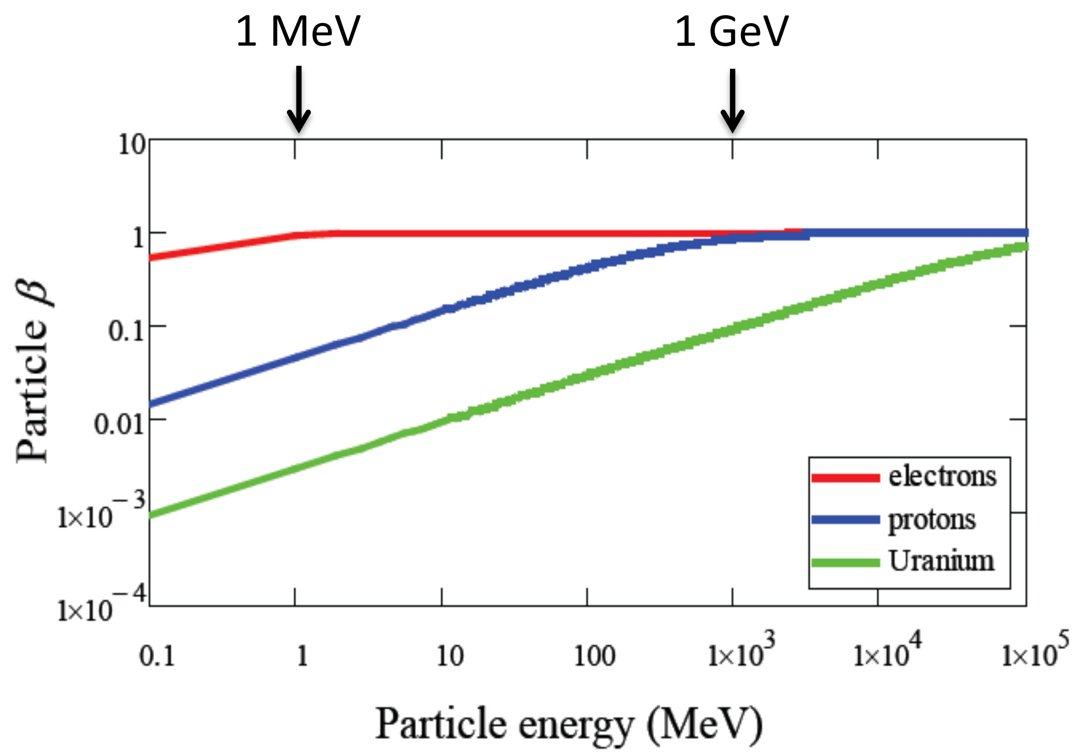
- Magnetic fields are used for control of trajectories

Energy vs velocity

- Acceleration increases velocity of a particle at small $\beta = v/c$.
- For high β velocity does not change. There is a change of momentum/ energy that can be expressed as a change of effective mass

Particle rest mass:

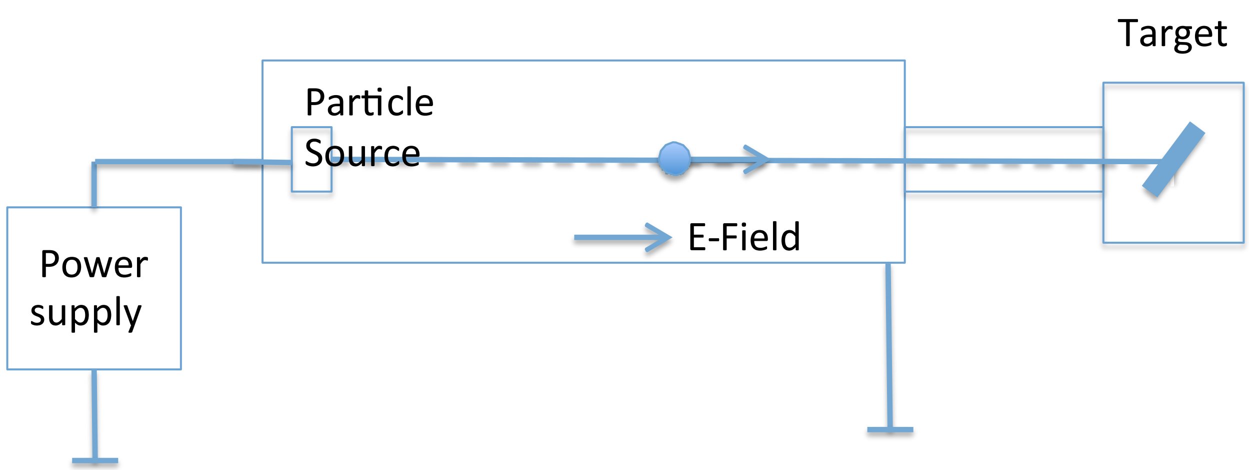
electron 0.511 MeV
proton 938 MeV
^{239}U ~220 000 MeV



$$\gamma = \frac{E}{E_0} = \frac{m}{m_0}$$

Electrons
Electrostatic accelerator

Set up electrostatic potential along particle trajectory.
Charged particles go through the accelerating voltage gap



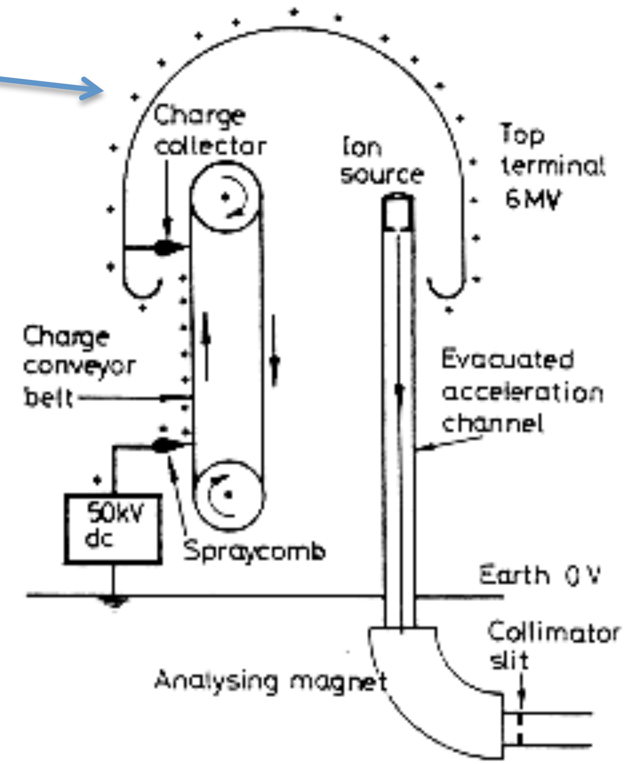
Limited by the maximum reachable voltage: ~ 10 MV

(For potentials above 10 MeV the electrostatic force may strip orbital electrons from the atoms of the material from which the accelerator is constructed creating sparks, breakdowns of the material and general mayhem).

van de Graaf Accelerator

Creation of high voltages by mechanical transport of charges

Terminal Potential: U ≈ 12 ...28 MV
(use of high pressure gas to suppress discharge)
•Particle energy limited by high voltage discharges
•High voltage can only be applied once per particle



1928-1932

Cockcroft-Walton Generator

Electric circuit that generates a high DC voltage from a low voltage AC or pulsing DC input

Technically: rectifier circuit, built of capacitors and diodes

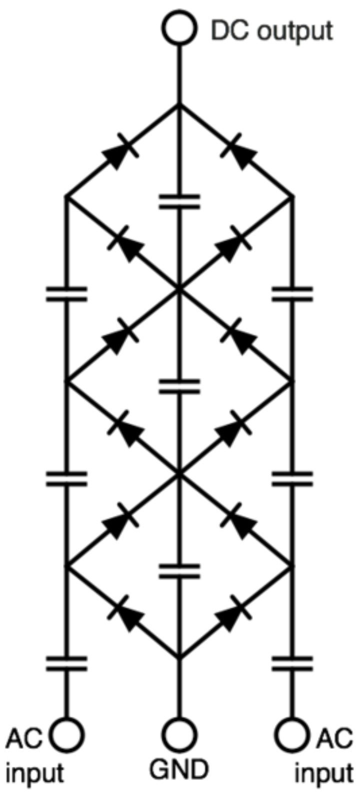
Particle source – hydrogen discharge tube at 400 kV

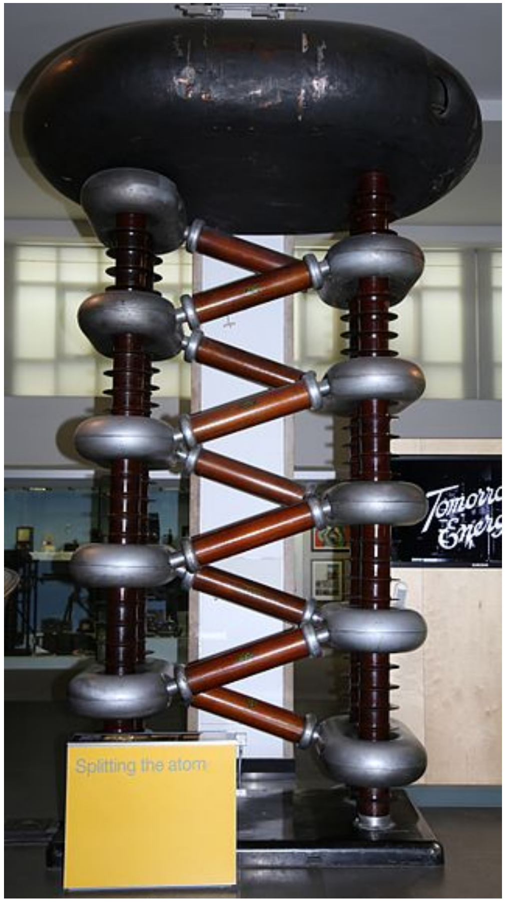
Used to split Li nuclei by bombardment with protons with 400 keV

Simple, cheap and robust.

Limitation: electric discharge
due to too high voltage.

Practical limit ~ 1MV



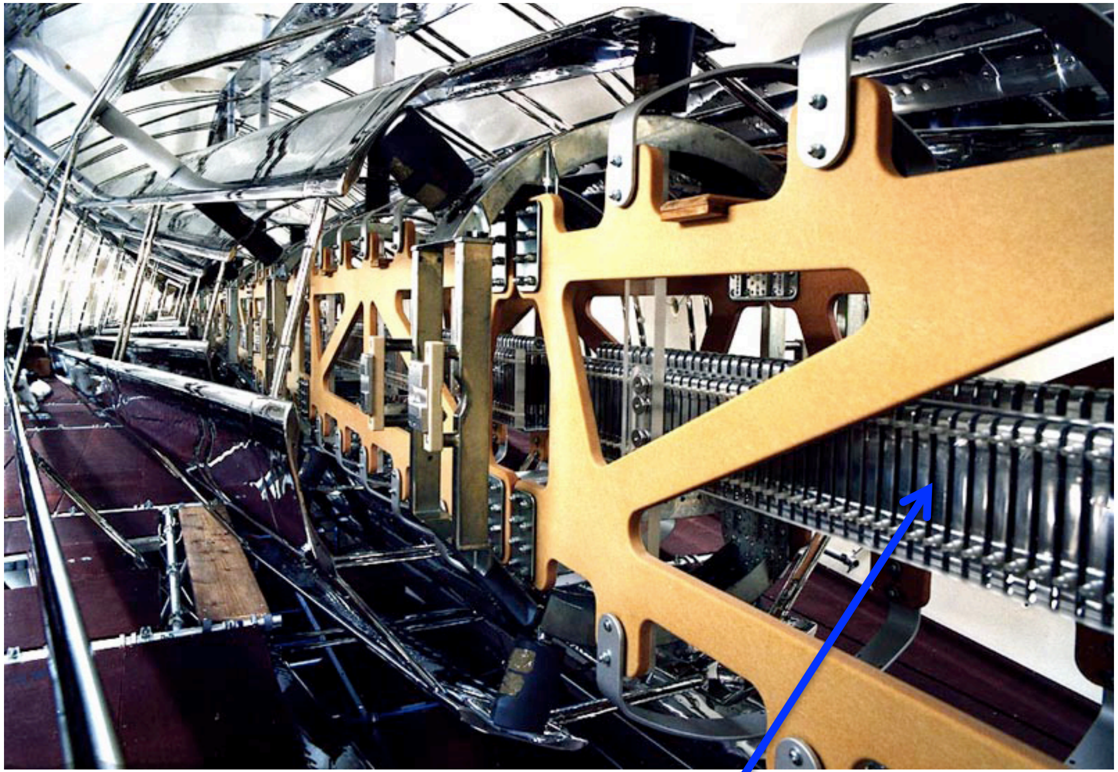


Tandem

Tandem principle: Apply the accelerating voltage twice by working with negative ions (e.g. H-) and stripping the electrons in the centre of the structure.

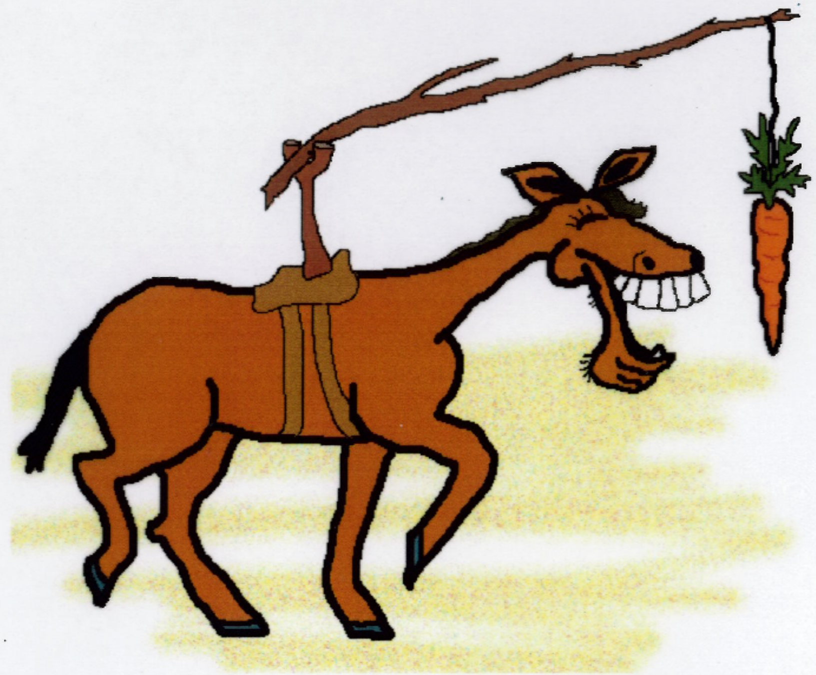
- negatively charged ions are accelerated through one potential difference before being stripped of two or more electrons inside a high voltage terminal and accelerated again

12 MV-Tandem van de Graaff accelerator at MPI Heidelberg



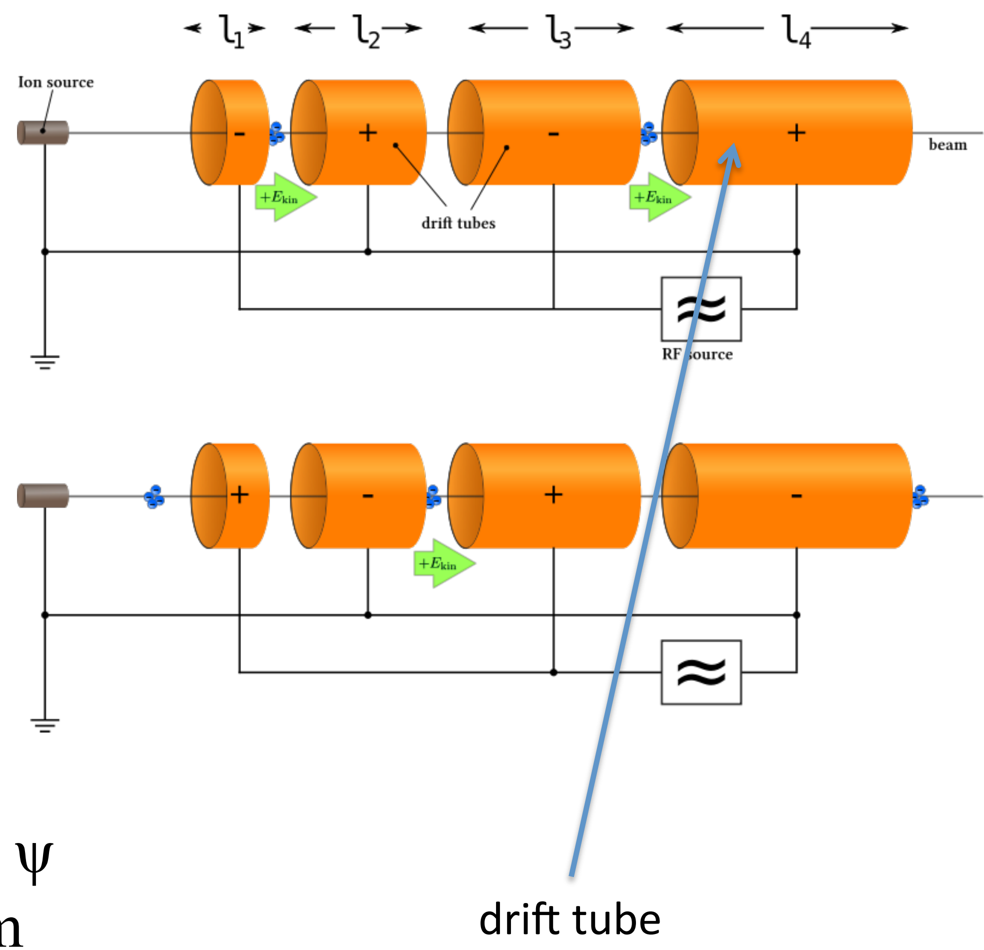
Accelerating structure

Principle of Acceleration Technique



1928 Wideroe **Linac – First RF Accelerator**

Acceleration voltage applied several times to the particle beam



- Acceleration in the gap
- RF – voltage has to be flipped to get the right sign in the next gap
- Shield particle in a drift tube during the negative half-wave of the RF voltage

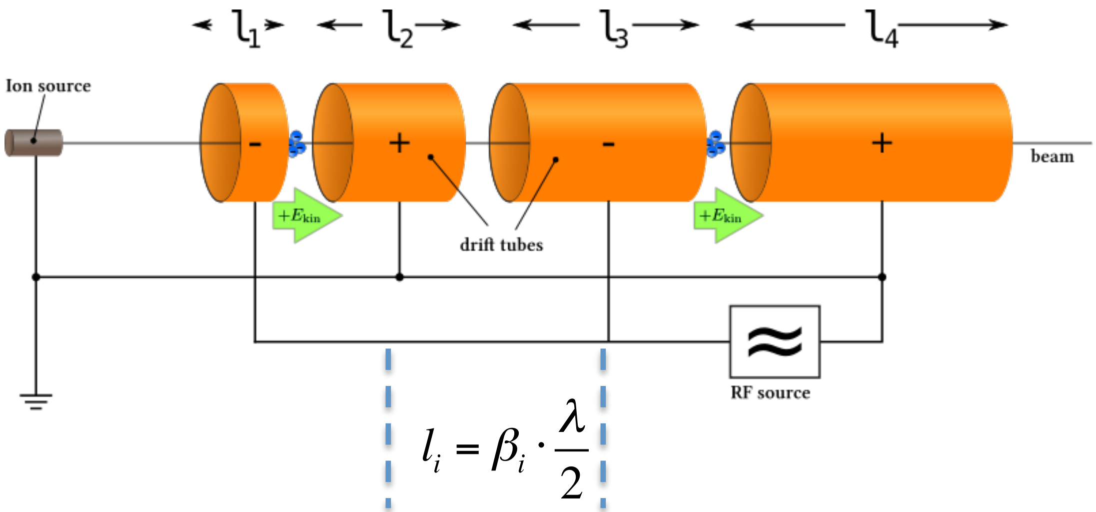
Energy after n gaps ~ n * q * U * ψ

U - peak voltage of the RF system

ψ – synchronous phase of the particle

Alternating RF Field

Apply the same voltage through acceleration gap many times.



Energy gain per gap:

$E = q\,\mathbf{V}_{RF}\sin(f_s)$

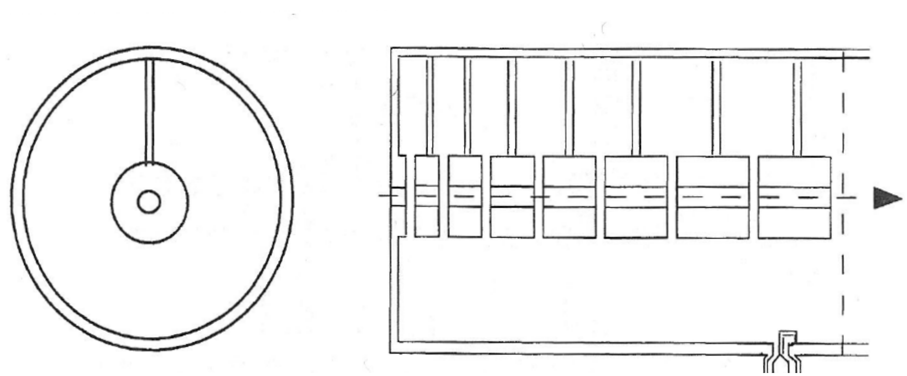
fs…phase wrt to RF field

- Particle synchronous with field. In shielding tube when field has opposite sign. Voltage across each cell the same.

- Shielding tubes have to become longer and longer, as particles become faster and faster or frequency must become higher $l = c/f_{RF}$
- Problem - radiation power loss: $P = \omega_{RF} C V_{RF}^2$, C - gap capacitance

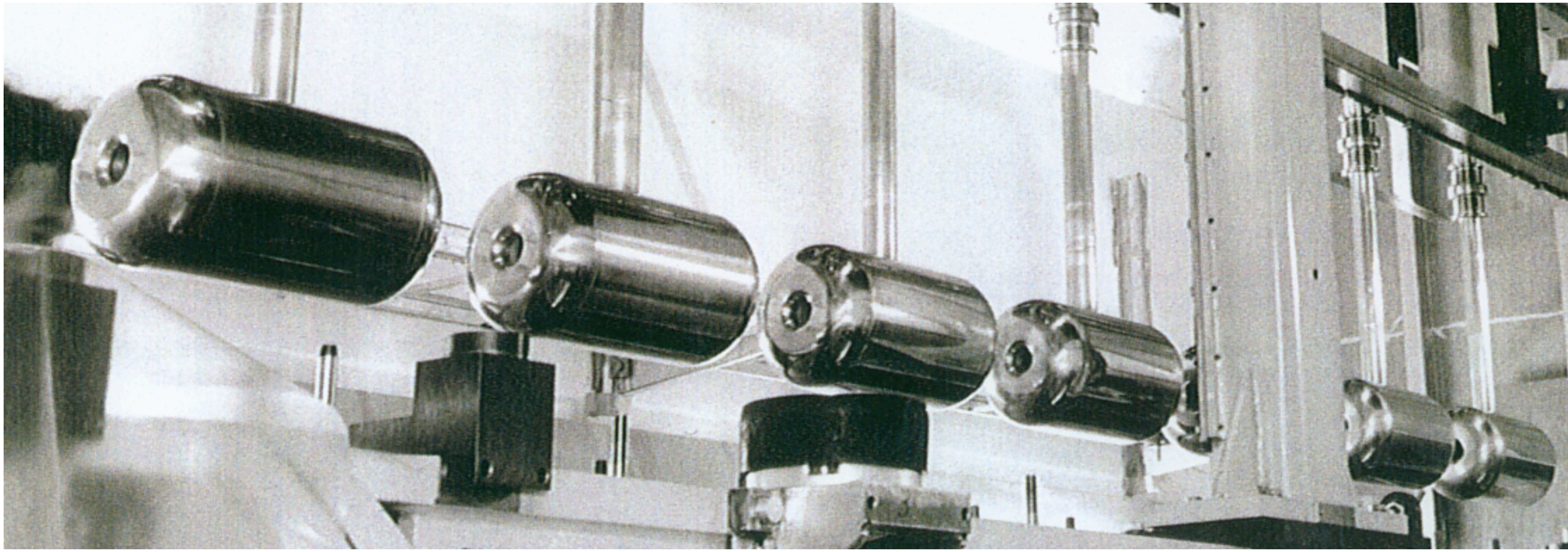
Drift Tube Linac (Alvarez)

- To eliminate power loss place drift tubes in a RF cavity
  - Electromagnetic field oscillating in cavity. Standing wave, TM mode
    TM – transverse magnetic mode of electromagnetic wave propagation imposed by the boundary condition – no magnetic field in the direction of propagation
    longitudinal E Field, transverse B Field
  - Resonant frequency of cavity = accelerating field frequency!
  - Reduces power loss
  - Exploit Farraday's law: $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$



Longest electron linac - SLAC 2 miles, can reach 50 GeV
Proposed International Linear Collider - ~20 miles can reach ~300 GeV

Accelerating structure of the proton linac at DESY



Proton rest mass $m_0c^2 = 938$ **MeV**

Total Energy $E_{tot} = 988$ **MeV**

Momentum $p = 310$ **MeV/c**

Kinetic energy $E_{kin} = E_{tot} - m_0c^2$ $E_{kin} = 50$ **MeV**

Relativistic relation: $E^2 = p^2c^2 + m_0^2c^4$

1930 –Livingston/Lawrence

Cyclotron

A cyclotron is a type of particle accelerator in which charged particles accelerate outwards from the center along a spiral path. The particles are held to a spiral trajectory by a static magnetic field and are accelerated by a rapidly varying electric field.

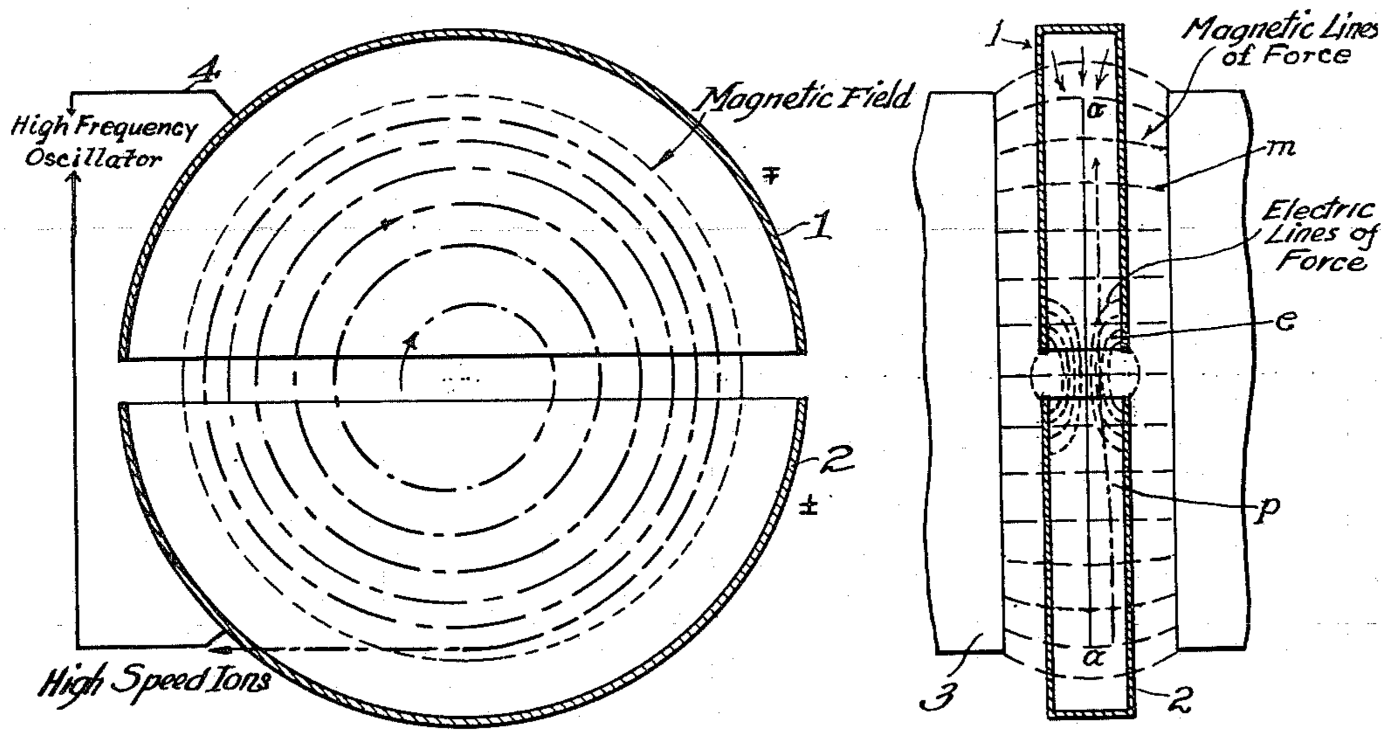


Diagram of cyclotron operation from Lawrence's 1934 patent. The "D" shaped electrodes are enclosed in a flat vacuum chamber, which is installed in a narrow gap between the two poles of a large magnet.

Lorentz force $$\vec{F} = q \bullet \left(\vec{v} \times \vec{B}\right) = qvB$$

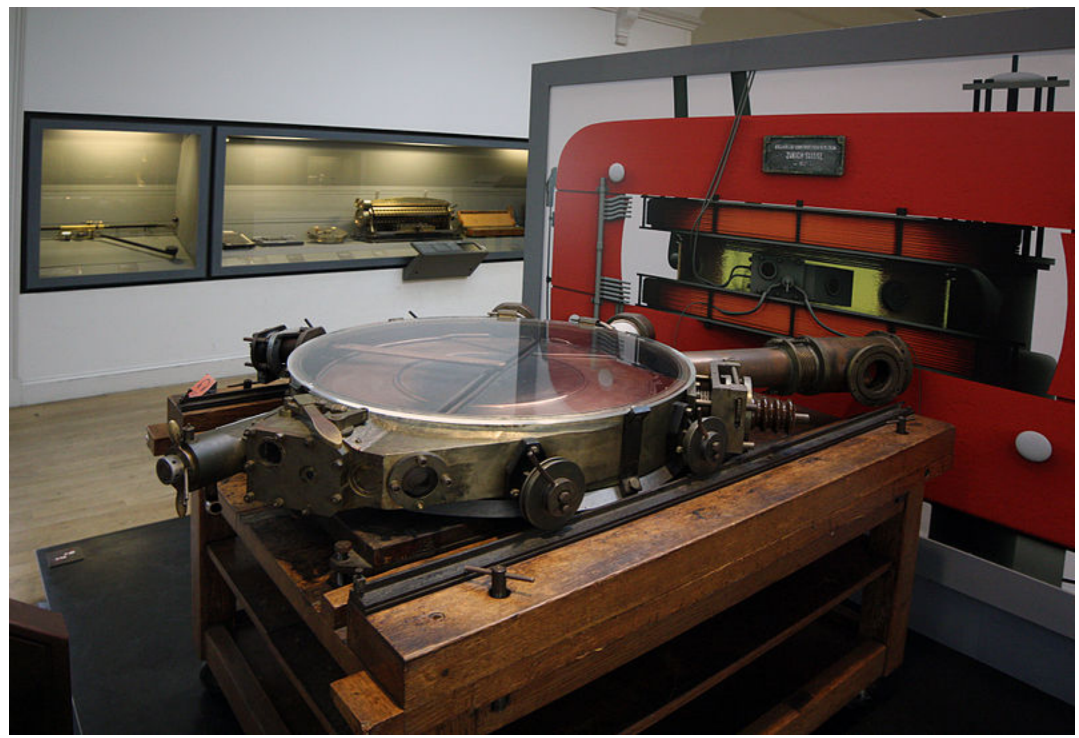
Circular orbit $qvB=mv^2/R$ implies $R= p/(qB)$ i.e., increasing radius for increasing momentum
-> spiral trajectory

Revolution frequency (rf) is independent of the momentum $$f = \frac{qB}{2\pi m}$$

Need huge magnets

proton/ion acceleration up to ~60MeV used for radiation therapy

At low energy frequency does not depend on the radius – mass is constant



1937 cyclotron in Zurich

Relativistic effects

Lorentz factor $$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Relativistic mass $m = \gamma m_0$

Relativistic cyclotron frequency $f = f_0/\gamma$

Here, m_0 is the rest mass and f_0 is the frequency in classical approximation.

To correct for relativistic effects **synchrocyclotron** has a frequency of the electric field varied to compensate for the apparent "increase" of mass.

Another solution is **isochronous cyclotron** where the magnetic field is increasing with radius and time.

1934 Wideroe, 1940 Kerst

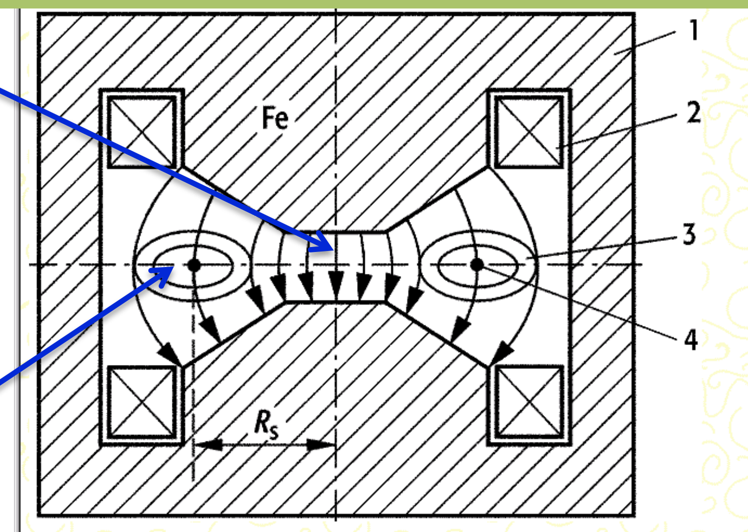
Betatron

Transformer with a torus-shaped vacuum tube as its secondary coil. An alternating current in the primary coils accelerates electrons in the vacuum around a circular path.
Time varying magnetic field induces a voltage that accelerates the particles. No RF system, just changing magnetic field.

Magnetic guide field

Accelerating electric field

schematic design



Betatron: Querschnitt durch ein Betatron. 1: Magnetkörper, 2: Spulenkörper, 3: Beschleunigerkammer, 4:

Initially used to accelerate electrons up to about 300 MeV (limited by synchrotron radiation).
Also used in medical applications: energetic electrons impinging on a metal plate are a source of hard X-rays.

1952 Courant, Livingston, Snyder

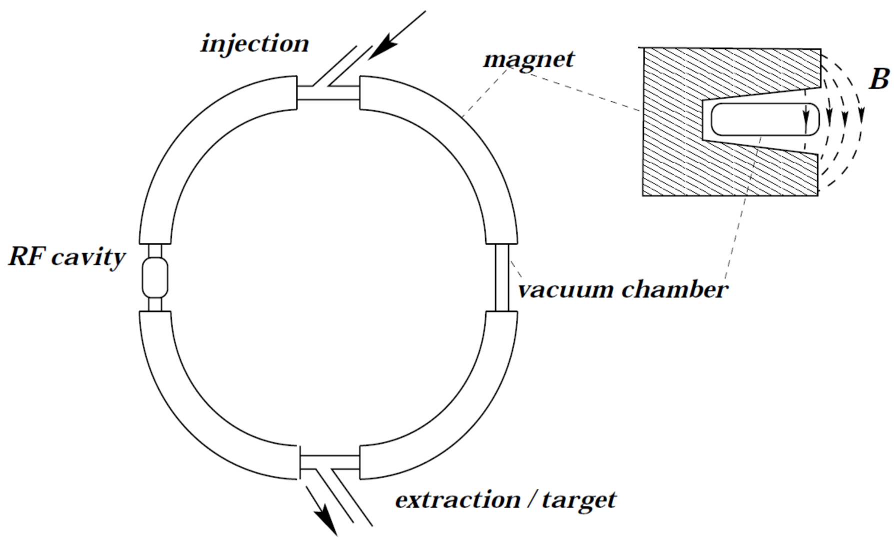
Synchrotron

The guiding magnetic field (bending the particles into a closed path) is time-dependent, being synchronized to a particle beam of increasing kinetic energy.

Idea: define a circular orbit of the particles, keep the beam there during acceleration, put magnets at this orbit to guide and focus.

Derivatives:

- Storage rings – synchrotron in which the kinetic energy of the particles is kept constant
- Colliders – combination of two storage rings with beams circulation in opposite directions
- Synchrotron light sources – combination of different accelerators to produce intense source of synchrotron radiation and X-rays.



Time Varying Fields

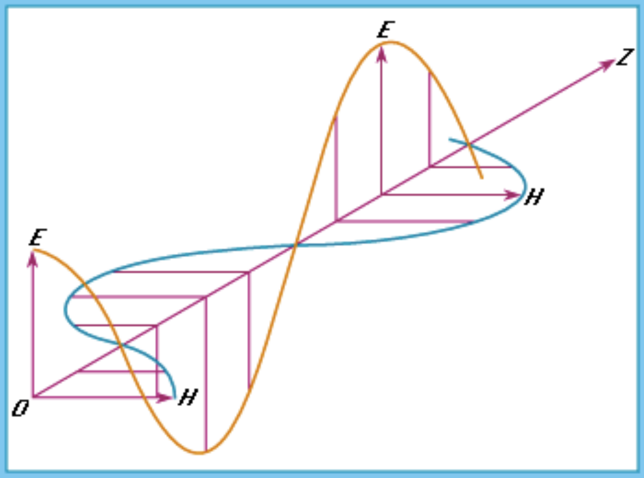
Plane electromagnetic wave

$$\vec{E} = \vec{E}_0 \cdot e^{i k \vec{n} x - \omega t}$$

$$\vec{B} = \vec{B}_0 \cdot e^{i k \vec{n} x - \omega t}$$

$$\vec{B}_0 = \sqrt{\mu\varepsilon} \cdot \vec{n} x \vec{E}_o$$

$$k = \frac{2\pi}{\lambda}$$



Wave equations

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{c^2}{\mu\varepsilon} \nabla^2 \vec{E} \qquad \frac{\partial^2 \vec{B}}{\partial t^2} = \frac{c^2}{\mu\varepsilon} \nabla^2 \vec{B}$$

No acceleration in the direction of propagation

Boundary conditions

Transverse electric waves (TE)

$E_z = 0$ everywhere

$$\frac{\partial B}{\partial n}\Big|_s = 0$$ at boundary

Transverse magnetic waves (TM)

$B_z = 0$ everywhere

$E_n|_s = 0$ at boundary