PHYS 5380: The Weak Interaction, Quark Mixing and the CKM Matrix

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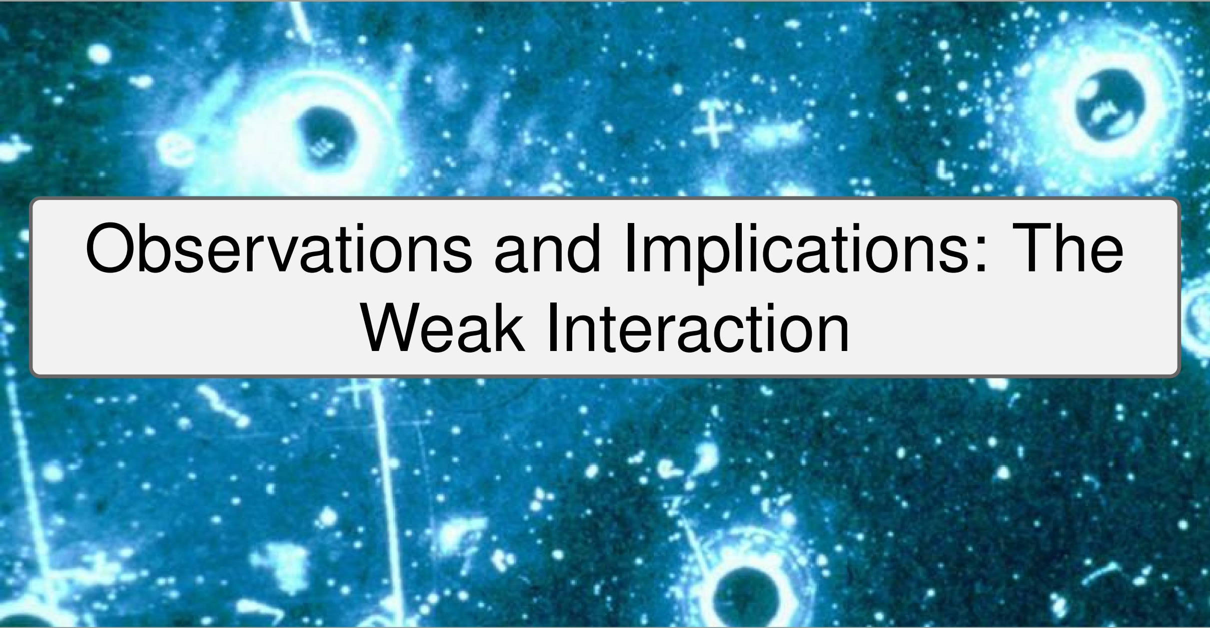
SMU | Dedman College of Humanities & Sciences

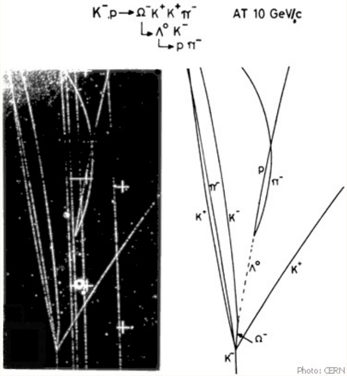
Outline



Observations and Implications: The Weak Interaction

Observations, 1963 (I)

- Heavy particles that are produced very readily in nuclear collisions but which decay very slowly despite their heavy mass are said to contain "strangeness," a new quantum number that constrains how such particles can decay. Examples: $K^{\pm}$, K^0, $\Sigma^{\pm}$, ...
- We can assume that strangeness (S) takes values of 0 for particles without this property and non-zero values for when it does contain this property.
- We can ask questions like: do particle interactions or decays where $|\Delta S| = 0$ happen at the same rate as those with $|\Delta S| \neq 0$?



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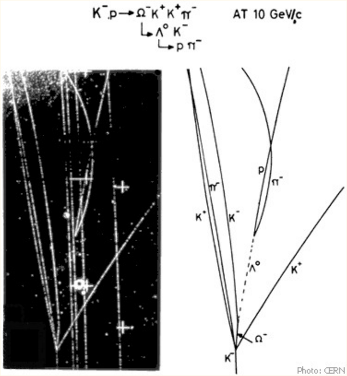

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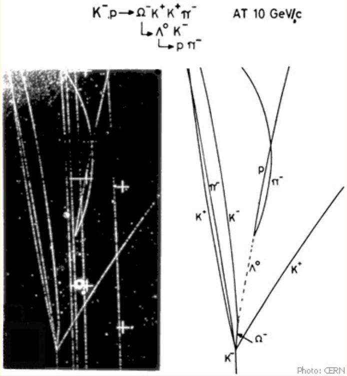

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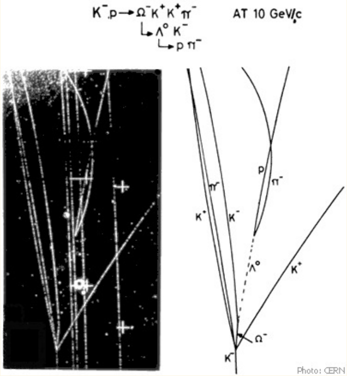

Observations, 1963 (II)

- Reactions in which kaons decay only to pions (or leptons) exhibit $|\Delta S| \neq 0$ (e.g. $K^0 \to \pi^+\pi^-$ or $K^+ \to \pi^+\pi^0$). Kaons contain strangeness, while pions do not. Therefore, the strangeness quantum number is not conserved in this reaction.
- Reactions in which pions decay to leptons (e.g. $\pi^+ \to \mu^+\nu_\mu$ or $\pi^+ \to e^+\nu_e$) exhibit $|\Delta S| = 0$, since neither pions not leptons possess "strangeness"
- Can we learn anything about the weak interaction by comparing the probabilities with which such processes occur?
  - For instance, how does the rate of $\pi^+ \to \mu^+\nu_\mu$ compare to the rate of the very similar process $K^+ \to \mu^+\nu_\mu$?
  - Alternatively, how does the rate of $\pi^+ \to \mu^+\nu_\mu$ compare to that of $\pi^+ \to e^+\nu_e$?
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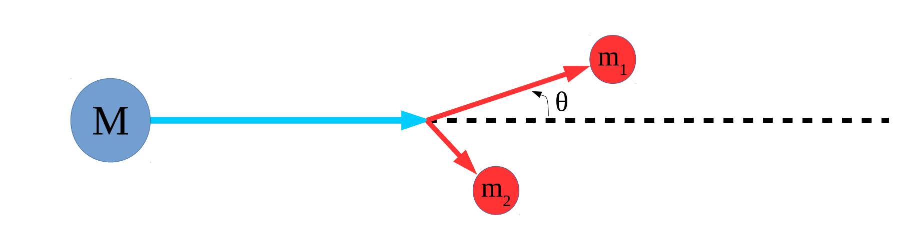
Important Aside: Rates of Decay

Let us consider a (nearly) fully classical physics process — the inelastic process of a single ball of mass M separating in flight into a pair of smaller masses, m_1 and m_2, subject to the constraint that $M \geq m_1 + m_2$.



- We can ask some critical questions at this point:
  - What is the probability of finding mass m_1 (or m_2) as some angle, θ, relative to the original mass' direction of flight?
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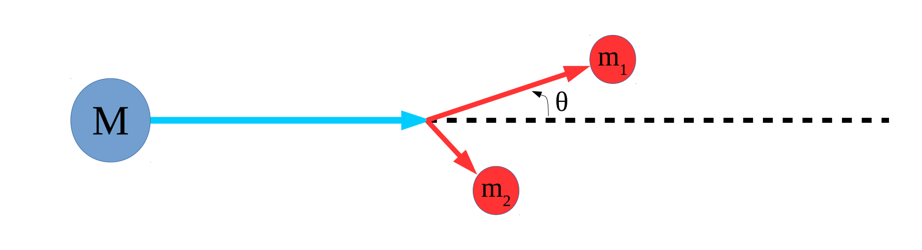

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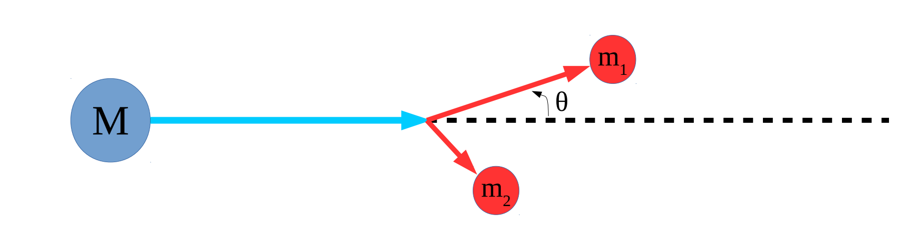

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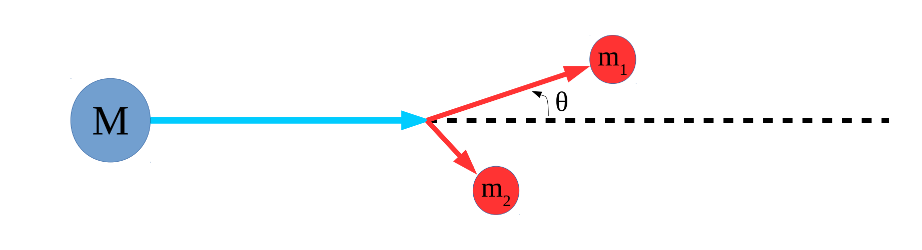

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Important Aside: Rates of Decay (II)

In a quantum theory, there is more to a reaction (like particle decay) than just kinematics (e.g. how energetically favorable or not is an outcome). There is also the inherent strength of the interaction.

- Think of electromagnetism. In a process that involves both charge and energy conservation, both factors play a role. If the strength of charge is reduced, the reaction or interaction becomes less likely.

We could imagine relating the probability of some outcome (P) to these two factors (a function of the coupling strength, $f(g)$, and kinematics, $K(p, m)$) schematically as follows:

$$P \propto f(g) \cdot E(p, m) \tag{1}$$

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Example: pion decay rate ratios

Without worrying too much about *why* the kinematic factors have the form you see below (it can be derived [1]), let us answer the following question: *does the weak interaction know the difference between* $\pi^+ \to \mu^+\nu_\mu$ *and* $\pi^+ \to e^+\nu_e$?

Quantity	Value [2]
$P(\pi^+ \to \mu^+\nu_\mu)$	0.9998770(4)
$P(\pi^+ \to e^+\nu_e)$	$1.230(4) \times 10^{-4}$
m_π	$139.57061(24)\ \mathrm{MeV/c^2}$
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$$P \propto f(g) \cdot E(p,m) \longrightarrow \frac{P(\pi^+ \to \mu^+\nu_\mu)}{P(\pi^+ \to e^+\nu_e)} = \left(\frac{g_{\mu\nu_\mu}}{g_{e\nu_e}}\right)^2 \times \frac{m_\mu^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2}{m_e^2\left(1-\frac{m_e^2}{m_\pi^2}\right)^2} \qquad (2)$$

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Plug in numbers and solve for $(g_{\mu\nu_\mu}/g_{e\nu_e})$ and you will find...

$$\frac{g_{\mu\nu_\mu}}{g_{e\nu_e}} \approx 1.021 \tag{4}$$

Wow! Once you correct for the kinematic effects of the different masses of the muon and electron, it seems that the process that decays the pion into these final states *doesn't have a strong preference for whether there is a muon (+neutrino) or electron (+neutrino) in the final state.*

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Example: kaon and pion decay ratios

Repeat the preceding example, but this time compare the rates of $K^+ \to \mu^+\nu$ and $\pi^+ \to \mu^+\nu$. Using this ratio, what question are we probing this time?

The inputs to the calculation change a little...

Quantity	Value [2]
$P(\pi^+ \to \mu^+\nu_\mu)$	0.998770(4)
m_π	139.57061(24) $\mathrm{MeV/c^2}$
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m_K	493.677(16) $\mathrm{MeV/c^2}$
Pion Lifetime	$2.6033(5) \times 10^{-8}\mathrm{s}$
Kaon Lifetime	$1.2380(20) \times 10^{-8}\mathrm{s}$

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Observations, 1963 (final)

- The weak interaction doesn't care if you make a transition between $u \leftrightarrow d$, $e \leftrightarrow \nu_e$, or $\mu \leftrightarrow \nu_\mu$ — the weak interaction appears to be "universal"
- However, the weak interaction does seem to know the difference between a strange-carrying particle and a non-strange carrying particle, and it matters which kind decays even when everything else is the same.
- **How can both of these statements be true? How can the weak interaction not care about flavor (what kind of particle it transitions between) but also care *sometimes* about flavor?**

This is a version of the puzzle that a young Nicola Cabibbo (pictured right) worked on, and solved, in 1963 [3] while working at the CERN Laboratory in Switzerland.

Nicola Cabbibo lecturing

Observations, 1963 (final)

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A Delicious Aside: Spaces and Bases

My Space. No! My Space!

- Imagine that you and a roommate are going to plan how to put furniture in a room. You each go off with pen and paper (or iPad and App) and layout the room, with dimensions and positions in exacting detail.
- But when you get together over coffee to compare proposals, you find that you have each assumed a different coordinate system for your designs!
- What can you do to reconcile your ideas without one of you starting over from scratch?

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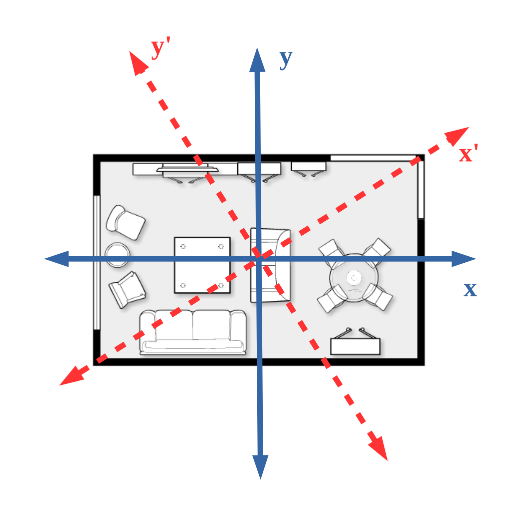

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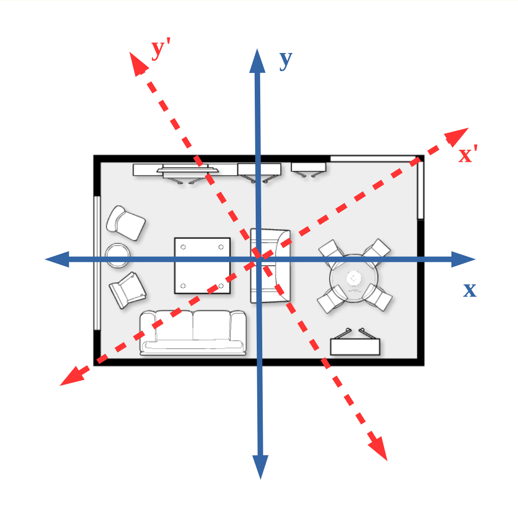

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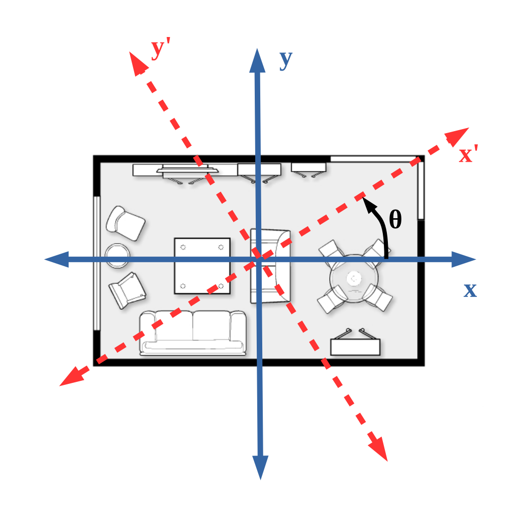
- Since each coordinate system is Cartesian with coinciding origins, there is an exact 90° angle between each x (x') and y (y') axis pair. That means the *orientation of one system with respect to the other is defined by a single angle*.
- Simply relate (x, y) to (x', y') as follows:

$$x' = x\cos\theta + y\sin\theta \quad (6)$$

$$y' = -x\sin\theta + y\cos\theta \quad (7)$$

- Your mutual hard work is not wasted — you can now map from one "basis" to the other "basis" using this simple transformation.



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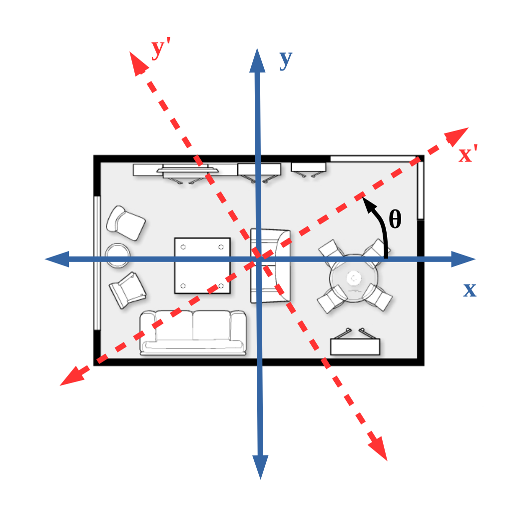

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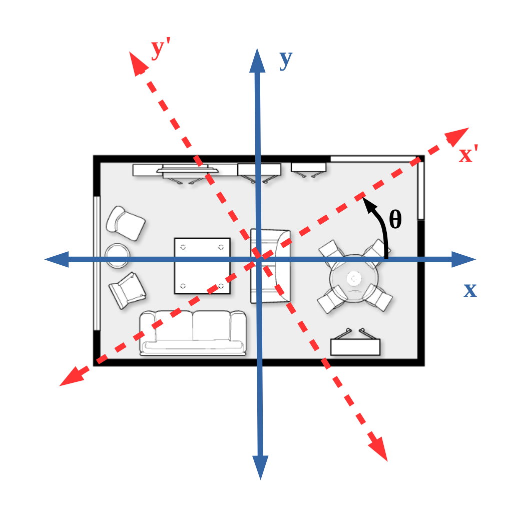

A more abstract example: real flavor and color

Imagine you have a bowl of jelly beans. How can you describe them? How is it that you can arrive at that description?

- You can look at them and see their color (red and green)
- You can taste (or smell) them and sense that there are cherry or lime flavors
- You can listen to them, but the atomic thermal vibrations of jelly beans at rest are faint and weak, probably unnoticeable
- You can touch them; they both feel smooth without obvious distinctions.

Using the above, make an analogy for the interactions that we have learned about so far in this course.

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What analogies can you make between these sense experiences with jelly beans and natural forces?

- Sight is a sense that detects a long-range effect (e.g. electromagnetism), and colors (red,green) are the distinct representations for this force.
- Taste (or smell) is a sense that detects a short-range effect (e.g. a nuclear force) because you have to be close to experience it, and the lime/cherry sensations are the distinct representations for this force.
- Hearing is a sense with long range, but the effect is so weak you cannot detect it (like gravity in subatomic interactions)
- Touch is a powerful short-range force, but the jelly beans have no distinct representation in touch — like being "uncharged" under that force

Predict: if you put a red jelly bean in your mouth, what will it taste like?

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Different Bases for Different Forces

The question, *"if you put a red jelly bean in your mouth, what will it taste like?"*, is of course loaded with cultural assumptions (red and green don't mean the same thing culturally to people from different backgrounds).

I tried to prime those of you with cultural backgrounds in the U.S. to assume that a red jelly bean will taste like a red cherry (color associated with taste) and a green jelly bean will taste like a green lime.

But in this experiment, when you put the red jelly bean in your mouth, it tastes like sweet cherry with a distinct touch of tart lime; the green ones taste like strong, tart lime with a distinct touch of sweet cherry.

How can we describe this strange reality using some of the ideas we've been developing: frames of reference and coordinate axes?

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Color and flavor: a mathematical basis (I)

- Represent color axes mathematically:

$$\hat{r}, \hat{g} \longrightarrow |r\rangle, |g\rangle \tag{8}$$

(in linear algebra, $|i\rangle$ represents a column vector, $\langle i|$ a row vector, with the property $|\langle i|i\rangle| = 1$)

- Represent flavor axes:

$$|c\rangle, |l\rangle \tag{9}$$

- Write one set in terms of the other:

$$|c\rangle = \cos\theta\,|r\rangle + \sin\theta\,|g\rangle \tag{10}$$
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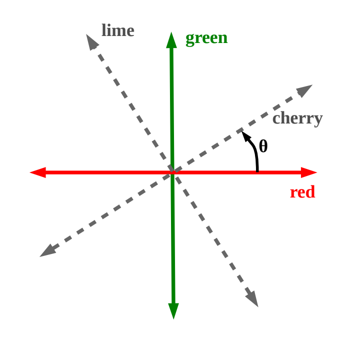

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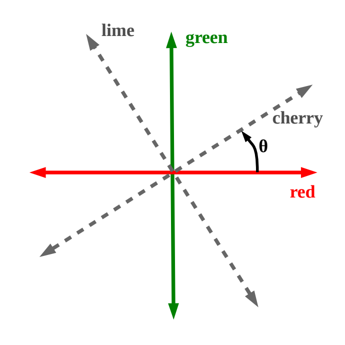

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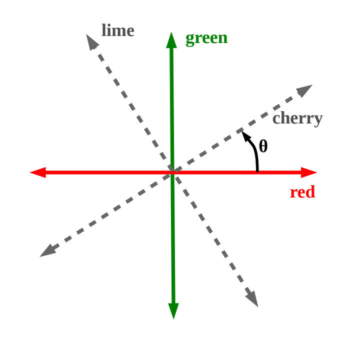

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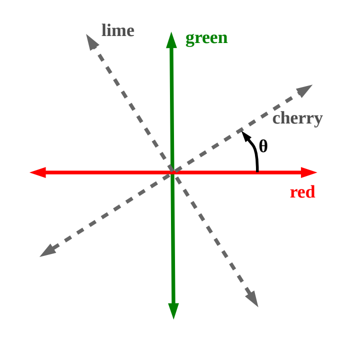

Color and flavor: a mathematical basis (II)

$$|c\rangle = \cos\theta\,|r\rangle + \sin\theta\,|g\rangle \tag{12}$$
$$|l\rangle = -\sin\theta\,|r\rangle + \cos\theta\,|g\rangle \tag{13}$$

For those of you with linear algebra experience, the above pair of equations will look more simple if represented as a 2×2 *rotation matrix* acting on a *column vector* to produce another column vector:

$$\begin{pmatrix} |c\rangle \\ |l\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |r\rangle \\ |g\rangle \end{pmatrix} \tag{14}$$

A comment on something you might hear in a later course: the column vector on the left (or the right) is not a simple pair of numbers; it's a pair of highly structured objects (each is itself a column vector). This kind of column vector is known as a "doublet".

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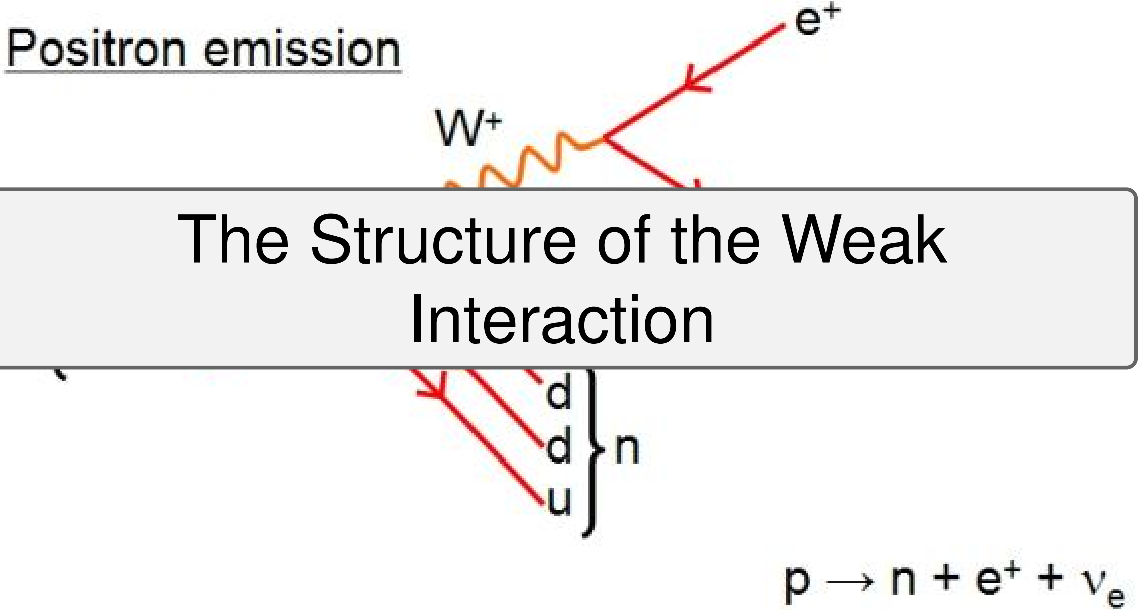
Color and Flavor: Answering Questions

Now that he have a mathematical description of a physical observation (that red and green jelly beans are not strictly cherry-flavored and lime-flavored, respectively), we can answer basic questions about measurements we might make on the beans:

- What is the probability that I pick up a red jelly bean and taste cherry?

$$\begin{aligned} P(\mathrm{cherry}|\mathrm{red}) &= |\langle c|r\rangle|^2 && (15)\\ &= |\langle c|(\cos\theta\,|c\rangle + \sin\theta\,|l\rangle)|^2 && (16)\\ &= |\cos\theta\,\langle c|c\rangle + \sin\theta\,\langle c|l\rangle|^2 && (17)\\ &= \cos^2\theta && (18)\end{aligned}$$

If $\theta = 13^\circ = 0.23\mathrm{rad.}$, then $P(\mathrm{cherry}|\mathrm{red})) = 0.95$. θ, in physics, is known as a "mixing angle" — how much the representation of one space (its "basis") is mixed up in the representation of another space.



The Structure of the Weak Interaction

Observations, 1963 (revisited)

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- However, the weak interaction does seem to know the difference between a strange-carrying particle and a non-strange carrying particle, and it matters which kind decays even when everything else is the same.
- **How can both of these statements be true? How can the weak interaction not care about flavor (what kind of particle it transitions between) but also care *sometimes* about flavor?**

Cabibbo was not working with quarks in 1963, but if we adapt his idea to the quark model there is a neat and elegant way of using the mixing concept to reconcile these seemingly disparate aspects of the weak interaction: its simultaneous universality of interaction strength with its suppression of strangeness-changing behaviors.

Nicola Cabbibo lecturing

Observations, 1963 (revisited)

- The weak interaction doesn't care if you make a transition between $u \leftrightarrow d$, $e \leftrightarrow \nu_e$, or $\mu \leftrightarrow \nu_\mu$ — the weak interaction appears to be "universal"
- However, the weak interaction does seem to know the difference between a strange-carrying particle and a non-strange carrying particle, and it matters which kind decays even when everything else is the same.
- **How can both of these statements be true? How can the weak interaction not care about flavor (what kind of particle it transitions between) but also care *sometimes* about flavor?**

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Quark Mixing

We can represent the three "known" quarks in the late 1960s/early 1970s in our notation for colors and flavors:

$$\mathrm{up} = |u\rangle\ ;\ \mathrm{down} = |d\rangle\ ;\ \mathrm{strange} = |s\rangle \tag{19}$$

Implicit in writing the above is the assumption that **there is only one basis in which we can represent the quarks.** But is that true?

- Possible bases (which may or may not be equivalent)

What if the "mass states" and the "weak states" do not coincide? What if they are separate, but equally valid, bases in which to consider quarks?

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Weak States and Mass States

Let the distinct states of the weak interaction be denoted $|u'\rangle$ and $|d'\rangle$. Let the distinct states of mass be denoted $|u\rangle$, $|d\rangle$, and $|s\rangle$.

Cabibbo's framework was to identify the up-type weak state $|u'\rangle = |u\rangle$ while identifying the down-type weak eigenstate as a *mixture*, $|d'\rangle = \cos\theta_c |d\rangle + \sin\theta_c |s\rangle$.

This solved the problem of universal weak interactions (transitions for $u' \leftrightarrow d'$, $e \leftrightarrow \nu_e$, and $\mu \leftrightarrow \nu_\mu$ are the same) while allowing for the observed suppression of strangeness-changing processes.

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Example: kaon and pion decay ratios (revisited)

Let us revisit $K^+ \to \mu^+\nu$ and $\pi^+ \to \mu^+\nu$ decay rate ratios in light of the Cabibbo theory.

Quantity	Value [2]
$P(\pi^+ \to \mu^+\nu_\mu)$	0.9998770(4)
m_π	139.57061(24) MeV/c^2
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Pion Lifetime	$2.6033(5) \times 10^{-8}$ s

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$P(K^+ \to \mu^+\nu_\mu)$	0.6356(11)
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Kaon Lifetime	$1.2380(20) \times 10^{-8}$ s

Map the weak interaction strength ratio onto the Cabibbo mixing angle idea to find:

$$\frac{P(K^+ \to \mu^+\nu_\mu)}{P(\pi^+ \to \mu^+\nu_\mu)} = \left(\frac{g_K}{g_\pi}\right)^2 \times \left[\frac{\Gamma_K}{\Gamma_\pi}\frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)^2}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2}\right] = \tan^2\theta_c \times [37.2] \longrightarrow \tan\theta_c \equiv \frac{g_K}{g_\pi} \approx 0.28 \quad (20)$$

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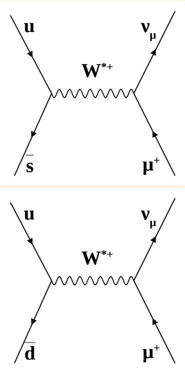
The Power of the Cabibbo Angle Approach

That's a nice idea, but does it tell us anything? We know now that weak interactions like $K^+ \to \mu^+\nu$ occur when quarks interact via a charged, weak boson (in this case, W^+) creating a virtual current that quickly decays to other things (in this case, $\mu^+\nu_\mu$). The Feynman Diagram pictorially representing this is:



In terms of ONLY the interaction strength (ignoring kinematic factors), the probability of this process in the Cabibbo framework goes like this:

$$P(\pi^+(u\bar{d}) \to \mu^+\nu_\mu) \quad \propto \quad \cos^2\theta \tag{22}$$
$$P(K^+(u\bar{s}) \to \mu^+\nu_\mu) \quad \propto \quad \sin^2\theta \tag{23}$$

Thus,

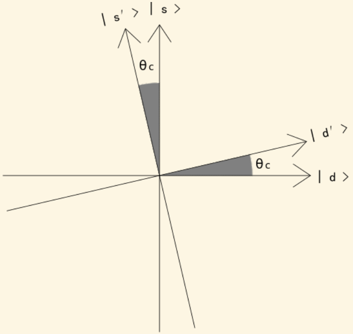
$$\frac{P(K^+(u\bar{s}) \to \mu^+\nu_\mu)}{P(\pi^+(u\bar{d}) \to \mu^+\nu_\mu)} \propto \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \tag{24}$$

Can we make new predictions?

Predicting a fourth quark: charm

By the early 1970s, there were reasons to believe that there might be a fourth quark. In the theory, this modifies our quark picture from a single doublet to two doublets as follows:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \longrightarrow \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \tag{25}$$



such that

$$|d'\rangle = \cos\theta_c\,|d\rangle + \sin\theta_c\,|s\rangle \tag{26}$$

$$|s'\rangle = -\sin\theta_c\,|d\rangle + \cos\theta_c\,|s\rangle \tag{27}$$

Then you can make predictions about charm mesons, e.g. $D^+(c\bar{d}) \to \mu^+\nu$. For instance, the transition $c \to s$ would go like $\cos^2\theta_c \approx 0.95$ while the transition $c \to d$ would go like $\sin^2\theta_c \approx 0.05$ — the latter would be "Cabibbo-suppressed". Indeed, the rate of $D^+_{(c\bar{s})} \to \mu^+\nu_\mu$ is about 0.55% while the rate for $D^+_{(c\bar{d})} \to \mu^+\nu_\mu$ is about 0.037%.

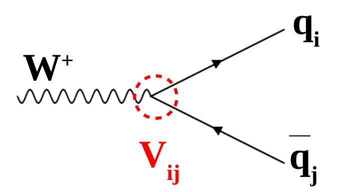
Indeed... is it highly suppressed and by a factor of about $\sin^2\theta_c/\cos^2\theta_c$!

The Cabibbo Mixing Matrix

$$\begin{pmatrix} |d'\rangle \\ |s'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \end{pmatrix} \quad (28)$$

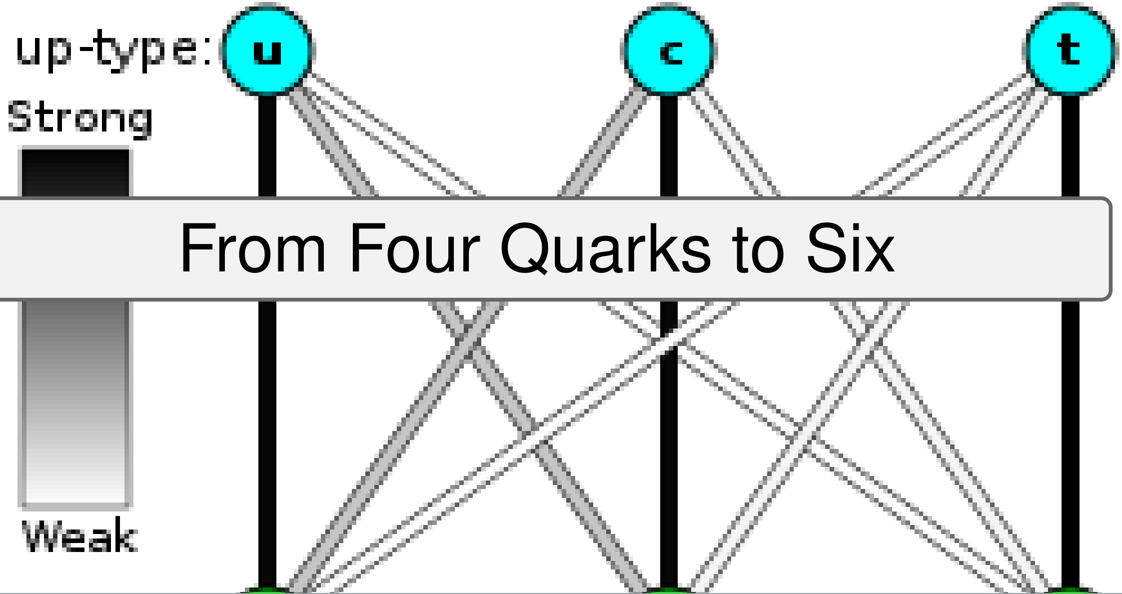
$$= \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \end{pmatrix} \quad (29)$$

This is the Cabibbo Mixing Matrix. Its elements, V_{ij}, encode the degree with which a transition will occur between quarks i, j in a natural process, as mediated by the weak interaction.



$$P(W^+ \to q_i \bar{q}_j) \propto |V_{ij}|^2 \quad (30)$$

The matrix elements can be complex numbers, but with just four quarks they are real numbers.



From Four Quarks to Six

Why Four Quarks Are Not Good Enough

Things moved fast in the 1970s. Before even the discovery of a fourth quark and continued success of the Cabibbo mixing angle there was motivation for even more than four quarks?

- The combination of Charge Conjugation Symmetry and Parity Symmetry, CP, was known to be violated as a symmetry of nature. However, *why it was violated at all was still a mystery*.

In 1973, Makoto Kobayashi and Toshihide Maskawa concluded work suggesting that if one added one more quark doublet (two more physical quarks), CP violation would naturally be explained as a consequence of the Cabibbo picture [4].

The Full Cabibbo-Kabayashi-Maskawa (CKM) Quark Mixing Matrix

$$\begin{pmatrix} |d'\rangle \\ |s'\rangle \\ |b'\rangle \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} |d\rangle \\ |s\rangle \\ |b\rangle \end{pmatrix} \quad (31)$$

Why extend to at least 6 quarks? Such a matrix is the minimum size needed to add a complex component to the otherwise real number components of the matrix. That complex component leads to CP violation, but that is a discussion for another time.

$$\theta_{12} = 13.04 \pm 0.05^\circ; \theta_{13} = 0.201 \pm 0.011^\circ; \theta_{23} = 2.38 \pm 0.06^\circ; \delta_{13} = 1.20 \pm 0.08\mathrm{rad}. \quad (32)$$

The first angle is the Cabibbo Angle. Note that it is about what we estimated. δ_{13} is the complex phase. These are not predicted; they must be measured.

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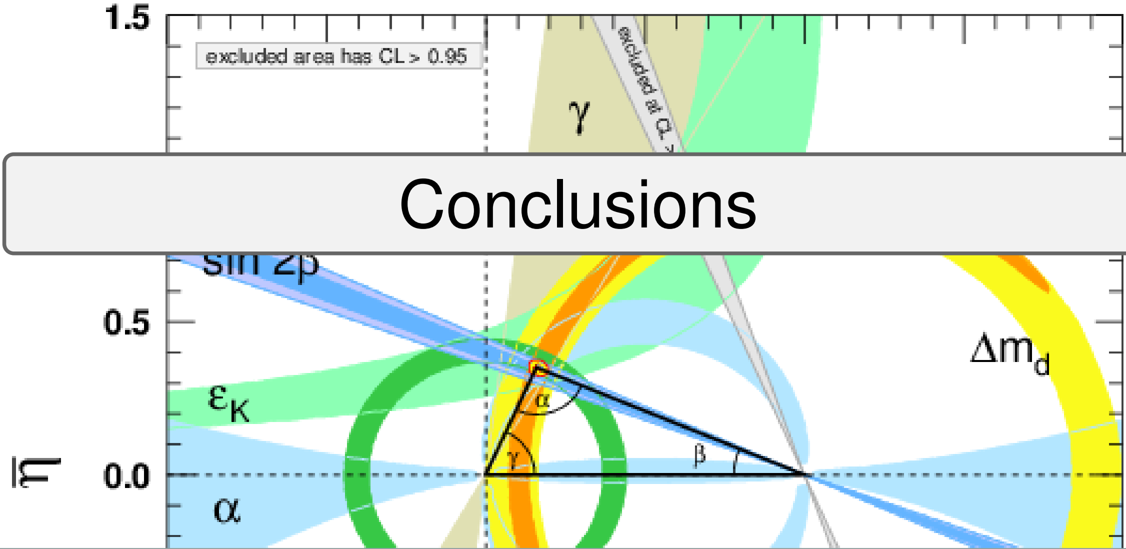

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- We have only scratched the surface today. There are many good references for learning more about quark mixing as a subject [5, 1, 6, 7].
- The weak interaction has a universal strength, usually written as the Fermi Coupling Constant, $G_F = 1.16639(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$
- However, that is modified by how the quark flavor changes in a weak interaction; the degree of modification is given by the CKM Matrix.
- This simple picture has remarkable power. It has been used since 1973 to successfully predict as-yet unmeasured parameters of the theory of nature (the standard model), specifically the parameters of the CKM matrix. Measured in a few processes, they have well-predicted then-unseen behaviors of quark matter that would later be observed and confirm this picture.
- But, there are so many mysteries: are there only 6 total quarks, and if so, why? What gave rise to the values of the 4 parameters of the 3×3 CKM matrix? Why is there more CP violation in nature writ large than is explained by this subatomic picture?

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