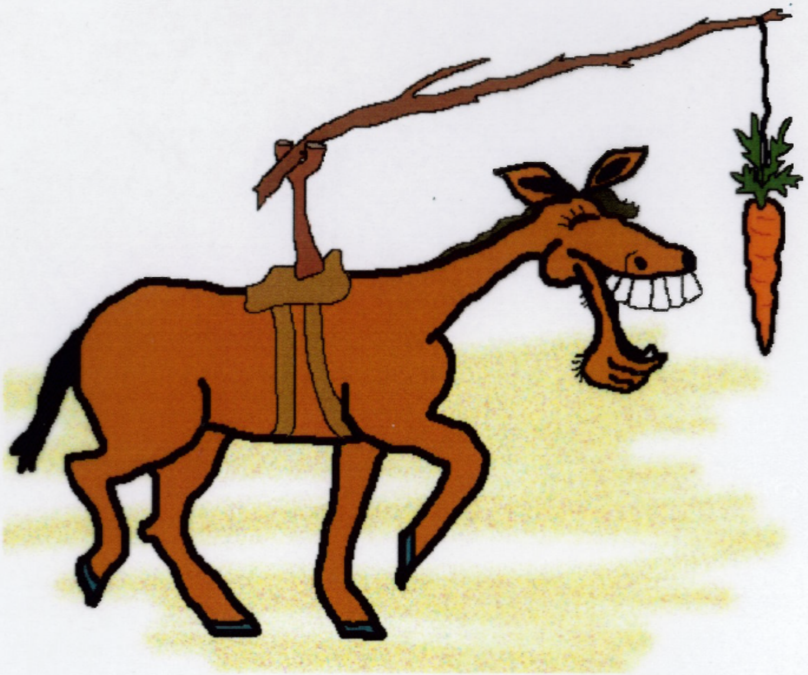
Lecture 14 − accelerators 2

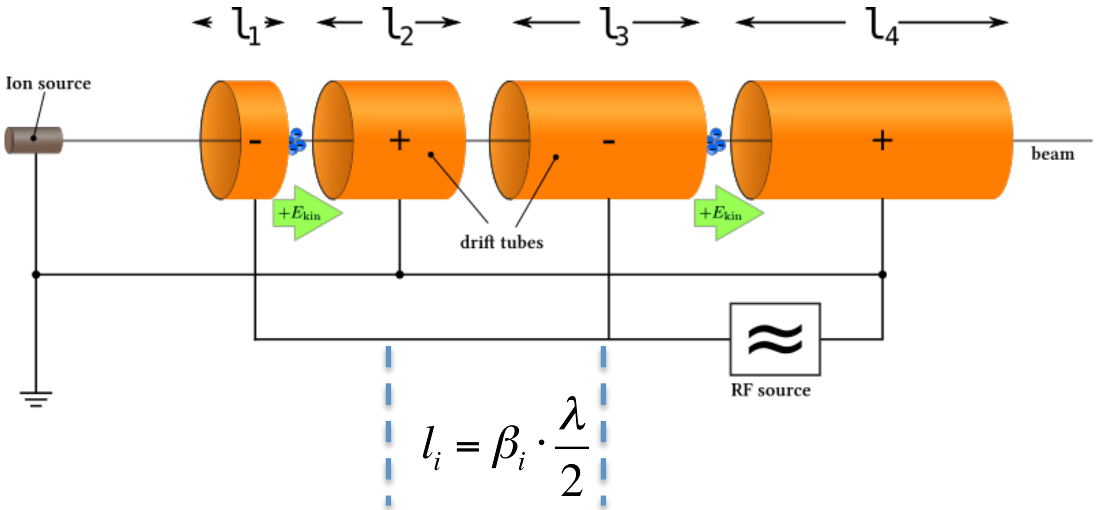
Principle of Acceleration Technique



Continuous acceleration - instead of segments of pipe with increasing length
use a moving e-m wave maximum accelerating potential
always ahead of the charged particle

Alternating RF Field

Apply the same voltage through acceleration gap many times.



$$l_i = \beta_i \cdot \frac{\lambda}{2}$$

Energy gain per gap:

$E = q\ \mathbf{V_{RF}} \sin(f_s)$

fs…phase wrt to RF field

- Particle synchronous with field. In shielding tube when field has opposite sign. Voltage across each cell the same.

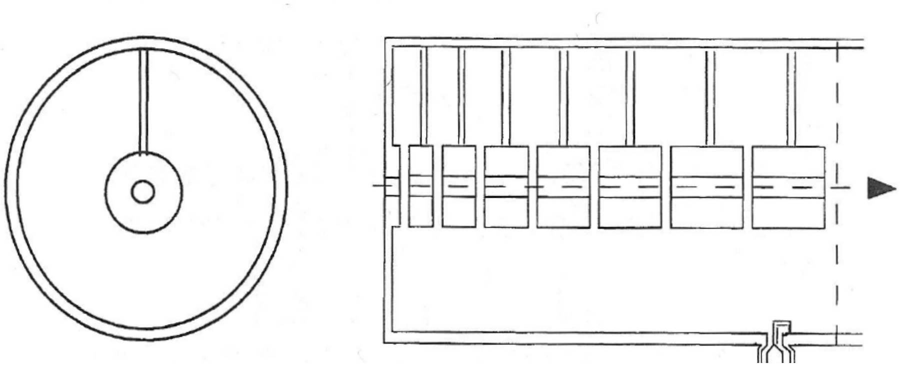
- Shielding tubes have to become longer and longer, as particles become faster and faster or frequency must become higher $l = c/f_{RF}$
- Problem - radiation power loss: $P = \omega_{RF} C V_{RF}^2$, C - gap capacitance

Drift Tube Linac (Alvarez)

- To eliminate power loss place drift tubes in a RF cavity
  - Electromagnetic field oscillating in cavity. Standing wave, TM mode
    TM – transverse magnetic mode of electromagnetic wave propagation imposed by the boundary condition – no magnetic field in the direction of propagation

    longitudinal E Field, transverse B Field
  - Resonant frequency of cavity = accelerating field frequency!
  - Reduces power loss
  - Exploit Farraday's law:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$



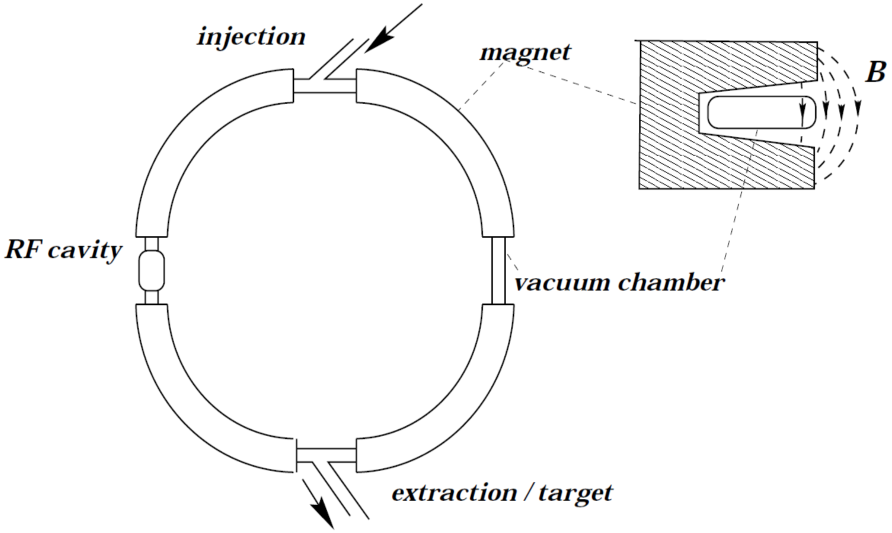
Longest electron linac - SLAC 2 miles, can reach 50 GeV
Proposed International Linear Collider - ~20 miles can reach ~300 GeV

1952 Courant, Livingston, Snyder

Synchrotron

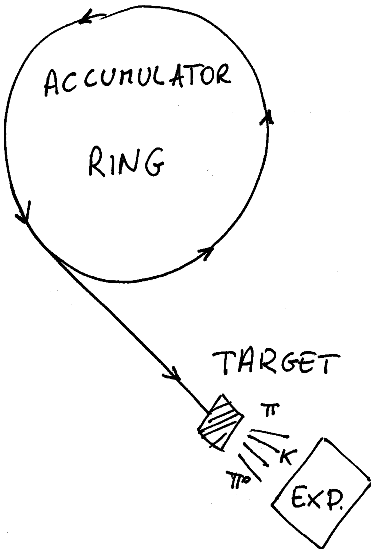
The guiding **magnetic field** (bending the particles into a closed path) **is time-dependent**, being synchronized to a particle beam of increasing kinetic energy.
Idea: define a circular orbit of the particles, keep the beam there during acceleration, put magnets at this orbit to guide and focus.
Derivatives:

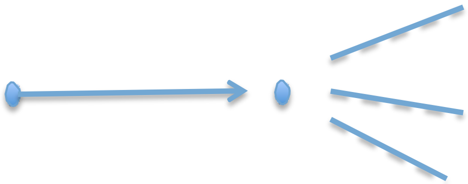
- **Storage rings** – synchrotron in which the kinetic energy of the particles is kept constant
- **Colliders** – combination of two storage rings with beams circulation in opposite directions
- **Synchrotron light sources** – combination of different accelerators to produce intense source of synchrotron radiation and X-rays.



Fixed target vs. Colliders

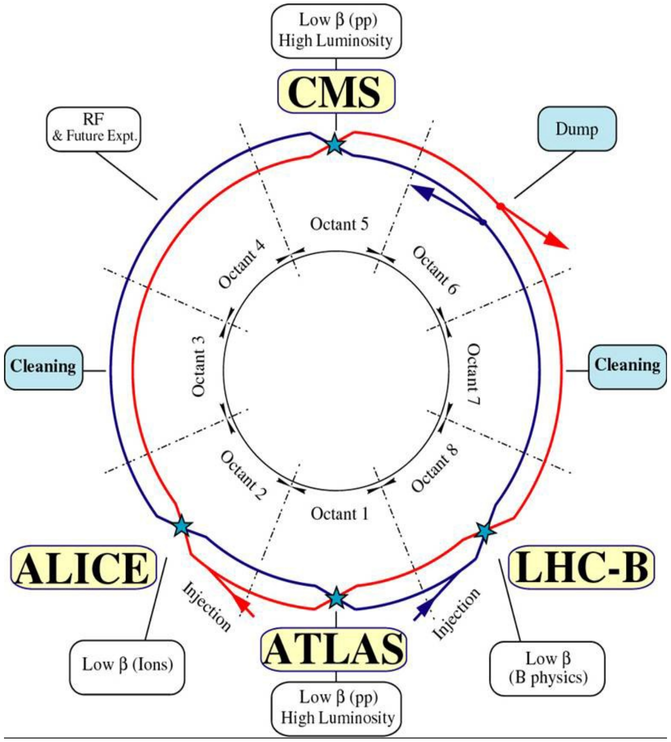
increase energy

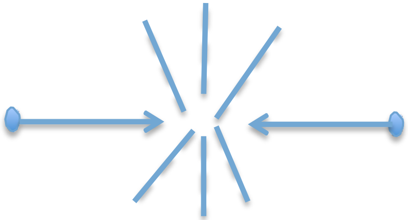




Approach used e.g., for test beams

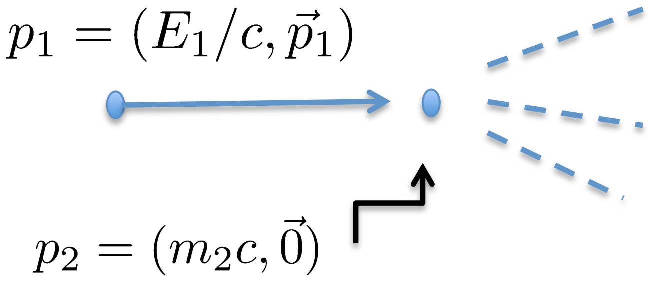
provide collisions





E_{CM} in Fixed Target Experiment vs Collider

Fixed target – target nucleon is stationary



$$p_1 = (E_1/c, \vec{p}_1)$$

$$p_2 = (m_2 c, \vec{0})$$

$$p_{tot} = (E_1/c + m_2 c, \vec{p}_1)$$
$$E_{CM}^2 = (m_1^2 + m_2^2)c^4 + 2E_1 m_2 c^2$$

$$\boxed{E_{CM} \propto \sqrt{E_1}}$$

Collider – laboratory frame = center of mass frame

$$p_1 = (E_1/c, \vec{p}_1)$$
$$p_2 = (E_2/c, -\vec{p}_1)$$

$$\boxed{E_{CM} = E_1 + E_2}$$

Circular Accelerators

- Linear accelerators can in principle accelerate to arbitrarily high energies, …….. but become longer and longer!

- Particles on circular paths to pass accelerating gap over and over again – circular accelerator

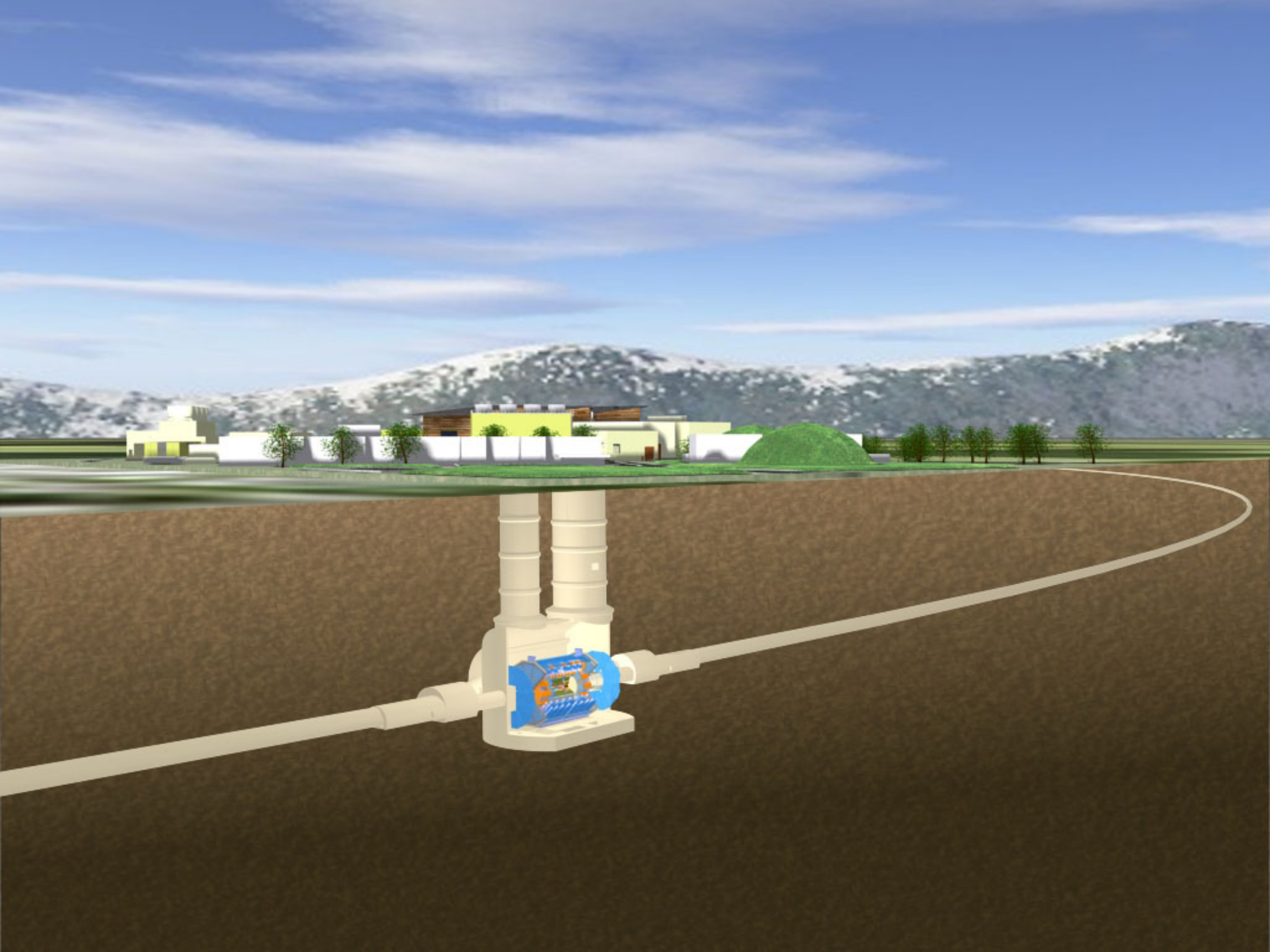
- First cyclotron proposed by E.O. Lawrence in 1929 and built by Livingston in 1931.

- Many conceptual modifications since then

- Synchrotron – constant radius, B field increases synchronously with beam energy. Single beam so magnets can be relatively simple.

- Collider more energy efficient but also more complex: two beams to be accelerated and to be brought into collision.
  Single beam ring for proton-antiproton colliders (opposite charges),
  Two beam rings for proton-proton colliders (same charges) – complex magnets

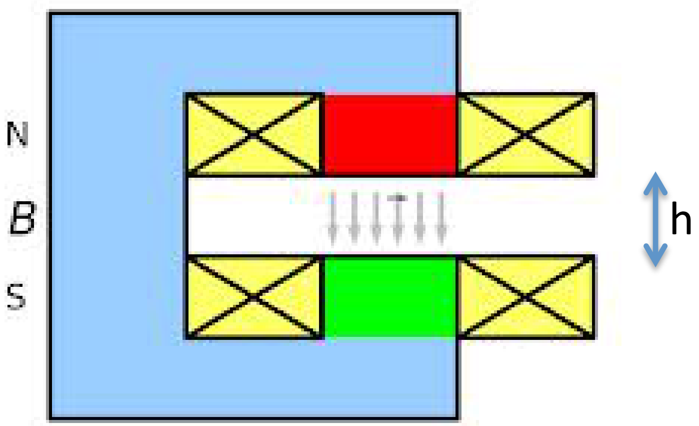
SLAC - Stanford Linear Accelerator



Dipole magnets: guiding magnets

- Vertical magnetic field to bend in the horizontal plane
- Dipole electro-magnets

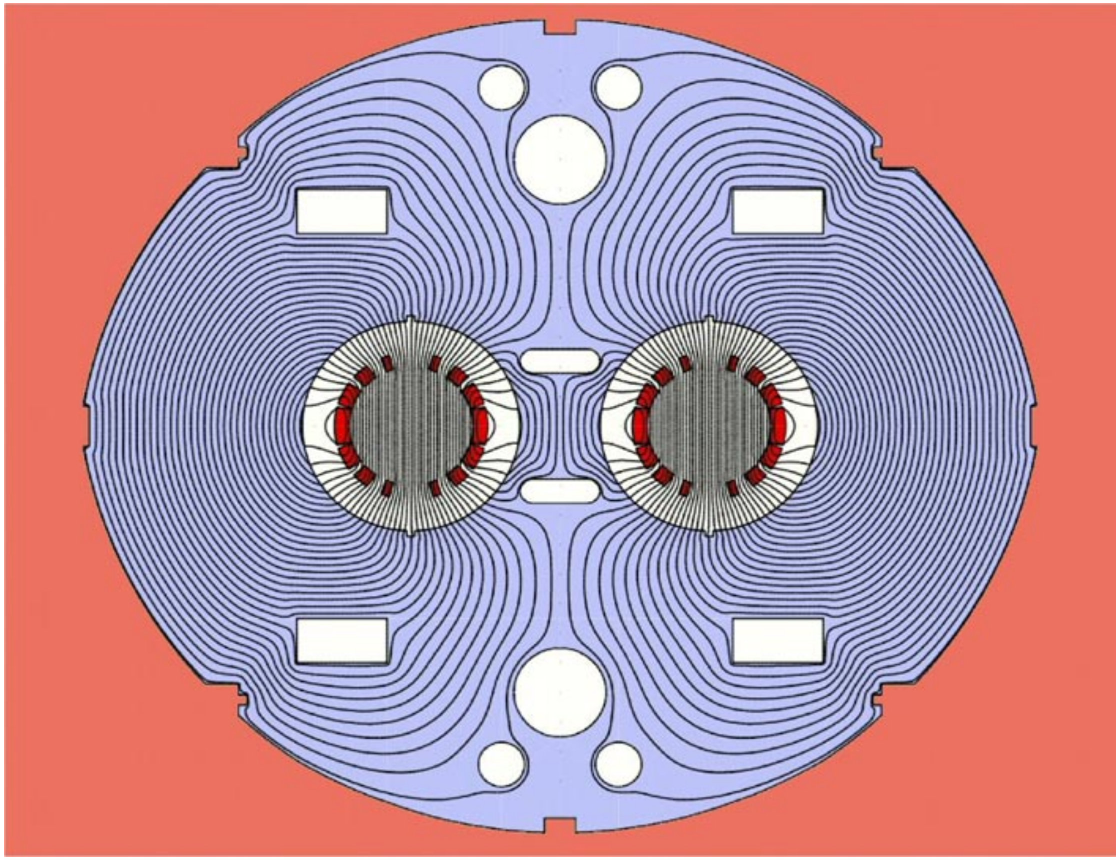
$$\boxed{\vec{F} = q \cdot \vec{v} \times \vec{B}}$$



$$B = \frac{\mu_0 n I}{h}$$

Fermilab

LHC dipoles



Dipole magnets: guiding magnets

- Circular accelerator: Lorentz Force = Centrifugal Force

$$\begin{aligned} F_L &= qvB \\ F_{centr} &= \frac{mv^2}{\rho} \end{aligned} \quad \longrightarrow \quad \frac{mv^2}{\rho} = qvB$$

-
- $\boxed{\frac{p}{q} = B\rho}$

Bρ - Beam rigidity
ρ – radius of the circular orbit

- Useful formula: $\boxed{\frac{1}{\rho[m]} \approx 0.3\frac{B[T]}{p[GeV/c]}}$

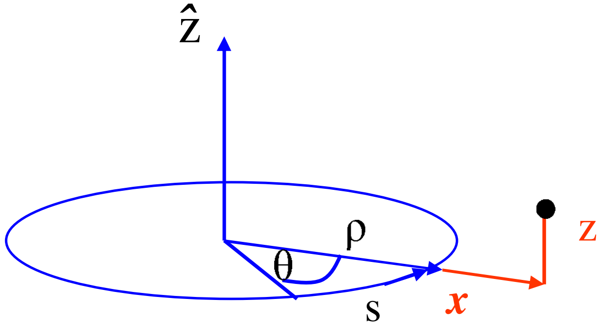
- Example for the LHC
  - p^+ @ 7 TeV/c
  - 8.3 T

$$\frac{1}{2.53\ km} = 0.3\frac{8.3}{7000}$$

Weak Focusing

Particles with deviations from the design trajectory need to feel restoring forces F(z) ~ -z otherwise the beam diverges

$$B_x = -const \cdot z \qquad \frac{\partial B_x}{\partial z} = -const$$



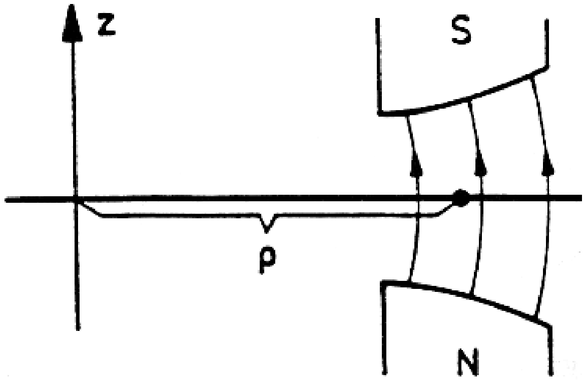
circular coordinate system

For deviations in the vertical plane we need either horizontal field component or a radially decreasing guide field

Maxwells equation:

$$\vec{\nabla} \times \vec{B} = 0$$

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \frac{\partial B_z}{\partial r} < 0$$



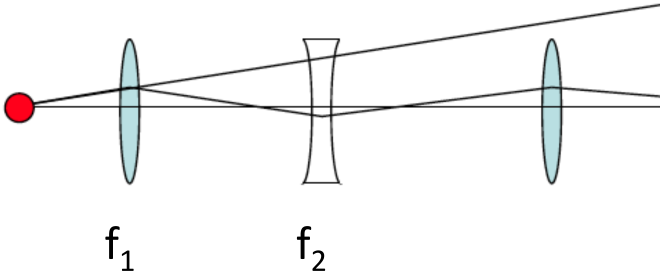
Similar considerations for horizontal plane

Pole shape in combined function magnet

Strong Focusing

Alternating gradient focusing

- Analogous to geometrical optics: a series of alternating focusing and defocusing lenses will focus and correct for chromatic divergences.
- No need to shape the poles of the magnets



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

For fixed focal length $f_1 = f$, $f_2 = -f$ → $F = d/f^2 > 0$

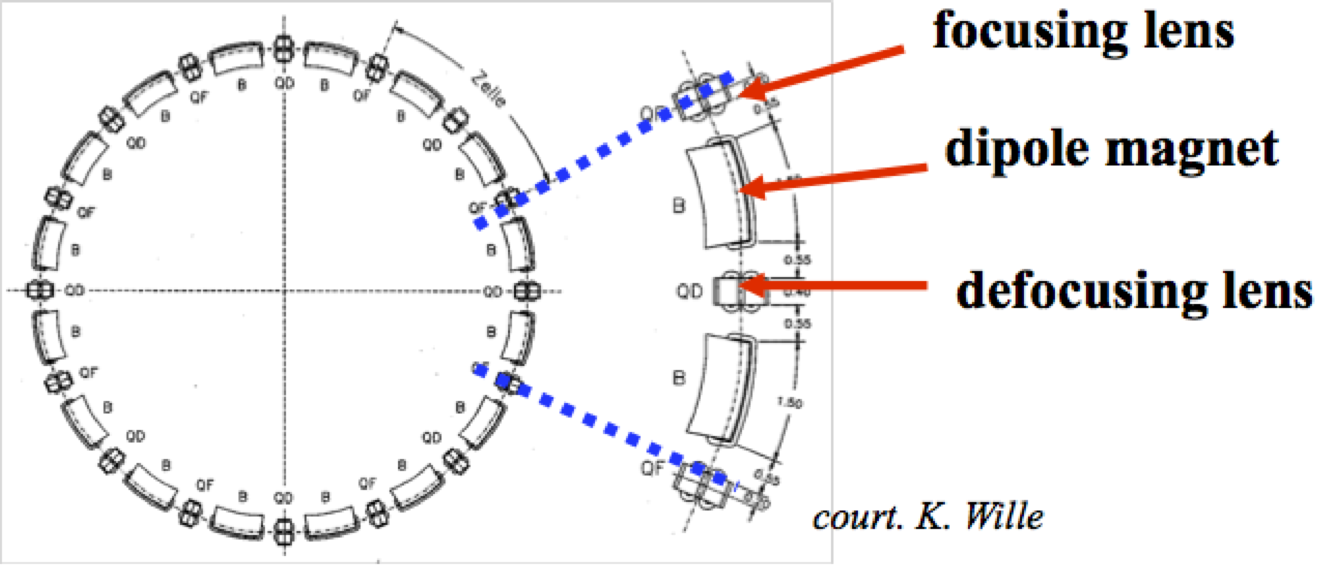
In this case the lenses are magnets with alternating gradients

Thin lens approximation – lens located in the middle of the magnet

FODO structure: O – bending magnets

All new (since 1960) accelerators are strong focusing synchrotrons

How to keep particles on a circular trajectory



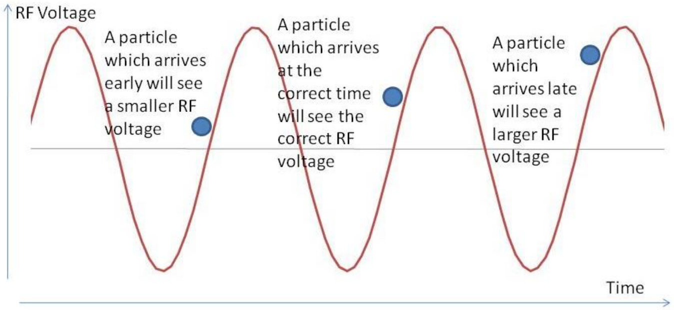
focusing lens

dipole magnet

defocusing lens

court. K. Wille

Acceleration and bunching provided by a traveling RF electromagnetic wave



A particle which arrives early will see a smaller RF voltage

A particle which arrives at the correct time will see the correct RF voltage

A particle which arrives late will see a larger RF voltage

Beam dispersion and effects of gravity

$\Delta y = \frac{1}{2} g t^2$: for $g = 9.8\ \mathrm{m/s^2}$ and $t = 60$ msec $\Delta y = 18$ mm

→Vertical focusing is required

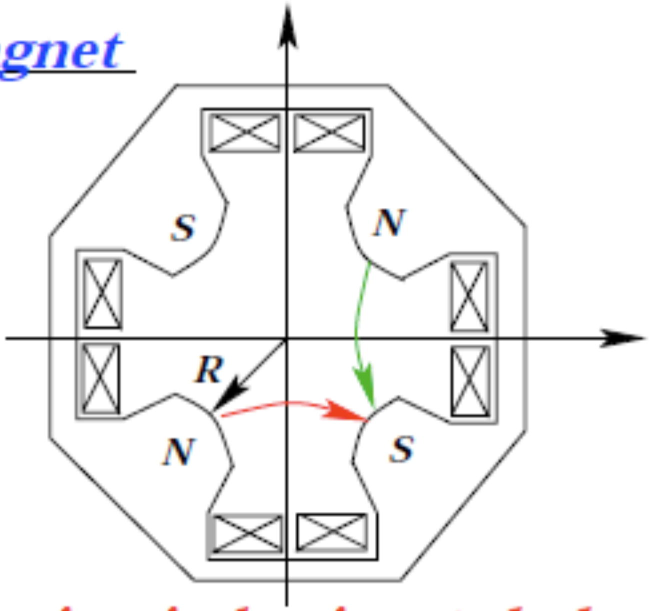
Quadrupole Magnet

$B_x = -g \cdot y$

$B_y = -g \cdot x$

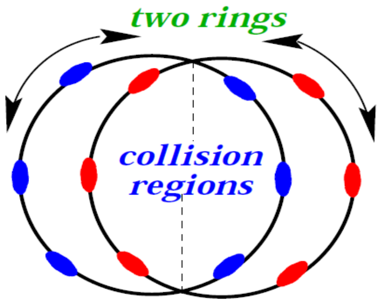
$F_x = g \cdot x$

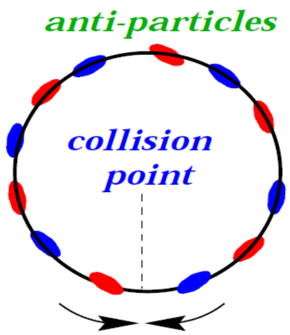
$F_y = -g \cdot y$



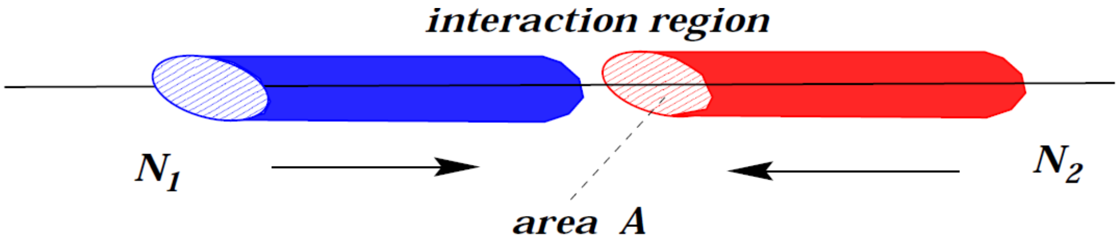
→ *defocusing in horizontal plane!*

Luminosity





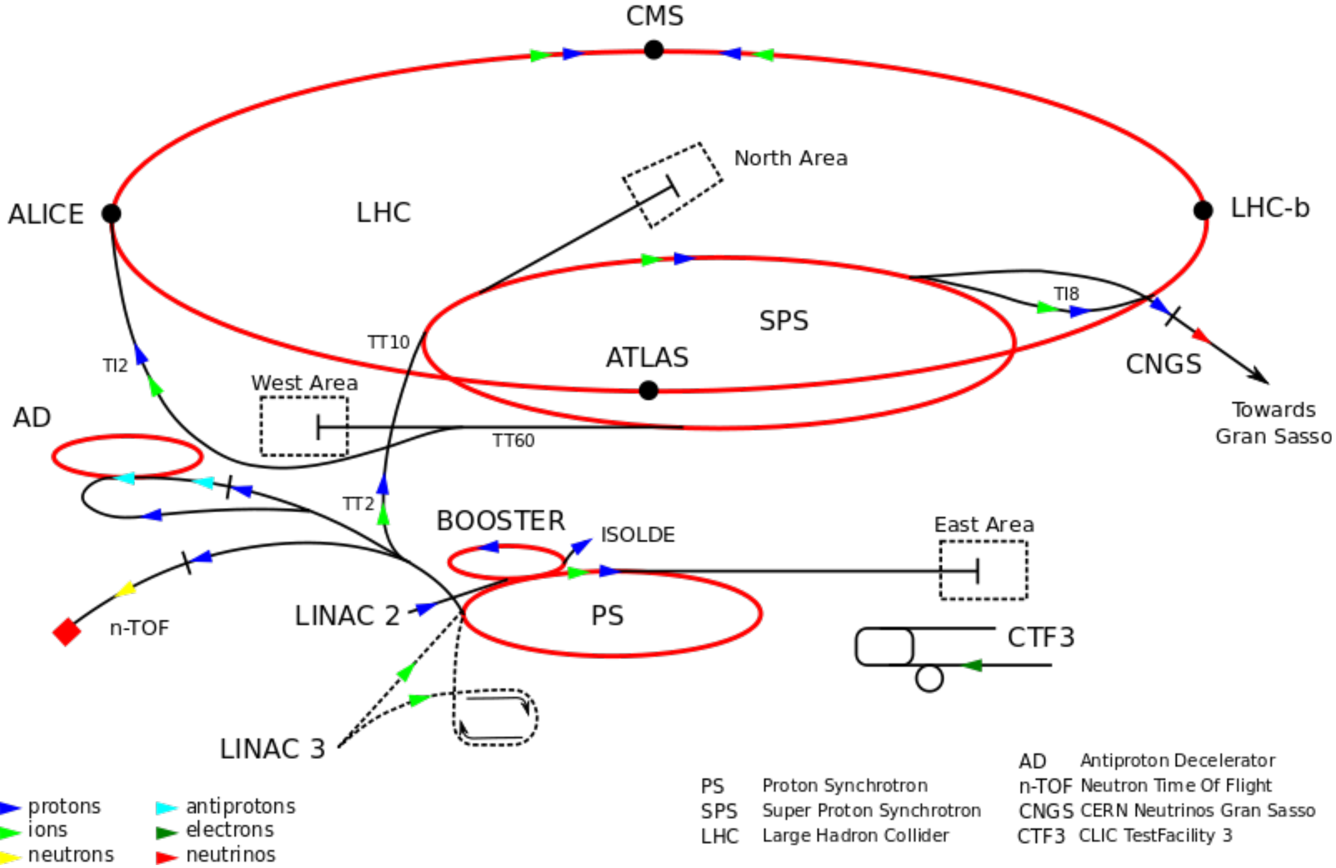
$N_{ev}/\mathrm{sec} = \sigma \times L \qquad L[\mathrm{cm}^{-2}\mathrm{s}^{-1}]$



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

A – beam size at the collision point

The CERN Accelerator Complex



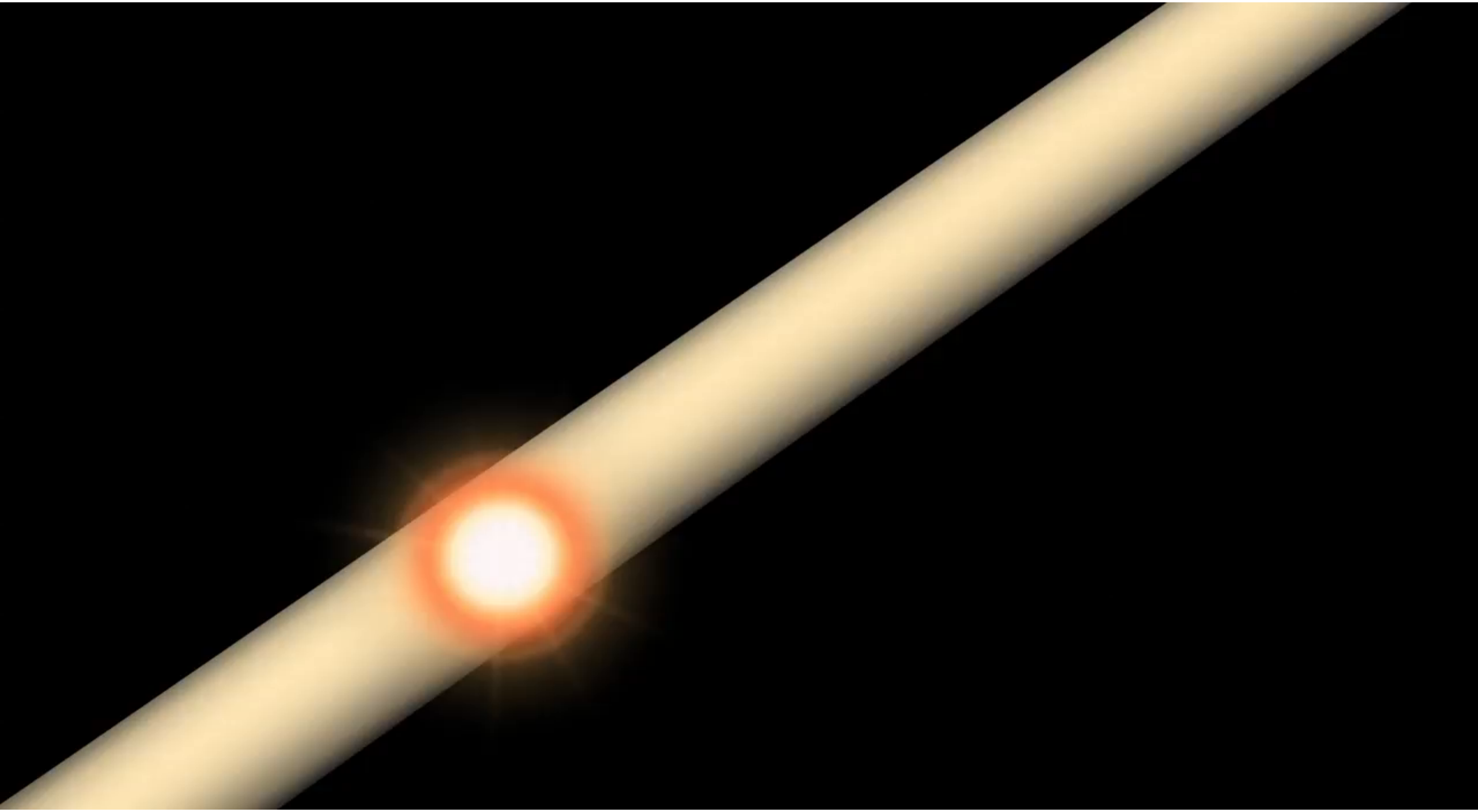
PS (Proton Synchrotron):
1959
LHC (Large Hadron Collider):
2008

Circumference:
PS ~ 628 m
LHC ~ 27 km

Energy:
PS 26 × 10^9 eV (GeV)
LHC 7 × 10^{12} eV (TeV)

Particles produced through:
PS: fixed target collisions
LHC: beam-beam collisions

Proton's view



Problems

Energy loss per turn
-> need compensate by acceleration

$$\Delta E = \frac{e^2}{3\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{R}$$

Emitted radiation hits and heats the beam pipe
-> need cooling
-> outgassing – release of molecules of air trapped in the metal of the beam pipe
requires continuous pumping

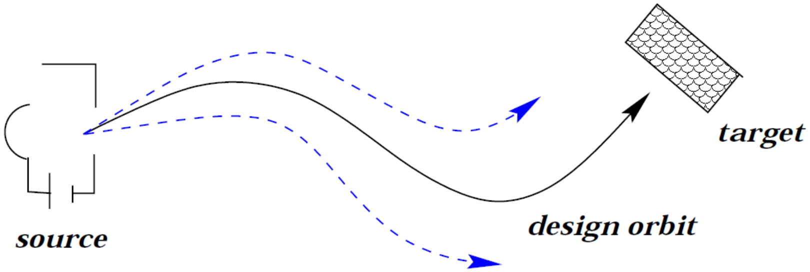
Alignment of magnetic elements

Stability of the trajectory

Beam-beam interactions

Beam size

Beam divergence



Geometrical focusing