Lecture 5

Follow up from last class

Nuclear reactors vs Sun

*If you assume that the cross section of your body is 1 m^2 and that the flux of reactors neutrinos is distributed uniformly on a surface of the sphere centered at the reactor then there are **~ 3.**7**×10^9** ν from that reactor crossing your body every second. Even with that number the rate of neutrino interactions in your body is much less than one during your lifetime.*

Flux of neutrinos from the Sun ~1000 larger and does not depend on the time of the day, as neutrino can path though the Earth with no difficulty.

These basic constituents interact via four fundamental forces:

Properties of the Interactions

f the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified d

	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interacti
	Mass – Energy	Flavor	Electric Charge	Color Char
xperiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Glu
nediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
t { 10^{-18} m	10^{-41}	0.8	1	25
3×10^{-17} m	10^{-41}	10^{-4}	1	60

Major unsolved problem is the theory if gravitational interactions. We know that the photon is affected by the gravitational field generated by massive objects (general relativity) but the quantum theory of gravitation does not yet exist. Many attempts have been made and there is progress in recent years. There are also astronomical observations consistent with expectations of quantum nature of gravitational field.

In present formulations, the carrier of gravitational field is called Graviton. It is massless, has spin =2 and the range of interactions is infinite.

LIGO observed last year events of gravtiational waves

These can be interpreted without invoking quantum theory.

Particle physics theorists are attempting to formulate a general theory of everything based on quantum field theoretical approach. An evidence for gravitons would indicate that to be possible..

Particle lifetime

- *A particle, once it exists, has no memory how and when it was produced*
- *Whenever allowed by energy conservation and not forbidden by some special rule, the heavy particle will decay into a lighter particles*
- *Each particle at rest has a mean lifetime. We cannot predict when the particle will decay. For a group of N particles, certain fraction of them will decay within a specific time interval.*
- *The probability per unit time is called* ***decay rate*** Γ.

In a group of N(t) particles that exist at time t, a number $N\Gamma dt$ *will decay within dt. The number of particle remaining will decrease by*

$$dN = -\Gamma N\, dt$$

$$N(t) = N(0) \exp(-\Gamma t)$$

We define mean lifetime as $\tau = 1/\Gamma$

$$N(t) = N(0) \exp^{-t/\tau}$$

If a particle has several decay modes, there is a Γ_i for each mode since they do not have to have a common probability of occurrence. Then total decay rate

$$\Gamma_{tot} = \Sigma\Gamma_i \quad \text{and} \quad \tau = 1/\Gamma_{tot}$$

The probability of having a particular decay mode is called

Branching Fraction *(or Branching Ratio)* = $\boldsymbol{\Gamma_i/\Gamma_{tot}}$

Cross section - σ - probability of interaction

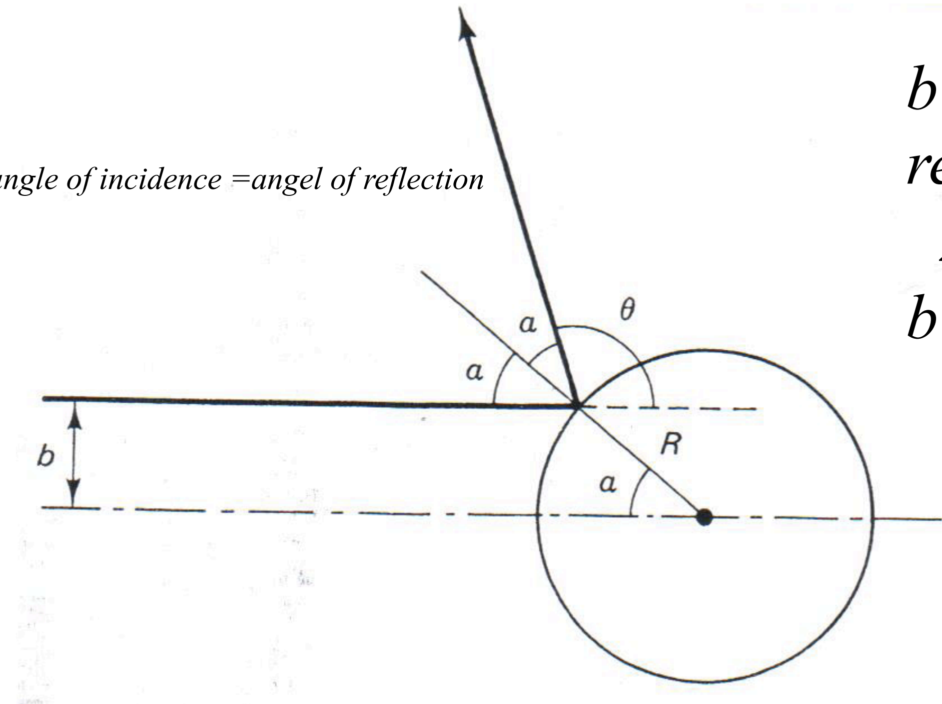
Shoot a particle at a target – what is the probability that you hit it? Depends on the size of the particle, the size of the target and the type of interactions.

For point like particle (e.g., electron) scattering on target resembling a billiard ball (e.g., proton) of radius R Griffith textbook goes through amusing, lengthy and precise geometrical derivation giving the cross section

$$\sigma = \pi R^2$$

Scattering on Hard Sphere

angle of incidence =angel of reflection



b – impact parameter

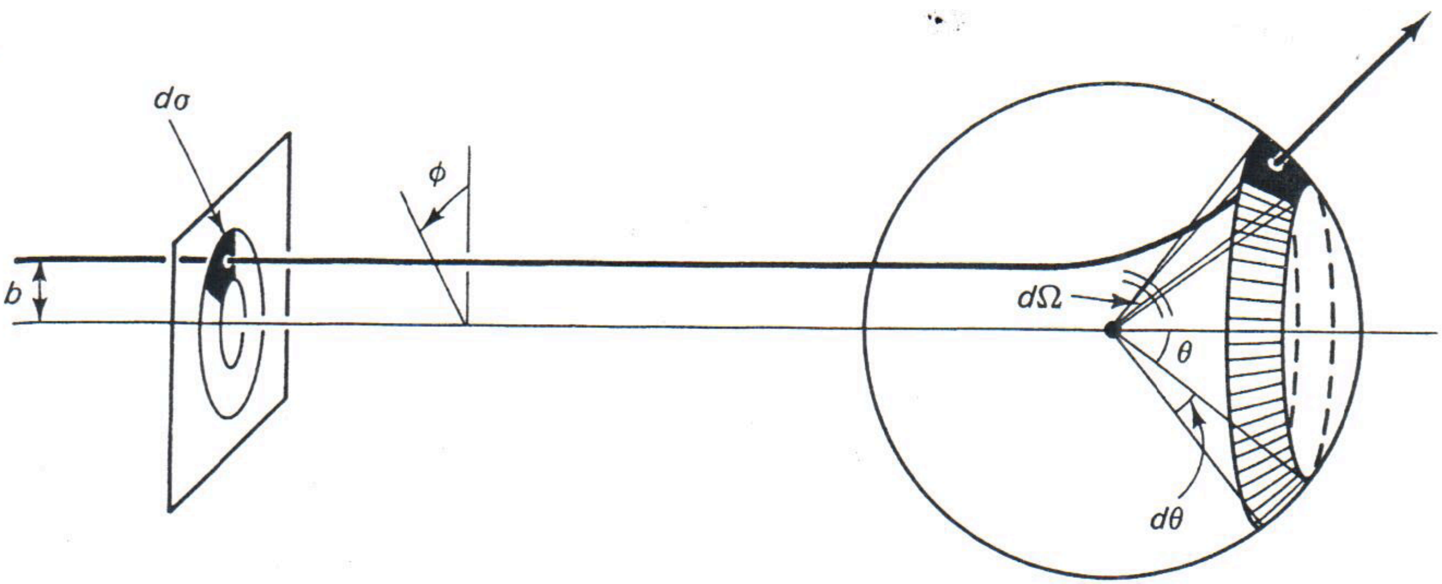
relation between angles:

$2\alpha + \theta = \pi \rightarrow \alpha = \pi/2 - \theta/2$

$b = R\cos(\theta/2)$

Particle with impact parameter between b and b + db will emerge
In the angular range between θ and θ + dθ

Particle hitting an area dσ will scatter into an angular range dΩ



Probability of scattering, $d\sigma = D(\theta)\, d\Omega$*, where the proportionality factor* $D(\theta)$ *is the differential scattering cross-section.*

with

$$d\sigma = |b\, db\, d\varphi|$$

$$d\Omega = |\sin\theta\, d\theta\, d\varphi|$$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \bullet \frac{db}{d\theta} \right|$$

$$\frac{db}{d\theta} = -\frac{R}{2}\sin\theta$$

$$D(\theta) = \frac{R\sin\frac{\theta}{2} \bullet b}{2\sin\theta} = \frac{R^2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\sin\theta} = \frac{R^2}{4}$$

$$\sigma = \int d\sigma = \int D(\theta)\,d\Omega = \int \frac{R^2}{4}d\Omega = \pi R^2$$

Resonances (late 1955 -1975)

(particles as waves)

- *New type of accelerators -proton synchrotrons - came into existence in the late 1950ties. These allowed for production of well controlled beams of stable and long-lived particles: protons, pions, kaons….*
- *Interactions of those beams with various targets led to discovery of resonances – particles with very short lifetimes.*
- *First observed as bumps in the scattering cross section as if at certain energy there would be a short-lived resonant intermediate state that then decays into the same system of particles.*
- *Measurements of effective mass of 2 or 3 particles*

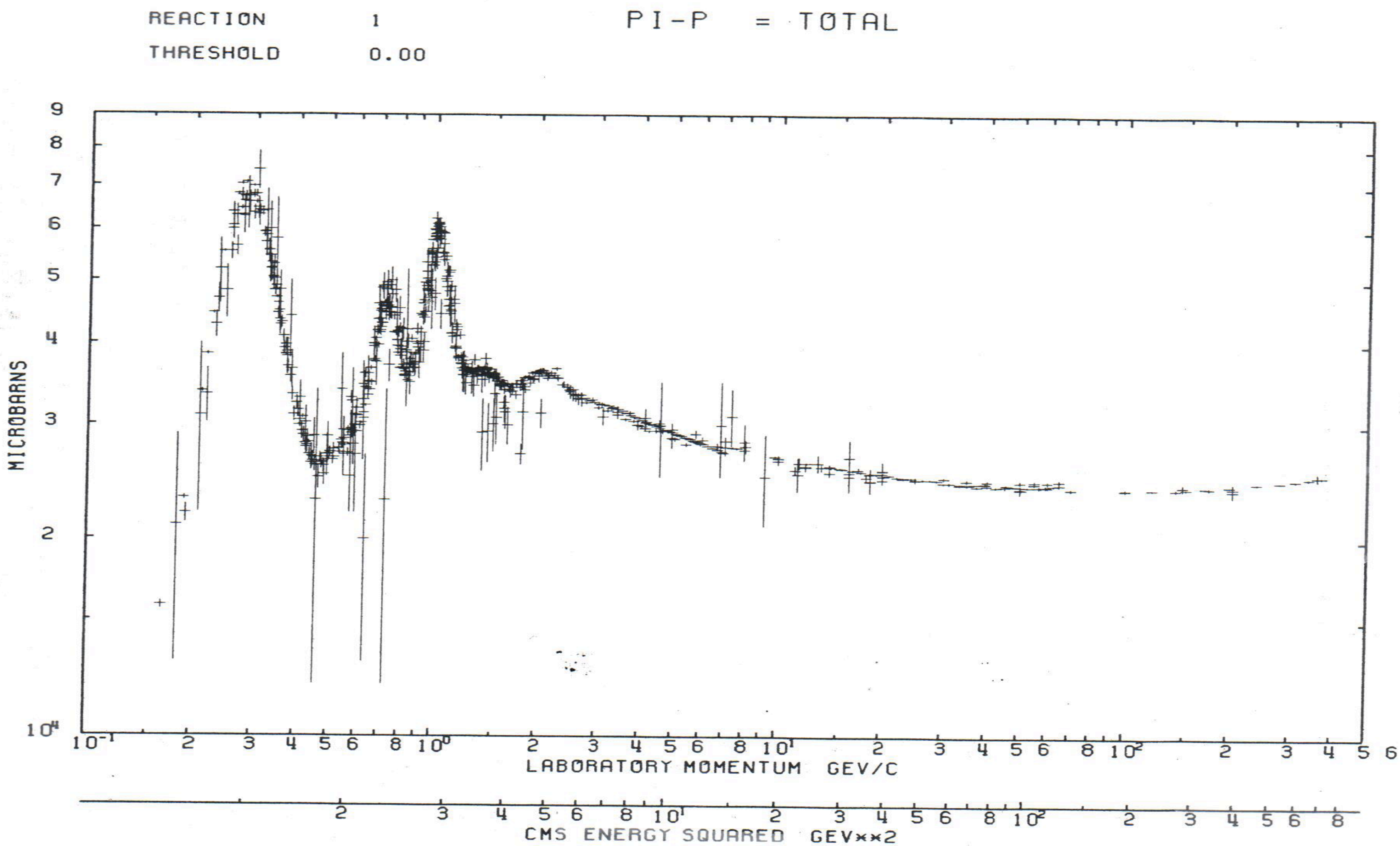
$$M^2c^4 = E^2 - (pc)^2$$

$$M^2 = E^2 - p^2 \text{ (in short-hand notation)}$$

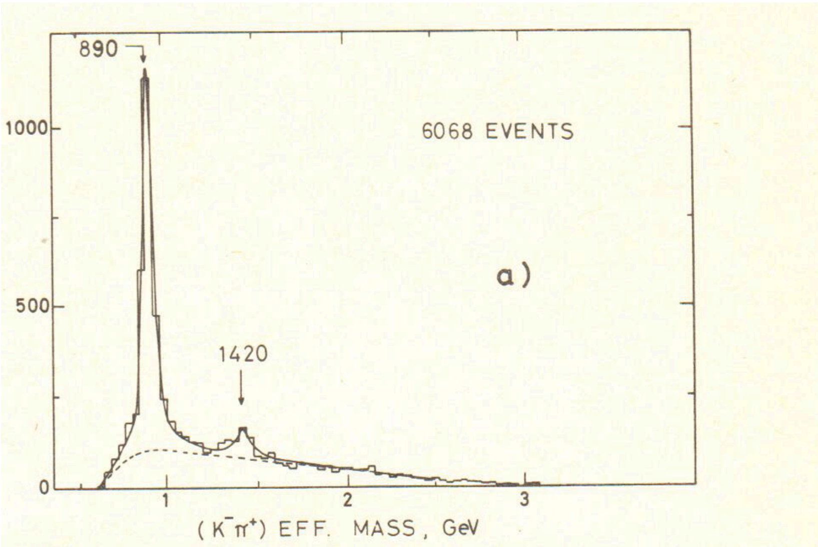
also has shown bumps in the invariant mass spectrum.

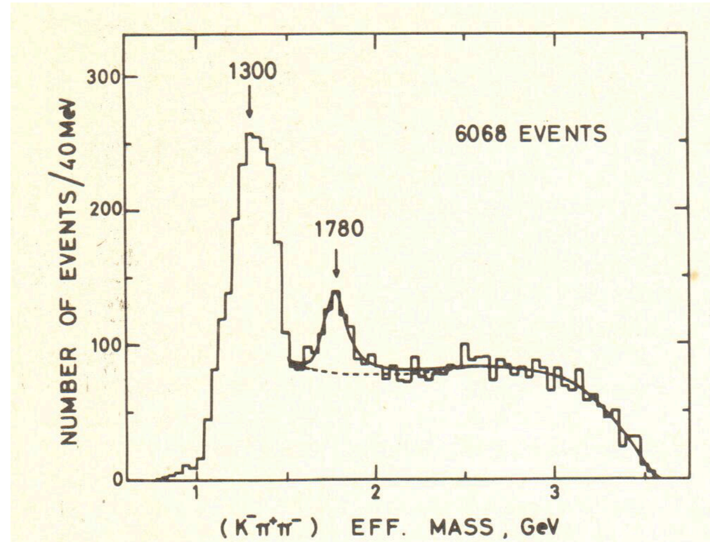
- *Effective mass is an invariant quantity – does not depend on the reference frame.→ We can always select a rest frame of the system of particles as most convenient.*

Resonance in total $\pi^- p$ cross section elastic scattering



Resonance states





$E^2 = (mc^2)^2 + (pc)^2$ → mc^2 - *invariant mass*

independent of reference frame

The Uncertainty Principle

Werner Heisenberg

CLASSICAL MECHANICS

Position and momentum of a particle can be measured independently and simultaneously with arbitrary precision

QUANTUM MECHANICS

Measurement perturbs the particle state → position and momentum measurements are correlated:

$$\Delta x \Delta p_x \approx \hbar$$

(also for *y* and *z* components)

Similar correlation for energy and time measurements:

$$\Delta E \Delta t \approx \hbar$$

Quantum Mechanics allows for a violation of energy conservation by an amount D*E* for a short time D*t* < *ħ* / D*E*

Numerical example: $\Delta E = 1\ \mathrm{MeV} \Longrightarrow \Delta t \approx 6.6 \times 10^{-22}\ \mathrm{s}$

Mass of the resonance is not well determined. It has a width that is inversely proportional to the lifetime.

A typical parameters can be seen for one of the common resonance $\rho^0(770)$

mass (central value) $m = 775.49 \pm 0.34$ *MeV*

Width $\Gamma = 149.1 \pm 0.8$ *MeV*

Branching fraction $\rho \rightarrow \pi^+ \pi^-$ $\Gamma_i/\Gamma \sim 100\%$

Challenge - estimate its mean lifetime