Lecture 7

Conserved quantities: energy, momentum, angular momentum
Conserved quantum numbers: baryon number, strangeness, ...
Particles can be produced by strong interactions

eg. pair of K mesons with opposite strangeness:

$$p + p \rightarrow p + p + K^{+} + K^{-}$$

the same particles can then decay via weak interactions

$$K^{+} \rightarrow \mu^{+} + \nu_{\mu}$$

both processes have characteristic associated time

Consequence – strong interactions conserve more "quantum numbers" then weak interactions

Symmetries and conservation laws

Mirror symmetry

Look at yourself in the mirror.
What is the role of your right hand in your reflection?

Right hand becomes left hand –
apparent left-right inversion, but – no top-bottom inversion
Now imagine yourself lying on one side on the floor in front of the mirror
-> your right hand is still reflected as left hand even though they are now top-bottom in the mirror

What happens in mirror symmetry?

Symmetry of the fields

Electrodynamics before Maxwell

$$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$ *Gauss law*

$$\vec{\nabla} \bullet \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$ *Faraday law*

$$\vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{J}$$ *Ampere law*

***Problem**: Divergence of curl = 0*

True for Faraday law:

$$\vec{\nabla} \bullet (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(-\frac{\partial \vec{B}}{\partial t}) = -\frac{\partial}{\partial t}(\vec{\nabla} \bullet \vec{B}) = 0$$

Not true for Ampere's law:

$$\vec{\nabla} \bullet (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \bullet \vec{J})$$

equal to zero for steady current only

Maxwell's fix – add extra term to the Ampere law

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \underbrace{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

Symmetry between E and B fields
Photon couples to both fields at the same time

Dirac's addition:
quantization predicts minimum quantum of an electric charge
quantization predicts minimum magnetic charge – monopole????

Emmy Noether's theorem
(proved 1915, published 1918)

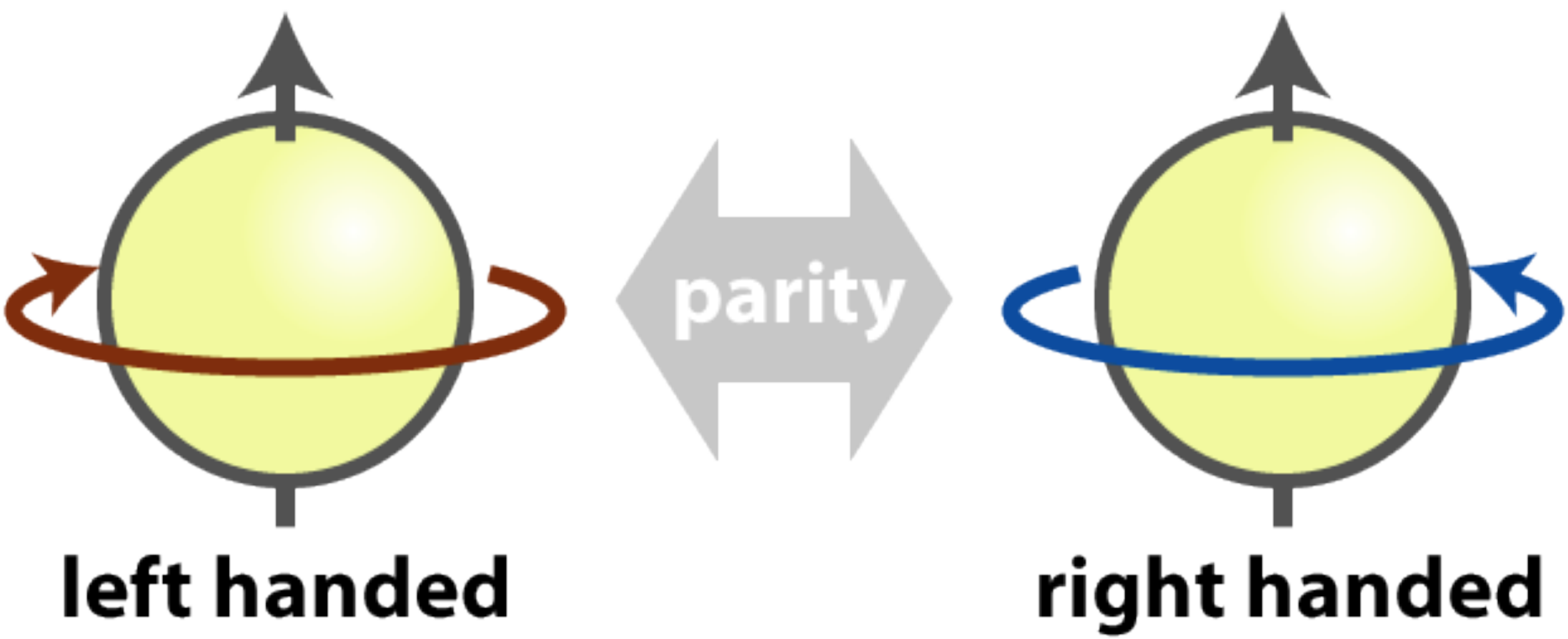
- Any differentiable symmetry of the action of a physical system has a corresponding conservation law. The action of a physical system is the integral over time of a Lagrangian function (which may or may not be an integral over space of a Lagrangian density function), from which the system's behavior can be determined by the principle of least action.
- Energy, momentum and angular momentum conservation laws are consequences of symmetries of space.

eg, if a physical system behaves the same regardless of how it is oriented in space -> Lagrangian is rotationally symmetric -> angular momentum of the system must be conserved

DISCRETE SYMMETRIES

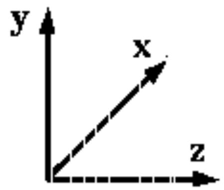
Mirror reflection

Mirror reverses forward-backward direction while maintaining the two other axes. It reverses left and right but not up and down

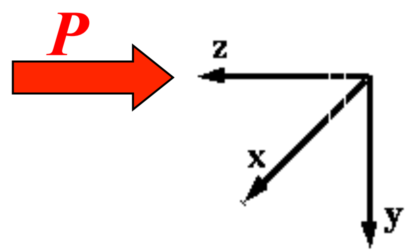


DISCRETE SYMMETRIES

PARITY: the reversal of all three axes in a reference frame



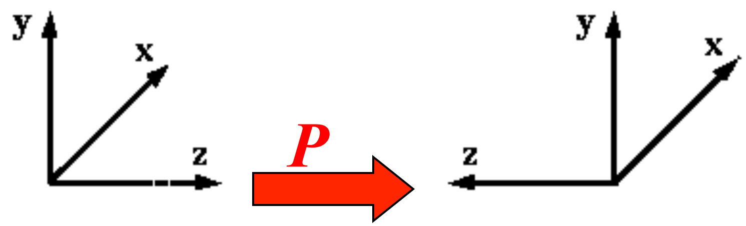
$$\left(\vec{u}_x \times \vec{u}_y\right)\cdot \vec{u}_z = 1$$



$$\left(\vec{u}_x \times \vec{u}_y\right)\cdot \vec{u}_z = -1$$

($\vec{u}$: unit vectors along the three axes)

P transformation equivalent to a mirror reflection



(first, rotate by 180° around the z – axis ; then reverse all three axes)

PARITY INVARIANCE:

All physics laws are invariant with respect to a *P* transformation;

For any given physical system, the mirror-symmetric system is equally probable;

In particle physics Nature does not know the difference between Right and Left.

Parity transformation, P, inverts every spatial coordinate

$$\mathbf{P(t, x) = (t, -x)}$$

$\mathbf{P^2 = 1}$, therefore eigenvalues of P are ± 1.

Ordinary vector v: $\mathbf{P(v) = -v}$

Scalar from v: $s = v \cdot v$

$$\mathbf{P(s) = P(v \cdot v) = (-v) \cdot (-v) = v \cdot v = +s}$$

Cross product of two vectors: $a = v \times w$

$$\mathbf{P(a) = P(v \times w) = (-v) \times (-w) = v \times w = +a}$$

Scalar from a and v: $p = a \cdot v$

$$\mathbf{P(p) = P(a \cdot v) = (+a) \cdot (-v) = - a \cdot v = -p}$$

<u>P transformations:</u>

Scalar	$P(s) = +s$
Pseudoscalar	$P(p) = -p$
Vector	$P(v) = -v$
Pseudovector (axial vector)	$P(a) = +a$

- *Two body system (a,b) has parity* $P(a)P(b)(-1)^l$
- *Particles and antiparticles have opposite parity*
- *Bound states have parity of* $(-1)^{l+1}$
- *Photons have parity of (-1) – this gives the* $\Delta l = \pm 1$ *selection rule in atomic transitions*

  l – orbital momentum

***Note**: Parity is a multiplicative quantum number – true for all discrete symmetries. Continuous symmetries have additive quantum numbers.*

- ***Parity is conserved in strong interactions, electromagnetism and gravity**. (not in weak interactions)*

Vector transformation under *P*

Radial (position) vector $\vec{r} \equiv (x, y, z) \Rightarrow (-x, -y, -z)$

Momentum vector $\vec{p} \equiv (p_x, p_y, p_z) \Rightarrow (-p_x, -p_y, -p_z)$

(all three components change sign)

Angular momentum $\vec{L} \equiv \vec{r} \times \vec{p} \Rightarrow \vec{r} \times \vec{p}$

(the three components do not change)

Spin $\vec{s}$: same behavior as for angular momentum ($\vec{s} \rightarrow \vec{s}$)

➡ **a scalar term of type $\vec{s} \cdot \vec{p}$ changes sign under *P***

If the transition probability for a certain process depends on a term of type $\vec{s} \cdot \vec{p}$, the process violates parity invariance

A puzzle in the early 1950's : the decays $K^+ \rightarrow \pi^+ \pi^\circ$ and $K^+ \rightarrow 3\pi$ ($\pi^+ \pi^+ \pi^-$ and $\pi^+ \pi^\circ \pi^\circ$)

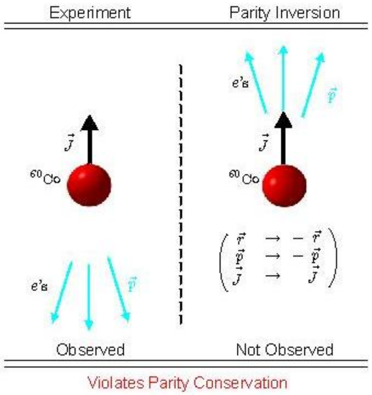
A system of two π– mesons and a system of three π– mesons, both in a state of total angular momentum =0, have OPPOSITE PARITIES

1956 C.S. Wu experiment - decay of polarized Co(60)

$$^{60}Co \rightarrow {}^{60}Ni + e^- + 2\gamma$$

- *electron from beta decay of Cobalt (weak interactions)*
- *photons from EM decays of Nickel*

observed result after changing polarization



expected result for parity conseravtion

Beta decay of Cobalt(60) – electron emitted in the direction of the nuclear spin

In the mirror image of the process –spin points upwards but electrons are still emitted downwards –in the direction opposite to spi

Radioactive decays (weak interactions) maximally violate parity P

1956: Suggestion (by T.D. Lee and C.N. Yang)
Weak interactions are NOT INVARIANT under Parity

$\pi^+ \rightarrow \mu^+ + \nu$ decay

Parity invariance requires that the two states

must be produced with equal probabilities → the emitted μ^+ is not polarized

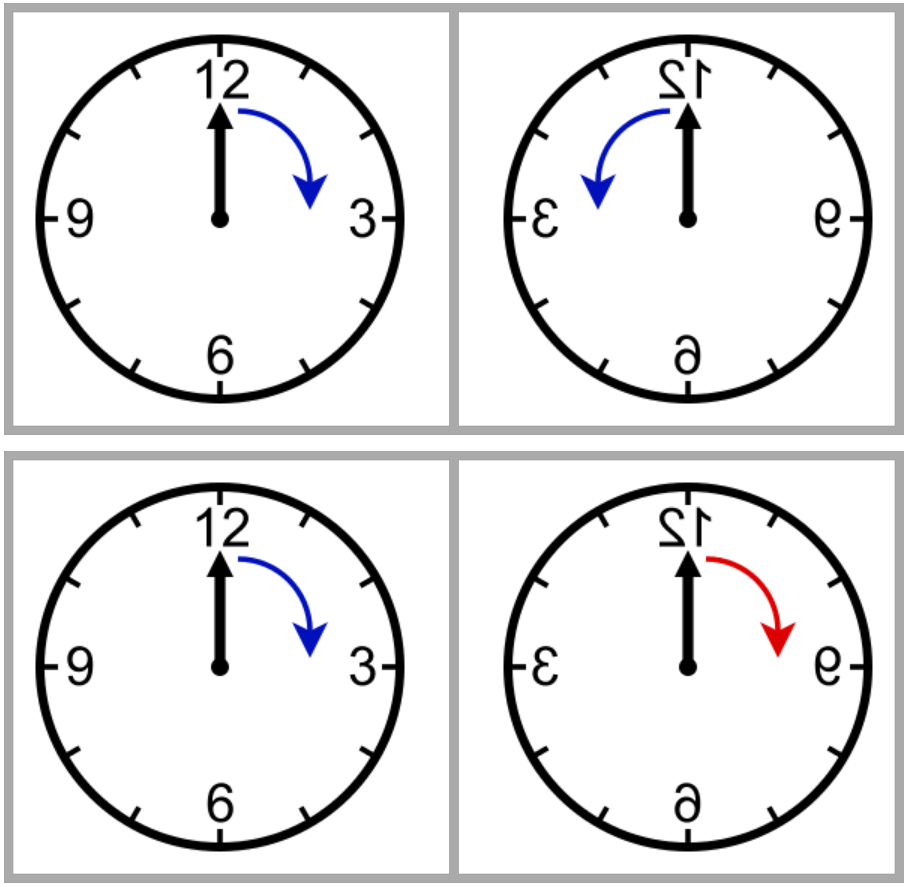
Experiments find that the μ^+ has full polarization opposite to the momentum direction → STATE A DOES NOT EXIST
→ MAXIMAL VIOLATION OF PARITY INVARIANCE

Parity and time reversal

P-symmetry: A clock built like its mirrored image

will behave like the mirrored image of the original clock.

P-asymmetry: A clock built like its mirrored image will not **behave like the mirrored image of the original clock.**



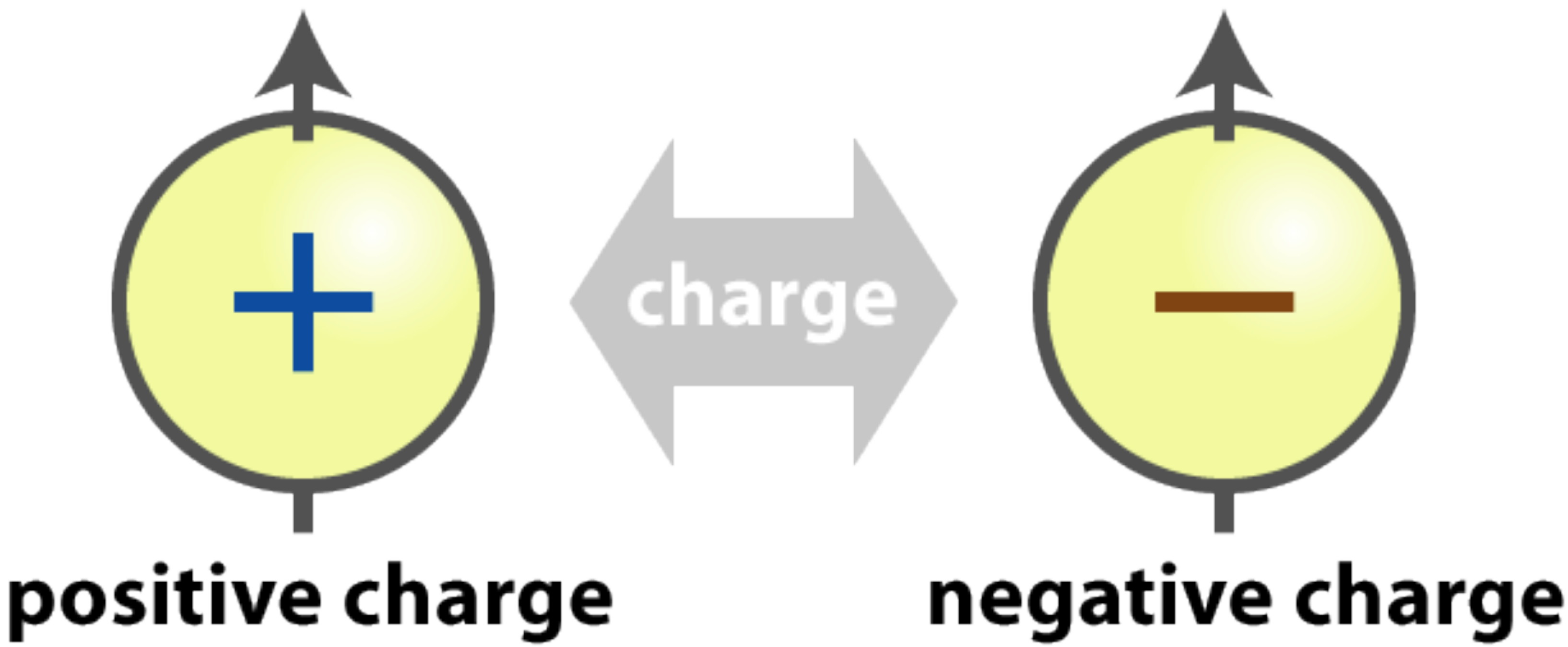
Parity operator - *reverses all three spatial coordinates in a reference frame*

- *equivalent to mirror reflection*

Another parity violation

Magritte

Charge Conjugation



Naively, one would expect that if we would swap all charges in the Universe the force between objects would be the same and nothing would change.

Charge Conjugation

• *The charge conjugation operator, C, converts a particle to its antiparticle.* $C \mid p > = \mid \bar{p} >$

In particular, C reverses ***every*** *internal quantum number (e.g. charge, baryon/lepton number, strangeness, etc.).*

• $C^2 = 1$ *implies that the only allowed eigenvalues of C are ±1.*

• *Unlike parity, very few particles are C eigenstates. Only particles that are their own antiparticles (π^0, η, γ) are C eigenstates.*

$C\,|\pi^+> = |\pi^-> \qquad C|\pi^0> = |\pi^0>$

•*The photon has C = -1* $\qquad C\,|\gamma> = -|\gamma>$

•*ff bound states (eg, two quarks) have* $C = (-1)^{\ell+s}$

• *Charge conjugation is respected by both the strong and electromagnetic interactions.*

• *Example: the* π^0 ($\ell = s = 0 \Rightarrow C = +1$) *can decay into 2γ but not 3γ*

$C\,|n\gamma> = (-1)^n\,|\gamma>$

$\pi^0 \rightarrow 2\gamma$ *is allowed (and observed)*

$\pi^0 \rightarrow 3\gamma$ *is not allowed (and not observed* $< 3.1 \times\sim 10^{-8}$*)*

Weak interactions violate C symmetry

Charge conjugation applies to all charges including "color" in strong interactions

Formalism of charge conjugation transform (field theory)

$$\psi \to -i(\bar{\psi}\gamma^0\psi^2)^T$$

$$\bar{\psi} \to -i(\gamma^0\gamma^2\psi)^T$$

$$A^\mu \to -A^\mu$$

does not change chirality of particles.
chirality – means the spin (left- or right-handedness)

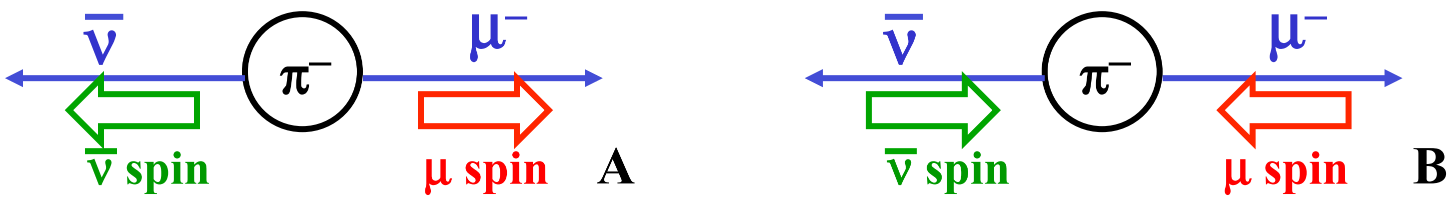
It has been believed that since both C and P parities are maximally violated in weak interactions – then perhaps the product of the two transformation CP will be conserved.

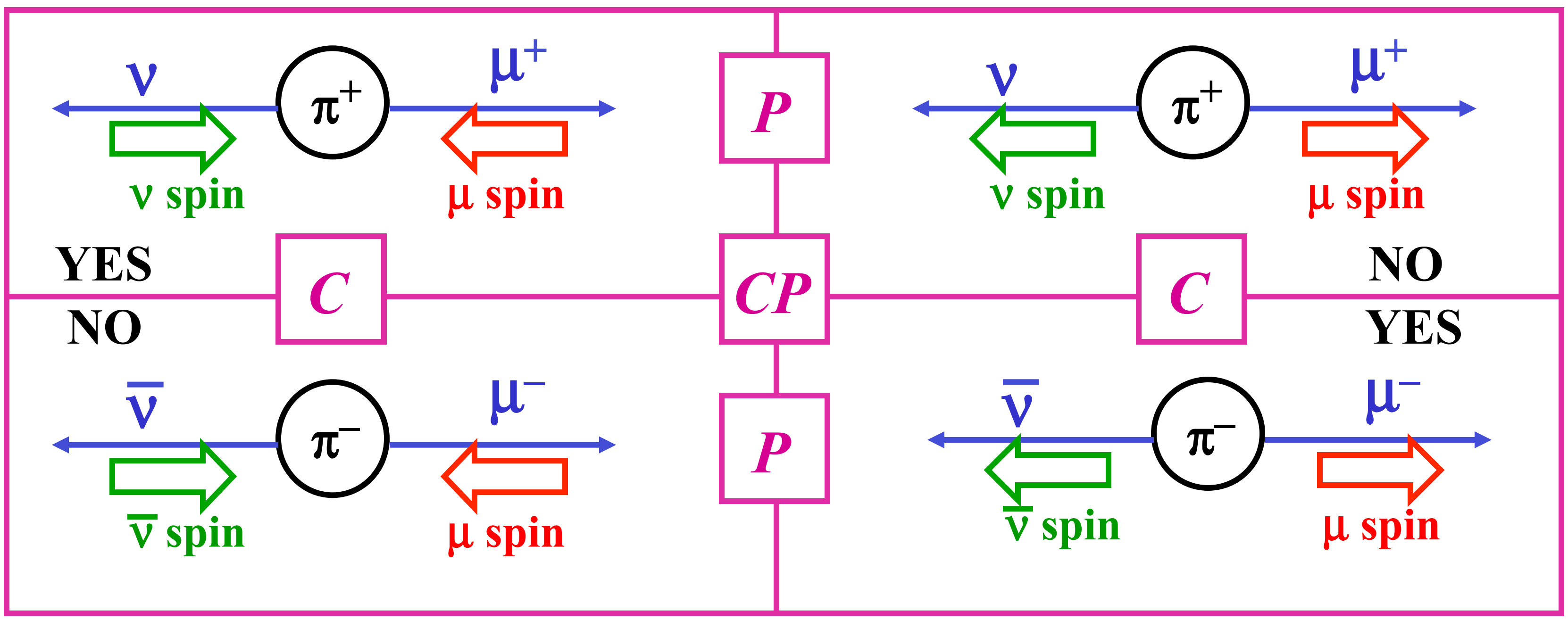
CHARGE CONJUGATION (*C*)

Particle → antiparticle transformation

$\pi^- \rightarrow \mu^- + \bar{\nu}$ decay

Experiments find that state B does not exist





➡ **π – meson decay violates maximally *C* and *P* invariance, but is invariant under *CP***

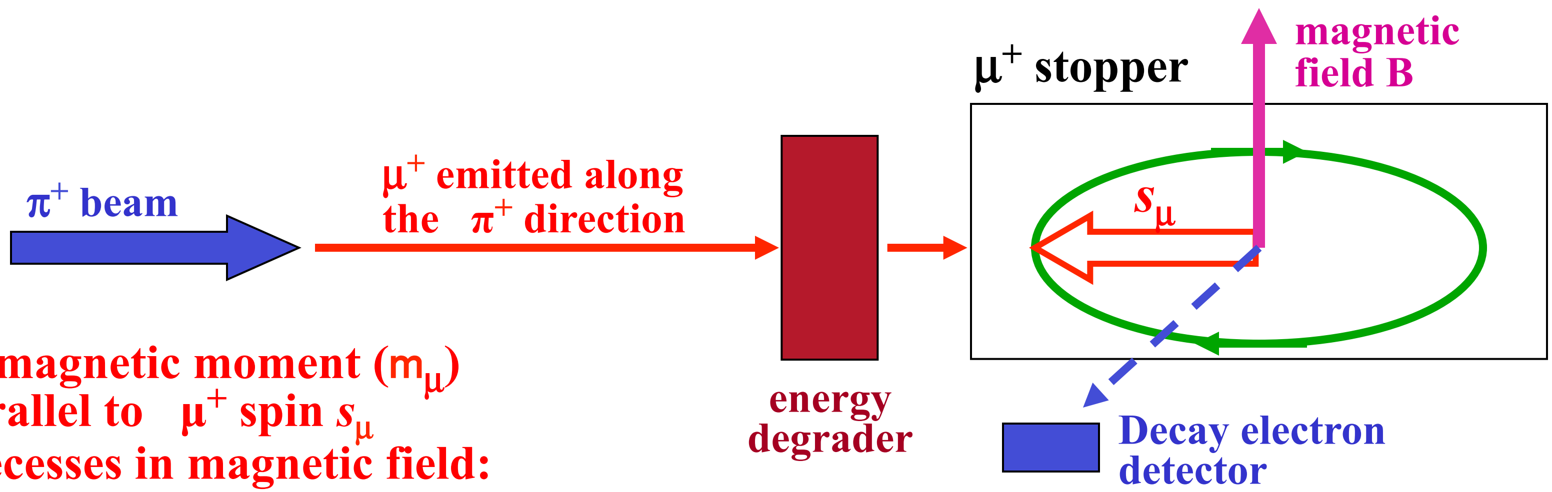
(a) Right-handed (b) Left-handed

All neutrinos are left handed and all antineutrinos are right handed
The right-handed neutrinos do not exist or do not interact with anything.
The left-handed antineutrinos do not exist or do not interact with anything

The observation of neutrino oscillations complicates the picture because that requires neutrinos to have non-zero (but very small) mass and the handedness is not a constant of relativistic motion.

Future of Fermilab projects lice MicroBOONE, DUNE and others will study neutrino interactions with high precision.

Method to measure the μ^+ polarization (R.L. Garwin, 1957)



μ^+ magnetic moment (m_μ) parallel to μ^+ spin s_μ precesses in magnetic field:

precession rate $\omega = 2 m_\mu B / \hbar$

Electron angular distribution from μ^+ decay at rest :

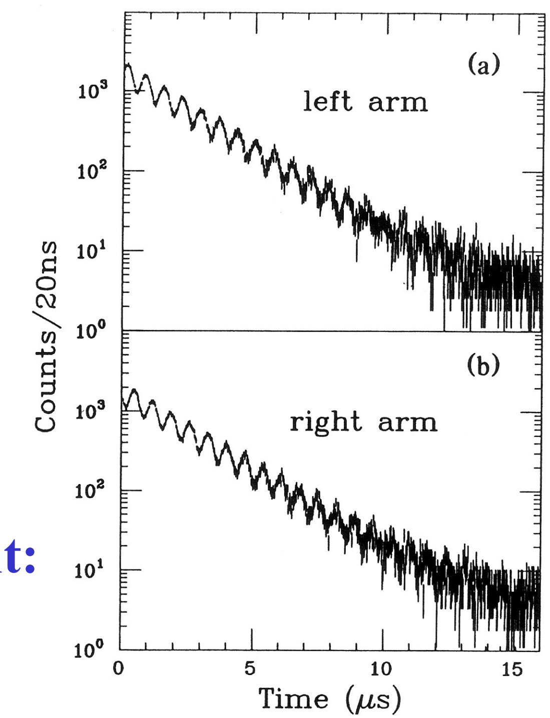
$dN / d\Omega = 1 + a \cos\theta$

θ - angle between electron direction and μ^+ spin s_μ

$\cos\theta \sim s_\mu \cdot p_e$ (term violating *P* invariance)

Spin precession: $\cos\theta \approx \cos(\omega t + \varphi)$

→ modulation of the decay electron time distribution



Experimental results:

- $a = -1/3$ → **evidence for *P* violation in μ^+ decay**
- **Simultaneous measurement of the μ^+ magnetic moment:**

$$\mu_\mu = \frac{e\hbar}{2m_\mu} \approx 2.79 \times 10^{-7} \text{ [eV/T]}$$