Accelerators Ideal Case

Goal of an accelerator: increase energy of CHARGED particles

- Increase energy

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q\int_{\vec{r}_1}^{\vec{r}_2} (\vec{E} + \vec{v} \times \vec{B})\, d\vec{r}$$

- The particle trajectory direction d**r** parallel to **v**

$$\Delta E = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\, d\vec{r} = q\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\, d\vec{r} = qU$$

- **…**increase of energy with electric fields

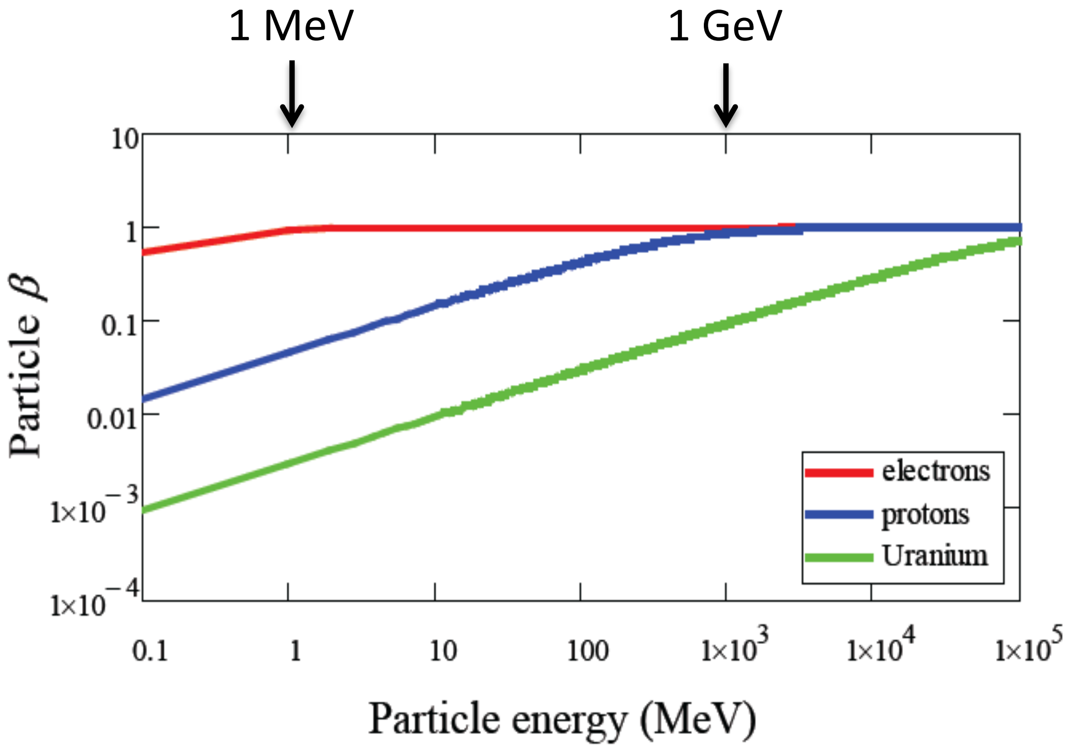
- Magnetic fields are used for control of trajectories

Energy vs velocity

- Acceleration increases velocity of a particle at small β = v/c.
- For high β velocity does not change. There is a change of momentum/ energy that can be expressed as a change of effective mass

Particle rest mass:

electron	0.511 MeV
proton	938 MeV
^{239}U	~220000 MeV

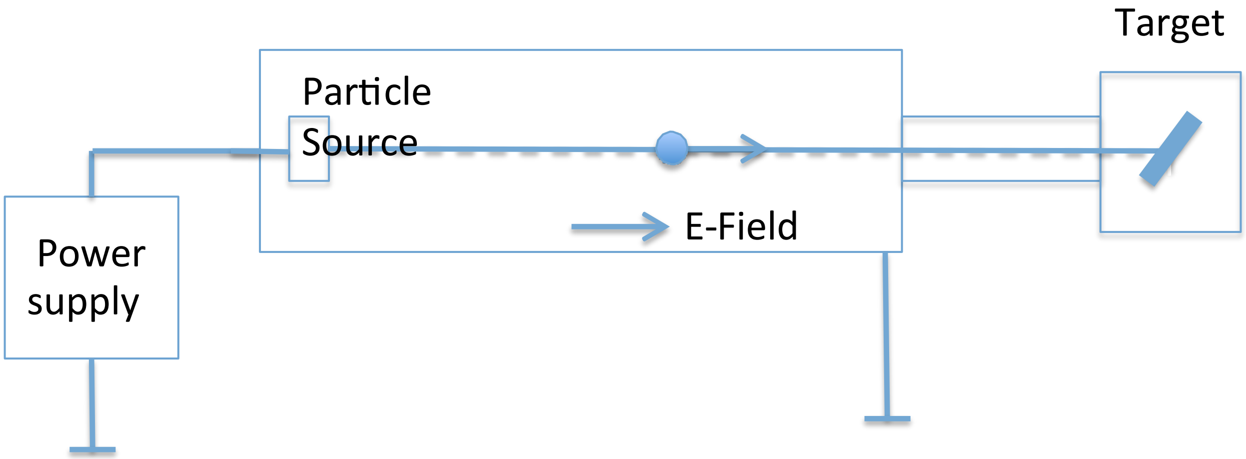

1 MeV

1 GeV

$$\gamma = \frac{E}{E_0} = \frac{m}{m_0}$$

Electrostatic accelerator

Set up electrostatic potential along particle trajectory.
Charged particles go through the accelerating voltage gap

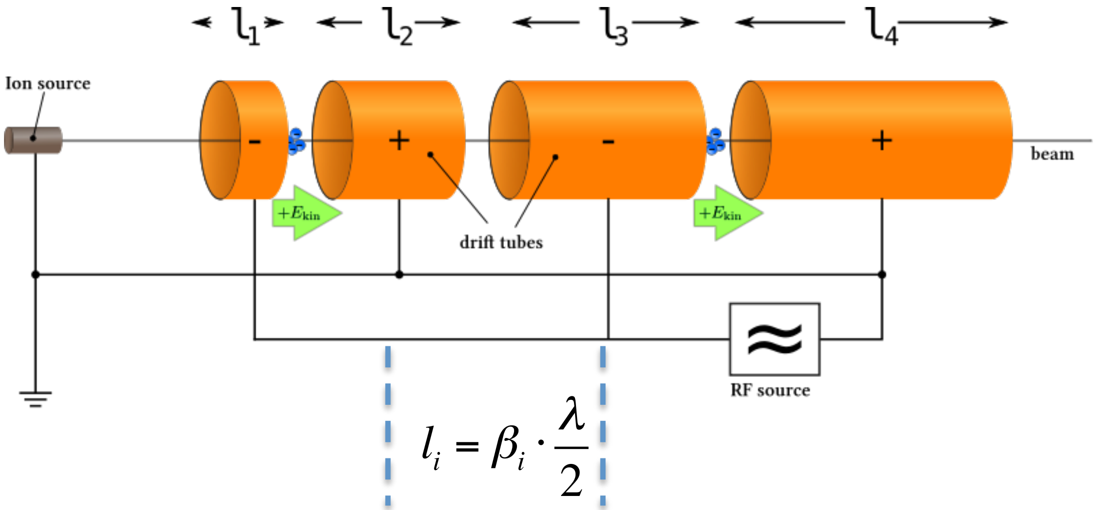


Limited by the maximum reachable voltage: ~ 10 MV

(For potentials above 10 MeV the electrostatic force may strip orbital electrons from the atoms of the material from which the accelerator is constructed creating sparks, breakdowns of the material and general mayhem).

Alternating RF Field

Apply the same voltage through acceleration gap many times.



$$l_i = \beta_i \cdot \frac{\lambda}{2}$$

Energy gain per gap:

$E = q\ \mathbf{V_{RF}} \sin(f_s)$

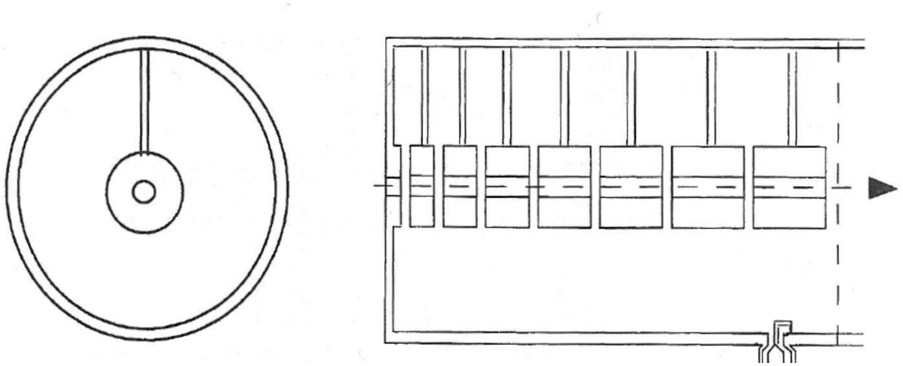
fs…phase wrt to RF field

- Particle synchronous with field. In shielding tube when field has opposite sign. Voltage across each cell the same.

- Shielding tubes have to become longer and longer, as particles become faster and faster or frequency must become higher $l = c/f_{RF}$
- Problem - radiation power loss: $P = \omega_{RF} C V_{RF}^2$, C - gap capacitance

Drift Tube Linac (Alvarez)

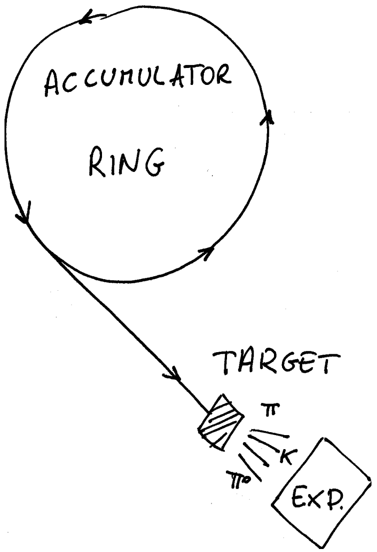
- To eliminate power loss place drift tubes in a RF cavity
  - Electromagnetic field oscillating in cavity. Standing wave, TM mode
    TM – transverse magnetic mode of electromagnetic wave propagation imposed by the boundary condition – no magnetic field in the direction of propagation
    longitudinal E Field, transverse B Field
  - Resonant frequency of cavity = accelerating field frequency!
  - Reduces power loss
  - Exploit Farraday's law: $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$

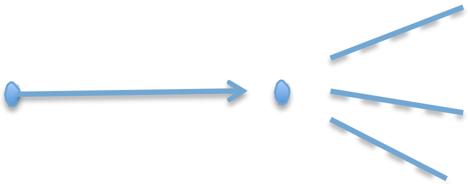


Longest electron linac - SLAC 2 miles, can reach 50 GeV
Proposed International Linear Collider - ~20 miles can reach ~300 GeV

Fixed target vs. Colliders

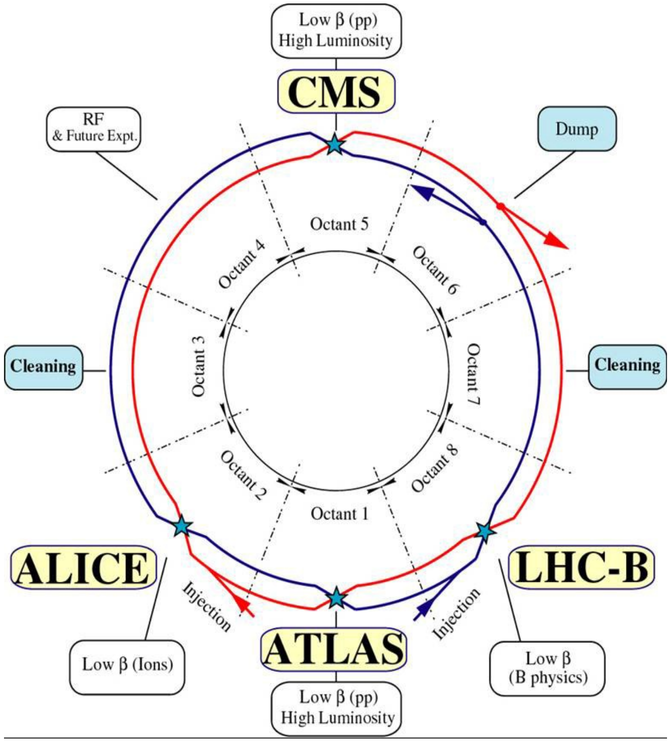
increase energy

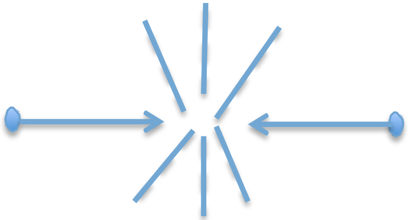




Approach used e.g., for test beams

provide collisions

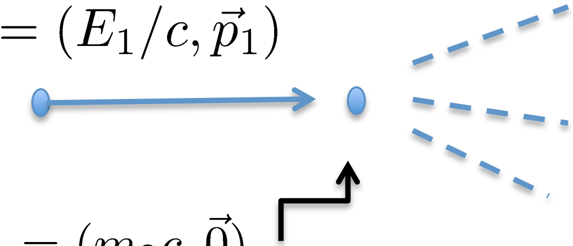




E_{CM} in Fixed Target Experiment vs Collider

Fixed target – target nucleon is stationary

$$p_1 = (E_1/c, \vec{p}_1)$$



$$p_2 = (m_2 c, \vec{0})$$

$$p_{tot} = (E_1/c + m_2 c, \vec{p}_1)$$

$$E_{CM}^2 = (m_1^2 + m_2^2)c^4 + 2E_1 m_2 c^2$$

$$\boxed{E_{CM} \propto \sqrt{E_1}}$$

Collider – laboratory frame = center of mass frame

$$p_1 = (E_1/c, \vec{p}_1)$$

$$p_2 = (E_2/c, -\vec{p}_1)$$

$$\boxed{E_{CM} = E_1 + E_2}$$

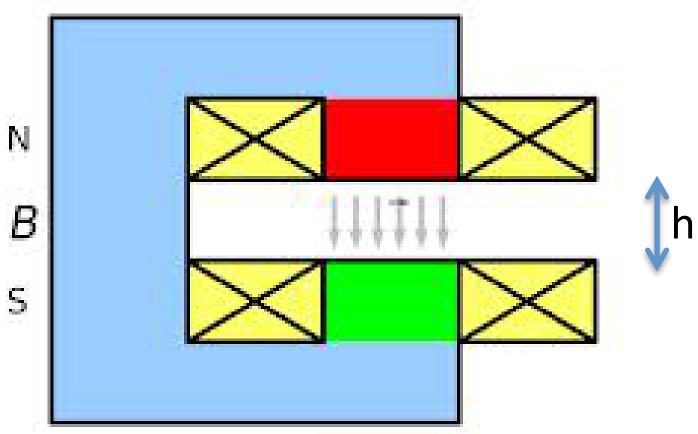
Circular Accelerators

- Linear accelerators can in principle accelerate to arbitrarily high energies, …….. but become longer and longer!
- Particles on circular paths to pass accelerating gap over and over again – circular accelerator
- First cyclotron proposed by E.O. Lawrence in 1929 and built by Livingston in 1931.
- Many conceptual modifications since then
- Synchrotron – constant radius, B field increases synchronously with beam energy. Single beam so magnets can be relatively simple.
- Collider more energy efficient but also more complex: two beams to be accelerated and to be brought into collision.
  Single beam ring for proton-antiproton colliders (opposite charges),
  Two beam rings for proton-proton colliders (same charges) – complex magnets

Dipole magnets: guiding magnets

- Vertical magnetic field to bend in the horizontal plane
- Dipole electro-magnets

$$\boxed{\vec{F} = q \cdot \vec{v} \times \vec{B}}$$



$$B = \frac{\mu_0 n I}{h}$$

Dipole magnets: guiding magnets

- Circular accelerator: Lorentz Force = Centrifugal Force

$$\begin{aligned} F_L &= qvB \\ F_{centr} &= \frac{mv^2}{\rho} \end{aligned} \longrightarrow \frac{mv^2}{\rho} = qvB$$

-
- $\boxed{\frac{p}{q} = B\rho}$

Bρ - Beam rigidity

ρ – radius of the circular orbit

- Useful formula:

$$\boxed{\frac{1}{\rho[m]} \approx 0.3\frac{B[T]}{p[GeV/c]}}$$

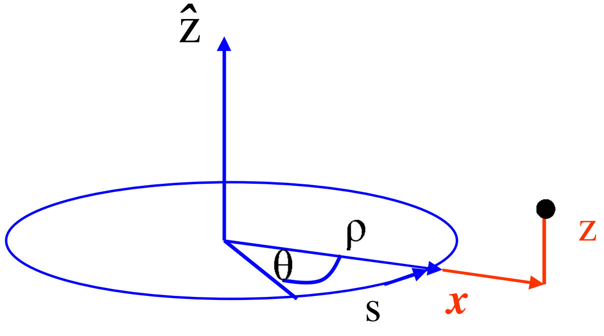
- Example for the LHC
  - p^+ @ 7 TeV/c
  - 8.3 T

$$\frac{1}{2.53\ km} = 0.3\frac{8.3}{7000}$$

Weak Focusing

Particles with deviations from the design trajectory need to feel restoring forces F(z) ~ -z otherwise the beam diverges

$$B_x = -const \cdot z \qquad \frac{\partial B_x}{\partial z} = -const$$



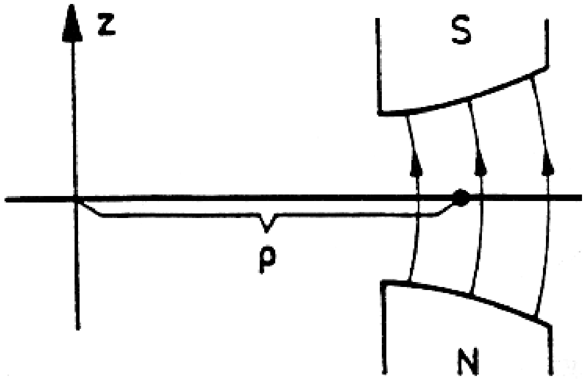
circular coordinate system

For deviations in the vertical plane we need either horizontal field component or a radially decreasing guide field

Maxwells equation:

$$\vec{\nabla} \times \vec{B} = 0$$

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \frac{\partial B_z}{\partial r} < 0$$



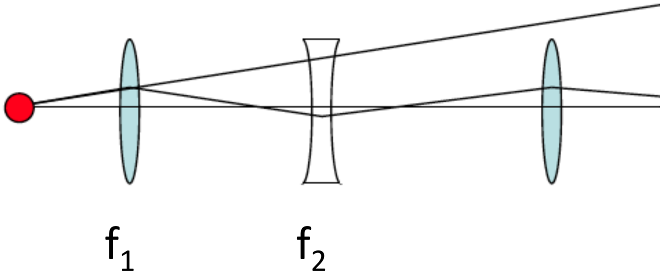
Similar considerations for horizontal plane

Pole shape in combined function magnet

Strong Focusing

Alternating gradient focusing

- Analogous to geometrical optics: a series of alternating focusing and defocusing lenses will focus.
- No need to shape the poles of the magnets



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

For fixed focal length $f_1=f$, $f_2 = - f$ → $F = d/f^2 > 0$

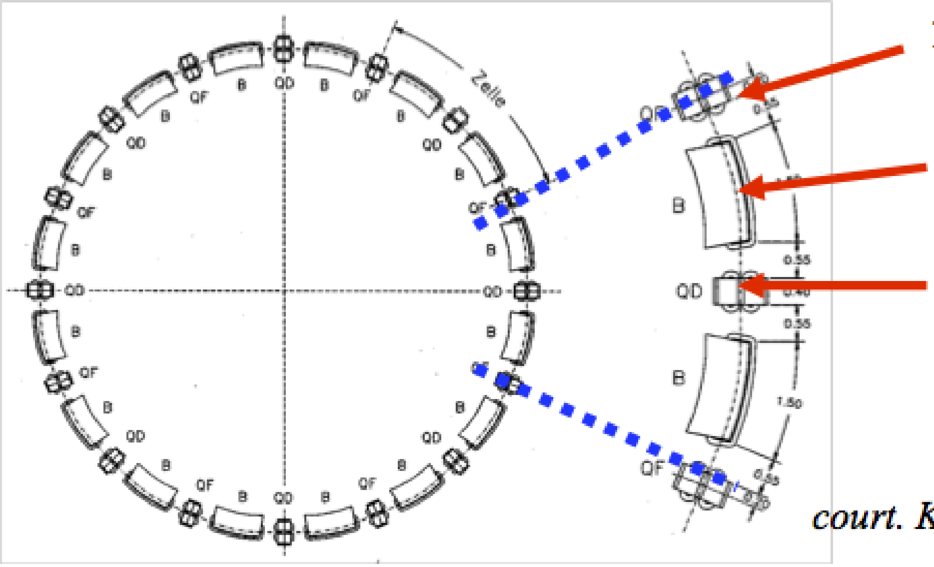
In this case the lenses are magnets with alternating gradients

Thin lens approximation – lens located in the middle of the magnet

FODO structure: O – bending magnets

All new (since 1960) accelerators are strong focusing synchrotrons

How to keep particles on a circular trajectory



focusing lens

dipole magnet

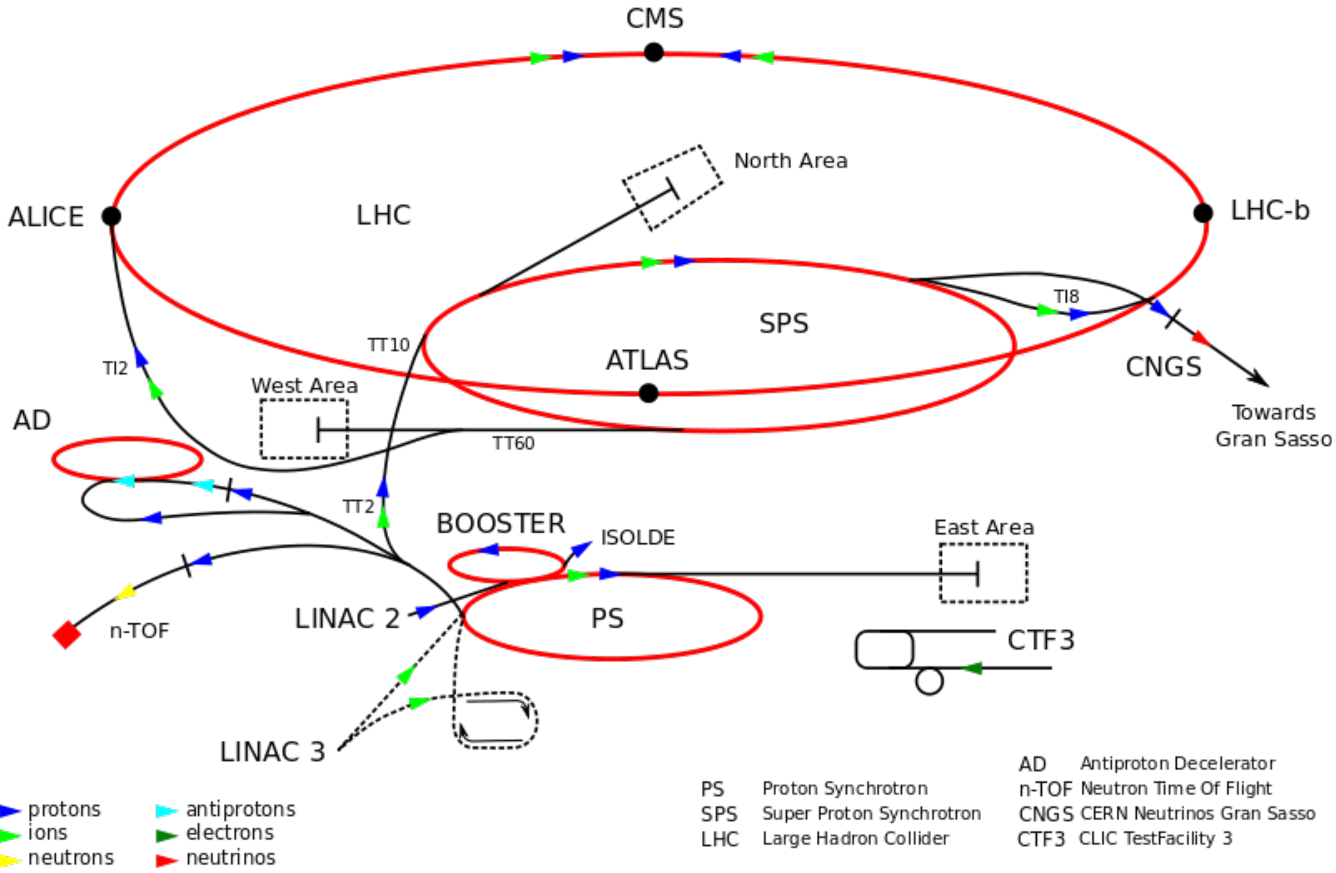
defocusing lens

court. K. Wille

Which particle to use?

e^- , e^+	**p^+**
Elementary particles with no internal structure	Consist of quarks held together by gluons
The total energy of the collider is transferred into the collision	The constituents of the protons collide. The energy available for the collision less than the collider energy
Precision measurements: beam energy can be exactly tuned to optimize the analysis	Discovery machine: with a single chosen beam energy different processes at different energies can be scanned
Disadvantage for circular colliders: low mass of these leptons. High power loss due to synchrotron radiation $P_{loss} \propto \frac{E^4}{m^4} \times \frac{current}{R}$ Solution: linear accelerators - long	

The CERN Accelerator Complex



PS (Proton Synchrotron):
1959
LHC (Large Hadron Collider):
2008

Circumference:
PS ~ 628 m
LHC ~ 27 km

Energy:
PS 26 × 10^9 eV (GeV)
LHC 7 × 10^{12} eV (TeV)

Particles produced through:
PS: fixed target collisions
LHC: beam-beam collisions