Chapter 1: Measurement

Dimensions

The following are different dimensions (physical natures):

length, area, volume, mass, density, time, speed, acceleration, ...

Some dimensions are chosen to be <u>fundamental</u>:

length	L
mass	M
time	T

The other dimensions are expressed in terms of these:

$$\text{area} = L^2$$
$$\text{volume} = L^3$$
$$\text{density} = M/L^3$$
$$\text{speed} = L/T$$
$$\text{acceleration} = L/T^2$$

Units

The fundamental set was chosen because standard units for L, M, and T can be defined very precisely:

$$[\text{meter}] = L$$
$$[\text{kilogram}] = M$$
$$[\text{second}] = T$$

It does not make sense to <u>add</u> quantities with different dimensions,

Ex. 2 apples + 3 oranges = 5 ???

2 meters + 3 seconds = ???

<u>but</u> different units with the same dimension can be added.

Ex. $2\,m + 3\,ft = 2.91\,m = 9.56\,ft$

The dimensions satisfy the same equality as the quantities.

Ex. $[2m] + [3\,ft] = [2.91\,m] = [9.56\,ft]$

$$L + L = L = L$$

Quantities with different dimensions can certainly be multiplied and divided.

Ex. $50\,\frac{\text{miles}}{\text{hour}} \times 3\text{ hours} = 150\text{ miles}$

$$\left[50\,\frac{\text{miles}}{\text{hour}}\right] \times [3\text{ hours}] = [150\text{ miles}]$$

$$\frac{L}{\cancel{T}} \times \cancel{T} = L$$

Dimensional Analysis

cannot tell you if the answer is right, but it can tell you if an answer is wrong.

Ex.

$$\rho v^2 \stackrel{?}{=} m a$$

$$[\rho][v^2] \stackrel{?}{=} [m][a]$$

$$\frac{M}{L^3}\left(\frac{L}{T}\right)^2 \stackrel{?}{=} M\frac{L}{T^2}$$

$$\frac{M}{LT^2} \stackrel{?}{=} \frac{ML}{T^2} \qquad X$$

The reason you can't be sure that you got an answer right by dimensional analysis:

p1.1a

Ex. $x = k\, a^n\, t^m$

x: distance; k: some number; a: acceleration; t: time

$$[x] = [k][a^n][t^m]$$

$$L = 1 \cdot \left(\frac{L}{T^2}\right)^n T^m$$

$$L^1 T^0 = L^n T^{m-2n}$$

$$\left.\begin{aligned} 1 &= n \\ 0 &= m - 2n \end{aligned}\right\} \quad \begin{aligned} n &= 1 \\ m &= 2 \end{aligned}$$

Dimensional analysis tells us <u>nothing</u> about the dimensionless constant k.

$$x = k a t^2 \qquad \left(\text{in fact, } k = \tfrac{1}{2}\right)$$

Unit Conversion

We are always free to multiply any quantity by 1.

Some odd names for 1 are:

$$1 = \left(\frac{1 \text{ meter}}{39.37 \text{ in}}\right) \qquad 1 = \left(\frac{39.37 \text{ in}}{1 \text{ meter}}\right)$$

$$1 = \left(\frac{1 \text{ hour}}{3600 \text{ sec}}\right) \qquad 1 = \left(\frac{1 \text{ hour}}{3600 \text{ sec}}\right)^2$$

Ex. Convert $50 \frac{\text{miles}}{\text{hour}}$ to $\frac{\text{feet}}{\text{second}}$

$$50 \frac{\text{miles}}{\text{hour}} (1)(1)$$

$$= 50 \frac{\cancel{\text{miles}}}{\cancel{\text{hour}}} \left(\frac{5280 \text{ feet}}{1 \cancel{\text{mile}}}\right)\left(\frac{1 \cancel{\text{hour}}}{3600 \text{ sec}}\right)$$

$$= 73\tfrac{1}{3} \frac{\text{feet}}{\text{sec}}$$

A more complicated unit conversion example:

Ex. The position of a certain particle varies in time as

$$x(t) = 9.1 + 2.4t + 3.7t^2$$

where x is in meters and t is measured in seconds.

(Common notation in the book!)

$$x(t) = 9.1\,m + (2.4\,\tfrac{m}{s})t + (3.7\,\tfrac{m}{s^2})t^2$$

Suppose that I wish to measure x in cm and t in minutes.

How do the coefficients change?

$$9.1\,m\,(1) = 9.1\,\cancel{m}\left(\frac{100\,cm}{1\,\cancel{m}}\right) = 910\,cm$$

$$2.4\,\frac{m}{s}(1)(1) = 2.4\,\frac{\cancel{m}}{\cancel{s}}\left(\frac{100\,cm}{1\,\cancel{m}}\right)\left(\frac{60\,\cancel{s}}{1\,min}\right)$$

$$= 14{,}400\,\frac{cm}{min}$$

$$3.7\,\frac{m}{s^2}(1)(1)^2 = 3.7\,\frac{\cancel{m}}{\cancel{s^2}}\left(\frac{100\,cm}{1\,\cancel{m}}\right)\left(\frac{60\,\cancel{s}}{1\,min}\right)^2$$

$$(3.7)(100)(60)^2\,\frac{cm}{min^2} = 1{,}332{,}000\,\frac{cm}{min^2}$$

$$x(t) = 910 + 14{,}400t + 1{,}332{,}000\,t^2$$

where x is in cm and t is in minutes.

Metric Prefixes

yotta	10^{24}
zetta	10^{21}
exa	10^{18}
peta	10^{15}
tera	10^{12}
giga	10^{9}
—mega	10^{6}
—kilo	10^{3}
hecto	10^{2}
deka	10^{1}
deci	10^{-1}
—centi	10^{-2}
—milli	10^{-3}
—micro	10^{-6}
nano	10^{-9}
pico	10^{-12}
femto	10^{-15}
atto	10^{-18}
zepto	10^{-21}
yocto	10^{-24}

Movie

Scientific Notation

- One non-zero numeral appears to the left of the decimal point.
- Any number of digits may follow the decimal point.
- This number is multiplied by 10 to some power.

Ex. $15 = 1.5 \times 10^{1}$

$4.1 = 4.1 \times 10^{0}$

$0.00325 = 3.25 \times 10^{-3}$

Stupid Calculator Tricks

A <u>very</u> common error is to enter a number like 10^6 incorrectly as:

$$10 \ \boxed{EE} \ 6$$

which, of course, is 10^7!

Remember: $10^6 = 1 \times 10^6$

should be entered as:

$$1 \ \boxed{EE} \ 6$$

Significant Figures

This concept is important in <u>measurement.</u>

<u>Ex.</u> I measure the thickness of Serway with a meter stick marked off in millimeters to be

$$53 \text{ mm} \pm 1 \text{ mm}$$

This answer is accurate to 2 significant figures.

In scientific notation:

$$5.3 \times 10^1 \text{ mm} \quad \text{or} \quad 5.3 \times 10^{-2} \text{ m}$$

Ex. I measure the width of Serway with the same meter stick to be 200 mm ± 1mm

This answer is accurate to 3 significant figures.

In scientific notation:

2.00×10^{2} mm a 2.00×10^{-1} m

The trailing zeroes must be kept in the answer! They are <u>significant</u>.

An ambiguity arises when you are given a measurement without an estimate of the error.

Ex. A contractor tells you that a stretch of road is 1500m long. How many significant figures are in that measurement?

Written in that form, there is no way to tell.

Scientific Notation to the rescue!

2 sig figs $\longrightarrow$ 1.5×10^{3} m

3 sig figs $\longrightarrow$ 1.50×10^{3} m

4 sig figs $\longrightarrow$ 1.500×10^{3} m

Zeroes that are used as placeholders are not significant.

Ex. How many sig figs in

0.000325 ? 3

Ex. How many sig figs in

0.0003250 ? 4

Much easier after conversion to scientifiz notation.

$0.000325 = 3.25 \times 10^{-4}$

$0.0003250 = 3.250 \times 10^{-4}$

Rule for sig figs in a product:

When multiplying or dividing several quantities, the number of sig figs in the final answer is equal to the smallest number of sig figs in any quantity.

Rule for sig figs in a sum:

When adding or subtracting several quantities, the number of decimal places in the final answer is equal to the smallest number of decimal places of any quantity.

- sig figs
- decimal places

Order of Magnitude Calculations

The spirit is conveyed by the other names:

- back-of-the-envelope calculations
- ball-park figures
- educated guesses
- within a factor of 10
- close enough for gov't work

No calculators required!

Only one significant figure should appear in the final answer.

Round-off errors should cancel each other.

Scaling

Ex. Consider a sphere

$$V(r) = \underbrace{\tfrac{4}{3}\pi}\, r^3 = k_1 r^3$$

$$A(r) = \underbrace{4\pi}\, r^2 = k_2 r^2$$

If the volume of a sphere increases by a factor of 64, by what factor does the surface area change?

$$V' = 64\,V$$

$$\cancel{k_1}(r')^3 = 64\,\cancel{k_1}(r)^3$$

$$r' = 4\,r$$

$$k_2(r')^2 = 16\,k_2(r)^2$$

$$\boxed{A' = 16\,A}$$

Chapter 3: Vectors

What is a vector?

A quantity with both a magnitude and a direction.

Ex. displacement, velocity, acceleration, momentum, force, ...

Vectors can be 1, 2, or 3 dimensional.