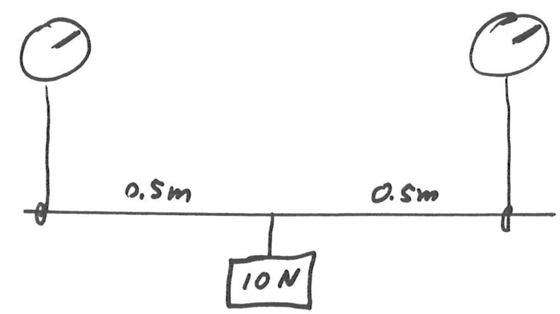
Chapter 12: Equilibrium

$$\sum \vec{F}_{ext} = 0 \quad \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

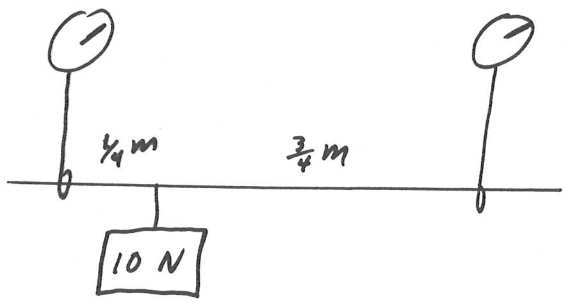
$$\sum \vec{\tau}_{ext} = 0 \quad \begin{cases} \sum \tau_x = 0 \\ \sum \tau_y = 0 \\ \sum \tau_z = 0 \end{cases}$$

About <u>every</u> origin.

Consider a 10 N weight (1 kg mass) on a massless rod suspended between two scales. What do the scales read?

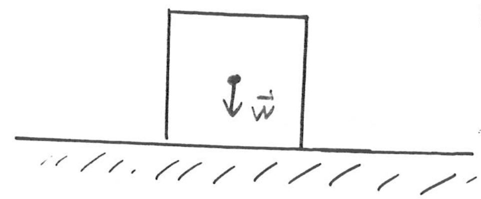


What about this arrangement?



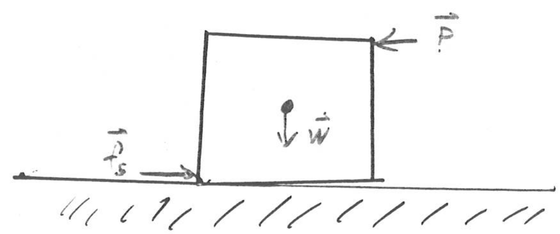
Consider a block on a table.

What forces act?
Where do they act?

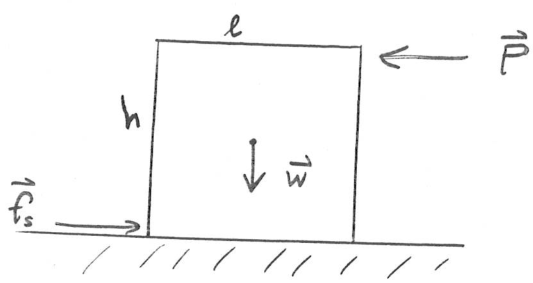


Now the block is tipped.

What forces act?
Where do they act?



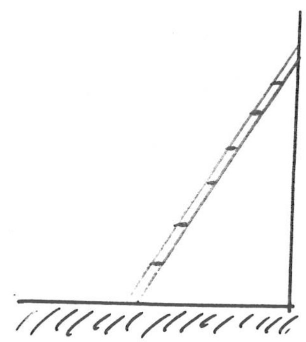
What happens to the normal force $\vec{N}$ as I push a block to topple it?



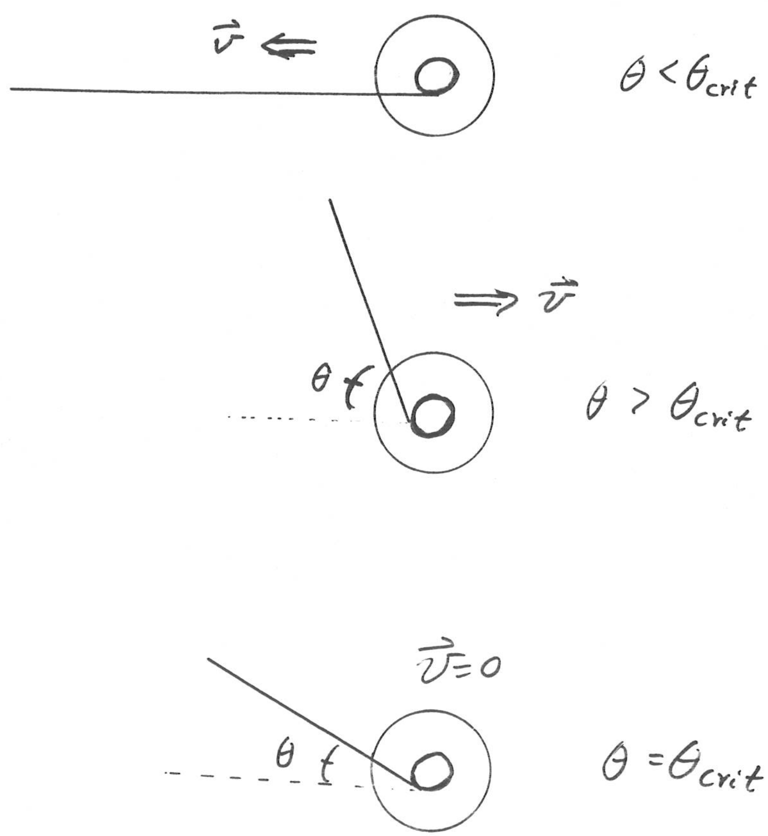
Find the distance from the corner at which $\vec{N}$ acts.

A Ladder Problem

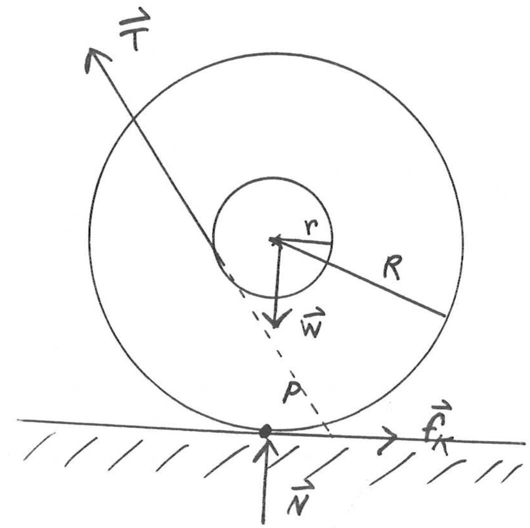
A uniform ladder of length $\underline{\ell}$ and mass $\underline{m}$ leans against a smooth wall. The floor is rough and has coefficient of static friction $\underline{\mu_s}$. At what critical angle will the ladder slip?



The yoyo effect:



Find θ_{crit}.

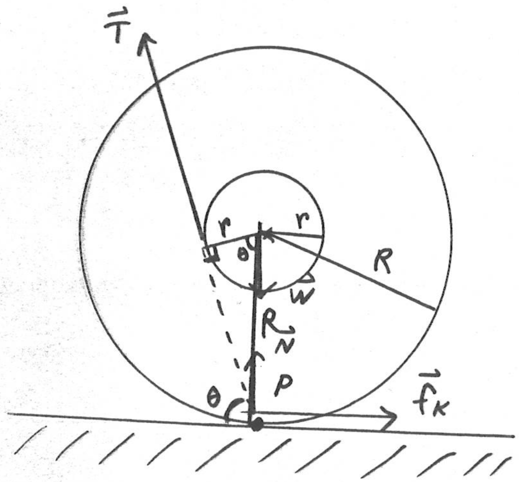


The torques due to all the forces must cancel so the yoyo rotates at constant ω $\Rightarrow$ $\alpha = 0$

The hard way: $\sum F_y = 0$ $\Rightarrow$ find N $\Rightarrow$ find f_x

$\sum F_x = 0$ $\Rightarrow$ find T_x

$\sum \tau_{cm} = 0$ $\Rightarrow$ find θ



The easy way: (No work required)

$\Sigma \tau_P = 0$ $\quad$ $\vec{W}$, $\vec{N}$, and $\vec{f}_K$ all pass through P $\Rightarrow$ no lever arms!
$\Rightarrow$ no torques

For no net torque, $\vec{T}$ must pass through P also. (An object cannot be in equilibrium under the action of one torque).

$$\cos\theta = \frac{adj}{hyp} = \frac{r}{R} \Rightarrow \theta_{critical} = \cos^{-1}\left(\frac{r}{R}\right)$$

The Center of Gravity

All of the weight of an object can be thought of as acting at a single point. This simplifies torque calculations enormously!

The center of gravity coincides with the center of mass as long as the gravitational field (acceleration) is constant.

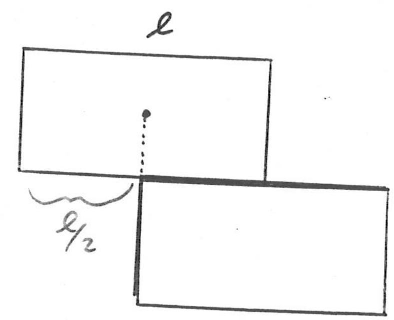
Ch. ~~14~~ 15 explores non-constant

When does an object fall over?

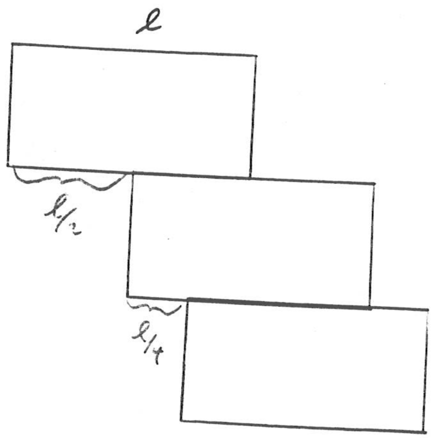
When the center of gravity is no longer over the base.

Demos

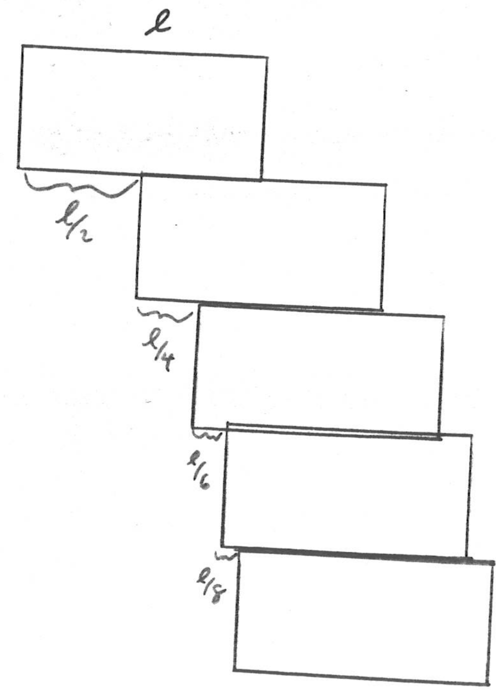
Cantilevered Blocks



Cantilevered Blocks



Cantilevered Blocks



$$\frac{\ell}{2}\underbrace{\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\right)}_{\text{Harmonic Series}} = \infty$$