Ch. 15 Fluid Mechanics

<u>Density</u> is the mass per unit volume.

$$\rho = \frac{m}{V}$$

<u>Ex.</u>
Which is worth more: gold or cotton?

1 mg of gold or 1000 metric tons of cotton?

We must compare the <u>same</u> amounts.
(Supermarkets display the unit price of items for comparison.)

"Price density"

gold: $2¢/mg$ cotton: $10^{-8}¢/mg$

saffron: $1.00/mg

<u>Ex.</u>
Which weighs more: gold or cotton?

We must specify the amount (volume).

1 m^3 of gold or 1 m^3 of cotton?

<u>Mass density</u>

$$\rho_{Au} = 19{,}300 \frac{kg}{m^3} \qquad \rho_{cotton} \approx 100 \frac{kg}{m^3}$$

Gold is <u>denser</u> than cotton. Given <u>equal amounts</u> of each, the gold weighs more than the cotton.

Water is a convenient standard for density

$$\rho_{water} = 1 \frac{g}{cc} = 1 \frac{\text{gram}}{\text{cubic centimeter}}$$

1 liter (1000cc) of water has a mass of 1 kg and a weight of 9.8 N.

<u>Ex</u> What is the mass of one cubic meter of water?

1000 kg = 1 metric ton

An object will float in a fluid if

$$\rho_{object} < \rho_{fluid}$$

and will sink if

$$\rho_{object} > \rho_{fluid}$$

Ex 12 oz. cans of soda have identical volumes, but...

Cans of regular and diet soda have different masses. Demo:

Compare their densities.

Compare their densities with ρ_{water}. Demo:

Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$[P] = \frac{[F]}{[A]} = \frac{\left(\frac{ML}{T^2}\right)}{(L^2)} = \frac{M}{LT^2}$$

MKS unit: $1 \text{ pascal} = 1 \text{ Pa} = 1 \frac{N}{m^2}$

Ex An elephant and a woman wearing high heeled shoes are walking in wet sand. Which leaves a deeper impression?

$$P_e = \frac{m_e g}{A_e} = \frac{(10{,}000 \text{ kg})(9.8 \text{ m/s}^2)}{4\,(0.5)^2} = 98{,}000 \text{ Pa}$$

$$P_w = \frac{m_w g}{A_w} = \frac{(70 \text{ kg})(9.8 \text{ m/s}^2)}{2\,(0.005 \text{ m})^2} = 13{,}720{,}000 \text{ Pa}$$

Atmospheric Pressure

The force per unit area at sea-level is

$$P_0 = 1.01 \times 10^5 \text{ Pa} \equiv 1.00 \text{ atm}$$
(atmosphere)

Ex What is the force of the atmosphere pushing down on a 1 m x 1 m sheet of paper?

$$F = P \cdot A = (1.01 \times 10^5 \text{ Pa})(1 \text{ m}^2)$$
$$= (1.01 \times 10^5 \tfrac{N}{m^2})(1 \text{ m}^2) = 1.01 \times 10^5 \text{ N}$$

(over 11 tons!)

With this much force acting on the surface area of your body, why aren't you crushed?

The same pressure exists <u>inside</u> your body. The pressure <u>difference</u> is zero.

Ex What force holds a suction cup to the wall?

Ex What force holds two steel hemispheres together?

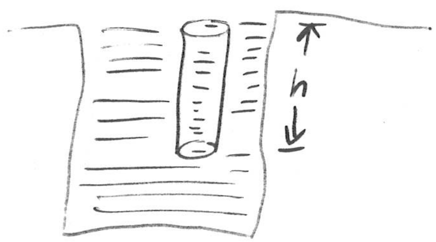
Pressure forces <u>do</u> <u>not</u> <u>suck</u>!

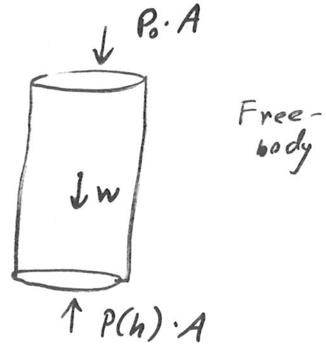
Variation of Pressure with Depth

Consider an incompressible fluid (one with constant density ρ)

Ex water, oil, not air

How does the pressure in this fluid change with distance below the surface?





$$W = mg = (\rho V)g = \rho(Ah)g$$

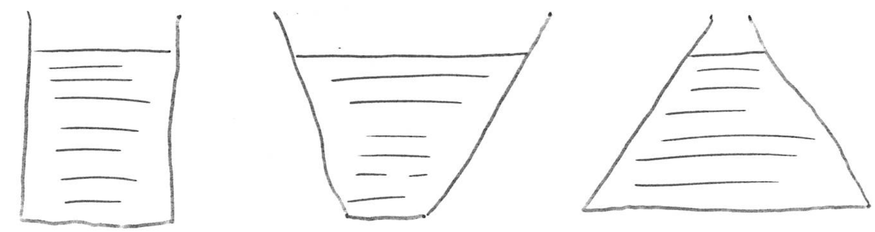
$$P(h) \cdot A - P_0 \cdot A - W = 0$$

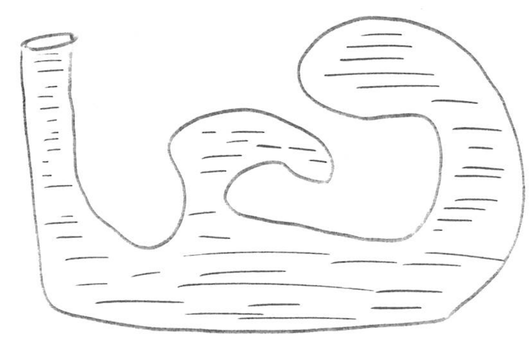
$$P(h) \cdot \cancel{A} - P_0 \cdot \cancel{A} - \rho \cancel{A} hg = 0$$

$$P(h) = P_0 + \rho g h$$

Pressure in the fluid varies linearly with depth.

Ex Compare the pressures at various points.





Ex What is the pressure at a depth of 20m in a fresh-water lake?

$$P(h) = P_0 + \rho_{water}\, g\, h$$

$$P(20m) = 1.01\times10^5\,Pa + (1000\,\tfrac{kg}{m^3})(9.8\,\tfrac{m}{s^2})(20m)$$

$$= 101{,}000\ Pa + 196{,}000\ Pa$$

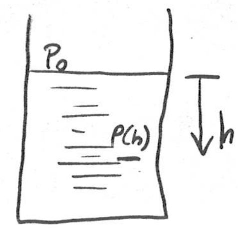
$$= 297{,}000\ Pa = 2.94\ atm$$

$P(h)$ is the <u>absolute</u> pressure.

$P(h) - P_0$ is the <u>gauge</u> pressure, since this is what a pressure gauge measures.

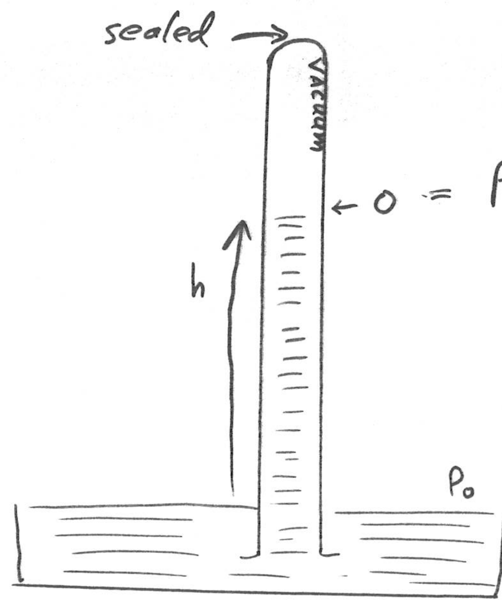
The Barometer

Pressure varies linearly with depth



$$P(h) = P_0 + \rho_{fluid}\, g\, h$$

The <u>same</u> formula holds for "negative depth"



$$\leftarrow 0 = P_0 - \rho_{fluid}\, g\, h$$

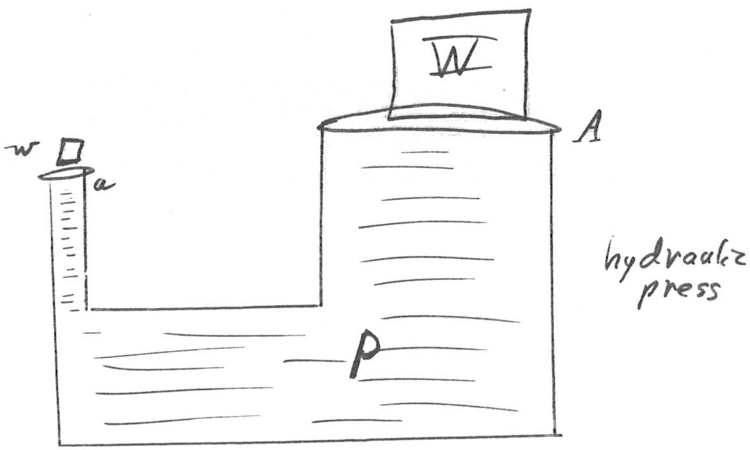
Given ρ_{fluid}, one can solve for h.

$\rho_{water} = 1\,\frac{g}{cc} \Rightarrow h = 10.3\,m$ $\sim 30\,ft$

$\rho_{Hg} = 13.6\,\frac{g}{cc} \Rightarrow h = 0.757\,m$ $= 757\,mm$

Pascal's Law

A change in the pressure applied to an enclosed <u>incompressible</u> liquid is transmitted undiminished to every point of the liquid and to the walls of the container.

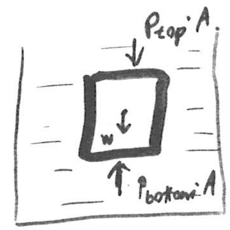


$$P_{top} = w/a = W/A$$

Archimedes' Principle

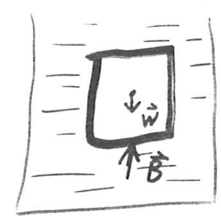
of Buoyant Forces

Any body completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid <u>displaced</u> by the body.



$$P_{bottom} = P_{top} + \rho_{fluid}\, g h$$

$$\Delta P = P_{bot} - P_{top} = \rho_{fluid}\, g h$$



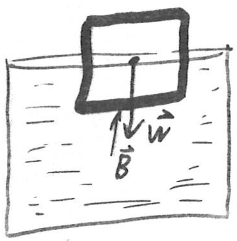
$$B = \underbrace{(\Delta P) \cdot A}_{= (P_{bottom} - P_{top}) A} = (\rho_{fluid}\, g h) \cdot A$$

$$= \rho_{fluid}\, g (hA) = \rho_{fluid}\, g V$$

$$= M_{\substack{fluid \\ in\ cube}}\, g = W_{\substack{fluid \\ in\ cube}}$$

$$W = m_{object}\, g$$

If the object is <u>less dense</u> than the fluid, then only a fraction of the object is submerged.



$\vec{W}$ = weight of object (wood)

$|\vec{B}|$ = weight of fluid (water) displaced

$$|\vec{W}| = |\vec{B}|$$

Ex

Suppose the object is half submerged.

$$W = m_{wood}\, g = (\rho_{wood} V)g$$

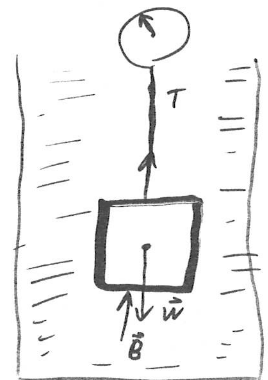
$$B = \text{weight of water displaced} = \left(\tfrac{1}{2}V\right)\rho_{water}\, g$$

$$W = B$$

$$\rho_{wood} \cancel{V} \cancel{g} = \tfrac{1}{2} \cancel{V} \rho_{water} \cancel{g}$$

$$\rho_{wood} = \tfrac{1}{2}\rho_{water} \quad \left(\begin{array}{c}\text{wood is}\\ \text{less dense}\end{array}\right)$$

If the object is <u>more dense</u> than the fluid, then it is totally submerged, but it is still buoyed up by a force B equal to the weight of fluid displaced.



$$\Sigma F_y = 0$$

$$T + B - W = 0$$

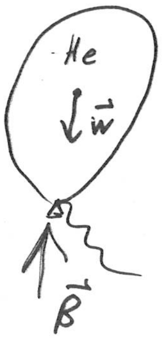
$$T = W - B$$

The scale reading T is the apparent weight (not true weight) of the object. T is less than the true weight W by the weight of fluid displaced.

$$T = W - B = \rho_{steel}\, g\, V - \rho_{water}\, g\, V$$

$$= (\rho_{steel} - \rho_{water})\, g\, V < W$$

Ex What volume of Helium is required to just get the balloon to float.



$$W = [m_{rubber} + m_{He}]\,g$$

$$B = m_{air}\,g$$

$$W = B$$

$$[m_{rubber} + \rho_{He} V]\cancel{g} = \rho_{air} V \cancel{g}$$

$$V = \frac{m_{rubber}}{\rho_{air} - \rho_{He}}$$

Specific Gravity

$$s.g. = \frac{\text{weight of object}}{\text{weight of water}}$$ same volumes

Ex The specific gravity of balsa wood is 0.15 (no units). What is the weight of $1m^3$ of balsa wood?

mass of $1m^3$ of water $= 1000\ kg$

weight of $1m^3$ of water $= 9800\ N$

weight of $1m^3$ of balsa $= (9800\ N)(0.15)$

$= \underline{1470\ N}$

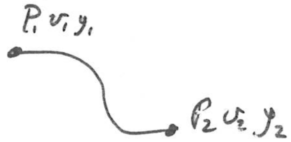
Bernoulli's Law

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

Consequence: Pressure is <u>least</u> where speed is <u>greatest</u>. (counter-intuitive)

Bernoulli's Law results from <u>energy</u> <u>conservation</u>.

$$W_{net} = \Delta K \qquad \text{work-energy theorem}$$



$$W_{pressure} + W_{gravity} = \Delta K$$

work done <u>on</u> system

$$-\int_1^2 (P\cdot A)\,dy - \int_1^2 mg\,dy = \frac{1}{2}m v_2^2 - \frac{1}{2}m v_1^2$$

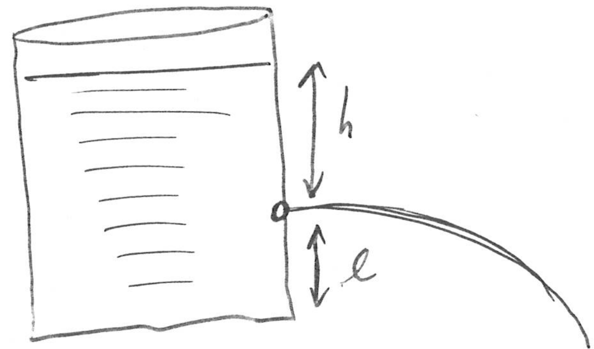
$$-\Delta P \cancel{\Delta V} - \rho g \cancel{\Delta V}\,\Delta y = \frac{1}{2}\rho \cancel{\Delta V}(v_2^2 - v_1^2)$$

$$-(P_2 - P_1) - \rho g(y_2 - y_1) = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

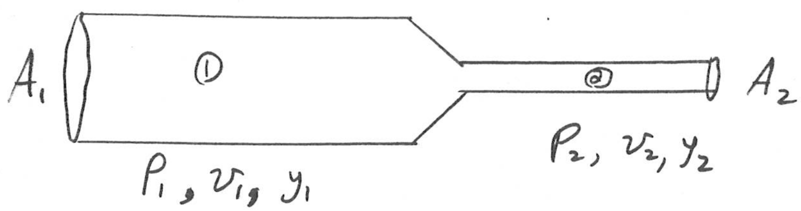
Applications

find speed, distance.



Applications

horizontal pipe



$$P_1 + \tfrac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} = P_2 + \tfrac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}$$

$y_1 = y_2$

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Conservation of Mass

Flow rate : $A_1 v_1 = A_2 v_2$ (3 gallons in $\Rightarrow$ 3 gallons out)

If A_1 is larger than A_2, then $v_2 > v_1$

Then Bernoulli's Law implies $P_2 < P_1$