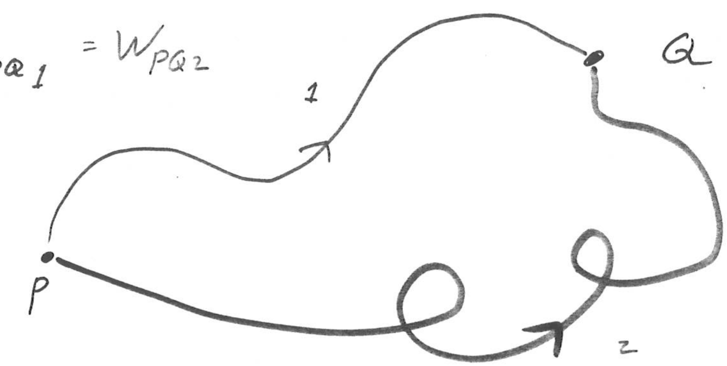
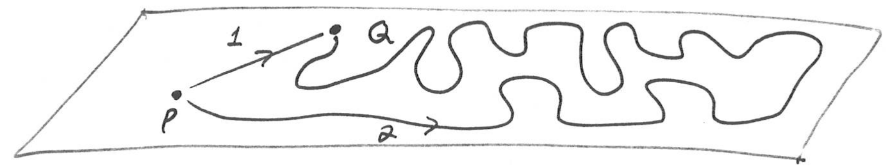
Chapter 8: Conservative Forces

A force is <u>conservative</u> if the work done by that force acting on a particle moving between any two points is independent of the path the particle takes between the points.

$W_{PQ1} = W_{PQ2}$



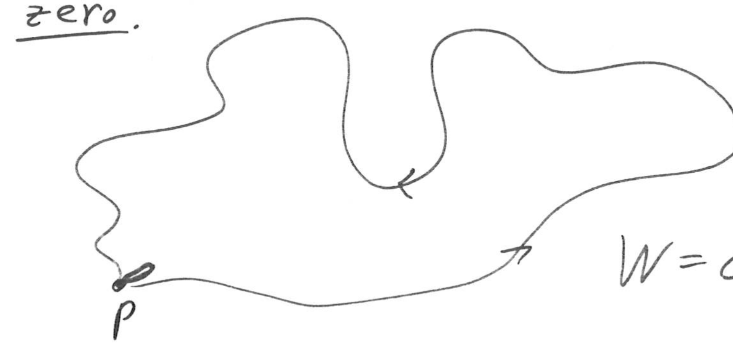
<u>Ex.</u> Clearly, friction is <u>not</u> conservati



$W_1 = -1\,J$ $\qquad$ $W_2 = -100\,J$

Another property of a <u>conservative</u> force:

The work done by a conservative force around any closed path is <u>zero</u>.



$W = 0$

Other equivalent statements:

- The work done by a conservative force depends <u>only</u> on the endpoints of the path.

- If the force is conservative, then

$$W_{P\to Q} = -W_{Q\to P}$$

Examples of non-conservative forces

$$\left.\begin{array}{l}\text{- kinetic friction}\\ \text{- Drag forces}\end{array}\right\}\ \text{dissipative forces}$$

Mechanical energy is converted into some other form of energy (heat).

The work done by a conservative force equals the negative of the change in potential energy $(-\Delta U)$.
$-(U_f - U_i)$

$$\Delta U = \underset{\uparrow\\ U_{(x)}}{U_f} - U_i = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{s} = -W$$

The <u>change</u> in U is a definite integral – there is no arbitrary constant.

The <u>function</u> $U(x) = -\int \vec{F}\cdot d\vec{s}$ is arbitrary up to a constant

There is <u>no</u> potential energy associated with friction or drag.

__Ex__ A 10 kg bowling ball is suspended 1 meter above the floor. What is its potential energy?

a) 98 J

b) - 98 J

c) 0 J

d) 2.45 J

e) All of the above

If it drops to the floor what is the change in P.E.? - 98 J

The "zero" of potential energy can be chosen anywhere.

__Ex__

y_i —— $U = 0$

$\Delta U = -98\ J$

y_f $U = -98\ J$

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y_i $U = +98\ J$

$\Delta U = -98\ J$

y_f —— $U = 0$

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y_i $U = +49\ J$

—— $U = 0$ $\Delta U = -98\ J$

y_f $U = -49\ J$

Remember:

y_f

y_i

$W_g = -mg(y_f - y_i)$

what is $\Delta U = U_f - U_i$?

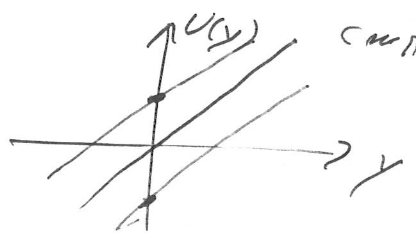
$\Delta U = -W = mg(y_f - y_i)$

What is the potential energy function $U(y)$?

$U(y) = -\int F_y \cdot dy = -\int(-mg)\, dy$

$= mgy + \underline{\text{constant}}$

Graph?

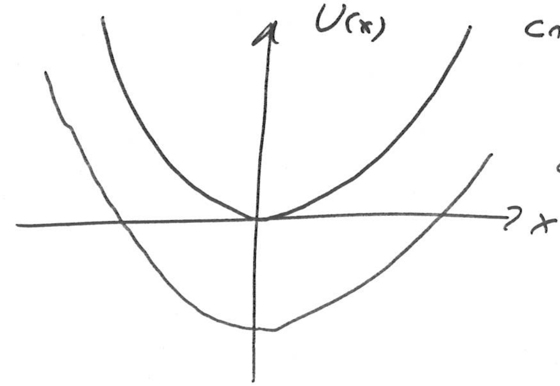


const = 0

Potential Energy Function of a Spring

$U(x) = -\int F_x\, dx = -\int(-kx)\, dx$

$= \frac{1}{2}kx^2 + \underline{\text{constant}}$



constant = 0

constant = -5 J

Conservation of Mechanical Energy

Work - Energy theorem:

$$W_{net} = \Delta K = K_f - K_i$$

$$W_{net} = \underbrace{W_{cons}}_{\text{Work done by conservative forces}} + \underbrace{W_{n.c.}}_{\text{work done by non-conservative forces}}$$

$$W_{cons} = -\Delta U_{total} = -(U_f - U_i)$$

$$-\Delta U + W_{n.c.} = \Delta K$$

$$W_{n.c.} = \Delta U + \Delta K$$

$$= (U_f - U_i) + (K_f - K_i)$$

$$= (U_f + K_f) - (U_i + K_i)$$

$$= E_f - E_i = \Delta E$$

E is the total <u>mechanical</u> energy of the system, that is kinetic energy + potential energy.

$$E_f = K_f + U_f$$

$$E_i = K_i + U_i$$

<u>Ex.</u> bowling ball - friction will reduce the mechanical energy.

If the work done by non-conservative forces is <u>zero</u> then mechanical energy is conserved.

$$W_{n.c.} = \Delta U + \Delta K = \Delta E$$

$$0 = E_f - E_i$$

$$E_f = E_i$$

mechanical energy does not change from i to f.

$W_{n.c.} = 0$ means that no mechanical energy is being converted into another form (heat) by dissipative forces (friction).

Force and Potential Energy

$$U_{(x)} = -\int F_x \, dx \qquad (1\text{-}0)$$

How can we invert this equation to get $F_{(x)}$ as a function of $U_{(x)}$?

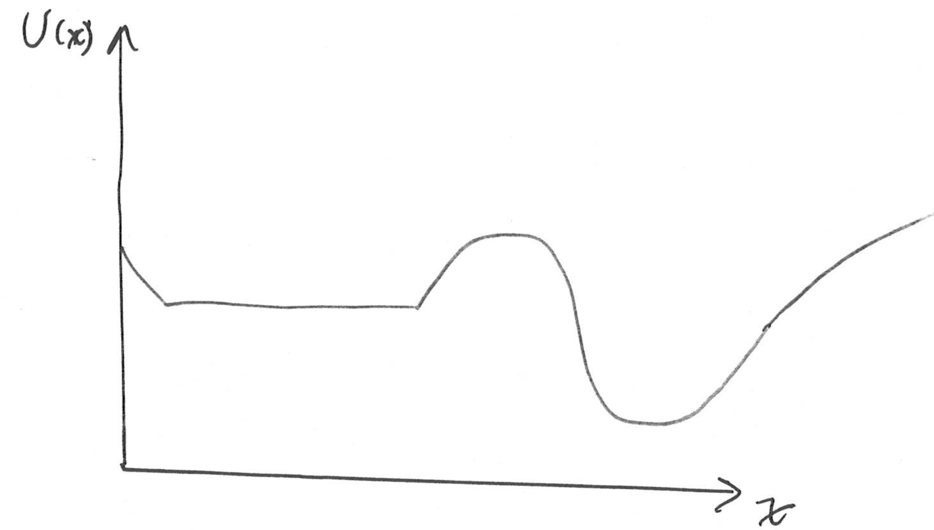
$$F_x = -\frac{d}{dx}\left[U_{(x)}\right]$$

On a plot of $U_{(x)}$ vs. x, this is the negative of the slope.

Equilibrium

An object is in equilibrium if the net force acting on it is zero.

If all of the forces are conservative then since $F_x = -\frac{dU}{dx}$ points of equilibrium are points where the slope on a $U(x)$ vs. x plot is flat.



3 kinds of Equilibrium

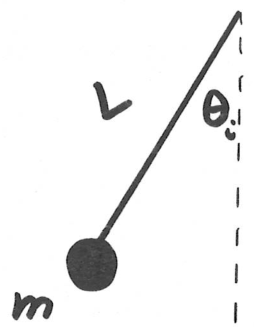
- stable - small displacements result in a <u>restoring</u> force.
- unstable - small displacements result in a force <u>away</u> from equilibrium
- neutral - small displacements result in <u>no</u> force.

An easy way to see this

Think of a real-life particle (like a tennis ball) on a real-life hill.

Demo:

Summary



$m = 5\ kg$

$L = 2.0\ m$

$\theta_i = 15°$

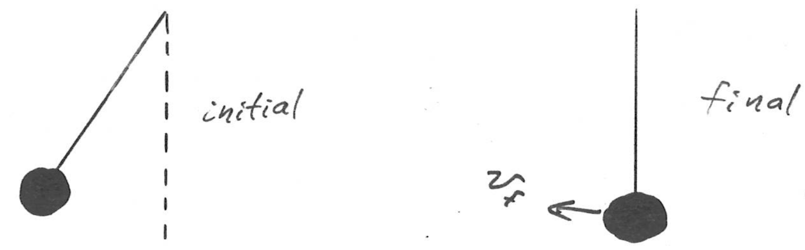
What is the velocity of the ball at $\theta = 0°$?

Approach: Conservation of Mechanical Energy

$$\Delta K + \Delta U = 0$$

$$K_i + U_i = K_f + U_f$$

$$E_i = E_f$$



$$\Delta K \equiv K_f - K_i$$

What is K_i ? $\underline{\quad 0 \quad}$

What is K_f ? $\underline{\frac{1}{2} m v_f^2}$

$$\Delta U = -\int_i^f \vec{F}_{total} \cdot \vec{ds}$$

$\vec{F}_{gravity} = mg$, downward

$\vec{F}_{tension} = T$, along the string

$$\Delta U = -\int_i^f \vec{F}_{gravity} \cdot \vec{ds} - \int_i^f \vec{F}_{tension} \cdot \vec{ds}$$

Does the tension contribute to the potential energy?

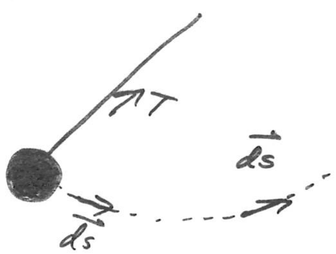
i.e. Does tension do any work in this problem?

The direction of $\vec{F}_{tension}$ changes, but it always lies

Perpendicular

to the increment $\vec{ds}$.

($\vec{ds}$ is in the same direction as $\frac{\vec{ds}}{dt}$ which is the velocity.)

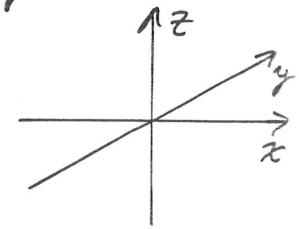


$$\vec{F}_{tension} \cdot \vec{ds} = 0$$

Tension does **NO** work in this problem. It does **NOT** change the potential energy.

What about gravity?

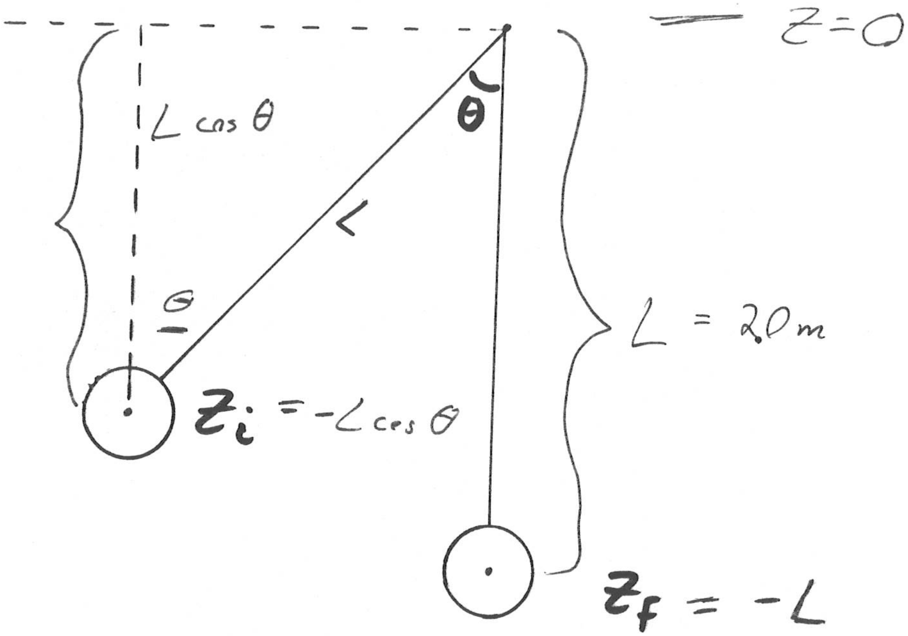
$$\Delta U = -\int_i^f \vec{F}_{gravity} \cdot \vec{ds}$$



$$\vec{F}_{gravity} = -mg\,\hat{k} \qquad \vec{ds} = dx\,\hat{\imath} + dy\,\hat{\jmath} + \underline{dz\,\hat{k}}$$

Recall a property of the dot product:

We only need to know the component of $\vec{ds}$ that lies along $\hat{k}$. $\Rightarrow$ $\underline{\underline{\text{change in height}}}$ dz



point

$$\Delta U = -\int_i^f \vec{F}_{gravity} \cdot \vec{ds} = -\int_{z_i}^{z_f} (-mg)\, dz$$

$$= mgz\Big|_{z_i}^{z_f} = mg(z_f - z_i)$$

$$= mgL(\cos\theta - 1)$$

Conservation of Energy

$$\Delta K + \Delta U = \cancel{W_{n.c.}}$$

$$\left(\tfrac{1}{2}\cancel{m}v^2 - 0\right) + \cancel{m}gL(\cos\theta - 1) = 0$$

$$v = \sqrt{2gL(1-\cos\theta)}$$

$$= \sqrt{2(9.8\ m/s^2)(20m)(1-\cos 15°)}$$

$$= \boxed{1.16\ m/s}$$

This is the velocity when the bowling ball is at its minimum height.

Ex A ball of mass 0.1 kg is whirled in a vertical circle of radius 0.5 m, such that its speed at the top is as low as can be. What is its speed at the bottom?