Chapter 13:

Oscillatory Motion

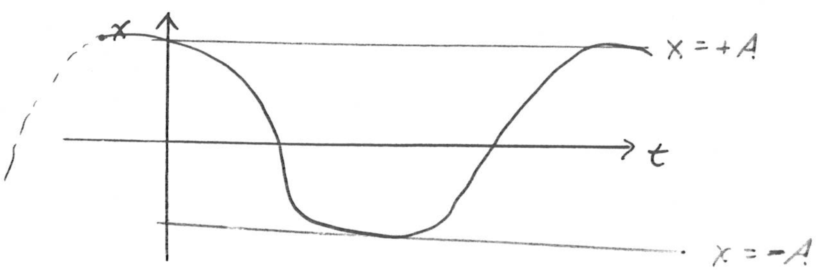
Simple Harmonic Motion (SHM):

$$x(t) = A \cos(\omega t + \delta)$$

A - amplitude (max. displacement)

ω - angular frequency

δ - phase constant



T is the period, the time it takes the system to go through one full cycle of oscillation.

$$\boxed{T = \frac{2\pi}{\omega}}$$

$$x = A\cos(\omega t + \delta) = A\cos(\omega[t+T] + \delta)$$
$$= A\cos(\omega t + 2\pi + \delta)$$
$$= A\cos(\omega t + \delta)$$

$[T]$ = Time ; unit = second

$[\omega] = \frac{1}{\text{Time}}$; unit = $\frac{\text{radian}}{\text{second}}$

f is the frequency, the number of cycles the system completes every second.

$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$

$$[f] = \frac{1}{\text{Time}} \qquad \text{unit} = \frac{\text{cycle}}{\text{second}} = Hz = \frac{1}{s}$$

(hertz)

Ex. The Earth in orbit around the Sun.

$$T = 1 \text{ year} = 3.1 \times 10^{7}\, s$$

$$f = \frac{1}{T} = 3.2 \times 10^{-8}\, Hz = \frac{\text{cycles}}{\text{sec}}$$

$$\omega = 2\pi f = 2.0 \times 10^{-7}\, \frac{\text{rad}}{\text{sec}}$$

Calculus Lesson

$$\frac{d}{dt}\cos(t) = -\sin(t)$$

$$\frac{d}{dt}\sin(t) = +\cos(t)$$

$$\frac{d}{dt}\cos(\omega t) = -\omega\sin(\omega t)$$

$\omega = \text{const.}$

$$\frac{d}{dt}\sin(\omega t) = +\omega\cos(\omega t)$$

$$\frac{d}{dt}\cos(\omega t + \delta) = -\omega\sin(\omega t + \delta)$$

$$\frac{d}{dt}\sin(\omega t + \delta) = +\omega\cos(\omega t + \delta)$$

chain-rule

$$x(t) = A\cos(\omega t + \delta)$$

take the time derivative of both sides

$$\boxed{\frac{dx}{dt} = v(t) = -\omega A \sin(\omega t + \delta)}$$

The velocity varies sinusiodally between $+\omega A$ and $-\omega A$.

The maximum speed is

$$\boxed{v_{max} = \omega A}$$

$$v(t) = -\omega A \sin(\omega t + \delta)$$

take the time derivative of both sides

$$\boxed{\frac{dv}{dt} = a(t) = -\omega^2 A \cos(\omega t + \delta)}$$

The acceleration oscillates between $+\omega^2 A$ and $-\omega^2 A$.

The maximum acceleration is:

$$\boxed{a_{max} = \omega^2 A}$$

$$a(t) = -\omega^2 A\cos(\omega t + \delta)$$

$$x(t) = A\cos(\omega t + \delta)$$

SHM $\boxed{a(t) = -\omega^2 x(t)}$ All times

$$a(t) = -\omega^2 x(t)$$

multiply both sides by the mass, m.

$$m\,a(t) = F(t) = -\omega^2 m\, x(t)$$

Dynamics

Simple Harmonic Motion

results whenever

$$\boxed{F(t) \propto -x(t)}$$

whenever the displacement causes a force in the opposite direction (restoring force) that returns the system to stable equilibrium.

Ex. Hooke's Law Spring

$$F = -kx \qquad \text{at all times}$$

$$F(t) = -k\,x(t)$$

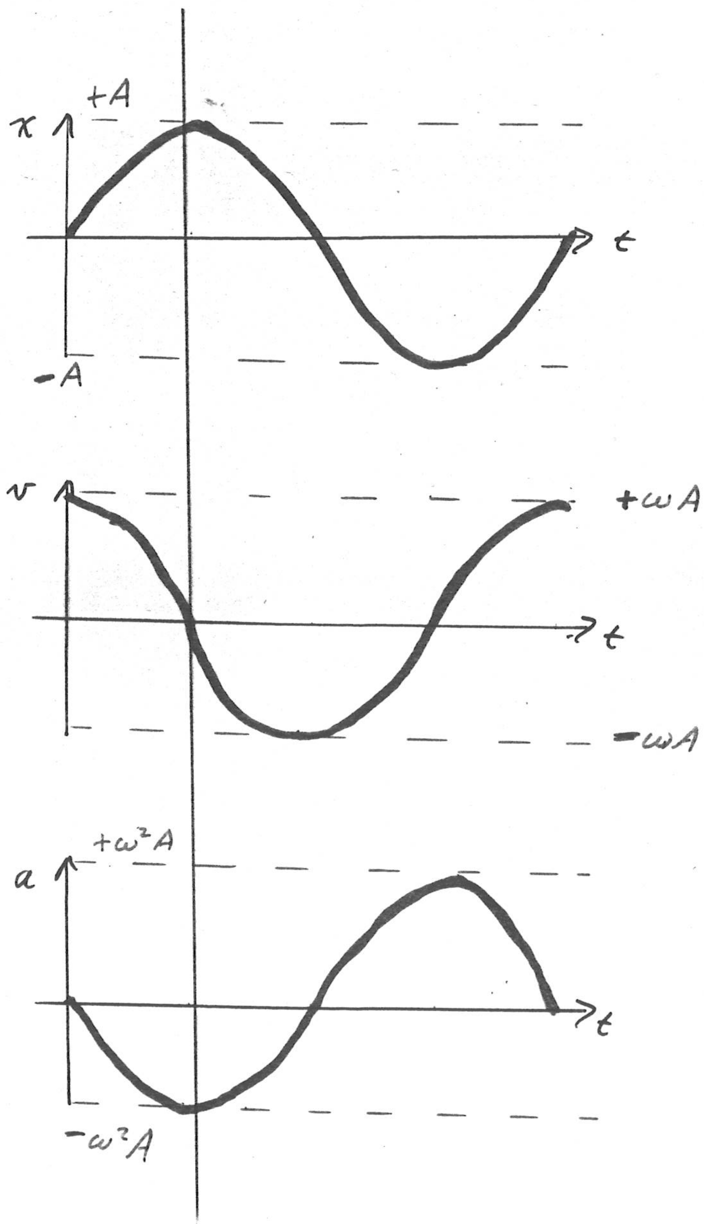
$$m\,a(t) = -k\,x(t)$$

$$a(t) = -\frac{k}{m}\,x(t) \Leftarrow$$

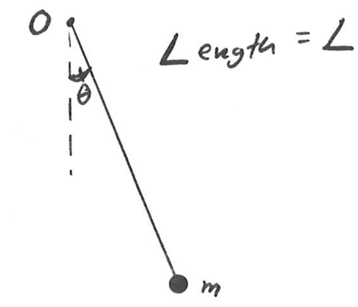
$$\Updownarrow$$

$$a(t) = -\omega^2\, x(t)$$

$$\omega^2 = \frac{k}{m} \qquad \omega = \sqrt{\frac{k}{m}}$$



The Simple Pendulum



Pendulum: $\omega = \sqrt{\frac{g}{L}}$

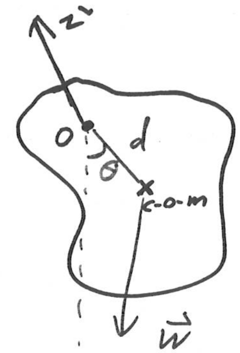
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \frac{1}{f}$$

The period of a pendulum is independent of the mass m and the Amplitude A.

Ex Does a pendulum clock run faster or slower on the Moon?

$$g_{moon} \approx \frac{1}{6} g_{Earth}$$

The Physical Pendulum



$$\Sigma\tau_o = I_o\alpha$$

$$-mgd\sin\theta = I_o\alpha$$

d is the distance from the pivot to the center of mass.

$F = -kx$

$\tau = -c\theta$

$\sin\theta \approx \theta$ θ in radians!

$$-mgd\,\theta = I_o\alpha$$

$$\alpha = -\left(\frac{mgd}{I_o}\right)\theta$$

$\alpha = -\omega^2\theta$

S.H.M.

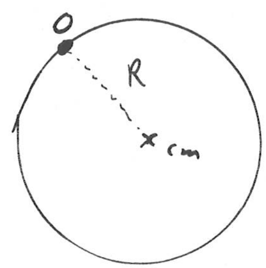
$$\omega^2 = \frac{mgd}{I_o}$$

$$\omega = \sqrt{\frac{mgd}{I_o}}$$

Ex A hoop is pivoted on a rod. (m, R)

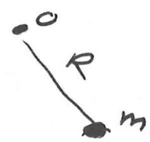
What is the period of oscillation, T?

$$\omega = \sqrt{\frac{mgd}{I_0}}$$



$$\omega = \sqrt{\frac{mgR}{2mR^2}}$$

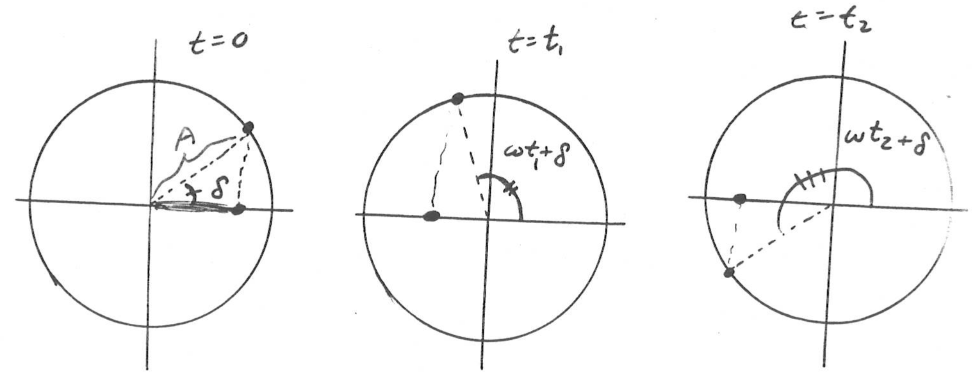
$$= \sqrt{\frac{g}{2R}}$$



$$\omega = \sqrt{\frac{g}{R}}$$

Simple Harmonic Motion vs. Uniform Circular Motion

Consider a particle moving around a circle with <u>constant</u> angular speed ω, radius A.



$x = A\cos(\delta)$ $\quad$ $x = A\cos(\omega t_1 + \delta)$ $\quad$ $x = A\cos(\omega t_2 + \delta)$

$$x(t) = A\cos(\omega t + \delta)$$

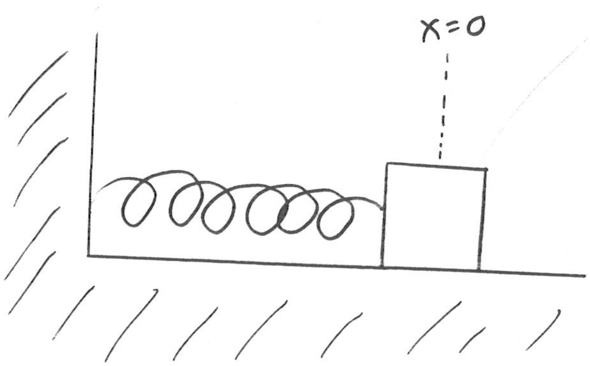
Energy in the Simple Harmonic Oscillator

$$x(t) = A\cos(\omega t + \delta)$$

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \delta)$$

$$a(t) = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \delta)$$

First, let us consider a mass m on a <u>horizontal</u> spring of constant k.



$$\omega = ? = \sqrt{\frac{k}{m}}$$

Kinetic Energy:

$$K_{(t)} = \frac{1}{2} m v^2 = \frac{1}{2}\underbrace{m\omega^2}A^2 \sin^2(\omega t + \delta)$$

$$= \frac{1}{2} k A^2 \sin^2(\omega t + \delta)$$

$$\omega = \sqrt{\frac{k}{m}}$$

Potential Energy:

$$U_{(t)} = \frac{1}{2} k x^2 + \cancelto{0}{\text{constant}}$$

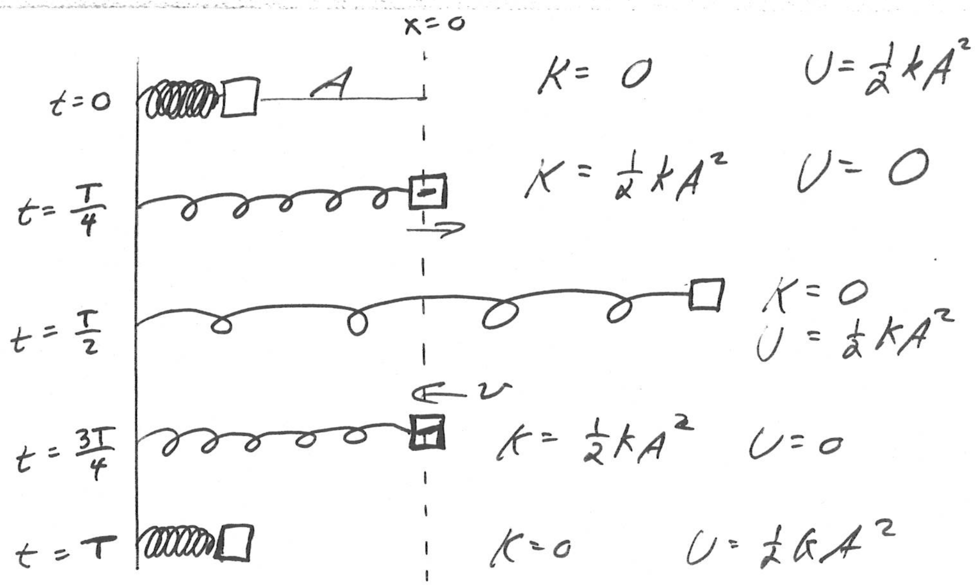
$$= \frac{1}{2} k A^2 \cos^2(\omega t + \delta)$$

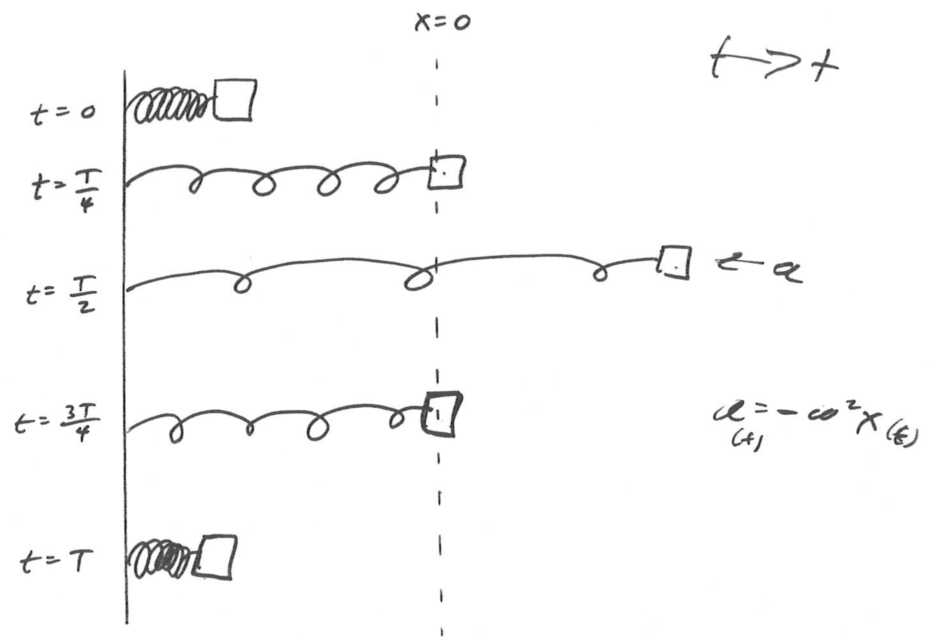
Total Mechanical Energy:

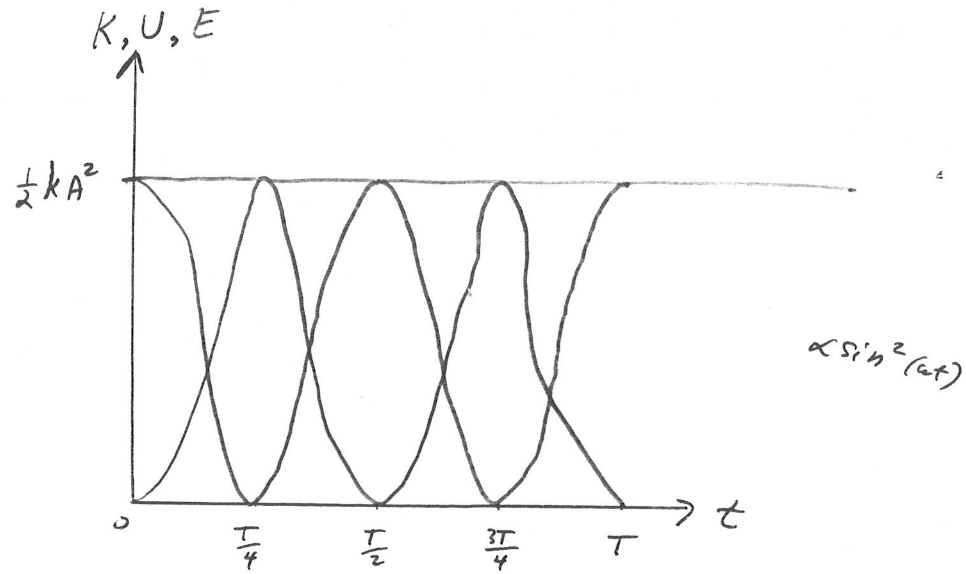
$$E = K + U = \frac{1}{2} k A^2 \left[\sin^2(\omega t + \delta) + \cos^2(\omega t + \delta)\right]$$

$$= \frac{1}{2} k A^2$$

E is <u>constant</u>! (No time dependence)







t	x	v	$a=-\omega^2 x$
0	$-A$	0	$\omega^2 A$
$T/4$	0	ωA	0
$T/2$	$+A$	0	$-\omega^2 A$
$3T/4$	0	$-\omega A$	0
T	$-A$	0	$\omega^2 A$

Resonance

All objects have a natural angular frequency, ω_0.

For a pendulum, $\omega_0 = \sqrt{\frac{g}{L}}$

For a mass on a spring, $\omega_0 = \sqrt{\frac{k}{m}}$

If a force is applied at a frequency close to ω_0, the amplitude of oscillation will increase dramatically.

A force applied at a frequency far from ω_0 will have little effect on the amplitude.

Chapter 14:
The Law of Universal Gravitati

A thought experiment:

Suppose that you climb a mountain and fire a projectile at different speeds horizontally.