Chapter 5: Forces

Kinematics is the study of motion in its own right. Dynamics is the study of motion and its causes.

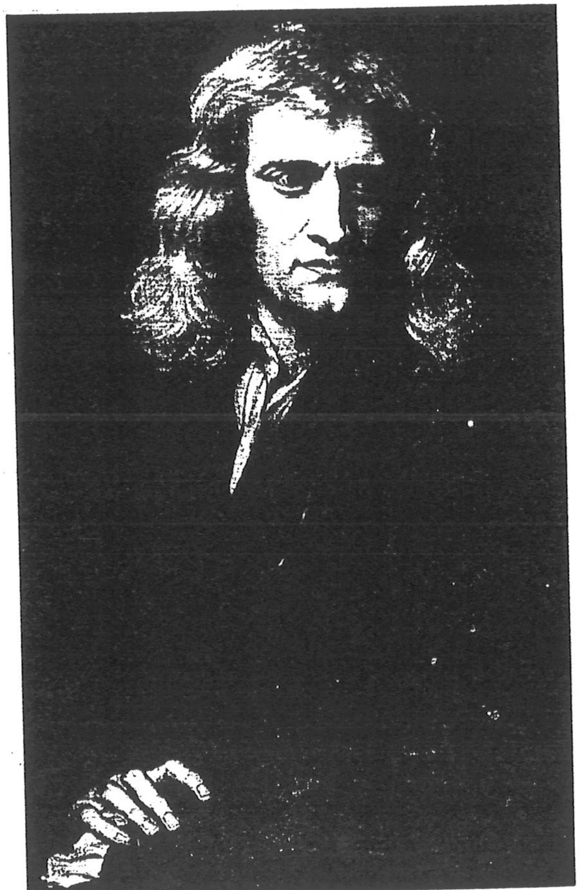
Forces cause the motion of a body to change.

Fundamental Forces

Strong Nuclear Force	Physics 7314
Electro-magnetism	Physics 1304
Weak Nuclear Force	Physics 6341
Gravitation	Physics 1303

Contact Forces

At the microscopic level, these are due to electro-magnetism acting between atoms



Newton's first portrait (1689) at age forty-six by Sir Godfrey Kneller. *(By permission of Lord Portsmouth and the Trustees of the Portsmouth Estates.)*

Newton's Second Law

$$\boxed{\sum_i \vec{F}_i = m\vec{a}}$$

the acceleration of the object

sum of all the forces acting on an object, Net Force, Total force, Resultant force

the mass of the object

Environment

Kinematics

Some comments:

$m\vec{a}$ is not a force, it describes the motion that results from all the forces acting on an object.

Many forces can act on an object:

$$\sum_i \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = \vec{F}_{total}$$

but the object has only one acceleration

$\sum_i \vec{F}_i = m\vec{a}$ is a vector equation.

$$\left\{ \begin{aligned} \sum_i (F_i)_x &= m\,a_x \\ \sum_i (F_i)_y &= m\,a_y \\ \sum_i (F_i)_z &= m\,a_z \end{aligned} \right.$$

Mass

Mass is the amount of "stuff" in an object.

Ex. Wood block vs. lead block

How can I compare two masses?

I can weigh them with a scale,

But...

But...

A scale does not directly measure mass. Instead, it records a special kind of force.

Weight

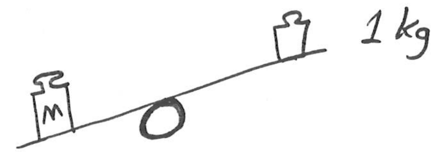
$$W = F_{gravity} = mg$$

direction: toward the center of the Earth

If you measure the weight, and you know g, then the mass is

$$\boxed{m = \frac{W}{g}}$$

Rather than a scale, I can use a balance to compare unknown masses with a standard kilogram.



This works on the surface of the Earth (where $g = 9.8\ m/s^2$) and on the surface of the Moon (where $g_{moon} = 1.68\ m/s^2$).

Now what can go wrong?

What if we are in outer space, far from a large gravitating mass?

$\boxed{\text{Mass is the resistance of a body to acceleration. (Inertia)}}$

The method

Apply a known force to an unknown mass. Measure the acceleration. then

$$m = \frac{F}{a}$$

The Slug

Ex. What is the weight of a 1 slug mass on the surface of the Earth?

Ex. What changes when we go to the Moon?

Dimensions + Units

$$\sum_i \vec{F}_i = m\,\vec{a}$$

$$\left[\sum_i \vec{F}_i\right] = [m]\,[\vec{a}]$$

$$[F] = M \cdot \frac{L}{T^2}$$

S.I. unit of F = newton (N)

$$1\ N = 1\ \frac{kg \cdot m}{s^2}$$

English unit of F = pound (lb)

$$1\ lb = 1\ \frac{slug \cdot ft}{s^2}$$

Proper Conversions

Force : Force $\begin{cases} 1\ N = 0.2248\ lb \\ 1\ lb = 4.448\ N \end{cases}$

kg : slug :: N : lb

Mass : Mass $\begin{cases} 1\ kg = 0.0685\ slug \\ 1\ slug = 14.59\ kg \end{cases}$

Improper Conversions

(on product labels, for example)

Mass: Force $\quad 1\ kg = 2.2\ lbs$

Force: Mass $\quad [1\ lb] = [0.456\ kg]$

$$\frac{ML}{T^2} \stackrel{?}{=} M$$

- These are only useful on the surface of the Earth.
- They will destroy any dimensional analysis check of your answers.

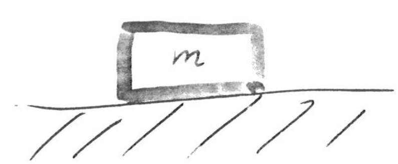
The Normal Force

This is a contact force that prevents one object from passing through another.

It can become as large as it has to be.

It can also shrink to zero.

Ex



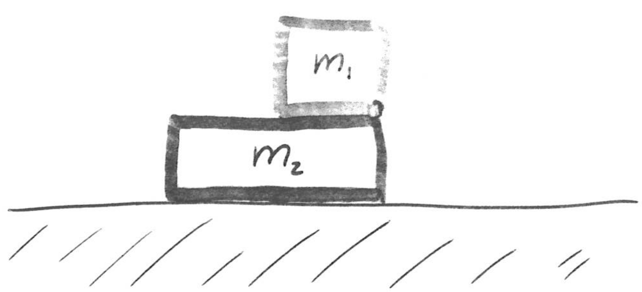
$W = mg$

$|N| = mg$

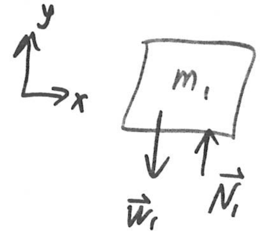
Free-Body Diagrams

- Isolate *one* object to study.
- Draw all the forces that act *on* this object.
- Choose coordinate axes.
- Apply Newton's Second Law.

Ex.



Free-body diagram for m_1



$$\sum_i \vec{F}_i = m_1 \vec{a}_1$$

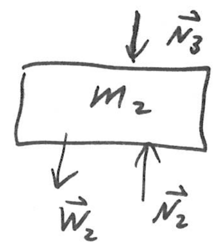
$$\vec{W}_1 + \vec{N}_1 = m_1 \vec{a}_1$$

y-components

$$-m_1 g + |\vec{N}_1| = m_1 \cancel{a_1}^{0}$$

$$\boxed{|\vec{N}_1| = m_1 g}$$

Free-body diagram for m_2



$$\sum_i \vec{F}_i = m_2 \vec{a}_2$$

$$\vec{W}_2 + \vec{N}_2 + \vec{N}_3 = m_2 \vec{a}_2$$

y-components

$$-m_2 g + N_2 - N_3 = m_2 \cancel{a_2}^{0}$$

$$N_2 - N_3 = m_2 g$$

Magnitudes only! All positive!

Newton's Third Law

Law of action and reaction.

When two bodies interact (change each other's motion)

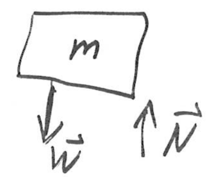
$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

on 1 due to 2

on 2 due to 1

equal in <u>magnitude</u>, opposite in <u>direction</u>.

★ The two forces in an action-reaction pair <u>never</u> act on the same object!



← force of <u>table</u> acting on m

$\vec{F}_{mt}$

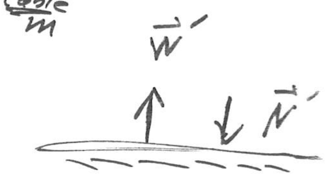
force of gravity due to Earth acting on m

$\vec{F}_{mE}$

$$\vec{W} = -\vec{W}'$$
$$\vec{N} = -\vec{N}'$$

Force of gr
due to m ac
on Earth

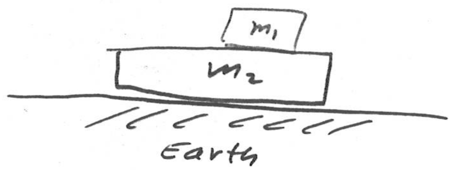
$\vec{F}_{Em}$

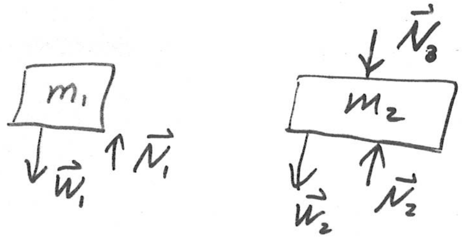


force of the mass m acti on the <u>table</u>

$\vec{F}_{tm}$

Ex Back to this problem:





$$N_2 - N_3 = m_2 g$$

$$N_2 - N_1 = m_2 g$$

$$N_2 - m_1 g = m_2 g$$

$$\boxed{N_2 = m_1 g + m_2 g}$$

$$\vec{N}_1 = -\vec{N}_3$$

$$N_1 = N_3$$

$$\boxed{N_3 = m_1 g}$$

$$\vec{W}_1 = -\vec{W}_1{}'$$

force on m_1 due to Earth

force on Earth due to m_1

$$\boxed{\vec{N}_1 = -\vec{N}_3}$$

force on m_1 due to m_2

force on m_2 due to m_1

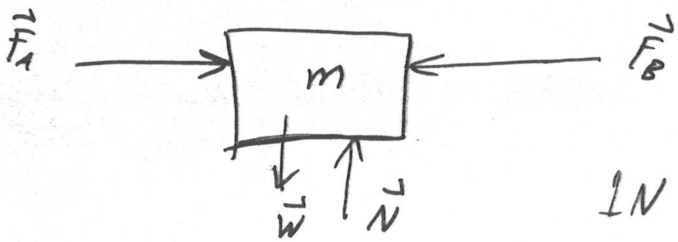
$$\vec{N}_2 = -\vec{N}_2{}'$$

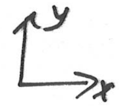
force on m_2 due to contact with Earth

force on Earth due to contact with m_2

Ex Student A pushes with a force $\vec{F}_A = 2N$ to the right on a box on rollers (no friction) while student B pushes with $\vec{F}_B = 3N$ to the left. The box has a mass $m = 10\,kg$. What is the acceleration of the box?

$\Rightarrow \vec{a}$





$1N = 1\,\frac{kg \cdot m}{s^2}$

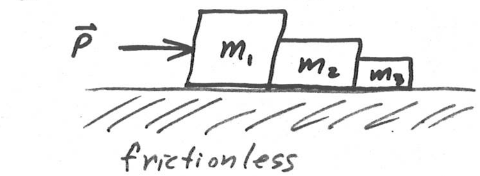
$\Sigma\vec{F} = m\vec{a}$

X- components	X - components
$N - W = m\cancel{a_y}^{\,0}$	$F_A - F_B = m\,a_x$
$N = W = mg$	$a_x = \frac{F_A - F_B}{m} = \frac{2N - 3N}{10kg}$
$= \lvert\vec{N}\rvert = \lvert\vec{w}\rvert$	$= -\frac{1}{10}\,m/s^2$

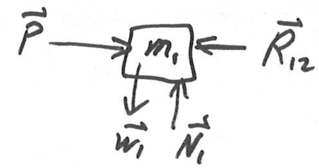
Ex

What is $\vec{a}$?

What are the contact forces?

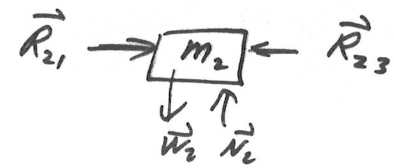


$\Rightarrow \vec{a}$



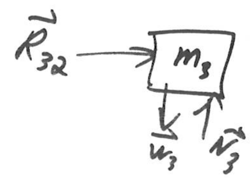
$\Sigma F_x = m_1\,a_{x_1}$

$\boxed{P - R_{12} = m_1\,a_1}$



$\Sigma F_x = m_2\,a_{2x}$

$\boxed{R_{21} - R_{23} = m_2\,a_2}$



$\Sigma F_x = m_3\,a_{3x}$

$\boxed{R_{32} = m_3\,a_3}$

$\vec{a}_1 = \vec{a}_2 = \vec{a}_3$

Action-Reaction Pairs

$$\vec{R}_{12} = -\vec{R}_{21} \qquad \vec{R}_{23} = -\vec{R}_{32}$$

$$R_{12} = R_{21} \qquad R_{23} = R_{32}$$

Add 3 boxed Equations

$$\begin{aligned} P - \cancel{R_{12}} &= m_1 a \\ \cancel{R_{21}} - \cancel{R_{23}} &= m_2 a \\ + \quad \cancel{R_{32}} &= m_3 a \\ \hline P &= (m_1 + m_2 + m_3) a \end{aligned}$$

$$\boxed{a = \frac{P}{m_1 + m_2 + m_3}}$$

$$R_{32} = m_3 a = \frac{m_3 P}{m_1 + m_2 + m_3} = R_{23}$$

$$R_{12} = P - m_1 a = P - \frac{m_1 P}{m_1 + m_2 + m_3} = \frac{(m_2 + m_3) P}{m_1 + m_2 + m_3}$$
$$= R_{21}$$

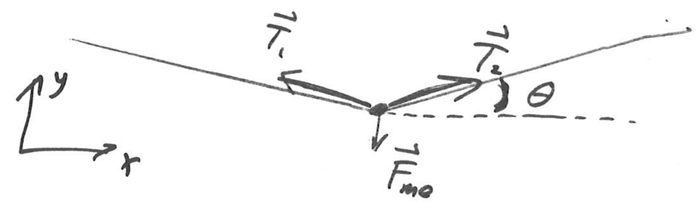
The game was rigged

Each team pulls with force F. The tension in the rope is F, not 2F!

Free-body diagram of the end of the rope:

The tension would be the same if one team pulled against a wall.

Free-body diagram for the middle of the rope:



$$\sum \vec{F}_i = \cancel{m}^{0} \cancel{\vec{a}}^{0} = 0$$

$$\vec{T}_1 + \vec{T}_2 + \vec{F}_{me} = \underline{0}$$

x-components

$$T\cos\theta - T\cos\theta = 0$$

y-components

$$-F_{me} + 2T\sin\theta = \underline{0}$$

$$F_{me} = 2T\sin\theta$$

$$\sin\theta = \frac{opp}{hyp} = \frac{0.5\ ft}{15\ ft} = 0.034$$

$$F_{me} = 0.068\,T \sim 6.8\%\,T$$

If μ between your shoes and the floor is less than 1, then the force you can exert is less than your weight.

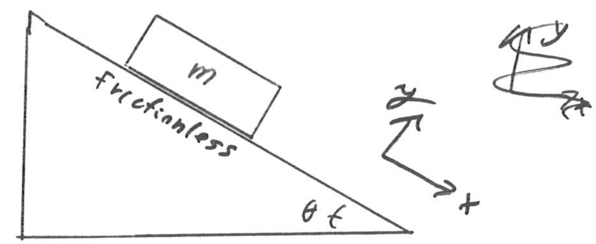
$$f = \mu N = \mu (mg)$$

$$T \leq 2 \times (150 \text{ lbs}) = 300 \text{ lbs}$$

$$F_{me} = 2T \sin\theta \leq 20 \text{ lbs}$$

<u>Corollary:</u> It takes an infinite tension to keep an ideal rope straight, even if the mass in the middle is tiny.

<u>Ex</u>

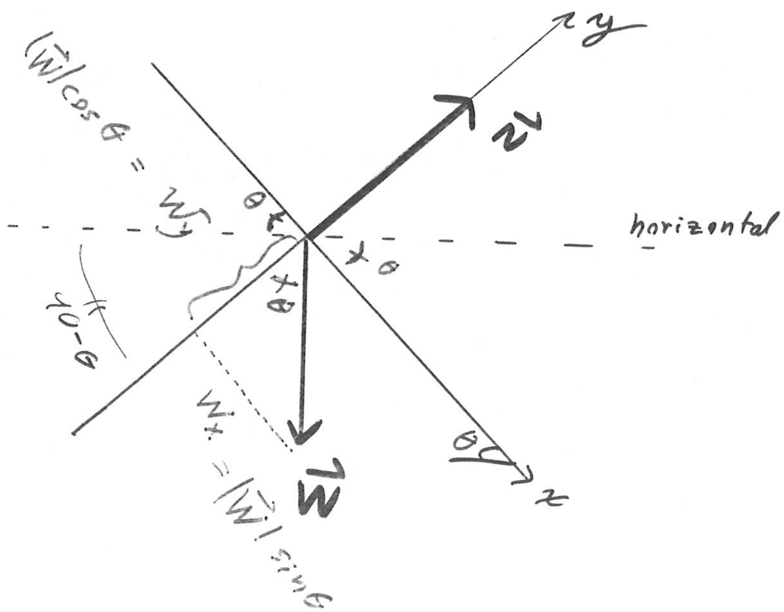


① What is the acceleration of the block down the slope?

② What is the magnitude of the normal force? $(N \neq mg)$

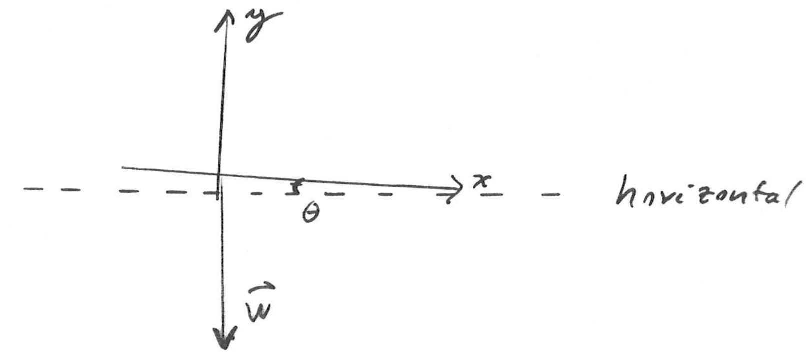
Why choose axes this way?

If there were any motion, that motion would be in one of the coordinate directions (x in this case), not both. This makes the motion 1-dimensional.



Trigonometry Short-Cut:

Imagine the angle θ shrinking to $0°$.



What happens to W_x?

It also shrinks to 0.

Which trig function shrinks to 0 as $\theta \to 0°$?

$\sin\theta \to 0$ as $\theta \to 0$

So... $W_x = W \sin\theta$ $\quad$ θ is not polar angle

Apply Newton's 2nd Law

x-components

$$\sum F_x = m a_x$$

$$W_x = m a_x$$

$$W \sin\theta = m a_x$$

$$\cancel{m} g \sin\theta = \cancel{m} a_x$$

$$\boxed{a_x = g \sin\theta}$$

diluted gravity

$$\boxed{W \equiv mg}$$

y-components

$$\sum F_y = m \cancelto{0}{a_y}$$

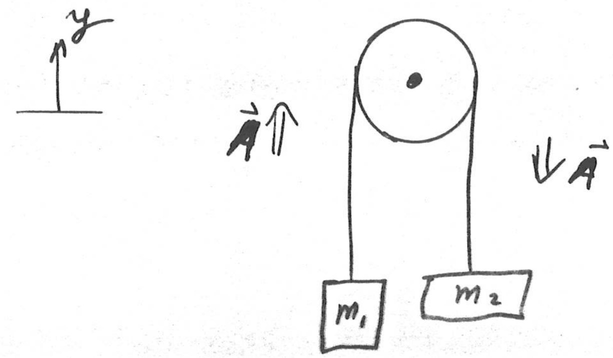
$$N - W\cos\theta = 0$$

$$N = W\cos\theta$$

$$\boxed{N = mg\cos\theta}$$

Atwood's Machine

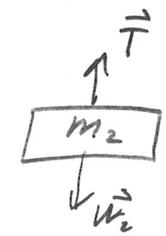
Another way to dilute gravity



$$\sum F_y = m_1 a_{1y}$$

$$T - W_1 = m_1 A$$

$$\boxed{T - m_1 g = m_1 A}$$



$$\sum F_y = m_2 a_{2y}$$

$$T - W_2 = m_2 (-A)$$

$$\boxed{T - m_2 g = -m_2 A}$$

$T - W_1 = m_1 A$ $\qquad$ $T - W_2 = -m_2 A$

$T = m_1 A + m_1 g$ $\qquad$ $T = -m_2 A + m_2 g$

$m_1 A + m_1 g = -m_2 A + m_2 g$

$m_1 A + m_2 A = m_2 g - m_1 g$

$A(m_1 + m_2) = g(m_2 - m_1)$

$A = g \frac{m_2 - m_1}{m_1 + m_2}$

check Limits $\quad m_1 = m_2 \Rightarrow A = 0$ ✓

$m_2 \neq 0$, $m_1 = 0$

Expect m_2 will acceler. at g

$A = g$ ✓

$A = g \frac{210 - 200}{200 + 210} = g \frac{10}{410} = \frac{1}{41} g$

$A = g \frac{m_2 - m_1}{m_2 + m_1}$

$T = m_1 A + m_1 g$

$= m_1 \left[g \left(\frac{m_2 - m_1}{m_2 + m_1} \right) \right] + m_1 g$

$= \frac{m_1 g (m_2 - m_1) + m_1 g (m_2 + m_1)}{m_2 + m_1}$

$= \frac{2 m_1 m_2 g}{m_2 + m_1}$ $\qquad$ Dimensions ✓

Limit $m_1 = m_2$

$T = \frac{2 m_1^2}{2 m_1} g = m_1 g$ ✓

Limit $m_1 = 0$ $\quad m_2 \neq 0$

$T = 0$

Friction

The magnitude of the maximum frictional force is proportional to the magnitude of the normal force.

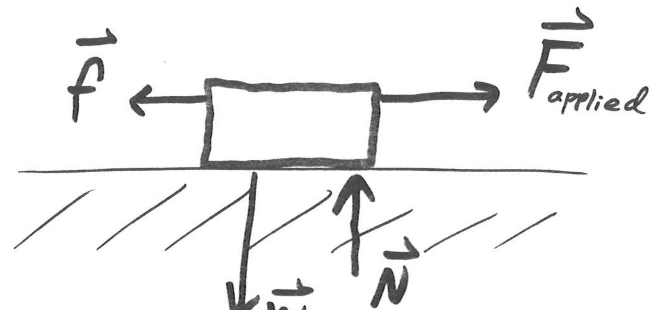
The constant of proportionality is called the coefficient of friction and has the symbol μ (mu).

$$\boxed{f_{max} = \mu N}$$

This is not a vector equation.

$\vec{N}$ always points perpendicular to the surface in contact with the object.

$\vec{f}$ always points parallel to the surface, in the direction that will resist motion.



There are two kinds of friction: static and kinetic.

If the surfaces in contact are moving <u>relative</u> to each other, then the force of kinetic friction acts.

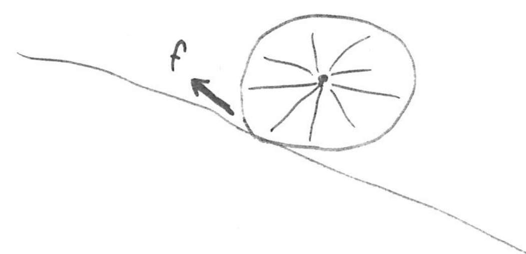
$$\boxed{f_k = \mu_k N}$$

Otherwise, the force of friction is static in nature.

$$\boxed{f_s \leq \mu_s N}$$

Ex. You are peddling a bicycle up a steep hill. The force of friction between the rubber tires and the road keeps you moving upward.

What kind of frictional force is this?



The static friction equation contains an inequality!

The magnitude of the static friction force has a range:

$$0 \leq f_s \leq \mu_s N$$

When is $f_s = 0$?

When there is no other force in the direction parallel to the surface for f_s to balance.

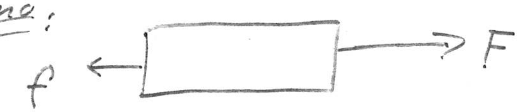
When is f_s a maximum ($\mu_s N$)?

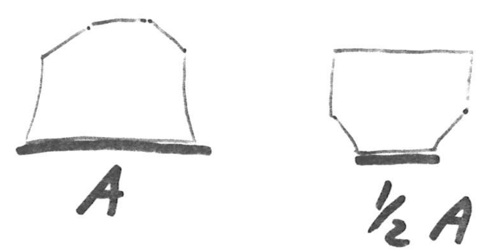
Just before the object slips.

The force of friction depends on the normal force and on the coefficient of friction only.

It does not depend on the area of contact between two surfaces, for example.

Demo:





Usually, $0.1 \le \mu \le 1$

some notable exceptions:

human synovial joints (elbows + knees) $\mu_s = 0.01$

$\mu_k = 0.003$

better than wet ice on wet ice

better than teflon!

---

racing tires $\mu_s = 3.34$

(this is Serway problem 5-90)

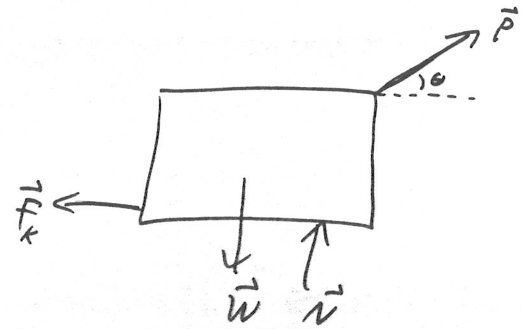
In general, $\mu_k < \mu_s$.

This means that the force of kinetic friction is smaller than the force of static friction.

A woman pulls her suitcase along a rough floor by a strap inclined at an angle θ from the horizontal. What is the magnitude of the normal force?

$v =$ const

$a = \frac{dv}{dt} = 0$



$$\Sigma F_x = m\cancel{a_x}^{\,0}$$

$$P\cos\theta - f_k = 0$$

$$f_k = P\cos\theta$$

$$\Sigma F_y = m\cancel{a_y}^{\,0}$$

$$-W + N + P\sin\theta = 0$$

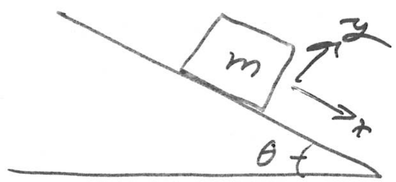
$$N = W - P\sin\theta$$

$$N = mg - P\sin\theta$$

Ex: 1) tilted coordinate axes

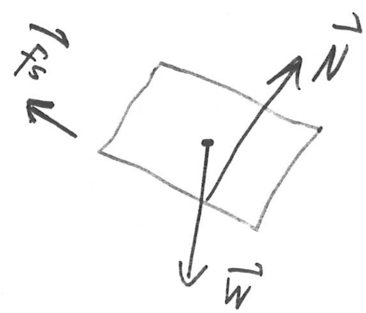
2) breaking forces into components

3) $N \neq mg$

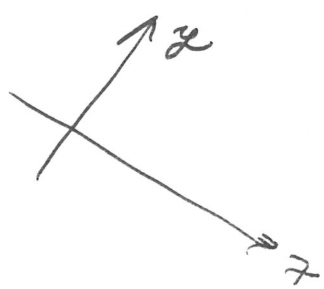


Given μ_s, what is the largest angle θ before the block slips?

Free-Body Diagram:



Choose Coordinates:

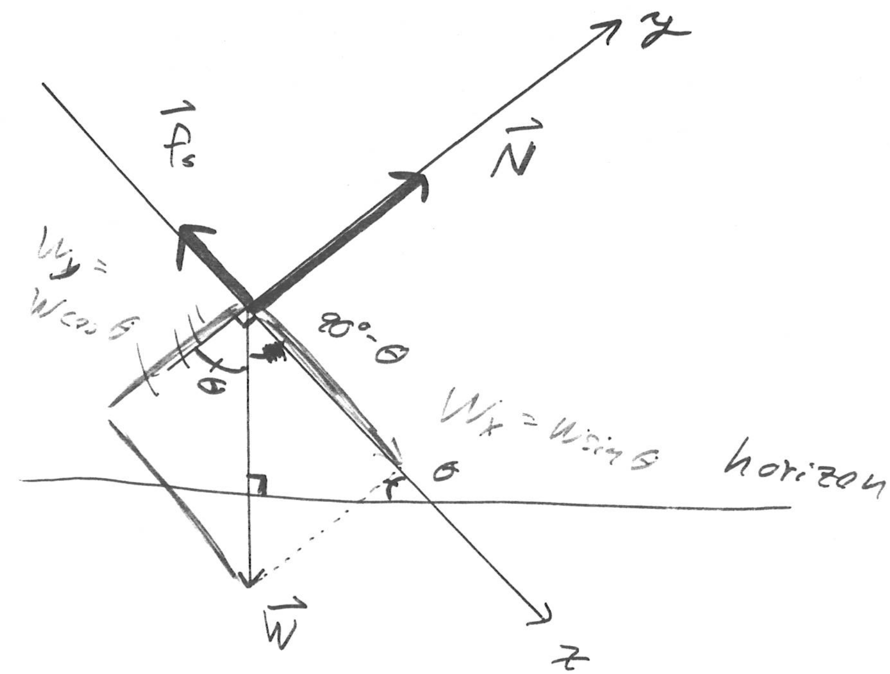


$a_y = 0$

Why choose coordinate axes this way?

If there were any motion, that motion would be in one of the coordinate directions (x in this case), not both. This makes the motion 1-D.

Vector Components:



Apply Newton's 2^{nd} Law

$$\sum \vec{F}_i = m\vec{a}$$

$$\vec{N} + \vec{f}_s + \vec{W} = m\vec{a}$$

x-components

$$\sum F_x = m a_x$$

not slipping yet

$$W\sin\theta - f_s = m\cancel{a_x}^{\,0}$$

$$f_s = W\sin\theta$$

$$f_s = mg\sin\theta$$

At critical angle only

$$f_s = \mu_s N \quad (\text{maximum})$$

$$\mu_s N = mg\sin\theta$$

y-components

$$\sum F_y = m\cancel{a_y}^{\,0}$$

$$N - W\cos\theta = 0$$

$$N = W\cos\theta$$

$$N = mg\cos\theta$$

$$\mu_s \cancel{mg}\cos\theta = \cancel{mg}\sin\theta$$

$$\mu_s\cos\theta = \sin\theta$$

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}(\mu_s)$$

Newton's First Law

An object at rest will remain at rest and an object in motion will continue in motion with a constant velocity (constant speed in a straight line) unless it experiences a net external force.

Isn't this just a special case of the second law:

$$\sum \vec{F} = m\vec{a}$$

when $\sum \vec{F} = 0$, the acceleration $\vec{a} = 0$?

Isn't the First Law just a special case of the Second, with $\vec{a} = 0$?

Yes ... and No!

The first law really tells you that such frames of reference exist.

A frame in which the first law holds true is called an

inertial frame of reference.

Best approximation: A frame that moves with constant (or zero) velocity with respect to distant stars.

Not:
- this room
- the Earth
- the Sun

⋮

coriolis

Force and Circular Motion

Ex. Consider a mass m in uniform circular motion, at speed v in radius r.

The mass accelerates toward the center of the circle.

$$a_r = \frac{v^2}{r} \qquad a_t = \frac{d|\vec{v}|}{dt} = 0$$

Where there is a mass and an acceleration, there is a net force.

The Centrifugal Force does not exist.

the Centripetal Force is not a Force.

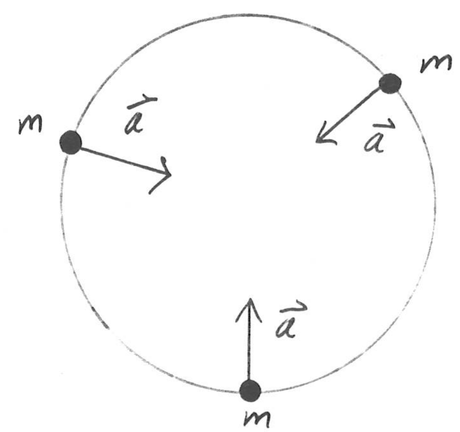
Centripetal Force

The net force in the radial direction is called the centripetal force.

$$\sum F_r = m a_r = m\frac{v^2}{r}$$

Recall:

$$\sum F_x = m a_x$$

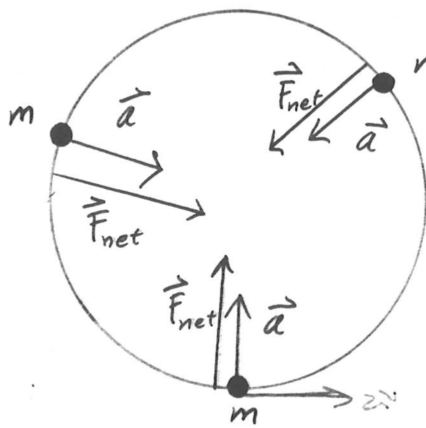


Centripetal Force

The <u>net</u> force in the radial direction is called the centripetal force.

$$\underline{\Sigma F_r} = m a_r = m\frac{v^2}{r}$$

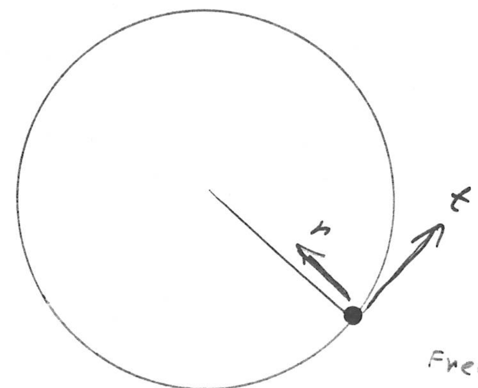
Recall: $\Sigma F_x = m a_x$



$\vec{F}_{net}$ points radially <u>inward</u>, same direction as $\vec{a}$

$$\Sigma \vec{F} = \vec{F}_{net} = m\vec{a}$$

<u>Ex</u>. Consider a mass on a string. Neglect gravity for now (top view).



Choose axes:

Free-Body Diagram

$$\Sigma \vec{F} = m\vec{a}$$

<u>r-comp</u>

$$\Sigma F_r = m a_r$$

$$T = m\left(\frac{v^2}{r}\right)$$

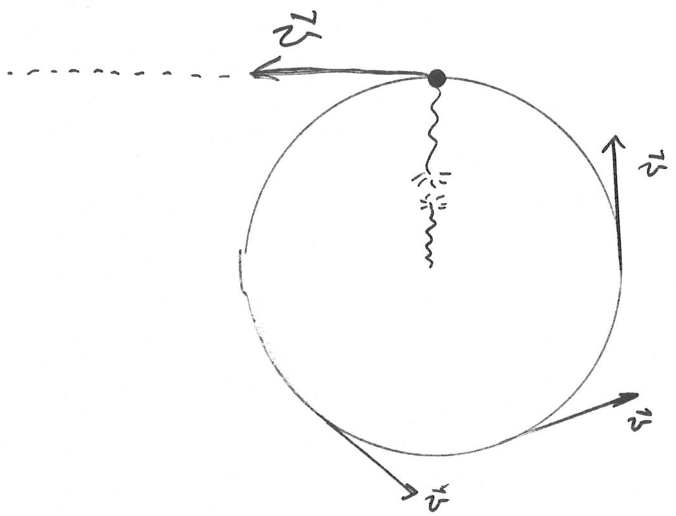
<u>t-comp</u>

$$\Sigma F_t = m a_t$$

$$0 = m a_t$$

$$\Rightarrow \boxed{a_t = 0}$$

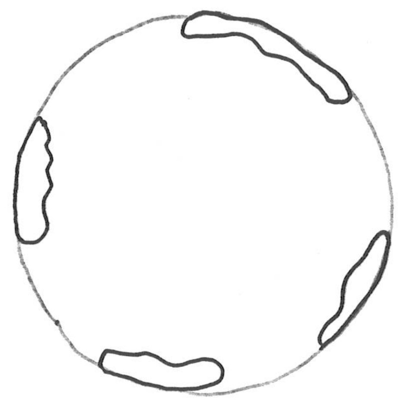
If the string breaks at the top, what is the path of the mass?



Free-Body Diagram: • no forces

What is the direction of $\vec{v}$ just before the string snaps?

Ex Washing machine spin cycle or salad spinner. Draw the water drops coming off.



What force holds the clothes (or lettuce) to the sides?

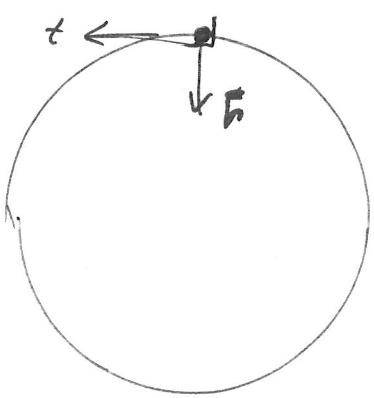
More Circular Motion

Ex A Ferris wheel rotating at a <u>constant</u> speed v.

(Include effects due to gravity.)

What speed will cause a rider to "feel" weightless at the top?

(Rider leaves seat.) $\vec{N} = 0$ $\quad W = mg$



$$\Sigma F_r = m a_r$$

$$W - \cancel{N}_{\to 0} = m\left(\frac{v^2}{r}\right)$$

$$\cancel{m}g = \frac{\cancel{m}v^2}{r}$$

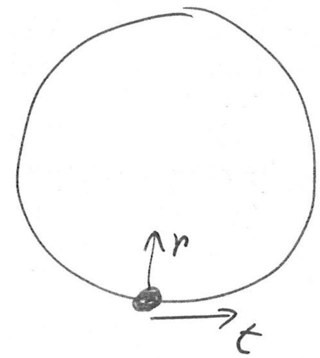
$$v = \sqrt{gr}$$

What is the centripetal force?

$\vec{W}$

② How heavy does the rider <u>feel</u> at the bottom when $v = \sqrt{gr}$

$N = ?$



$$\Sigma F_r = m a_r$$

$$+N - W = m\left(\frac{v^2}{R}\right)$$

$$N = W + \frac{mv^2}{R}$$

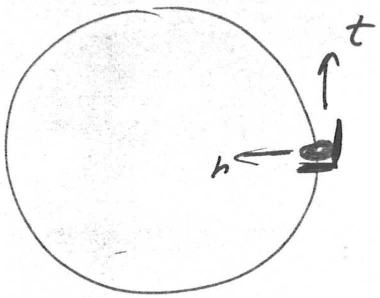
$$N = mg + \frac{mv^2}{R}$$

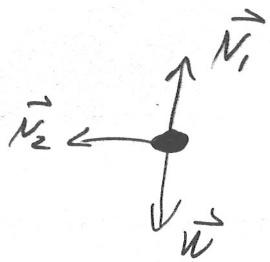
$$N = mg + \frac{m}{\cancel{R}}(\cancel{R}g) = 2mg$$

$N - W$

What is the centripetal force?

③ Between top and bottom:





r-components

$\Sigma F_r = m a_r$

$N_2 = m\left(\frac{v^2}{R}\right)$

$N_2 = \frac{m}{R}(Rg) = mg$

t-components

$\Sigma F_t = m\cancel{a_t}^{\,0}$

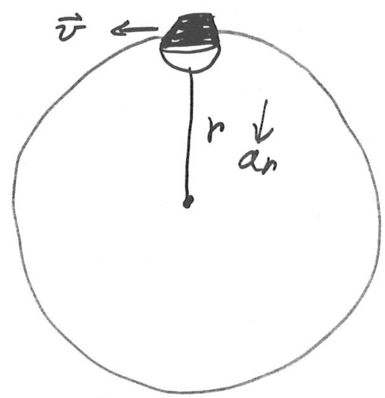
$N_1 - W = 0$

$N_1 = mg$

what is the centripetal force?

N_2

Ex What is the minimum speed at which I can whirl a pail of water over my head in a circle without getting wet?



$\Sigma F_r = m a_r$

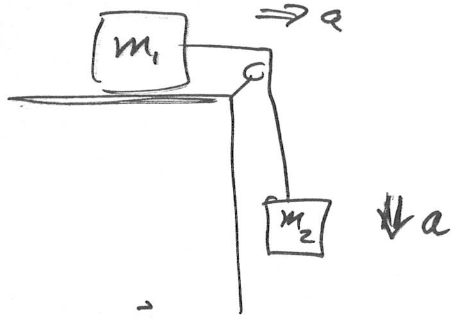
$W + N = m\left(\frac{v^2}{r}\right)$

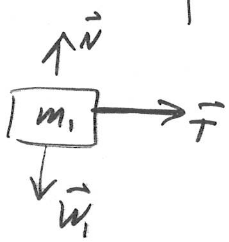
water falls out $\rightarrow$ $N = 0$

$W = m\frac{v^2}{r}$

$\cancel{m}g = \cancel{m}\frac{v^2}{r}$

$v = \sqrt{gr}$





$$\sum F_x = m_1 a_x$$

$$T = m_1 a_x$$
$$= m_1 (a)$$

$$\sum F_y = m_2 a_y$$

$$T - W_2 = m_2 (-a)$$

$$T = m_1 a \qquad T = m_2 (g - a)$$

$$m_1 a = m_2 (g - a)$$

$$\boxed{a = \frac{m_2 g}{m_1 + m_2}}$$

$$\boxed{T = \frac{m_1 m_2 g}{m_1 + m_2}}$$