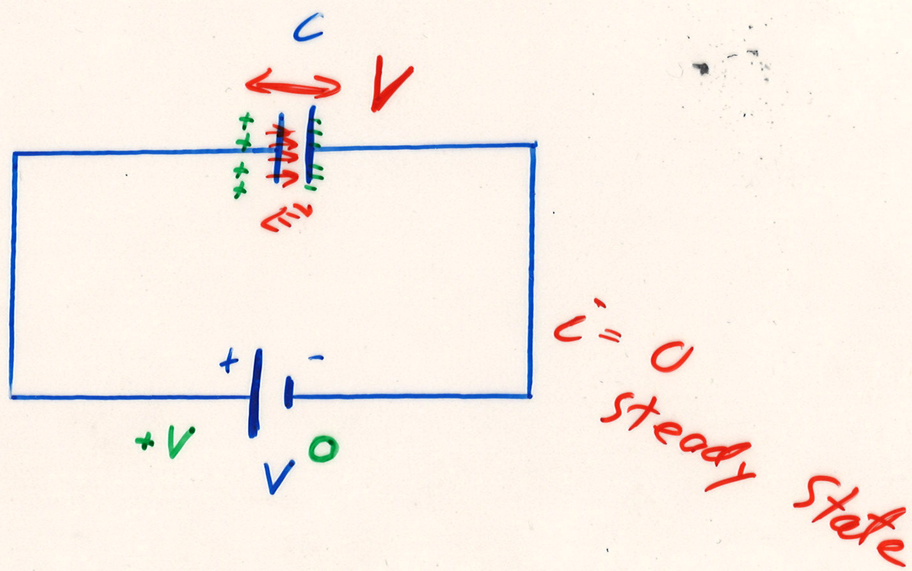
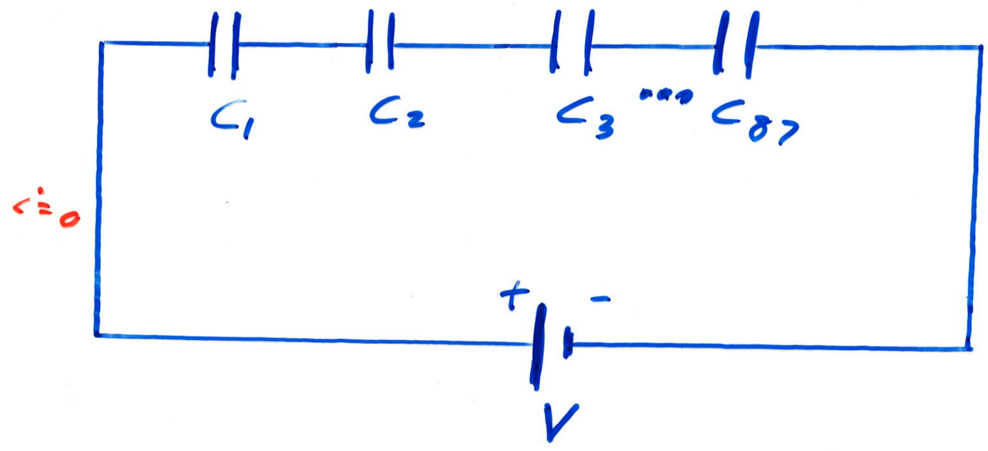
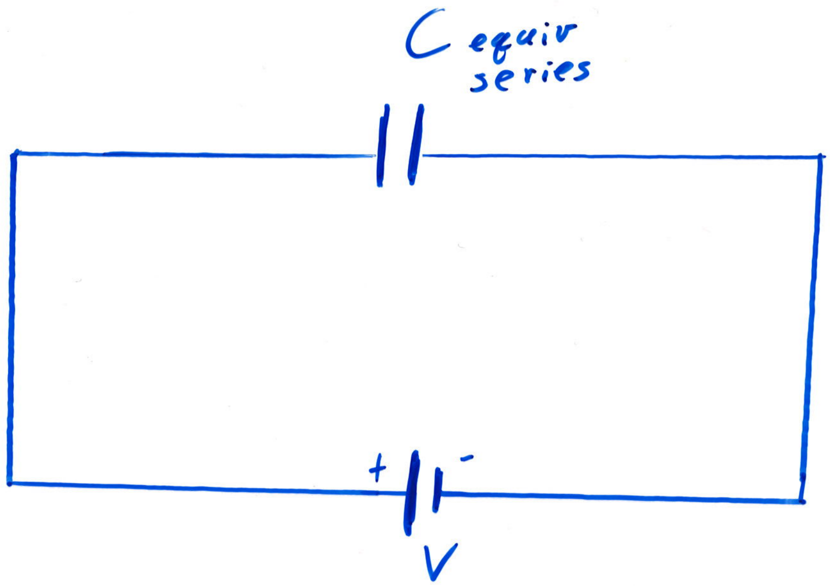
Circuits

In a circuit containing batteries and capacitors, current flows until all of the capacitors are fully charged. Then no more current flows. This is the steady state. The current is not changing; it is zero (a constant).



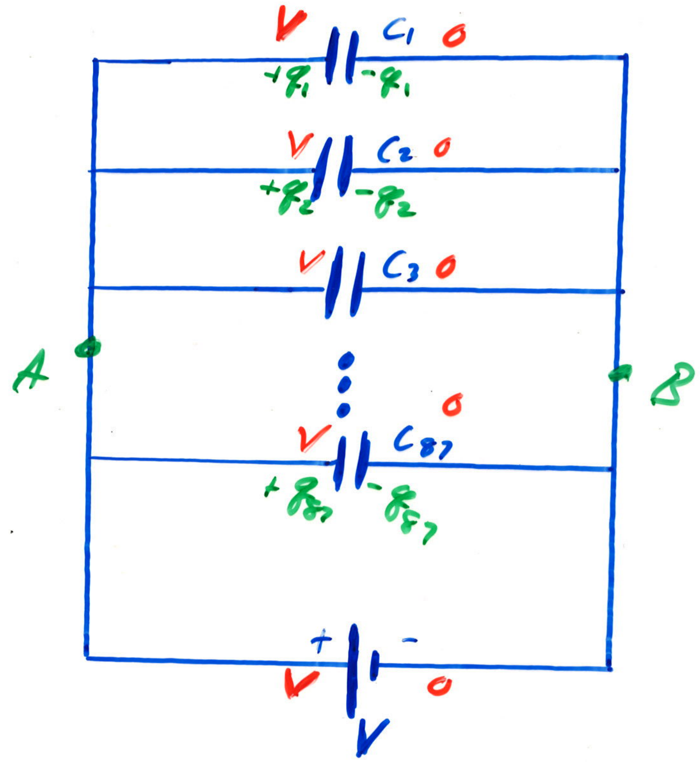
Capacitors in Series





$$\boxed{\frac{1}{C_{\substack{equiv \\ series}}} = \sum_{i=1}^{N} \frac{1}{C_i}}$$

Capacitors in Parallel



$q_1 = C_1 V_1$

$q_2 = C_2 V_2$

$\vdots$

$$V_1 = V_2 = \cdots = V_{87} = V_{\text{Battery}} = V$$

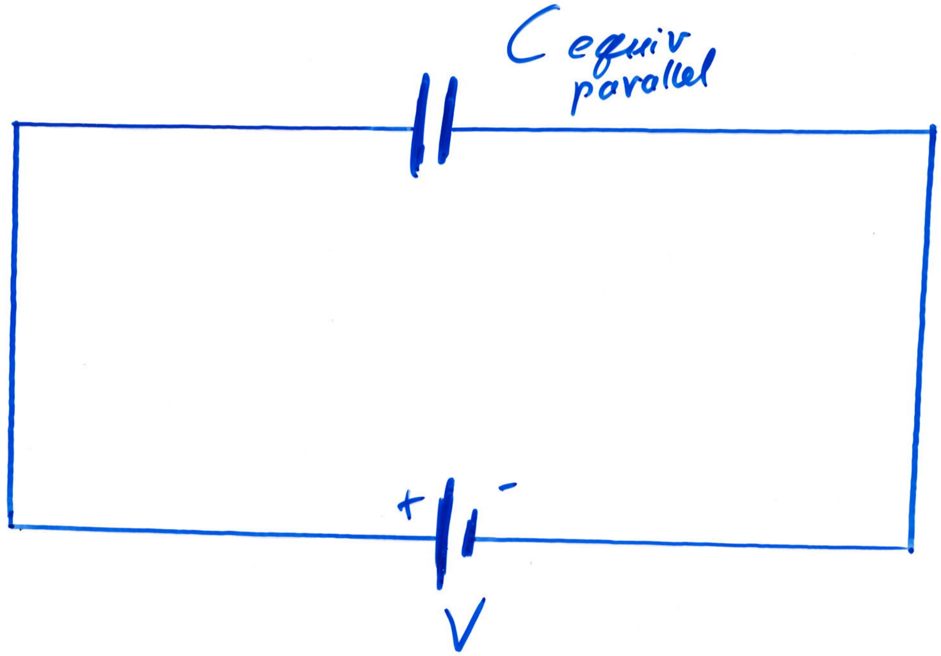
Total charge held by all capacitors

$$Q_{TOT} = q_1 + q_2 + \cdots + q_{87}$$

$$= C_1 V_1 + C_2 V_2 + \cdots + C_{87} V_{87}$$

$$= (C_1 + C_2 + \cdots + C_{87})\, V$$

$$Q_{TOT} = C_{\substack{\text{equiv}\\ \text{parallel}}} V$$



$$C_{\substack{equiv\\parallel}} = \sum_{i=1}^{N} C_i$$

What idea did we use?

The potential difference between two points, A and B, in the circuit can be obtained by following <u>any</u> path through the circuit.

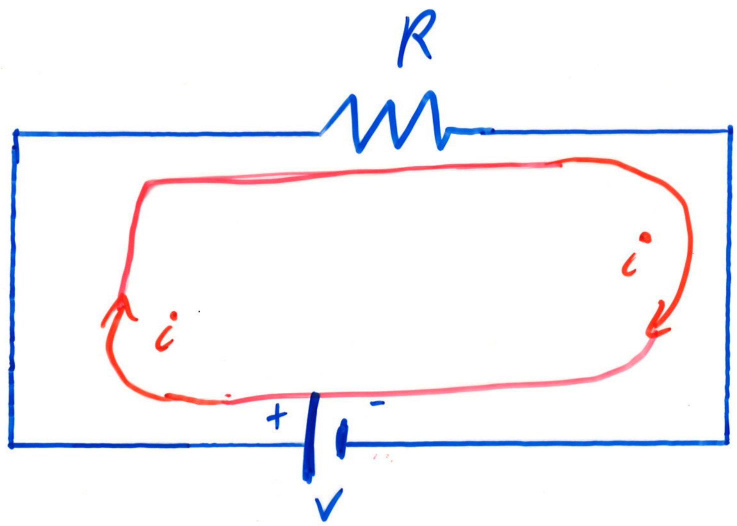
Kirchhoff's Loop Rule:

The algebraic sum of the changes in potential encountered in a complete traversal of any circuit must be zero.

Now consider circuits containing batteries and resistors, but no capacitors. In the steady state, a non-zero constant current flows through the circuit forever.

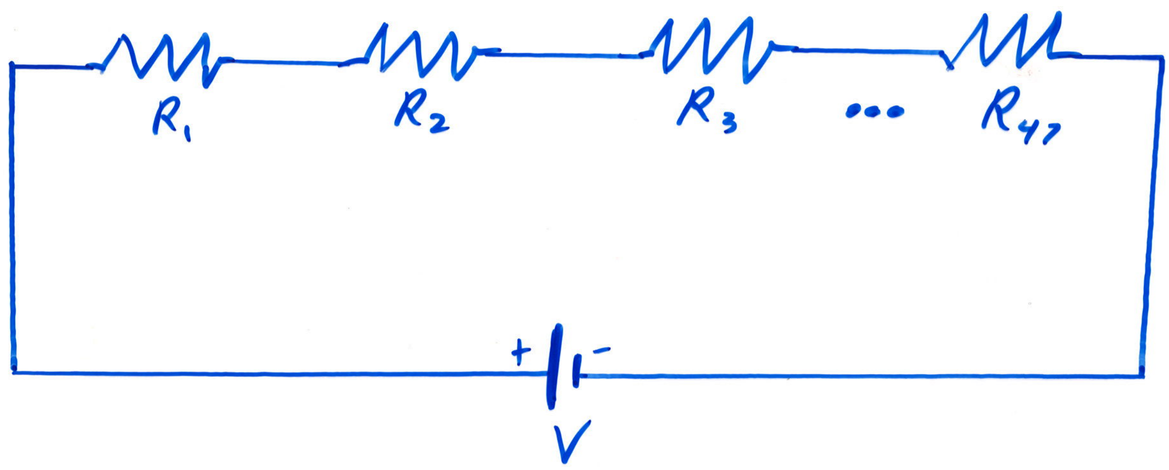
$V = iR$

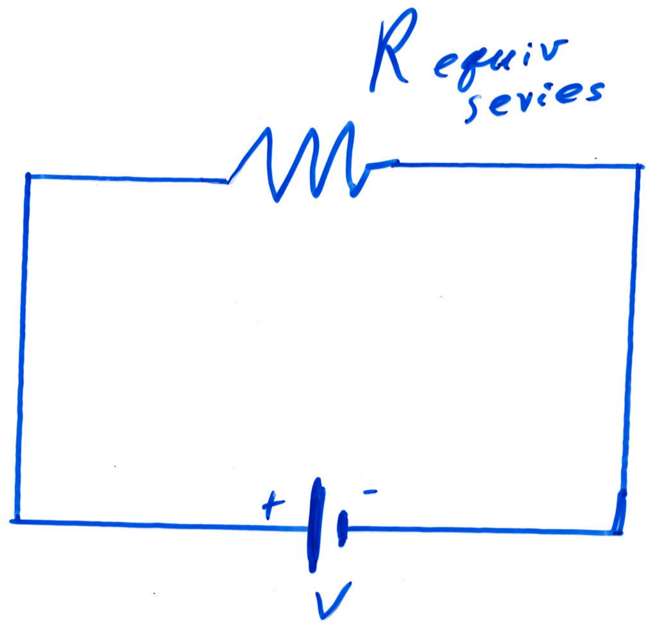
"Ohm's Law"



The current must be the same on both sides of the resister because charge can not "pile up" anywhere in the circuit.

Resistors in Series



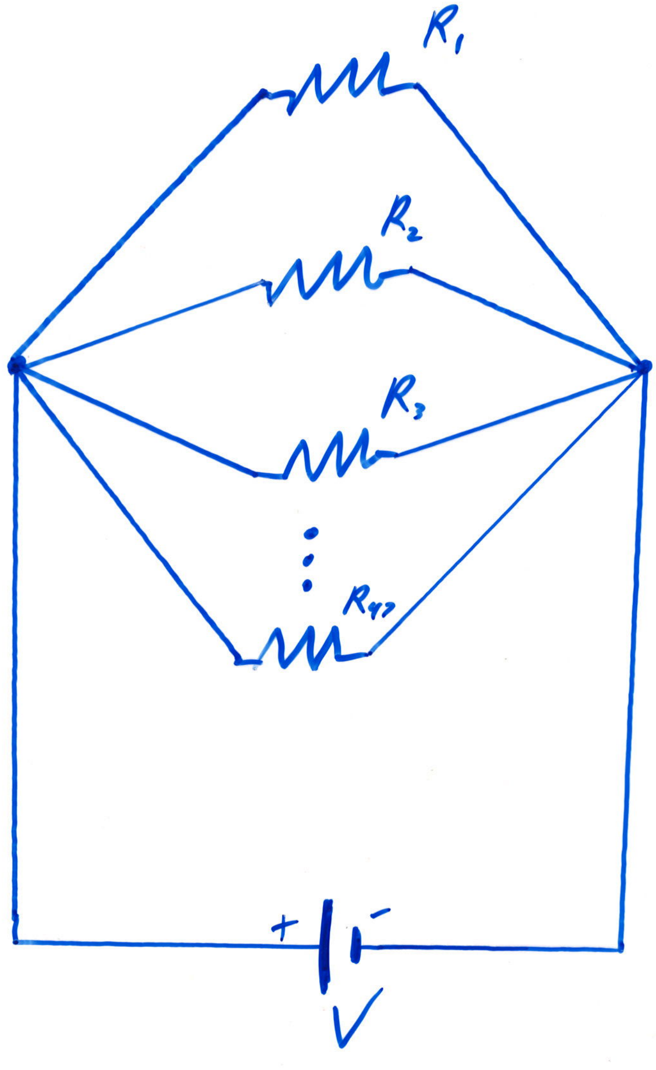


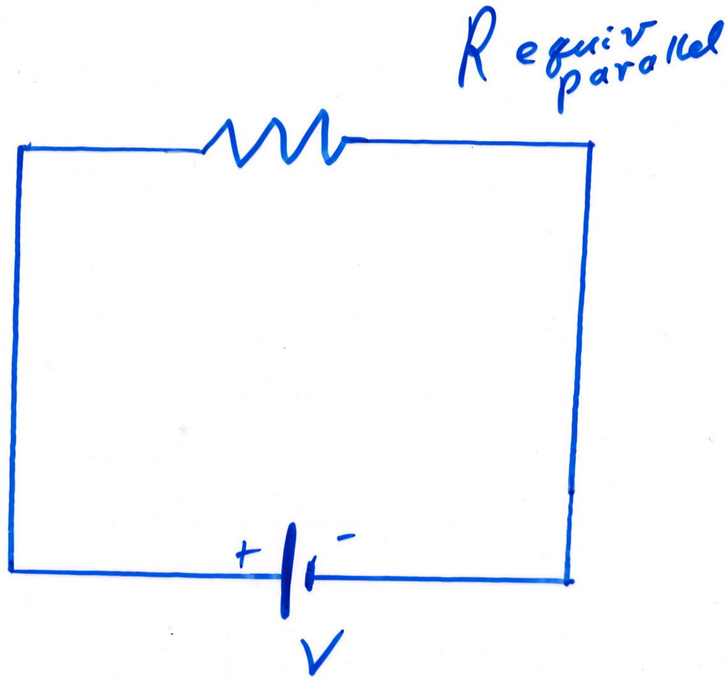
$$R_{\substack{equiv\\series}} = \sum_{i=1}^{N} R_i$$

looks like

$$C_{\substack{equiv\\parallel}} = \sum_{i=1}^{N} C_i$$

Resistors in Parallel





$$\frac{1}{R_{\substack{equiv\\parallel}}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

looks like

$$\frac{1}{C_{\substack{equiv\\series}}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

What principle did we use this time?

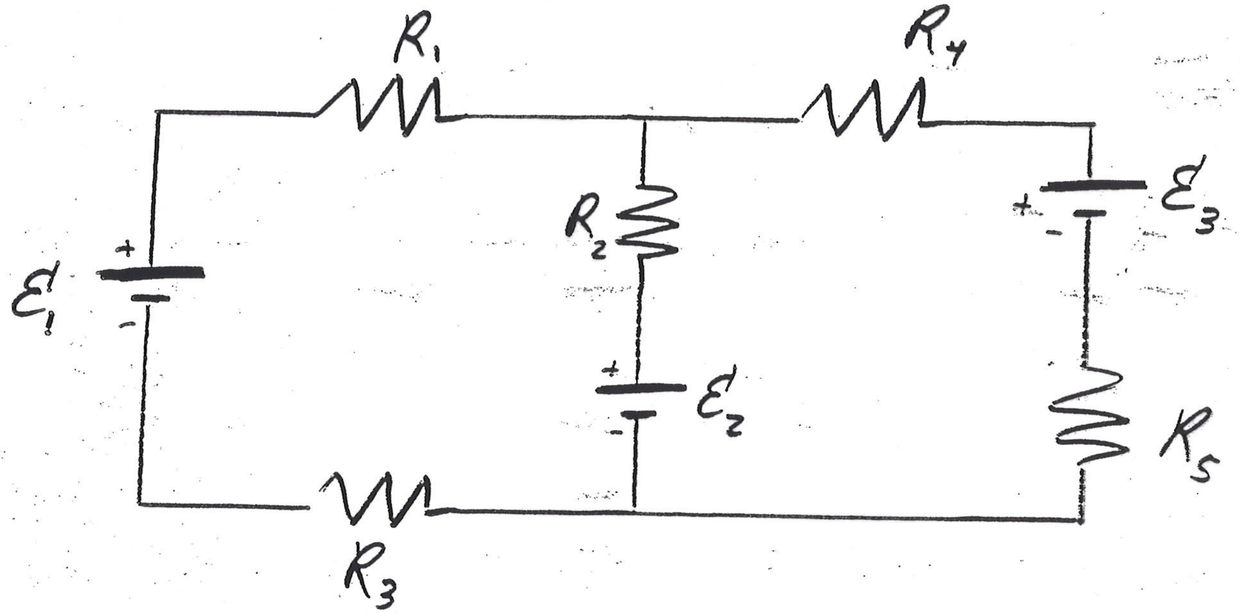
Electric charge is conserved, so it can't "pile up" anywhere in the circuit. Whatever charge flows <u>into</u> a junction must flow <u>out</u>.

Kirchhoff's Junction Rule:

The sum of the currents approaching any junction must be equal to the sum of the currents leaving that junction.

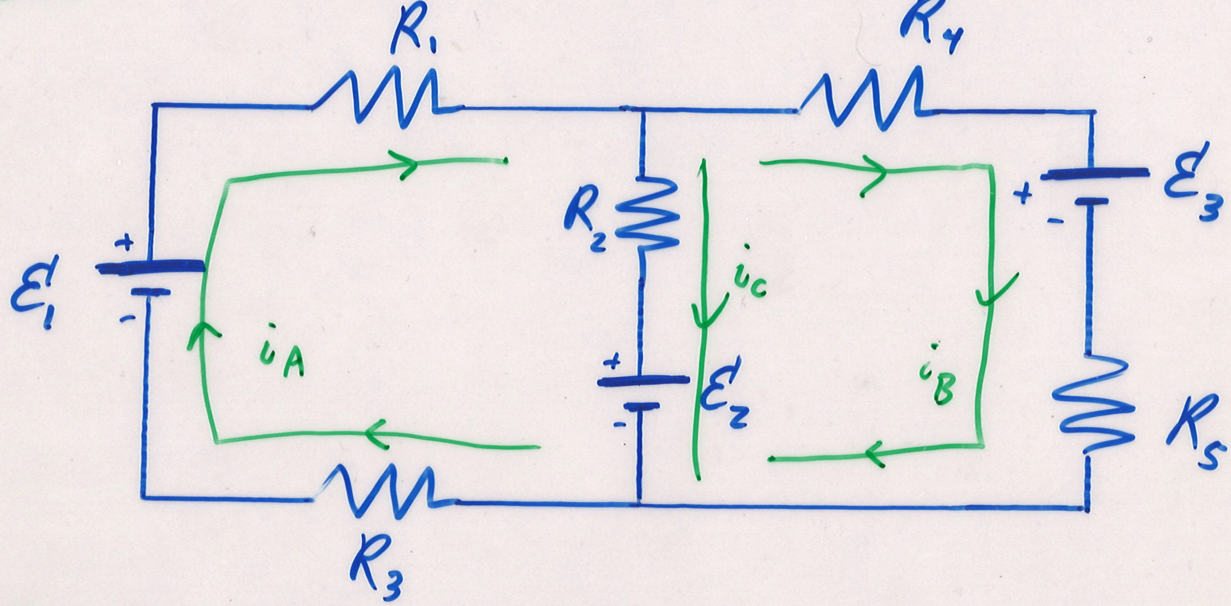
Steady state: Charge Conservation implies Current Conservation

Circuits containing more than one Battery



Circuits containing more than one Battery

P.800



1) Try to reduce the circuit by replacing R's in series and R's in parallel with their equivalents.

2) Draw currents with directions.

3) Kirchhoff's Junction Rule

4) Kirchhoff's Loop Rule

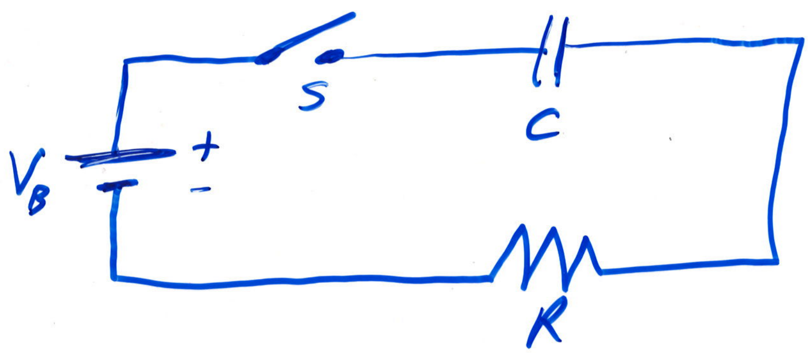
5) Solve same number of simultaneous equations.

None of the resistors are in series or parallel. Now what?

series ⇒ same current   parallel ⇒ same voltage

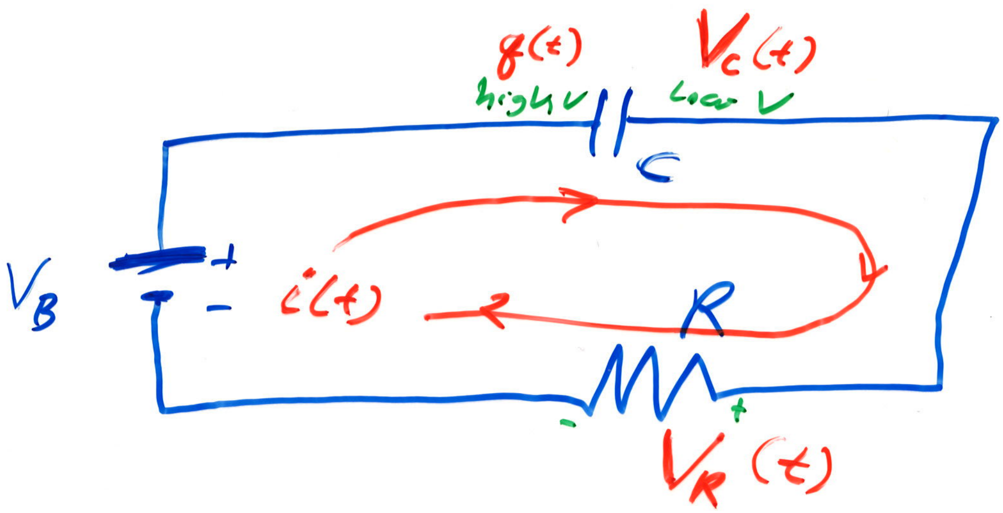
RC Circuits

Circuits containing a Resistor, a Capacitor, and possibly a battery, in series.



If you wait long enough, the steady state is boring: $i = 0$.

But this is our first opportunity to study a time-dependent current $i(t)$.



At <u>any</u> time t, Kirchhoff's Loop Rule gives:

$$0 = V_B - V_c(t) - V_R(t)$$

$$0 = V_B - \frac{q(t)}{C} - i(t)R$$

$q(t) = C\,V_c(t)$

$V_R(t) = i(t)R$

but the current is the time rate of change of the charge:

$$i(t) = \frac{d}{dt}\left[q(t)\right]$$

This is a differential equation.

A solution will tell you the charge at any time. That is, the solution is not a number, but rather a <u>function</u> of time $q(t)$.

$$0 = V_B - \frac{q(t)}{C} - R\frac{d}{dt}[q(t)]$$

Guess!

Try a solution of the form:

$$q(t) = \underbrace{A}_{\text{constants}} e^{-t/RC} + B$$

$$i(t) = \frac{d}{dt}[q(t)] = -\frac{A}{RC}\, e^{-t/RC}$$

$$\frac{d}{dt} e^{-\frac{t}{\tau}} = \left(\frac{-1}{\tau}\right) e^{-t/\tau}$$

where τ is a constant

Substitute :

$$0 = V_B - \frac{Ae^{-t/RC} + B}{C} + -R\left[\frac{-A}{RC} e^{-t/RC}\right]$$

$$0 = V_B \underbrace{- \frac{A}{c} e^{-t/RC}} - \frac{B}{c} + \underbrace{\frac{A}{c} e^{-t/RC}}$$

$$V_B = \frac{B}{c} \implies B = CV_B$$

"A" is still not fixed

$$q(t) = Ae^{-t/RC} + CV_B$$

Fix A with a so-called boundary condition : the charge on the capacitor at $t=0$ is zero.

The capacitor is neutral before the switch is closed.

$$q(0) = 0$$

$$0 = q(0) = Ae^{-\frac{0}{RC}} + CV_B = A + CV_B$$

$$\Rightarrow A = -CV_B = -B$$

Solution:

$$q(t) = CV_B\left(1 - e^{-t/RC}\right)$$

$$i(t) = \frac{d}{dt}[q(t)] = \frac{CV_B}{RC}e^{-t/RC} = \frac{V_B}{R}e^{-t/RC}$$

$$V_C(t) = \frac{q(t)}{C} = V_B\left(1 - e^{-t/RC}\right)$$

$$V_R(t) = i(t)R = V_B e^{-t/RC}$$

Check the solution!

$$0 \stackrel{?}{=} V_B - V_C(t) - V_R(t)$$

$$= V_B - V_B\left(1 - e^{-t/RC}\right) - V_B e^{-t/RC} = 0 \checkmark$$

Consider some special times:

$\boxed{t=0}$

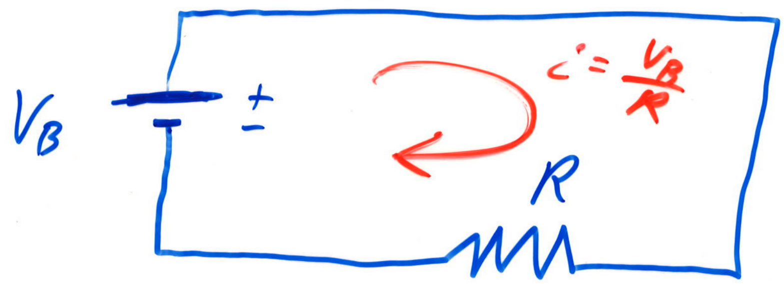
$q(0) = 0$

$i(0) = \frac{V_B}{R}$

$V_C(0) = 0$

$V_R(0) = V_B$

What does this mean?



no capacitor!

$\boxed{t=\infty}$

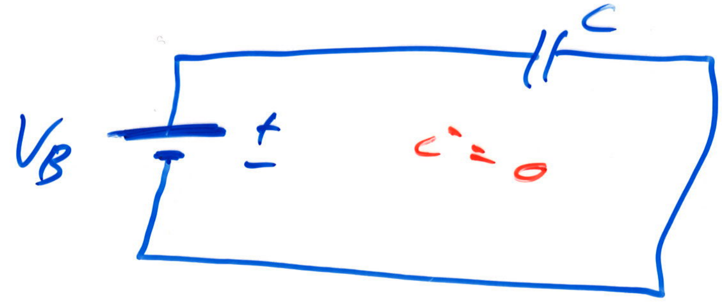
$q(\infty) = C V_B$

$i(\infty) = 0$

$V_C(\infty) = V_B$

$V_R(\infty) = 0$

What does this mean?



no resistor!

Capacitive Time Constant

RC has the dimension of time

$$[RC] = T$$

In MKS units: $1\,\Omega \cdot F = 1$ second

In the time RC, the charge on the capacitor has increased from

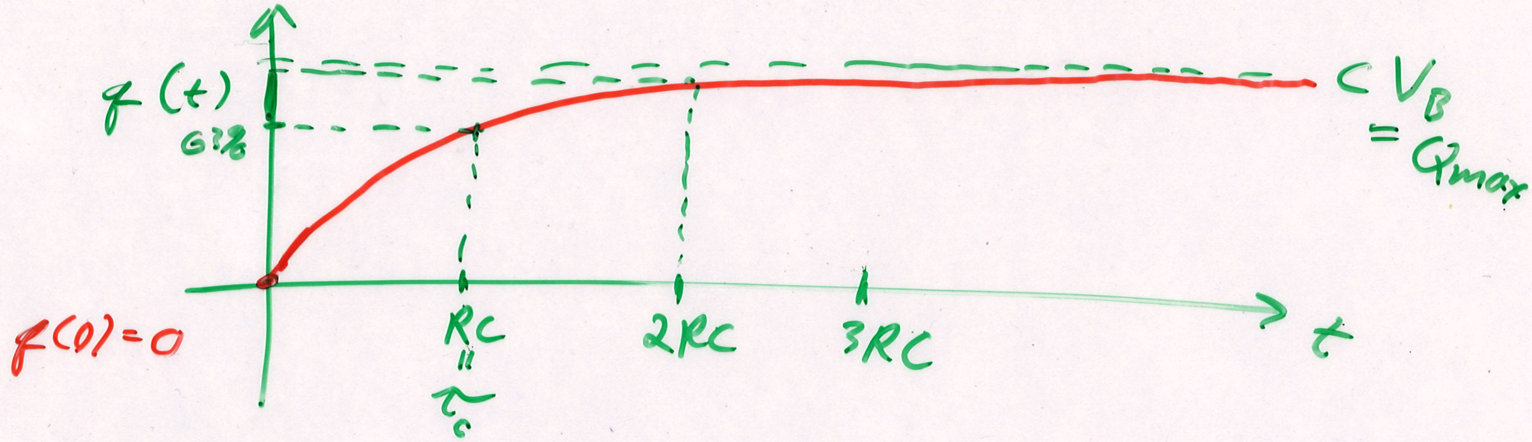
$q(0) = 0$ to

$$q(RC) = CV_B\left(1 - e^{-\frac{RC}{RC}}\right)$$

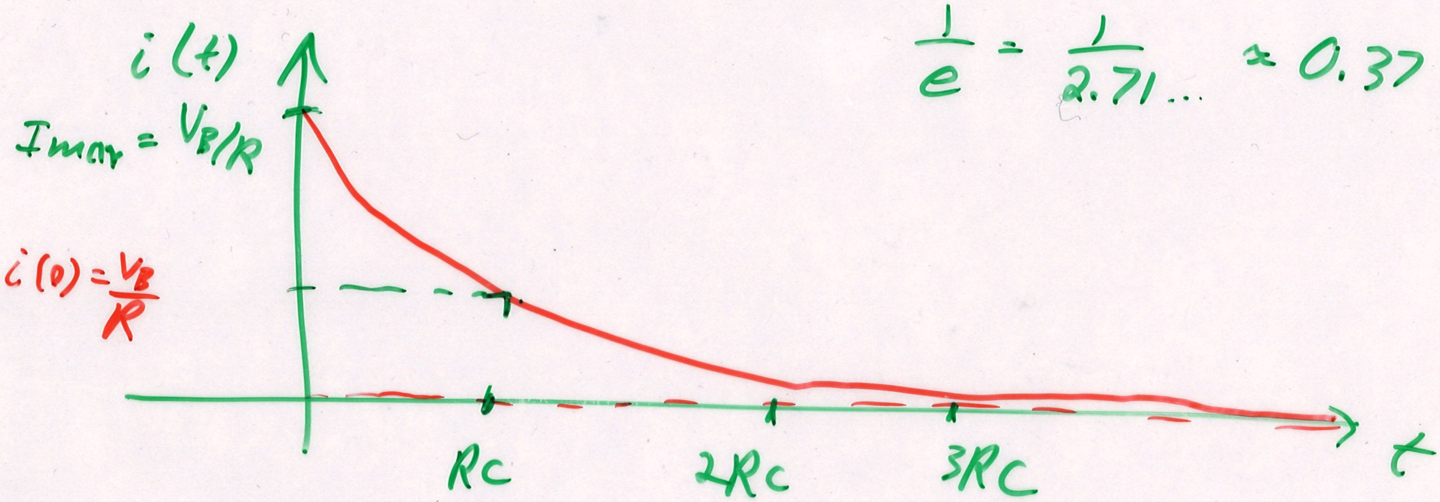
$$= CV_B\left(1 - e^{-1}\right)$$

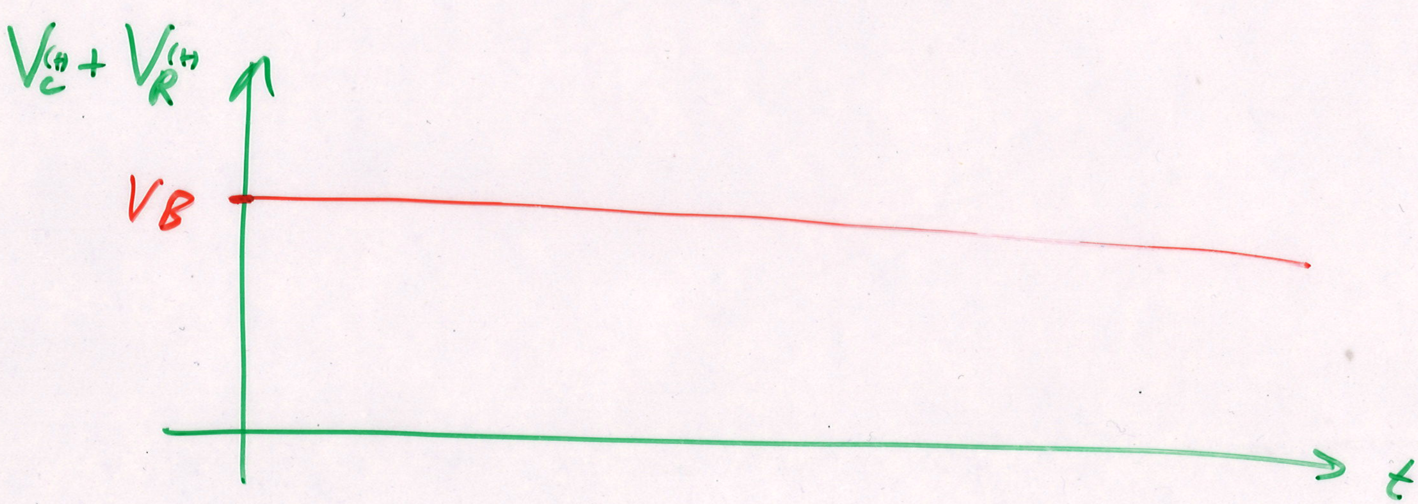
$$\approx 63\%\ CV_B$$

that is 63% of its fully charged value.



$$\frac{1}{e} = \frac{1}{2.71\ldots} \approx 0.37 = 37\%$$





Capacitors easily pass changing currents (they block steady currents)

Filter (Surge Protector)