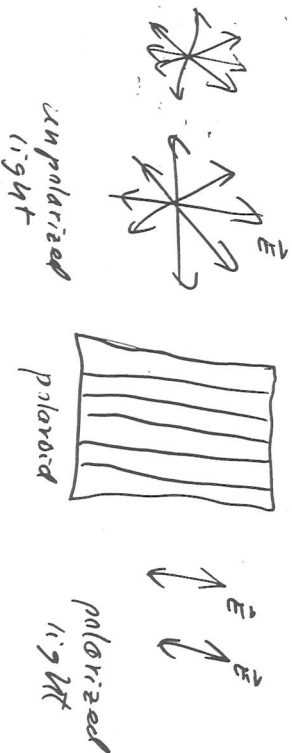


Polaroids

Light from the Sun, a light bulb, a match, etc. is unpolarized, that is, it contains \vec{E} fields pointing in all directions of random.

Think of a polaroid as a picket fence that only allows those \vec{E} fields aligned along the pickets to pass through.



If the polaroid direction (picket direction) and the \vec{E} field direction make an angle θ , then only

$$E_{\max} \cos \theta \text{ gets through}$$

and the component of \vec{E} that does get through is now polarized along the polaroid direction.

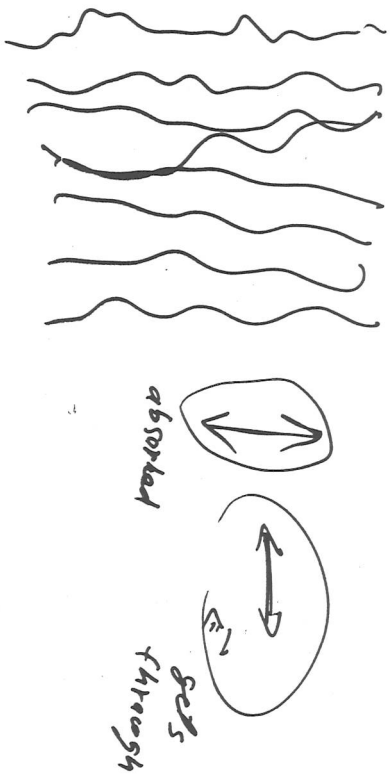
The intensity of radiation is proportional to $|\vec{E}|^2$

$$I_{\max} = \frac{1}{c\mu_0} E_{\text{rms}}^2 = \frac{1}{c\mu_0} \frac{E_{\max}^2}{2}$$

so the intensity (what your eye detects) that gets through the polaroid is

$$I_{\max} \cos^2 \theta$$

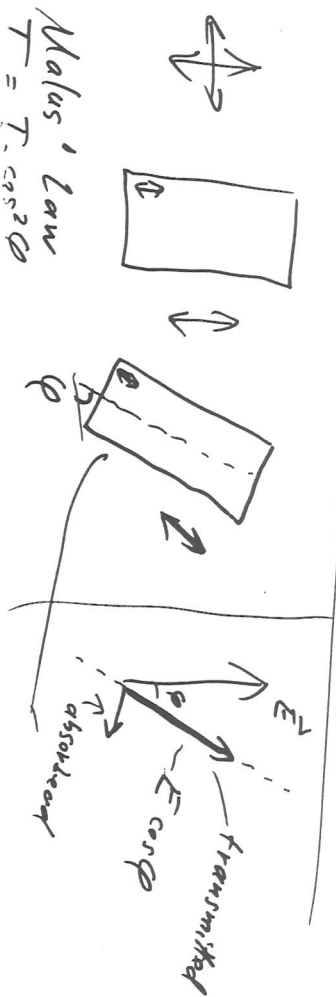
Polarization: direction of Electric field perpendicular to direction of travel



$$I = \vec{E}^2 = E_x^2 + E_y^2$$

unpolarized

$$I_{\text{polarized}} = E_x^2 = \frac{1}{2} I_{\text{unpolarized}}$$



Malus' Law

$$\frac{I}{I_0} = \cos^2 \phi$$

One polaroid

What happens to the original intensity I_{max} after unpolarized light passes through one polaroid?

$$\frac{I_{\text{max}}}{2}$$

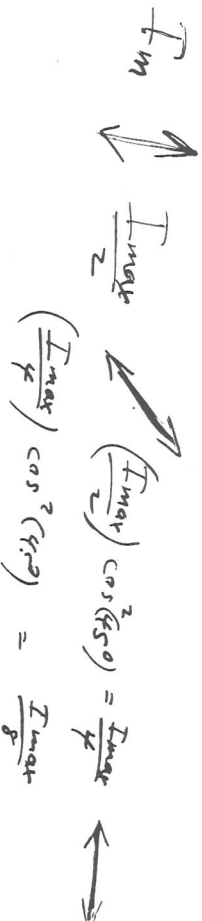
Crossed polaroids

What happens to I_{max} as unpolarized light is passed through 2 polaroids set at 90° to each other?

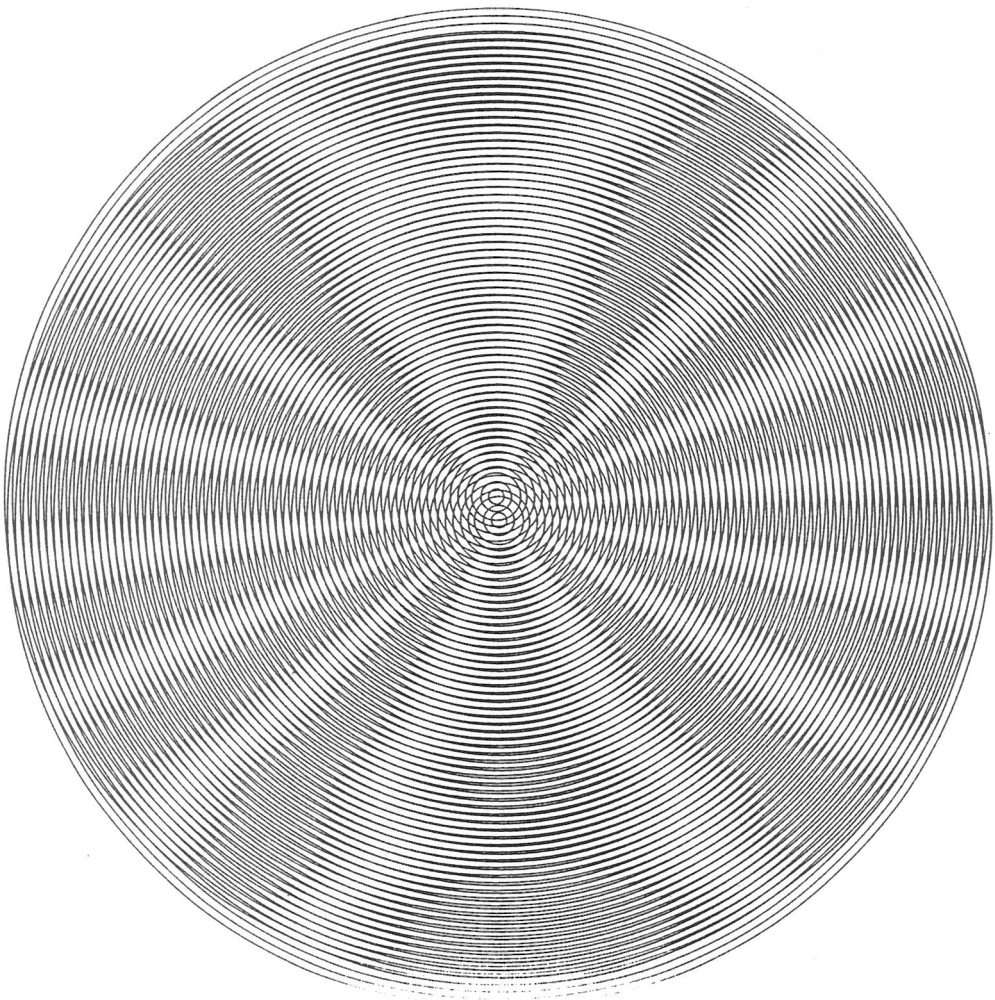
0

Stacked polaroids

A third sheet is inserted between 2 crossed polaroids 45° from each. I_{max} ?



$$\left(\frac{I_{\text{max}}}{2}\right) \cos^2(45^\circ) = \frac{I_{\text{max}}}{8}$$

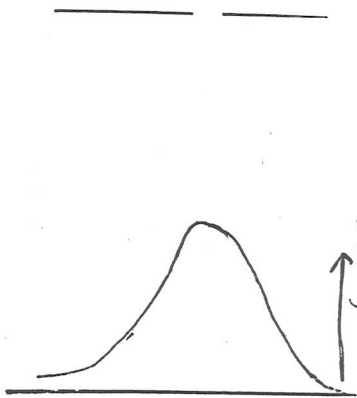


Interference

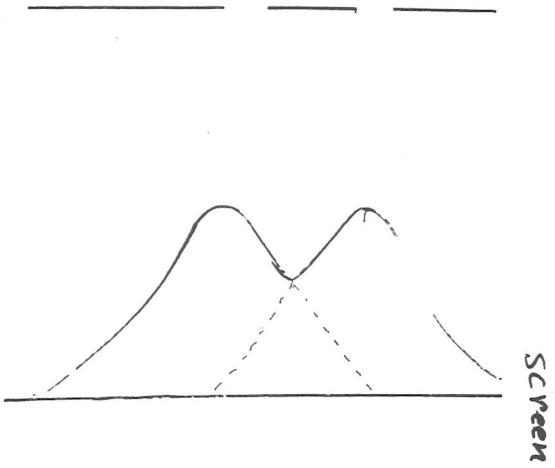
Newton: Light is made of particles or corpuscles.

Young: Light is made of waves.

One slit



two slits



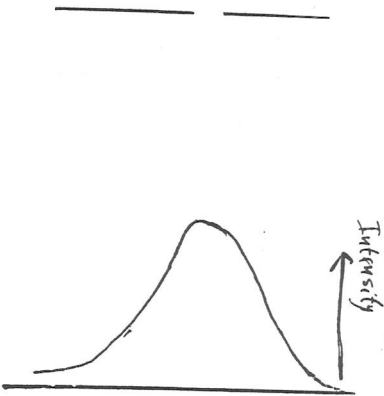
Expected pattern for corpuscles

Interference

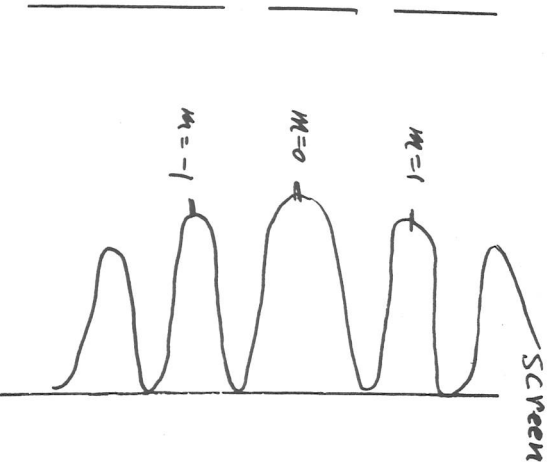
Newton: Light is made of particles or corpuscles.

Young: Light is made of waves.

One slit

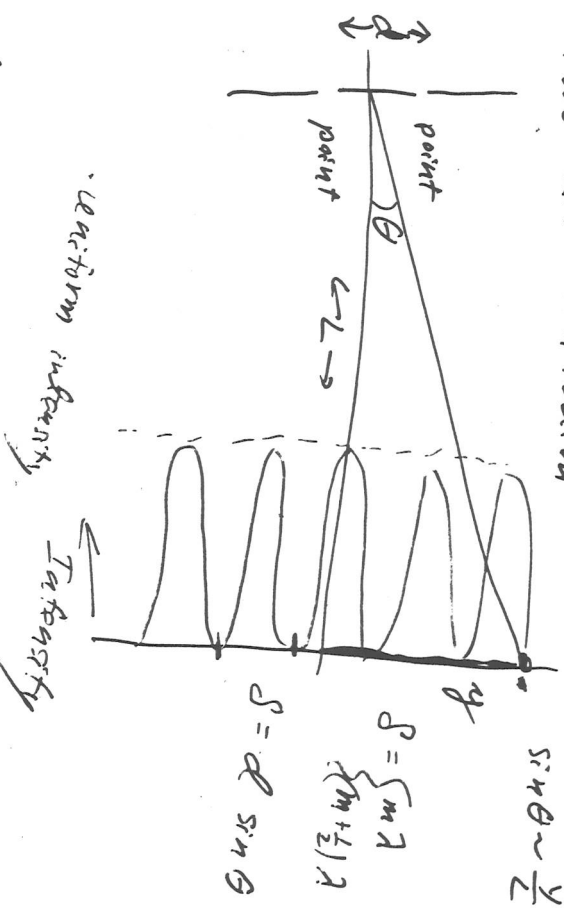


Two slits



actual pattern for interfering waves.
(particles do not interfere.)

Two slit diffraction



path difference δ tells you where bright spots occur

Intensity function — requires

$$E_{tot} = E_1 + E_2$$

$$\sin \theta \sim \frac{y}{L}$$

$$\delta = \sum_{m=1}^M \lambda$$

$$\delta = d \sin \theta$$

Path difference $\delta = d \sin \theta$

Constructive Interference
Bright $\delta = d \sin \theta = m \lambda$
 $m = 0, \pm 1, \pm 2, \dots$

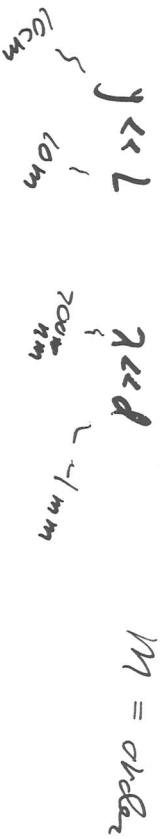
Destructive Interference
Dark $\delta = d \sin \theta = (m + \frac{1}{2}) \lambda$
 $m = 0, \pm 1, \pm 2, \dots$

Small angle approximation

for $\theta \approx 0.1 \text{ rad } (\approx 5^\circ)$

$$\theta \approx \sin \theta \approx \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{L} \quad (\text{radian})$$

$$y = L \sin \theta = \begin{cases} L m \lambda / d & \text{const. bright} \\ L (m + \frac{1}{2}) \lambda / d & \text{destr. dark} \end{cases}$$



Phase Difference ϕ

When $\delta = 0$, $\phi = 0$
When $\delta = \lambda$, $\phi = 2\pi$

$$\frac{\delta}{\lambda} = \frac{\phi}{2\pi} \Rightarrow \phi = \frac{2\pi \delta}{\lambda} = \frac{2\pi d \sin \theta}{\lambda}$$

Intensity

$$I = S_{\text{ave}} = \frac{E_0^2}{2\mu_0 c}$$

\leftarrow Poynting vector

$$E_{\text{tot}} = E_1 + E_2$$

$$= E_0 \sin(\omega t) + E_0 \sin(\omega t + \varphi)$$

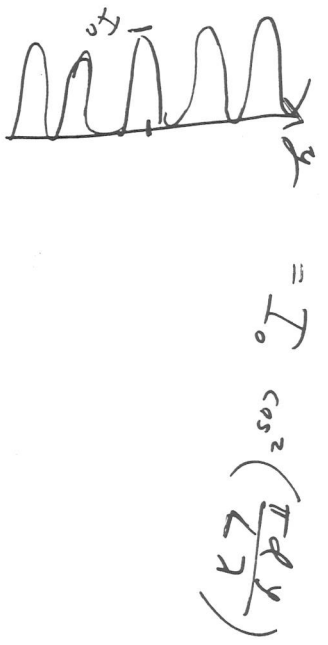
$$= 2E_0 \cos\left(\frac{\varphi}{2}\right) \sin\left(\omega t + \frac{\varphi}{2}\right)$$

max is $2E_0$ - const. - $\varphi = 0, 2\pi, 4\pi, \dots$

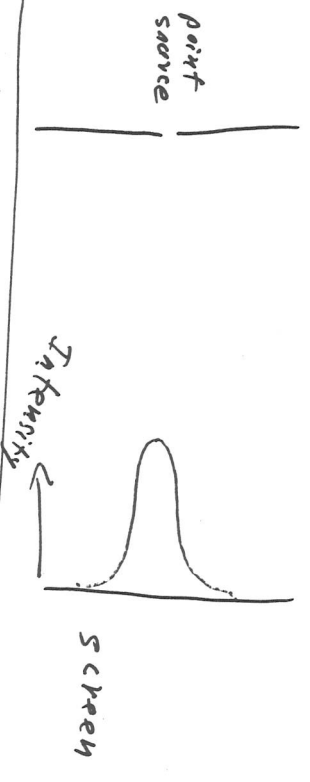
min is 0 - destr. - $\varphi = 180^\circ = \pi$ rad

$\frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

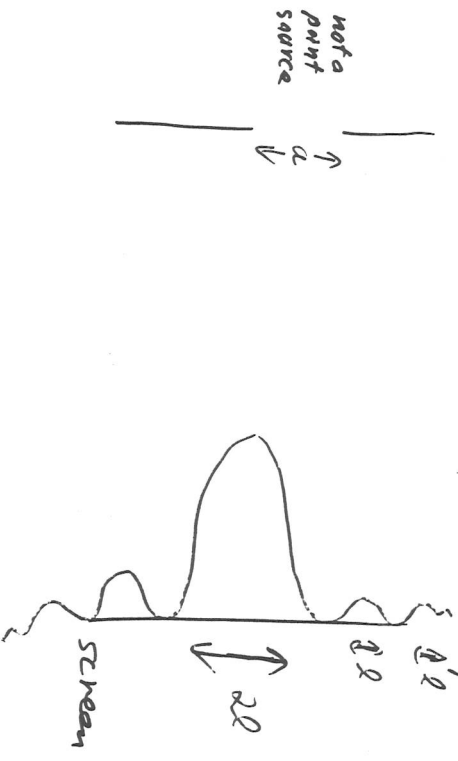
$$I = I_0 \cos^2\left(\frac{\varphi}{2}\right) = I_0 \cos^2\left(\frac{\pi d \sin \theta}{\lambda}\right)$$



Previously, single slit pattern



Single slit Diffraction pattern

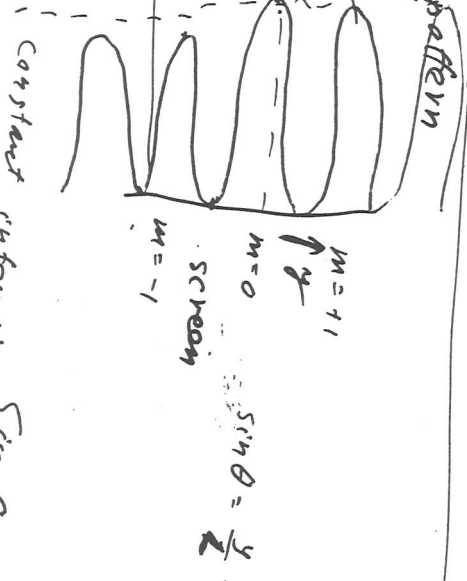


Observations

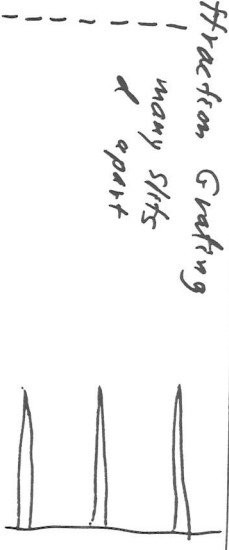
- side peaks are not as bright as the central peak.
- central maximum is twice as wide as other maxima.

$$E_{\text{tot}} = E_1 + E_2 = E_0 \sin^2(\alpha) + E_0 \sin^2(\alpha + \phi)$$

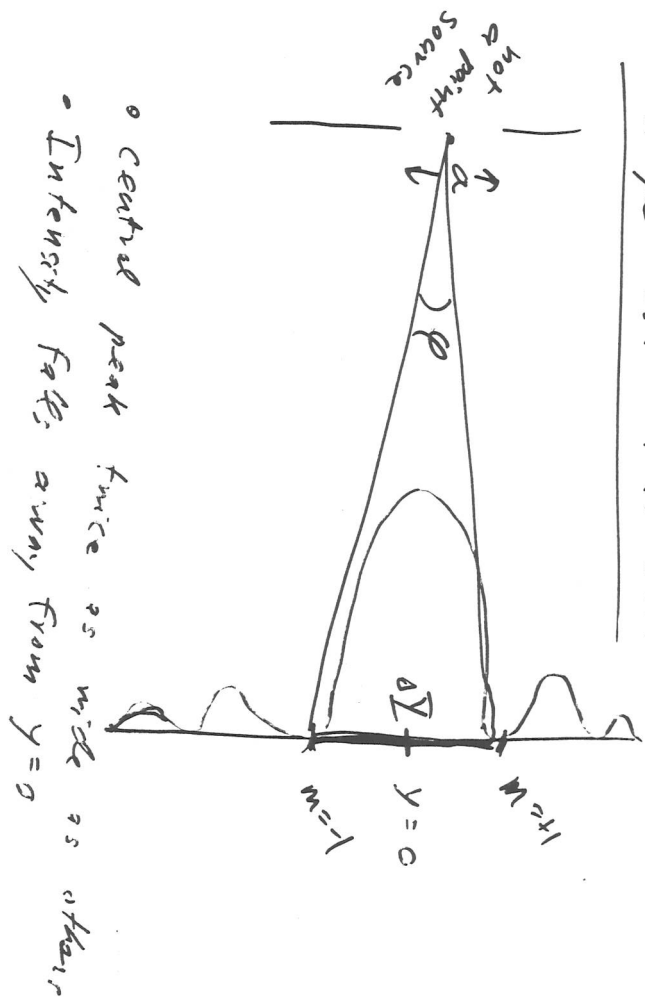
$I \propto E_{\text{tot}}^2$



Diffraction Grating



Single slit Diffraction

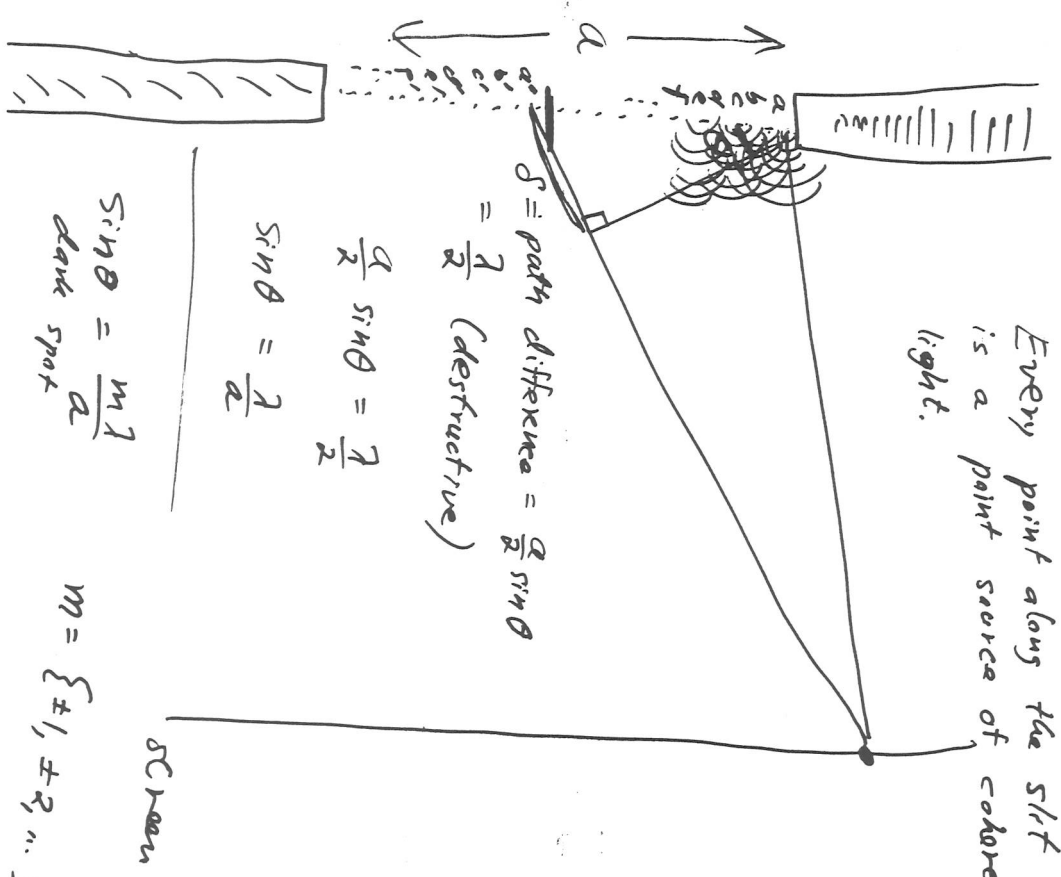


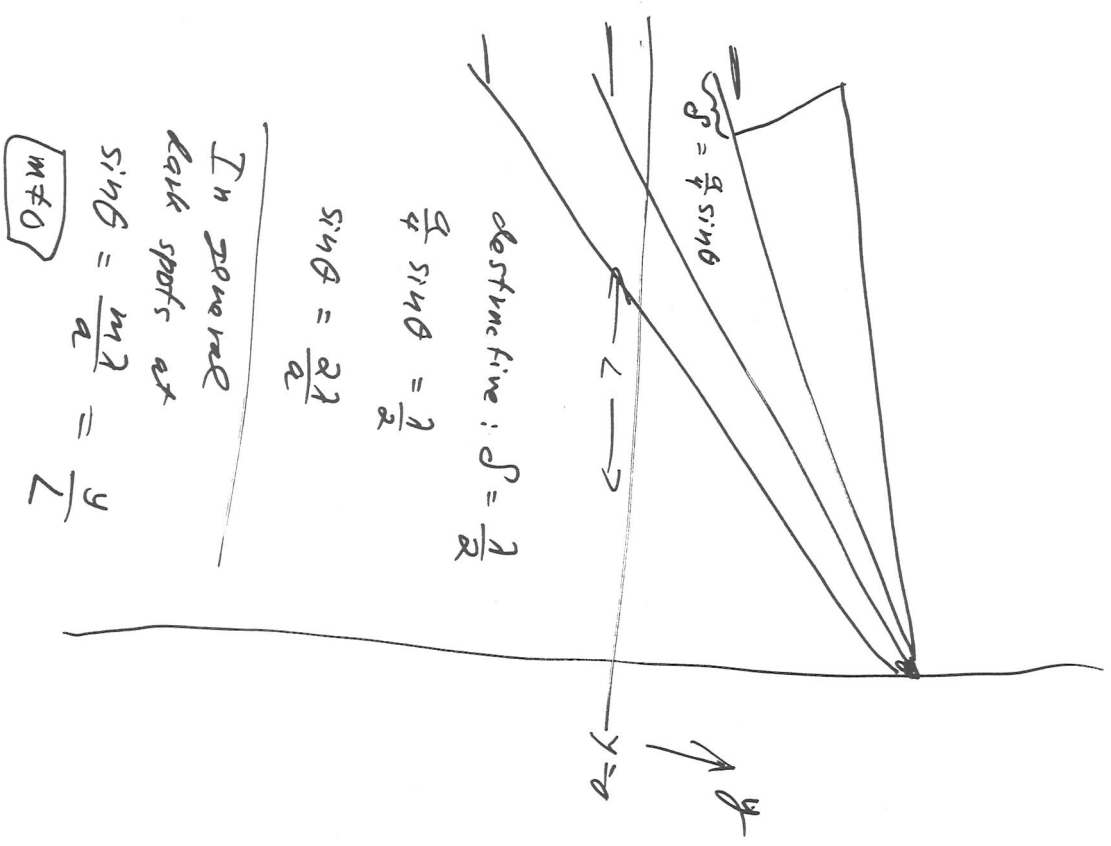
$$\phi = 2 \sin^{-1}\left(\frac{\lambda}{a}\right)$$

$$\Delta Y = 2\phi$$

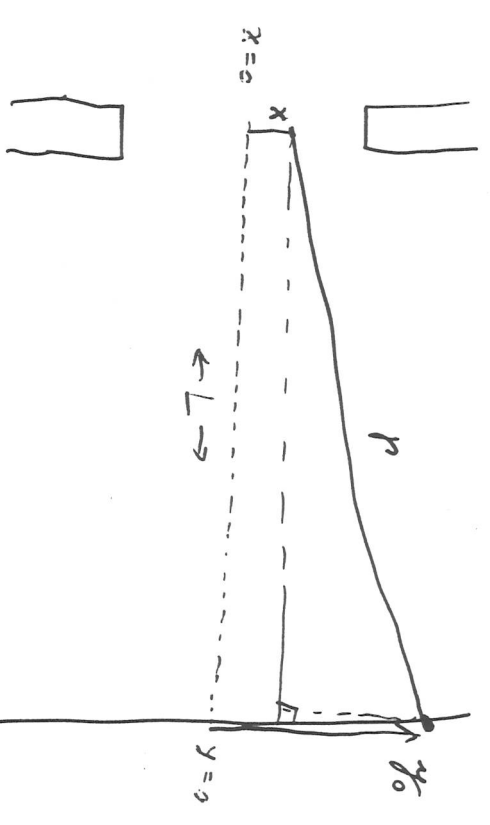
Huygen's Principle

Every point along the slit is a point source of coherent light.





In general
 look spots at
 $\sin \theta = \frac{m\lambda}{a} = \frac{y}{L}$
 $m \neq 0$



$$r^2 = (y_0 - x)^2 + L^2$$

$$= R^2 + x^2 - 2x y_0$$

$$= R^2 \left(1 + \frac{x^2}{R^2} - \frac{2x y_0}{R^2} \right)$$

$$= R^2 \left(1 - \frac{2x}{R} \sin \theta \right)$$

$\left(\frac{x}{R}\right)^2 \approx 0$ $\frac{y_0}{R} \sim \sin \theta$

$$r = R \sqrt{1 - \frac{2x}{R} \sin \theta}$$

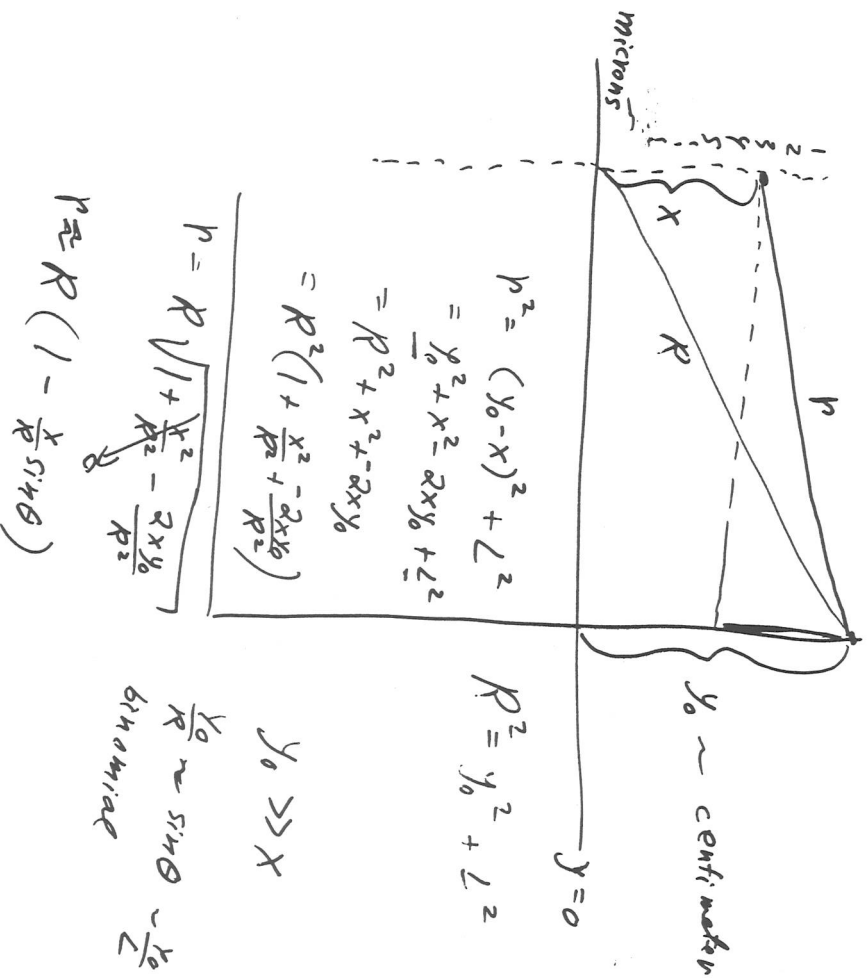
Binomial Theorem $(1-z)^n \approx 1 - nz + \dots$ $z \ll 1$

$r \approx R \left(1 - \frac{x}{R} \sin \theta \right)$

Intensity

$$E_{\text{tot}} = dE_1 + dE_2 + dE_3 + \dots$$

$$= \sum_n dE_n \Rightarrow \int dE$$



$$E_{\text{tot}}^{\text{at screen}} = dE_1 + dE_2 + \dots = \sum_n dE_n = \int dE$$

$$dE = \frac{E_0}{a} dx \sin(kr - \omega t) \quad K = \frac{2\pi}{\lambda}$$

$$dE = \frac{E_0}{a} \sin(kR - \omega t - kx \sin \theta) dx$$

$$E_{\text{tot}} = \int_{-a/2}^{+a/2} \frac{E_0}{a} \sin(kR - \omega t - kx \sin \theta) dx$$

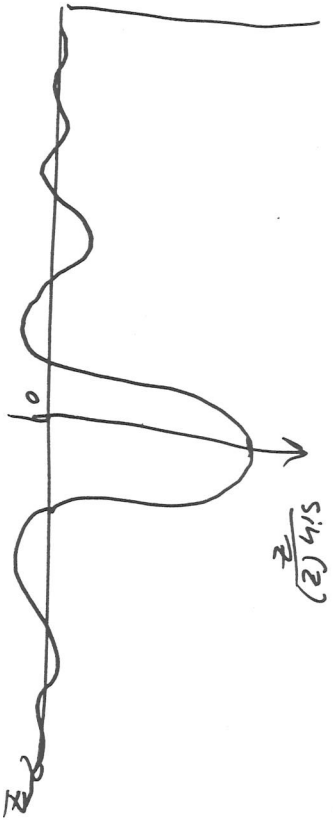
$$= \frac{E_0}{a} \frac{-\cos(kR - \omega t - kx \sin \theta)}{-k \sin \theta} \Bigg|_{-a/2}^{+a/2}$$

$$= \frac{E_0}{a k \sin \theta} \left[\cos(kR - \omega t - \frac{ka \sin \theta}{2}) - \cos(kR - \omega t + \frac{ka \sin \theta}{2}) \right]$$

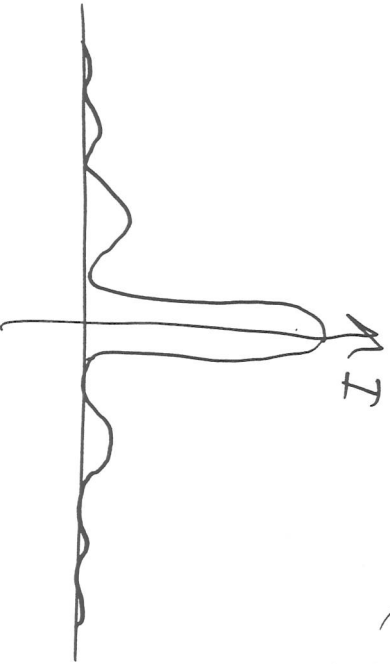
$$= \frac{E_0}{a k \sin \theta} 2 \sin(kR - \omega t) \sin\left(\frac{ka \sin \theta}{2}\right)$$

$$= \left[E_0 \sin(kR - \omega t) \right] \frac{\sin\left(\frac{ka \sin \theta}{2}\right)}{\frac{ka \sin \theta}{2}}$$

$$E_{\text{tot}} = E_0 \sin(kR - \omega t) \frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\pi a \sin\theta}$$

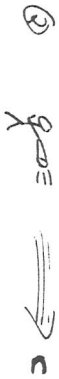


Intensity : $I \propto E^2 \propto \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}$



Michelson and Morley ...

"... cannot find the aether, so the speed of light is measured to be the same in all inertial reference frames.



Observer ② measures the speed of his light beam to be c , and observer ③ measures the speed of ①'s light beam to be c as well!

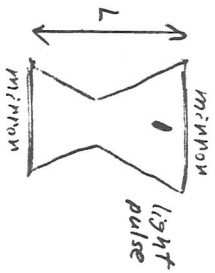
Consequence: ① and ③ cannot have work as and meter sticks that agree.

Space and time are not absolute.

Different observers will get different measurements.

The Light Clock

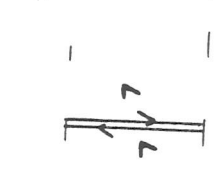
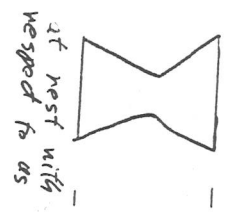
This is a theoretical device used in Gedanken (or thought) experiments.



The period T is the time it takes the light pulse to complete one full cycle (tick-tock).
The path length covered by the pulse in one period T is $2L$.

We will derive results using the light clock, but the results will be valid for any clock (springs, pendula, biological, etc.).

Time Dilation



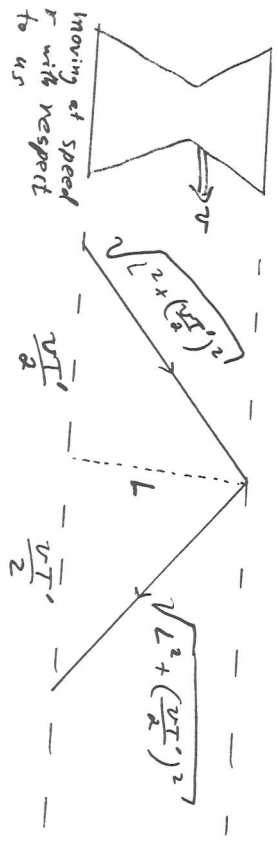
at rest with respect to us

Speed = $c = \frac{\text{Total path length}}{\text{Total time}}$

Period T

$$c = \frac{2L}{T} \Rightarrow L = \frac{cT}{2}$$

We will assume that the length of the clock perpendicular to the direction of motion is unchanged. We will prove this shortly.



Period of this moving clock = T'

speed of the light pulse = $c = \frac{\text{Total path length}}{\text{Total time}}$

$$c = \frac{2\sqrt{L^2 + \left(\frac{vT'}{2}\right)^2}}{T'}$$

A Little Algebra

$$c = \frac{2\sqrt{L^2 + \left(\frac{vT'}{2}\right)^2}}{T'}$$

$$c^2 = \frac{4L^2 + v^2 T'^2}{T'^2}$$

$$L = \frac{cT}{2}$$

← not T'

$$c^2 = \frac{c^2 T^2 + v^2 T'^2}{T'^2}$$

$$c^2 T'^2 = c^2 T^2 + v^2 T'^2$$

$$T'^2 (c^2 - v^2) = c^2 T^2$$

$$T' = \sqrt{\frac{c^2 T^2}{c^2 - v^2}} \quad T = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} T'$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \quad (\text{gamma factor})$$

dimensionless

$$T' = \gamma T$$

v	γ
$0.5c$	1.155
$.75c$	1.511
$.9c$	2.294
$.99c$	7.10
$.999c$	22.36
$.999999c$	707.1

$$T' = \gamma T$$

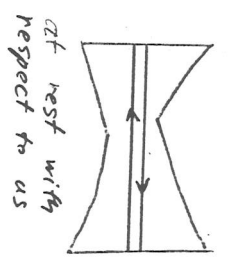
In sequence: Moving clocks run slow compared to clocks at rest with respect to the observer. The faster a clock moves, the slower it runs.

Proper Time \equiv time recorded by a clock at rest. Proper time passes quickest of all.

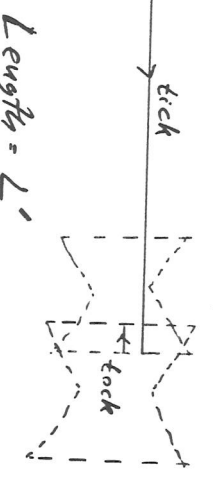
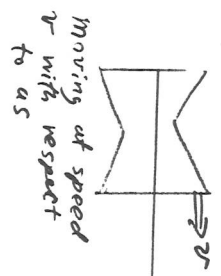
Paradox #1: Suppose that you and I are moving relative to one another. I observe your clock to be running slow, but you observe my clock to be running slow.

(By the way, I see you moving in slow motion and you see me moving in slow motion.)

Length Contraction



Length = L
 period = T
 speed of pulse = $c = \frac{L}{T}$
 $\Rightarrow T = \frac{L}{c}$



Length = L'
 period = $T' = \gamma T = \frac{L'}{c} \gamma$
 the clock is moving, so it is running slow.

speed of light pulse = c

$t_{tick} = \frac{L'}{c-v}$
 $t_{tock} = \frac{L'}{c+v}$
 } observed by us

period: $T' = t_{tick} + t_{tock} = \frac{L'}{c-v} + \frac{L'}{c+v}$

More Algebra

$$T' = \frac{L'}{c-v} + \frac{L'}{c+v}$$

$$T' = \frac{L'}{c^2 v^2} [c(v) + (c-v)] = \frac{2cL'}{c^2 v^2}$$

$$\frac{T}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2cL'}{c^2 v^2} \quad \text{but } T = \frac{2L}{c}$$

$$\frac{2L}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{2cL'}{c^2 v^2}$$

$$L' \left(\frac{c^2}{c^2 v^2} \right) = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L' \left(\frac{1}{1-\frac{v^2}{c^2}} \right) = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L' = L \sqrt{1-\frac{v^2}{c^2}} = \frac{L}{\gamma}$$

$$T' = \gamma T$$

$$L' = \frac{L}{\gamma}$$

$$L' = \frac{L}{\gamma}$$

Consequence: Moving objects shrink in the direction of motion by the gamma factor.

Eg. A meter stick moving at 0.999999c along its length is $\frac{1\text{m}}{707.1} = 1.414 \text{ mm}$ long as observed by us. Of course, if an observer were moving at 0.999999c with the meter stick, that observer would record a length of 1 meter.

$L =$ Proper Length \equiv the length recorded by an observer not moving with respect to the object.

Proper length is the longest of all.

Paradox #2: Suppose that you and I are moving relative to one another. I observe you to be shrunk in the direction you're moving. You observe me to be shrunk in the same direction.

Proof ...

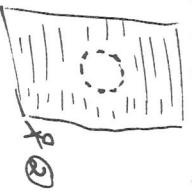
... that dimensions perpendicular to the direction of motion are unaffected.

Wata's readings and meter stick measurements may disagree among moving observers, but all observers can agree on the result of a yo-yo experiment (this is independent of space + time coords).

Questions like: Which occurred first?, or which is longer? cannot be answered consistently by all observers.

Suppose that you are traveling in a cylindrical space ship with a radius just small enough to fit through a hole in the Great Barrier.

(Measurements at ship and hole radii at rest.)



If perpendicular dimensions are shrunk (like the parallel dimension) then:

- ② will observe a shrinking cylinder that will easily pass through the hole.
- ① will observe a shrinking hole that is too tiny to fit through.

The question is: Does the ship crash into the barrier?

- ② Predicts no crash
- ① Predicts a crash

But both observers ① and ② must agree.

The only way to resolve this problem is to say that perpendicular dimensions do not change.

Proof of Special Relativity

An elementary particle called the "muon" has an average lifetime of 2 μ s before it decays. (measured at rest - proper lifetime)

If special Relativity were incorrect, then the average distance that a muon could travel would be

$$(3 \times 10^8 \text{ m/s})(2 \times 10^{-6} \text{ s}) = 600 \text{ m}$$

But muons are created in the upper atmosphere by cosmic ray collisions, and muons reach the Earth's surface

100,000 m below!

How does Special Relativity explain this?

Explanation #1

From the point of view of an Earth-bound observer:

The speed of the muons is close to (but less than) c .

The muon lifetime is time dilated.

Suppose the gamma factor is 100.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 100$$

Then the moving muon lifetime is 2 ms = 100 (2 μ s) on average.

Can these reach the Earth's surface?

$$d_{\text{avg}} = (3 \times 10^8 \text{ m/s})(2 \text{ ms}) = 600,000 \text{ m}$$

Yes!

Explanation #2

From the point of view of the moon:

The Earth is approaching at close to (but less than) c .

The moon lifetime is 2 yrs.

But the distance to the Earth's surface is length contracted.

If $\gamma = 1000$, then the distance from the top of the atmosphere to the surface

$$\text{is } \frac{100,000 \text{ m}}{\gamma} = 100 \text{ m}$$

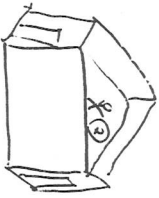
Can the muons reach the Earth's surface

$$d_{\text{avg}} = (3 \times 10^8 \frac{\text{m}}{\text{s}})(2 \text{ yrs}) = 600 \text{ m} > 100 \text{ m}$$

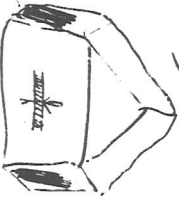
Yes!

The Loss of Simultaneity

Observer ① is carrying a ladder of proper length 20 ft at a high speed toward a barn of proper width 10 ft. Observer ② is at rest with respect to the barn.

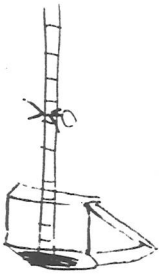


② will observe the ladder to be Lorentz contracted so that it will easily fit completely within the barn. While the shrunken ladder is inside, ② closes both doors at the same time, then reopens them to let ① run through the barn.

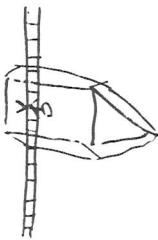


① will observe the barn to be Lorentz contracted so that it will not contain the entire ladder. How does ① observe the closing doors?

Answer: ① observes the far barn door close first.



Then ① observes the far barn door open again.



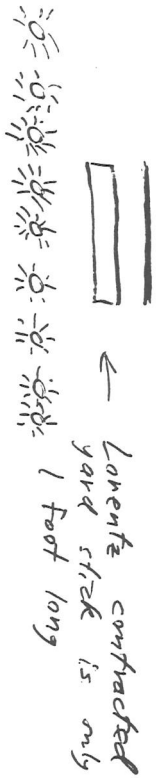
Then ① observes the first barn door close.



Then ① observes the first barn door open again.

In sequence: Events that are simultaneous in one frame will not be simultaneous in a frame moving relative to the first.

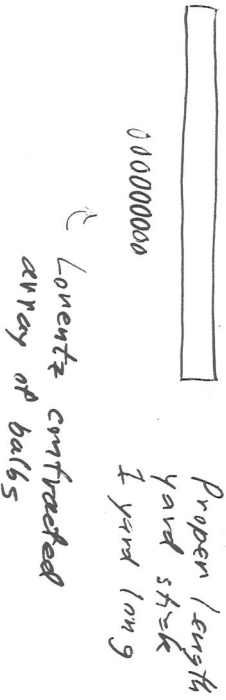
If we are in the rest frame of the bulbs, what do we observe?



All the bulbs go off at the same time.

Result: a 1 foot shadow on film.

If we are in the rest frame of the yard stick, what do we observe?



If the bulbs go off at the same time in this frame, the shadow image will be 3 yards long. But this is a contradiction — there is only the one photo.

Answer: As observed from the rest frame of the yard stick, the right most bulb goes off first, then the next, until finally the leftmost bulb goes off last.

Result: The shadow image on film is $\frac{1}{3}$ yard long. This is the same result obtained in the rest frame of the bulbs.

Relativistic Velocity Addition

Galilean

$$v_{ac} = v_{ab} + v_{bc}$$

Einsteinian

$$v_{ac} = \frac{v_{ab} + v_{bc}}{1 + \frac{(v_{ab})(v_{bc})}{c^2}}$$

$$v_{ac} = v_{ab} - v_{cb}$$

$$v_{ac} = \frac{v_{ab} - v_{cb}}{1 - \frac{(v_{ab})(v_{cb})}{c^2}}$$

① If two velocities less than c are added, then the resultant velocity is also less than c .

② If either velocity is equal to c , then the resultant velocity is also equal to c .

E.g. A spaceship flying away from Earth at $0.8c$ fires a missile forward at $0.8c$ with respect to the spaceship. What is the speed of the missile with respect to Earth?

X Galilean: $v_{m,E} = v_{m,S} + v_{S,E}$
 $= 0.8c + 0.8c = 1.6c$

✓ Einsteinian: $v_{m,E} = \frac{v_{m,S} + v_{S,E}}{1 + \frac{(v_{m,S} \times v_{S,E})}{c^2}}$

$$= \frac{0.8c + 0.8c}{1 + \frac{(0.8c)(0.8c)}{c^2}} = \frac{1.6c}{1 + (0.8)^2}$$

$$= \frac{1.6c}{1 + 0.64} = \frac{1.6}{1.64} c = \boxed{0.975c}$$

less than c !

E.g. A spaceship flying away from Earth at $0.8c$ turns on its headlights. The pilot sees the light speed away at c . What is the speed of the light with respect to the Earth?

Galilean: $v_{p,E} = v_{p,S} + v_{S,E}$
 $= c + 0.8c = 1.8c$

Einsteinian: $v_{p,E} = \frac{v_{p,S} + v_{S,E}}{1 + \frac{(v_{p,S} \times v_{S,E})}{c^2}}$

$$= \frac{c + 0.8c}{1 + \frac{(c)(0.8c)}{c^2}} = \frac{1.8c}{1 + 0.8} = \frac{1.8}{1.8} c = \boxed{c}$$

Light always travels at the speed of light, regardless of the speed of the source or of the observer.