The Laws

of Electricity + Magnetism <u>so far</u>

Gauss' Law: $\displaystyle\oiint_S \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ ← by S

closed surface

Ampere's law: $\displaystyle\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc}$ ← by C

closed curve

Faraday's law:

$$-\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{A} = \oint_C \vec{E}\cdot d\vec{\ell}$$

↑ open surface bounded by C

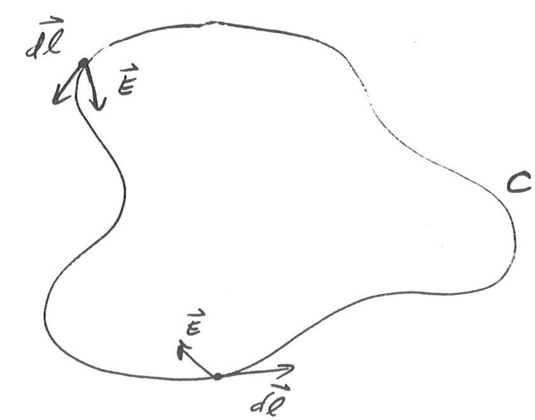
↑ closed curve

Meaning of Faraday's Law

$$\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\Phi_B$$

$$= -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{A}$$ ← Isn't this = 0?

S ← open

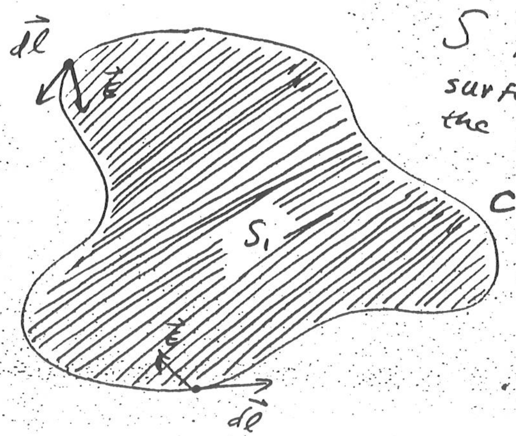


A changing magnetic ~~field~~ flux gives rise to an electric field on the curve C.

Meaning of Faraday's Law

$$\oint_C \vec{E}\cdot\vec{d\ell} = -\frac{d}{dt}\Phi_B$$

$$= -\frac{d}{dt}\iint_S \vec{B}\cdot\vec{dA}$$



S is *any* open surface bounded by the closed curve C

A changing magnetic field gives rise to an electric field on the curve C.

Meaning of Faraday's Law

$$\oint_C \vec{E}\cdot\vec{d\ell} = -\frac{d}{dt}\Phi_B$$

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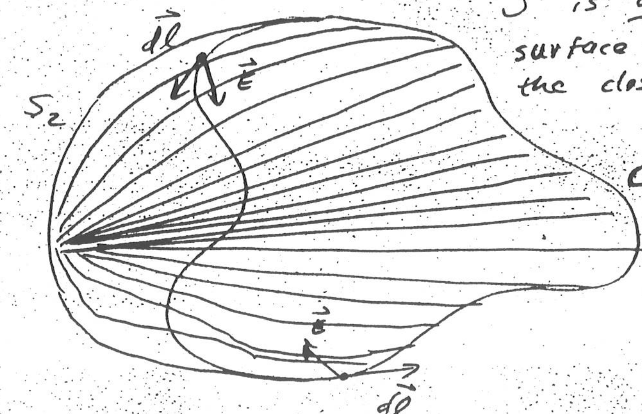


S is *any* open surface bounded by the closed curve C

A changing magnetic field gives rise to an electric field on the curve C.

Faraday's Law

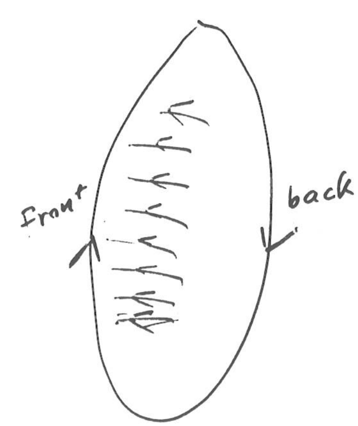
$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{A}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\mathcal{E} = -\frac{d}{dt}\Phi_B$$

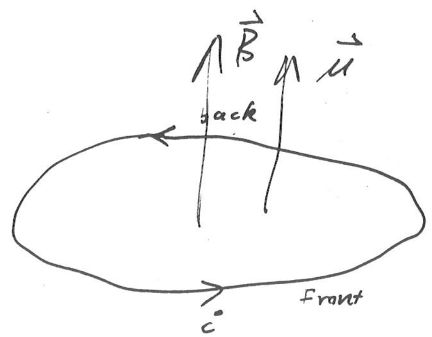
Lenz's Law

An induced current in a closed conducting loop will create a magnetic field that opposes the change in the external magnetic field

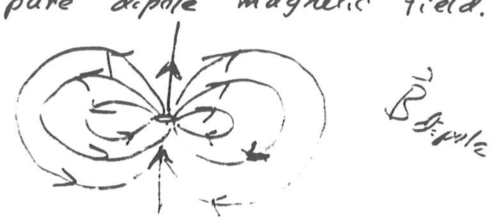


Magnetic Dipole Moment

For a flat current loop, the magnetic dipole moment is a vector $\vec{\mu}$ with magnitude $|\vec{\mu}| = i \cdot Area$ and direction given by the right-hand rule

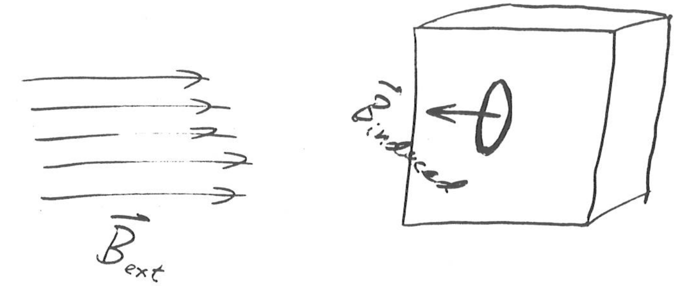


If $Area \to 0$ and $i \to \infty$ with $i \cdot Area = const.$ we get a pure dipole magnetic field.



Diamagnetism

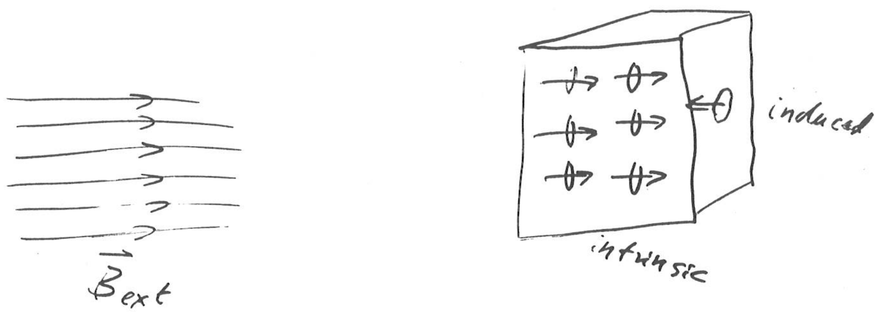
This is a consequence of Lenz's Law.



Magnetic dipoles are induced in the sample. This effect occurs in all substances to some extent. The result is a repulsion from the pole of a magnet.

Contrast this with the electrostatic case, where $\vec{E}_{ext}$ Fields induce electric dipole moments in the sample and attract uncharged bits of paper and neutral metal objects.

Paramagnetism



If the molecules of the sample <u>already</u> have an intrinsic magnetic dipole moment (not induced by the external $\vec{B}$ field), then those magnetic dipoles align with the external field and <u>attraction</u> results. This attraction is usually stronger than the diamagnetism repulsion.

Heating the sample will randomize the dipoles and destroy the paramagnetism.

Ferromagnetism

This effect is like paramagnetism, but 10,000 → 100,000 times stronger.

In certain substances, the intrinsic magnetic dipole moments of the molecules are enormous <u>and</u> they tend to align with each other in "domains"

Again, heating will destroy the alignment and the ferromagnetism. The temperature at which all of the ferromagnetism disappears is called the "Curie temperature."
~1000 K

Ferromagnetic Elements

Fe Co Ni

APPENDIX C

Periodic Table of the Elements*

Appendix C

Group I	Group II	Transition elements										Group III	Group IV	Group V	Group VI	Group VII	Group 0
H 1 1.0080 $1s^1$																H 1 1.0080 $1s^1$	He 2 4.0025 $1s^2$
Li 3 6.94 $2s^1$	Be 4 9.012 $2s^2$											B 5 10.81 $2p^1$	C 6 12.011 $2p^2$	N 7 14.007 $2p^3$	O 8 15.999 $2p^4$	F 9 18.998 $2p^5$	Ne 10 20.18 $2p^6$
Na 11 22.99 $3s^1$	Mg 12 24.31 $3s^2$											Al 13 26.98 $3p^1$	Si 14 28.09 $3p^2$	P 15 30.97 $3p^3$	S 16 32.06 $3p^4$	Cl 17 35.453 $3p^5$	Ar 18 39.948 $3p^6$
K 19 39.102 $4s^1$	Ca 20 40.08 $4s^2$	Sc 21 44.96 $3d^1 4s^2$	Ti 22 47.90 $3d^2 4s^2$	V 23 50.94 $3d^3 4s^2$	Cr 24 51.996 $3d^5 4s^1$	Mn 25 54.94 $3d^5 4s^2$	Fe 26 55.85 $3d^6 4s^2$	Co 27 58.93 $3d^7 4s^2$	Ni 28 58.71 $3d^8 4s^2$	Cu 29 63.54 $3d^{10} 4s^1$	Zn 30 65.37 $3d^{10} 4s^2$	Ga 31 69.72 $4p^1$	Ge 32 72.59 $4p^2$	As 33 74.92 $4p^3$	Se 34 78.96 $4p^4$	Br 35 79.91 $4p^5$	Kr 36 83.80 $4p^6$
Rb 37 85.47 $5s^1$	Sr 38 87.62 $5s^2$	Y 39 88.906 $4d^1 5s^2$	Zr 40 91.22 $4d^2 5s^2$	Nb 41 92.91 $4d^4 5s^1$	Mo 42 95.94 $4d^5 5s^1$	Tc 43 (99) $4d^5 5s^2$	Ru 44 101.1 $4d^7 5s^1$	Rh 45 102.91 $4d^8 5s^1$	Pd 46 106.4 $4d^{10}$	Ag 47 107.87 $4d^{10} 5s^1$	Cd 48 112.40 $4d^{10} 5s^2$	In 49 114.82 $5p^1$	Sn 50 118.69 $5p^2$	Sb 51 121.75 $5p^3$	Te 52 127.60 $5p^4$	I 53 126.90 $5p^5$	Xe 54 131.30 $5p^6$
Cs 55 132.91 $6s^1$	Ba 56 137.34 $6s^2$	57-71*	Hf 72 178.49 $5d^2 6s^2$	Ta 73 180.95 $5d^3 6s^2$	W 74 183.85 $5d^4 6s^2$	Re 75 186.2 $5d^5 6s^2$	Os 76 190.2 $5d^6 6s^2$	Ir 77 192.2 $5d^7 6s^2$	Pt 78 195.09 $5d^9 6s^1$	Au 79 196.97 $5d^{10} 6s^1$	Hg 80 200.59 $5d^{10} 6s^2$	Tl 81 204.37 $6p^1$	Pb 82 207.2 $6p^2$	Bi 83 208.98 $6p^3$	Po 84 (210) $6p^4$	At 85 (218) $6p^5$	Rn 86 (222) $6p^6$
Fr 87 (223) $7s^1$	Ra 88 (226) $7s^2$	89-103**	Unq 104 (261) $6d^2 7s^2$	Unp 105 (262) $6d^3 7s^2$	Unh 106 (263)	Uns 107 (262)	Uno 108 (265)	Une 109 (256)									

Symbol — Ca 20 — Atomic number; Atomic mass † — 40.08; $4s^2$ — Electron configuration

*Lanthanide series

La 57 138.91 $5d^1 6s^2$	Ce 58 140.12 $5d^1 4f^1 6s^2$	Pr 59 140.91 $4f^3 6s^2$	Nd 60 144.24 $4f^4 6s^2$	Pm 61 (147) $4f^5 6s^2$	Sm 62 150.4 $4f^6 6s^2$	Eu 63 152.0 $4f^7 6s^2$	Gd 64 157.25 $5d^1 4f^7 6s^2$	Tb 65 158.92 $5d^1 4f^8 6s^2$	Dy 66 162.50 $4f^{10} 6s^2$	Ho 67 164.93 $4f^{11} 6s^2$	Er 68 167.26 $4f^{12} 6s^2$	Tm 69 168.93 $4f^{13} 6s^2$	Yb 70 173.04 $4f^{14} 6s^2$	Lu 71 174.97 $5d^1 4f^{14} 6s^2$

**Actinide series

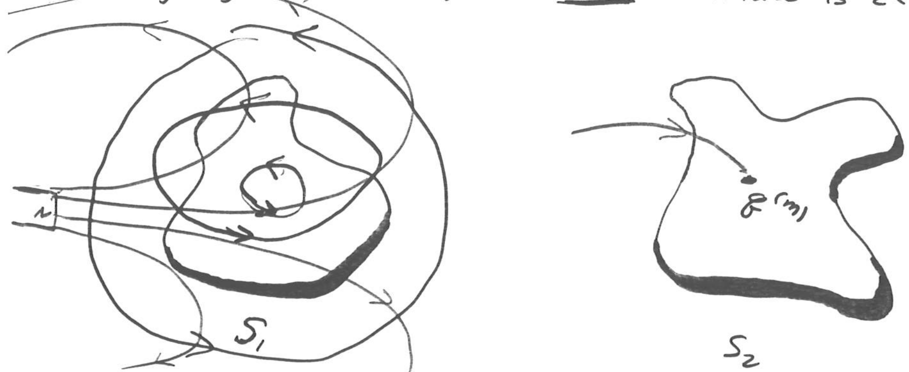
Ac 89 (227) $6d^1 7s^2$	Th 90 (232) $6d^2 7s^2$	Pa 91 (231) $5f^2 6d^1 7s^2$	U 92 (238) $5f^3 6d^1 7s^2$	Np 93 (239) $5f^4 6d^1 7s^2$	Pu 94 (239) $5f^6 6d^0 7s^2$	Am 95 (243) $5f^7 6d^0 7s^2$	Cm 96 (245) $5f^7 6d^1 7s^2$	Bk 97 (247) $5f^8 6d^1 7s^2$	Cf 98 (249) $5f^{10} 6d^0 7s^2$	Es 99 (254) $5f^{11} 6d^0 7s^2$	Fm 100 (253) $5f^{12} 6d^0 7s^2$	Md 101 (255) $5f^{13} 6d^0 7s^2$	No 102 (255) $6d^0 7s^2$	Lr 103 (257) $6d^1 7s^2$

‡ Atomic mass values given are averaged over isotopes in the percentages in which they exist in nature.
† For an unstable element, mass number of the most stable known isotope is given in parentheses.

Gauss' Law for Magnetism

$$\oiint_S \vec{B}\cdot d\vec{A} = 0$$

The net number of magnetic field lines going out through a closed surface is zero.



James Clerk Maxwell, *1831–187*
Scottish physicist. He was professor a
King's College, London, and later

Maxwell's Equations

(so far)

$$\left.\begin{aligned} \oint\!\!\oint_S \vec{E}\cdot d\vec{A} &= \frac{q_e^{\text{enclosed by } S}}{\epsilon_0} \\ \oint\!\!\oint_S \vec{B}\cdot d\vec{A} &= 0 \end{aligned}\right\} \text{Gauss' Laws}$$

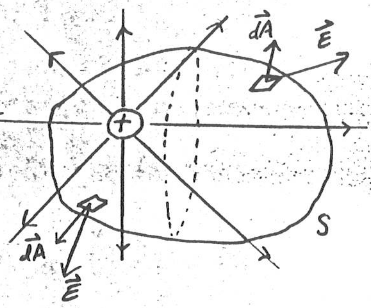
$$\oint_C \vec{E}\cdot d\vec{l} = \frac{-d}{dt}\Phi_B = \frac{-d}{dt}\iint_S \vec{B}\cdot d\vec{A} \qquad \text{Faraday's Law of Induction}$$

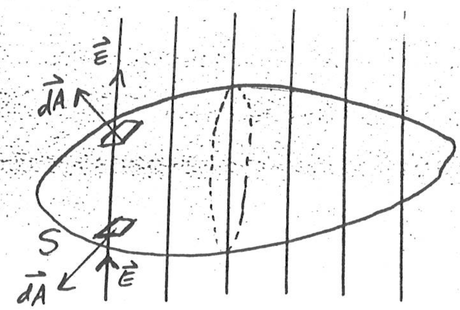
$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0\, i_{enc}^{\text{enclosed by } C} \qquad \text{Ampere's Law}$$

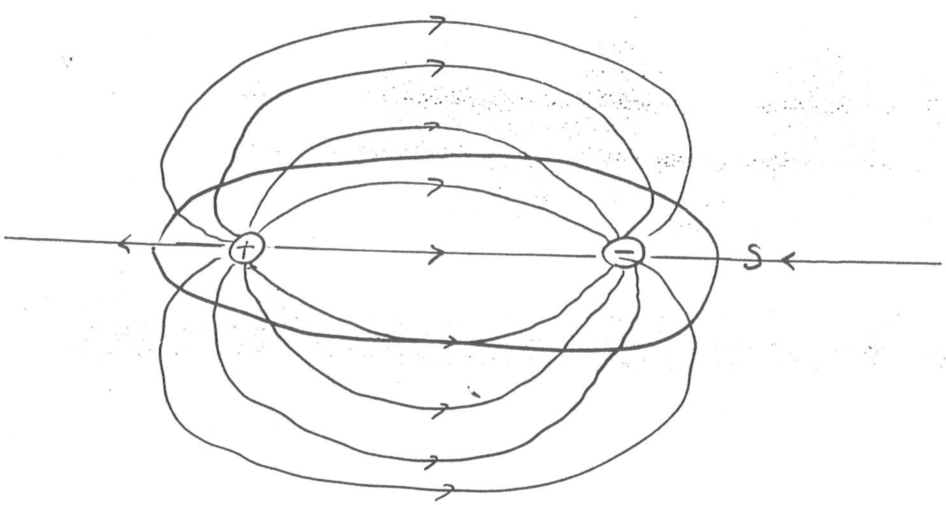
Why are these called Maxwell's equations?

Meaning of Gauss' Laws

$$\oiint_S \vec{E}\cdot d\vec{A} = \frac{q_e}{\epsilon_0}$$

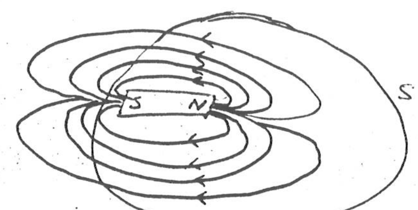






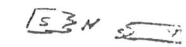
An Apparent Asymmetry

$$\oiint_S \vec{B}\cdot d\vec{A} = 0$$



There is no magnetic charge —

no "magnetic monopoles."

You can't isolate a north pole. 

$$\oiint_S \vec{B}\cdot d\vec{A} = \mu_0 \, q_m^{\text{enclosed by } S}$$

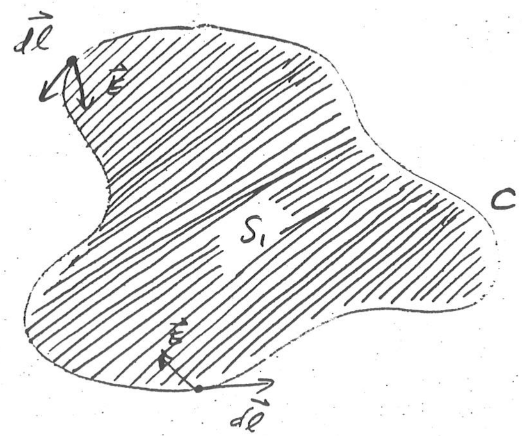
$q_m = 0$ is Nature's choice

Meaning of Faraday's Law

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B$$

$$= -\frac{d}{dt}\iint_{S} \vec{B}\cdot d\vec{A} \quad \leftarrow \text{Isn't this} = 0?$$

$\leftarrow$ open



A changing magnetic ~~field~~ gives rise to an electric field on the curve C.

Meaning of Faraday's Law

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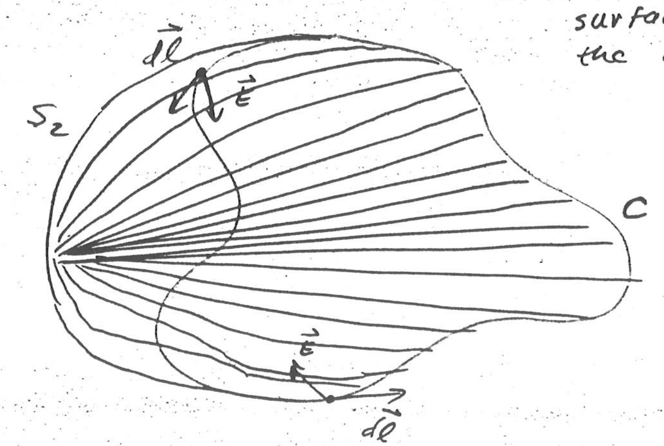
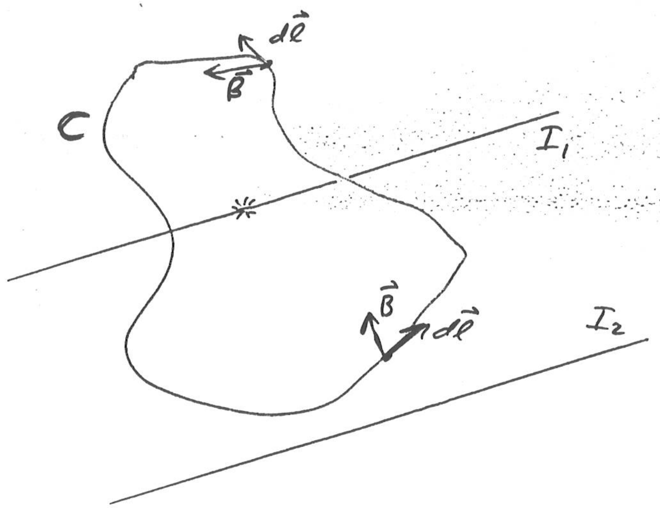
S is <u>any</u> open surface bounded by the closed curve C



A changing magnetic ~~field~~ flux gives rise to an electric field on the curve C.

Meaning of Ampere's Law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_e \quad \text{enclosed by } C$$



Another Apparent Asymmetry

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_e$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B + \frac{I_m}{\epsilon_0}$$

Since there are no magnetic charges, they cannot be put in motion to create "magnetic currents"

$$I_m = \frac{dq_m}{dt} = 0$$

again, this is Nature's choice

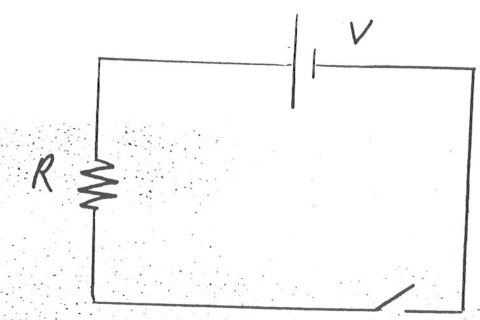
A real asymmetry

$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B + \cancelto{0}{\frac{I_m}{\epsilon_0}}$$

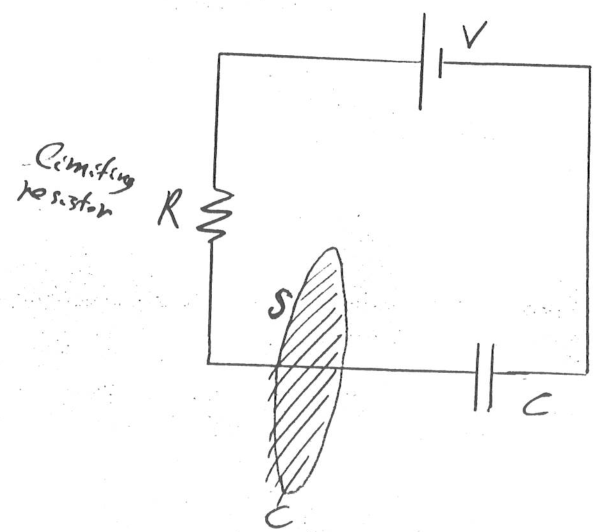
$$\oint_c \vec{B}\cdot d\vec{l} = ??? + \mu_0 I_e$$

If a changing $\vec{B}$ field creates an $\vec{E}$ field (Faraday), then shouldn't a changing $\vec{E}$ field create a $\vec{B}$ field?

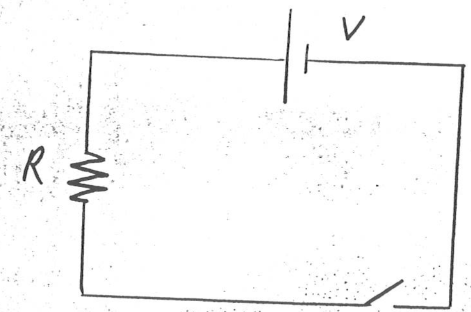
Enter James Clerk Maxwell ...



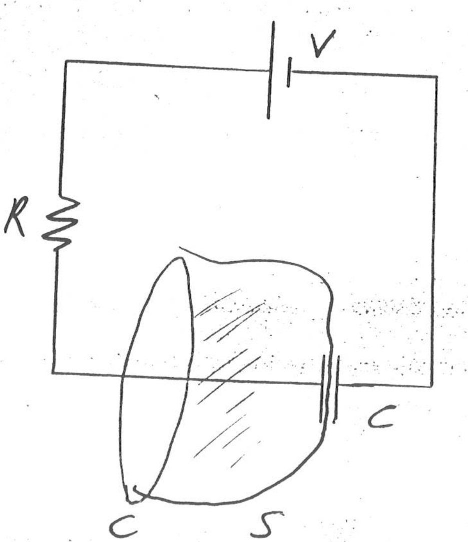
open circuit



$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I$$



open circuit



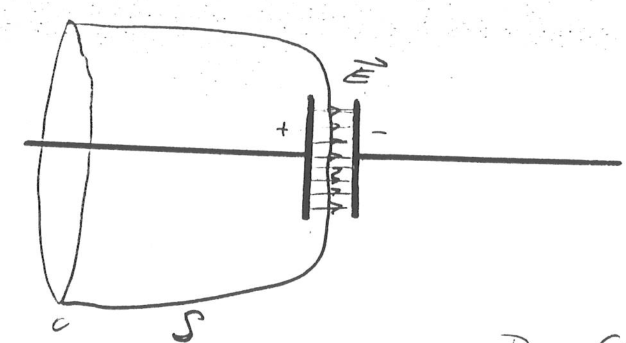
limiting resistor

$$\oint_c \vec{B}\cdot d\vec{l} = 0$$

What if we include the term demanded by symmetry?

electric flux

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_e + \mu_0\left(\epsilon_0 \frac{d}{dt}\Phi_e\right)$$



$$\Phi_e = \iint_S \vec{E}\cdot d\vec{A}$$

For a capacitor,

$$|\vec{E}| = \frac{Q}{\epsilon_0 A} \qquad \Phi_e = |\vec{E}|A = \frac{Q}{\epsilon_0}$$

$$\mu_0\left(\epsilon_0 \frac{d}{dt}\Phi_e\right) = \mu_0 \frac{dQ}{dt} = \mu_0 I$$

$$\left(\epsilon_0 \frac{d}{dt} \Phi_e\right)$$

is called the "displacement current," but it is ~~not~~ a real current.

Ampere - Maxwell Law

$$\oint_c \vec{B} \cdot d\vec{\ell} = \mu_0 \left[I_e + \epsilon_0 \frac{d}{dt} \Phi_e \right]$$

A changing electric field creates a magnetic field.

Maxwell's Equations

(complete set)

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\oiint_S \vec{B} \cdot d\vec{A} = 0 \quad \cancel{\mu_0 q_m} \to 0$$

$$\oint_c \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \Phi_B + \cancel{\frac{I_m}{\epsilon_0}} \to 0$$

$$\oint_c \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d}{dt} \Phi_e + \mu_0 I$$

A changing $\vec{B}$ field creates a changing $\vec{E}$ field,
which creates a changing $\vec{B}$ field,
which creates a changing $\vec{E}$ field,
which ...

This is an electro-magnetic wave.

Speed?

Permittivity of free space:

$$\epsilon_0 = 8.85 \times 10^{-12}\ \frac{F}{m} \qquad \frac{C}{Vm}$$

Permeability of free space:

$$\mu_0 = 1.26 \times 10^{-6}\ H/m \qquad \frac{V \cdot s^2}{C\,m}$$

$$\frac{1}{\epsilon_0 \mu_0} = 9 \times 10^{16}\ \frac{m^2}{s^2}$$

$$\sqrt{\frac{1}{\epsilon_0 \mu_0}} = 3 \times 10^{8}\ \frac{m}{s} = c$$

the speed of light!

Plane Wave solutions to Maxwell's Equations:

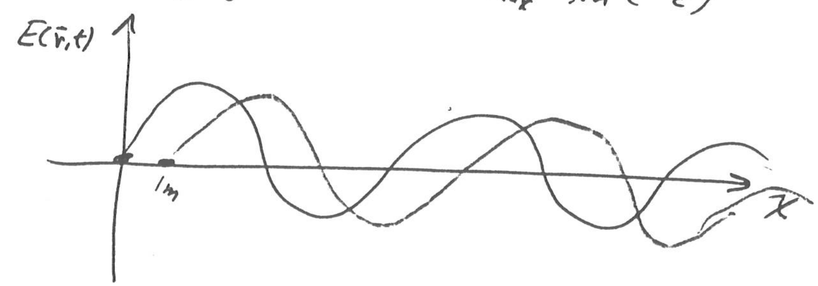
Suppose the wave is travelling along the x-axis from $-\infty$ to $+\infty$.

The function

$$E(\vec{r},t) = E_{max} \sin\left[\omega\left(\frac{x}{c} - t\right)\right]$$

satisfies Maxwell's Eq's in free space and describes a wave moving along the x-axis with speed c.

$t=0$ $\qquad \bar{E} = E_{max} \sin\left(\frac{\omega x}{c}\right)$



$$t = \frac{1m}{c} = \frac{1m}{3\times10^{8}\ m/s} = \frac{1}{3}\times 10^{-8}\ s$$

$\left(\frac{x}{c} - t\right) = 0$ when $x = 1\,m$

Notice that there is no y or z dependence in $E(\vec{r},t)$.

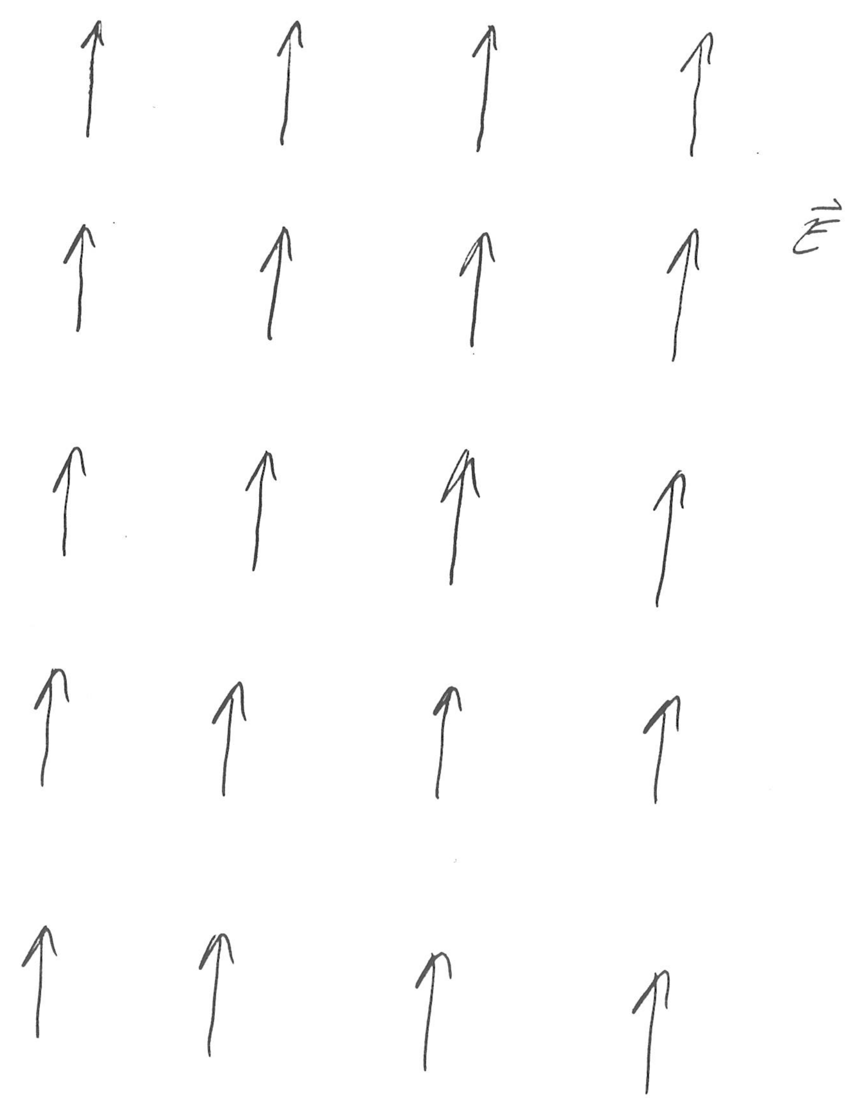
- What does this look like?

- What about the magnetic field?

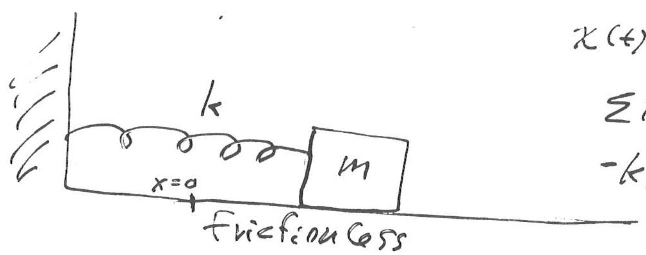
- $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ are <u>vector</u> fields.

  They are perpendicular to each other <u>and</u> to the direction of propagation.

<u>For example</u>: $\vec{E}$ along y
$\vec{B}$ along z
motion along x



A trip down memory lane



$x(t)$, $v(t)$

$\Sigma F = ma$

$-kx = m\frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$ Differential equation

X $x(t) = 3t^2+5$

$\frac{dx}{dt} = v(t) = 6t$

$\frac{d^2x}{dt^2} = a(t) = 6$

$6 + \frac{k}{m}(3t^2+5) = 0$

must be true at all times

$x(t) = A\sin(\omega t + \varphi)$

arbitrary

$v(t) = \frac{dx}{dt} = A\omega\cos(\omega t+\varphi)$

$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t+\varphi)$

$-A\omega^2\sin(\omega t+\varphi) + \frac{k}{m}A\sin(\omega t+\varphi) = 0$

$\omega = \sqrt{\frac{k}{m}}$

ω is angular frequency

units of $\frac{rad}{s}$ → MKS

f is linear frequency

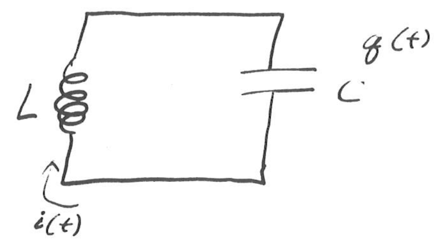
unit of $Hz = \frac{1}{sec} = \frac{cycles}{sec} = \frac{rev}{sec}$

$$f = \frac{\omega}{2\pi}$$

T is period of oscillation

$$f = \frac{1}{T}$$

Deja vu all over again



$q = CV$

$V = \frac{q}{C}$

$i(t) = \frac{dq}{dt}$

$\frac{di}{dt} = \frac{d^2 q}{dt^2}$

Kirchhoff's Loop Rule

$$V_c + V_L = 0$$

$$\frac{q(t)}{C} + L\frac{di}{dt} = 0$$

$$\frac{d^2 q}{dt^2} + \frac{1}{LC} q = 0$$

solution: $q(t) = A\cos(\omega t + \varphi)$

$$i(t) = \frac{dq}{dt} = -A\omega \sin(\omega t + \varphi)$$

$$\frac{di}{dt} = \frac{d^2 q}{dt^2} = -A\omega^2 \cos(\omega t + \varphi)$$

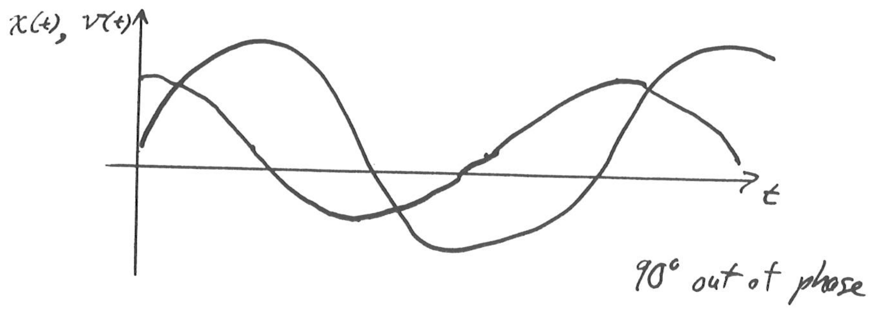
$$= -\omega^2 q(t)$$

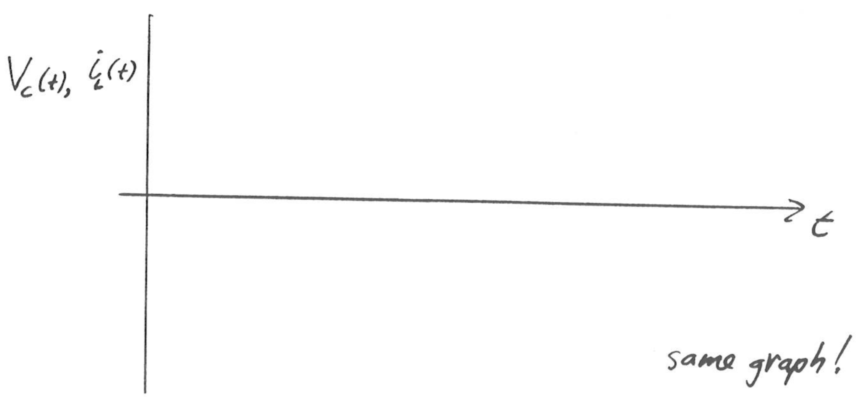
$$\frac{d^2 q}{dt^2} + \frac{1}{LC} q = 0$$

$$-\omega^2 q + \frac{1}{LC} q = 0$$

$$\Rightarrow \omega = \sqrt{\frac{1}{LC}}$$ natural angular frequency

$[\omega] = \frac{1}{T}$ units $\frac{rad}{s}$





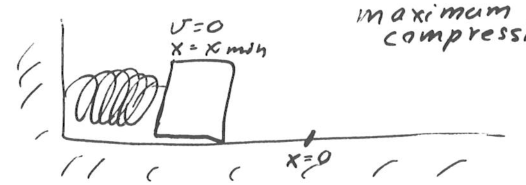
Resistance

If you add a resistor to an LC circuit then some energy from the electric and magnetic fields is converted into heat by the resistor. The oscillations die out exponentially.

Mechanical - Electrical Correspondences

x	q
$v = \frac{dx}{dt}$	$i = \frac{di}{dt}$
m	L
k	$1/C$
large k $\leftrightarrow$ stiff spring	$\leftrightarrow$ small capacitor

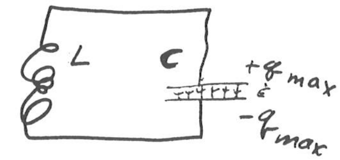
maximum compression



$U = \frac{1}{2}k\, x_{min}^2 = \frac{1}{2}kA^2$

$K = 0$

$t=0$



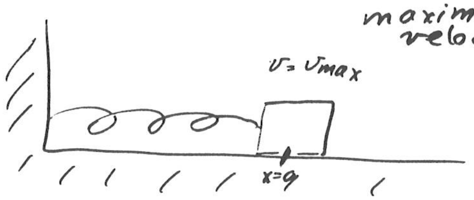
$U_E = \frac{1}{2}\frac{q_{max}^2}{C}$

$U_B = 0$

Capacitor fully charged (positive on the top plate)

$t=0$

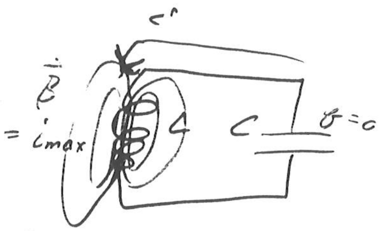
maximum velocity



$U = 0$

$K = \frac{1}{2}m\, v_{max}^2$

$t = \frac{1}{4}$ cycle



$U_E = 0$

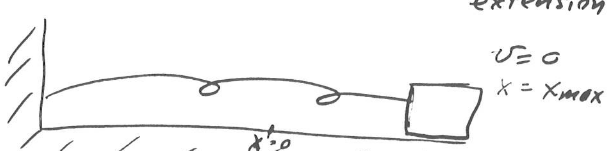
$U_B = \frac{1}{2}L\, i_{max}^2$

Capacitor discharged

$t = \frac{1}{4}$ cycle

$t = \frac{1}{2}$ cycles

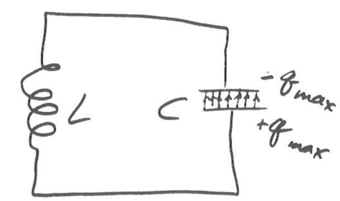
maximum extension



$U = \frac{1}{2}k\, x_{max}^2$

$K = 0$

$t = \frac{1}{2}$ cycle



$U_E = \frac{1}{2}\frac{q_{max}^2}{c}$

$U_B = 0$

Capacitor fully charged (but positive on the bottom plate)

AC Circuits

(Alternating ~~Current~~ Voltage)

Consider a source of potential difference (seat of emf, voltage supply) whose voltage varies sinusoidally with time

$$V(t) = \underbrace{V_0}_{\text{maximum voltage}} \sin(\omega t - \varphi)$$

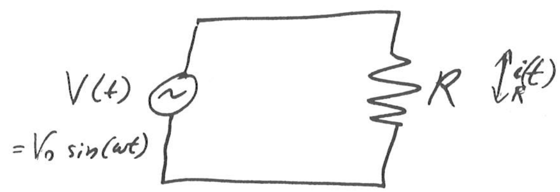
Put a resistor in a circuit with this Alternating Voltage. Demo

In the U.S. $V_0 = 163$ volts

$f = 60\ Hz = 60\ \frac{\text{cycles}}{\text{sec}}$

$\omega = 2\pi f = 377\ \frac{\text{rad}}{\text{sec}}$

A purely resistive circuit



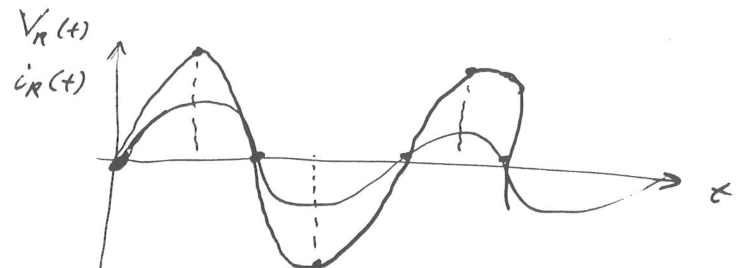
Voltage across the resistor:

$$V_R = V_0 \sin(\omega t)$$

Current through the resistor:

$$i_R = \frac{V_R}{R} = \frac{V_0 \sin(\omega t)}{R}$$

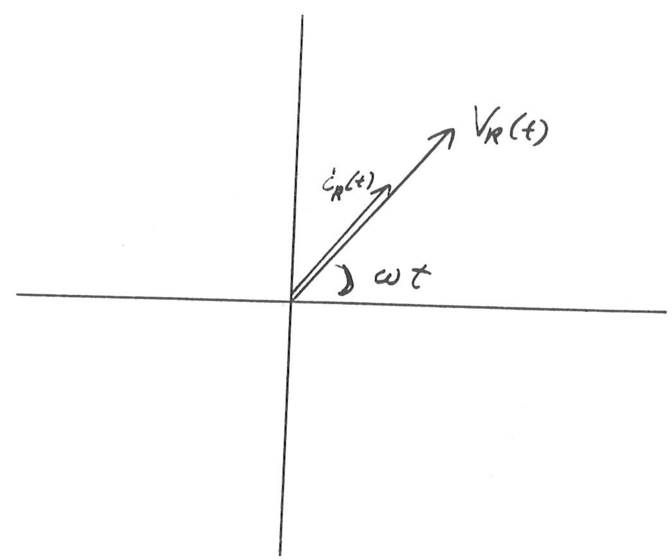
$V_{(t)} = i_{(t)} R$



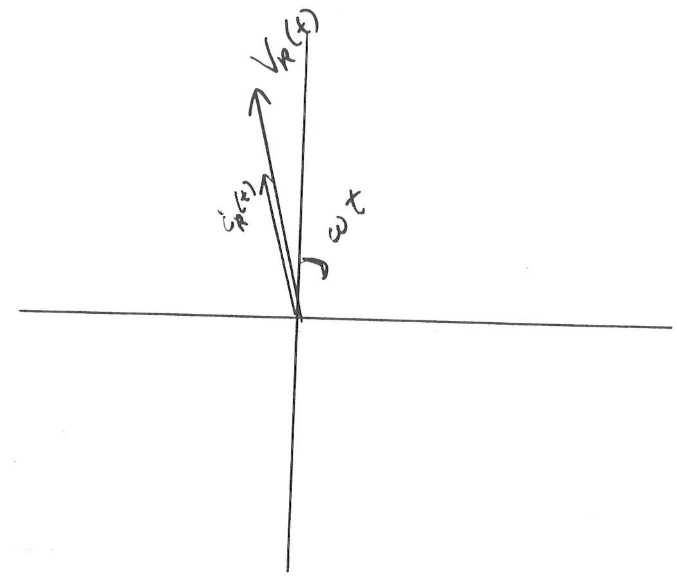
In phase

$i(t)$ and $V_R(t)$ in phase.

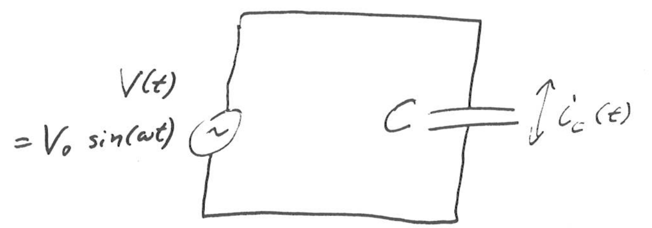
Phasors



Phasors



A purely capacitive circuit



Voltage across the capacitor:

$$V_c(t) = V_0 \sin(\omega t)$$

$q(t) = C\,V(t)$

Charge on capacitor plate:

$$q_c(t) = C\,V_c(t) = C\,V_0 \sin(\omega t)$$

Current through the capacitor

$$i_c(t) = \frac{dq}{dt} = \omega C V_0 \cos(\omega t)$$

$$= \omega C V_0 \sin(\omega t + 90°)$$

This will look similar to the purely resistive result: $i_R = \frac{V_R}{R} = \frac{V_0}{R}\sin(\omega t)$

If we define the "capacitive reactance"

$$X_c \equiv \frac{1}{\omega C}$$

then $i_c = \omega C V_0 \sin(\omega t + 90°)$

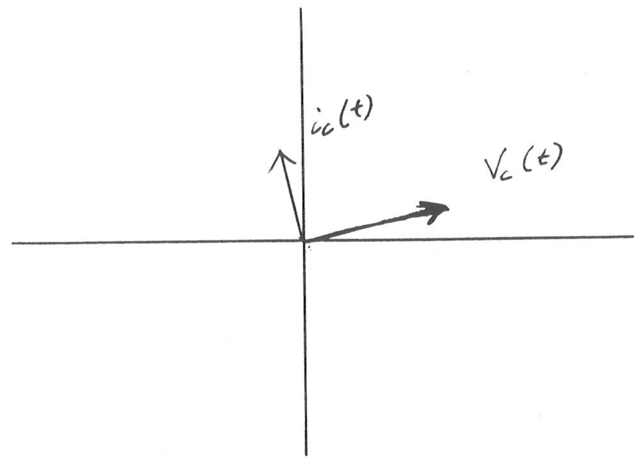
$$= \frac{V_0}{X_c}\sin(\omega t + 90°)$$

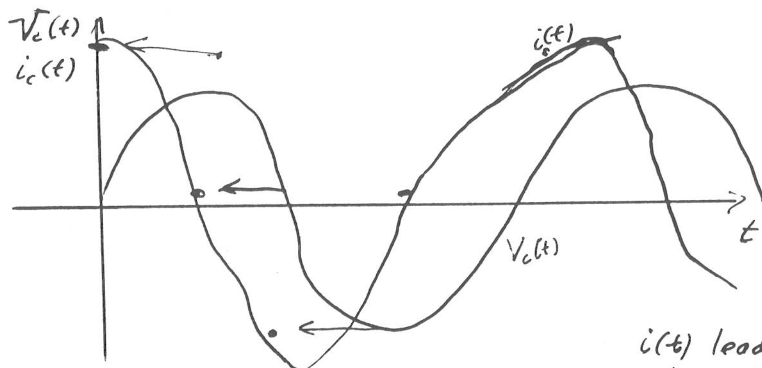
Think of the capacitive reactance as the "resistance" of a capacitor to alternating current flow.

Also, note that the current i_c is 90° <u>ahead</u> of the voltage V_c.

$\boxed{\text{ICE}}$

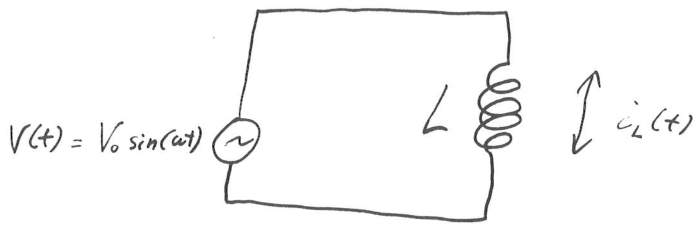
Phasors ready, Captain.





$i(t)$ leads $V_c(t)$ by $90°$

A purely inductive circuit



Voltage across the inductor:

$$V_L(t) = V_0 \sin(\omega t)$$

For an inductor

$$V_L = L \frac{di}{dt} \qquad \frac{di}{dt} = \frac{V_0 \sin(\omega t)}{L}$$

Current through the inductor

$$i_L = \int \left(\frac{di}{dt}\right) dt = \frac{V_0}{L} \int \sin(\omega t)$$

$$= -\frac{V_0}{L\omega} \cos(\omega t)$$

$$= \frac{V_0}{L\omega} \sin(\omega t - 90°)$$

We can make this resemble the purely resistive result $i_R = \frac{V_R}{R} = \frac{V_0}{R} \sin(\omega t)$

if we define the "inductive reactance"

$$X_L \equiv \omega L$$

then $i_L = \frac{V_0}{L\omega} \sin(\omega t - 90°)$

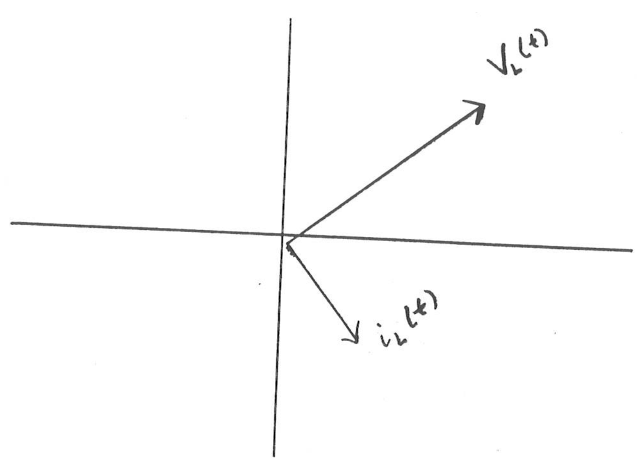
$$= \frac{V_0}{X_L} \sin(\omega t - 90°)$$

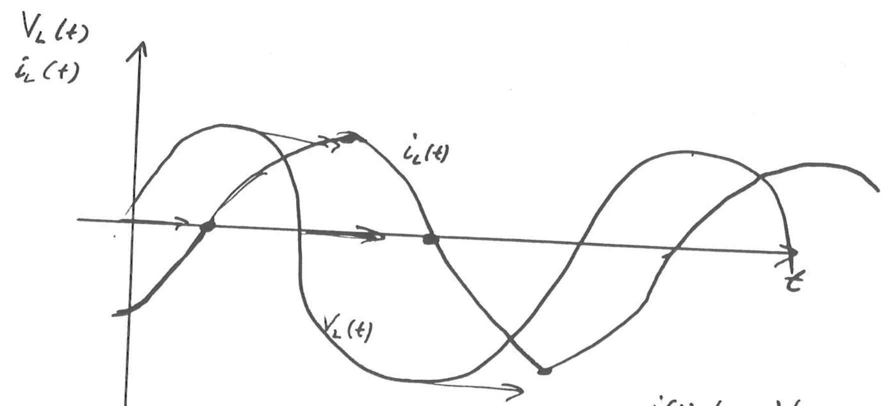
Think of the inductive reactance as the "resistance" of an inductor to alternating current flow.

Note that this time, the current i_L is 90° <u>behind</u> the voltage V_L.

$\boxed{ELI}$

Phasor Diagram





$i(t)$ lags $V_L(t)$ by $90°$

Reactance

$X_C = \frac{1}{\omega C}$ This is small for large angular frequencies

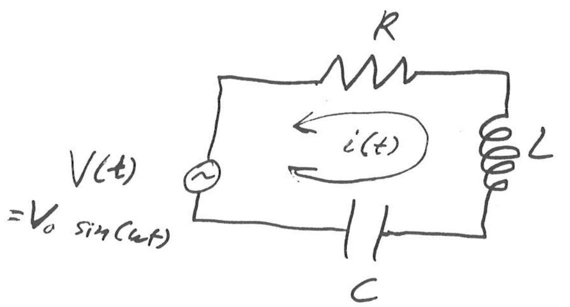
A capacitor offers almost no resistance to high-frequency AC flow, but a capacitor offers infinite resistance to very low-frequency AC (that is, DC) flow.

$\omega = 0$

$X_L = \omega L$ This is large for large angular frequencies

For DC flow, an inductor looks like a resistance-less piece of wire. However, an inductor offers considerable resistance to high-frequency AC flow.

A series RLC circuit



Kirchhoff's Loop Rule is valid at any instant.

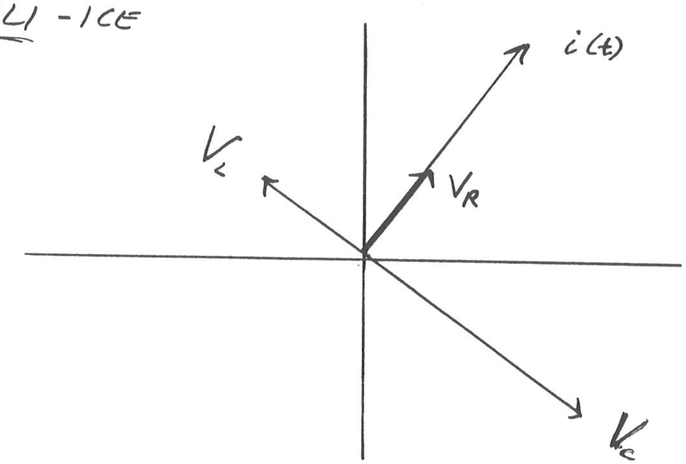
$$V_{supply}(t) - V_R(t) - V_L(t) - V_C(t) = 0$$

At t_1) $100 - 50 - 25 - 25 = 0$

At t_2) $0 - 0 - (-25) - 25 = 0$

Phasor Diagram

ELI - ICE



The three voltages : V_R, V_L, V_C add like vectors

$$V_0^2 = V_R^2 + (V_L - V_C)^2$$

$$= (iR)^2 + (iX_L - iX_C)^2$$

$$V_0 = i\sqrt{R^2 + (X_L - X_C)^2} \equiv iZ$$

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

is called the <u>impedance</u> of the AC circuit. It is frequency-dependent.

$$X_C = \frac{1}{\omega C}$$
$$X_L = \omega L$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

<u>Resonance</u> occurs when Z is a minimum;

when $\omega L - \frac{1}{\omega C} = 0$ or $\omega L = \frac{1}{\omega C}$

or $\omega^2 = \frac{1}{LC}$ or $\omega = \sqrt{\frac{1}{LC}}$

$Y = \frac{1}{Z}$ is called the <u>Admittance</u>

$$Y = \frac{1}{\sqrt{R^2 + (X_C - X_L)^2}}$$

At resonance Z is a minimum
Y is a maximum

AC Power

purely resistive

$$P_{inst} = [i(t)]^2 R = [i_{max} \sin(\omega t)]^2 R$$

$$= (i_{max})^2 R \sin^2(\omega t)$$

$P_{AVG} = ?$

Average value of $\sin^2(\omega t) = ?$

$$\langle \sin^2 \rangle = \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta = \frac{1}{2\pi}\pi = 1/2$$

$$= \frac{1}{4\pi}\int_0^{4\pi} \sin^2\theta \, d\theta = \frac{1}{4\pi} 2\pi = \boxed{1/2}$$

$$P_{AVG} = \frac{1}{2}(i_{max})^2 R = \left(\frac{i_{max}}{\sqrt{2}}\right)^2 R \equiv i_{RMS}^2 R$$

$$i_{RMS} = \frac{i_{max}}{\sqrt{2}}$$

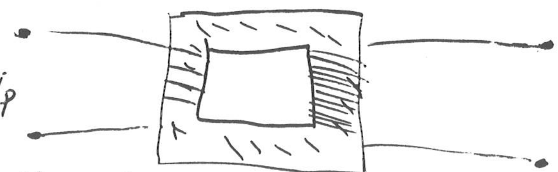
RMS $\rightarrow$ root mean square

$$i_{RMS} = \frac{i_{max}}{\sqrt{2}} \qquad V_{RMS} = \frac{V_{max}}{\sqrt{2}}$$

$V_{RMS} = 120$ volts AC

$V_{max} = \sqrt{2}\, V_{RMS} = \sqrt{2}\, 120$ volts $= 170$ vol

Transformer



V_p 120V

I_p = ~~0.01 Amp~~ 1 Amp

V_p turns primary

V_s turns secondary

$V_s = 10$ KV

I_s = ~~1 Amp~~ 0.01 Amp

Faraday's law

$$\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{A}$$

$$V = -\frac{d}{dt}\Phi_B \leftarrow \text{total magnetic flux}$$

$$V_p = N_p \left(\text{Flux through one loop}\right)$$

flux through the iron

$$V_s = N_s \left(\text{flux through one loop}\right)$$

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$

$\frac{\text{Energy}}{\text{time}}$ Conservation = Power Cons.

$$P_p = I_p V_p = P_s = I_s V_s$$

$$\frac{V_p}{N_p} = \frac{V_s}{N_s} \Rightarrow I_p N_p = I_s N_s$$

Plane Wave solutions to Maxwell's Equations:

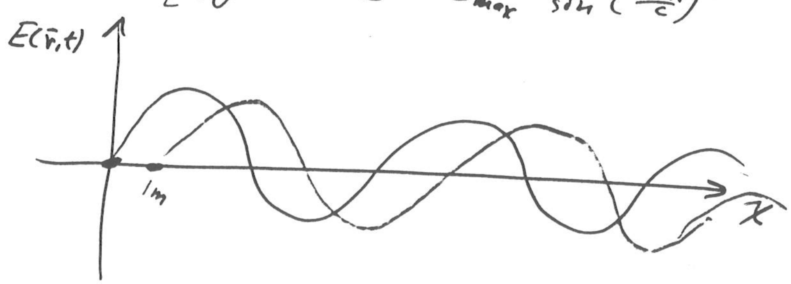
Suppose the wave is travelling along the x-axis from $-\infty$ to $+\infty$.

The function

$$E(\vec{r},t) = E_{max} \sin\left[\omega\left(\frac{x}{c} - t\right)\right]$$

satisfies Maxwell's Eq's in free space and describes a wave moving along the x-axis with speed c.

$t = 0$ $\quad E = E_{max} \sin\left(\frac{\omega x}{c}\right)$



$$t = \frac{1m}{c} = \frac{1m}{3\times10^{8}\,m/s} = \frac{1}{3}\times10^{-8}\,s$$

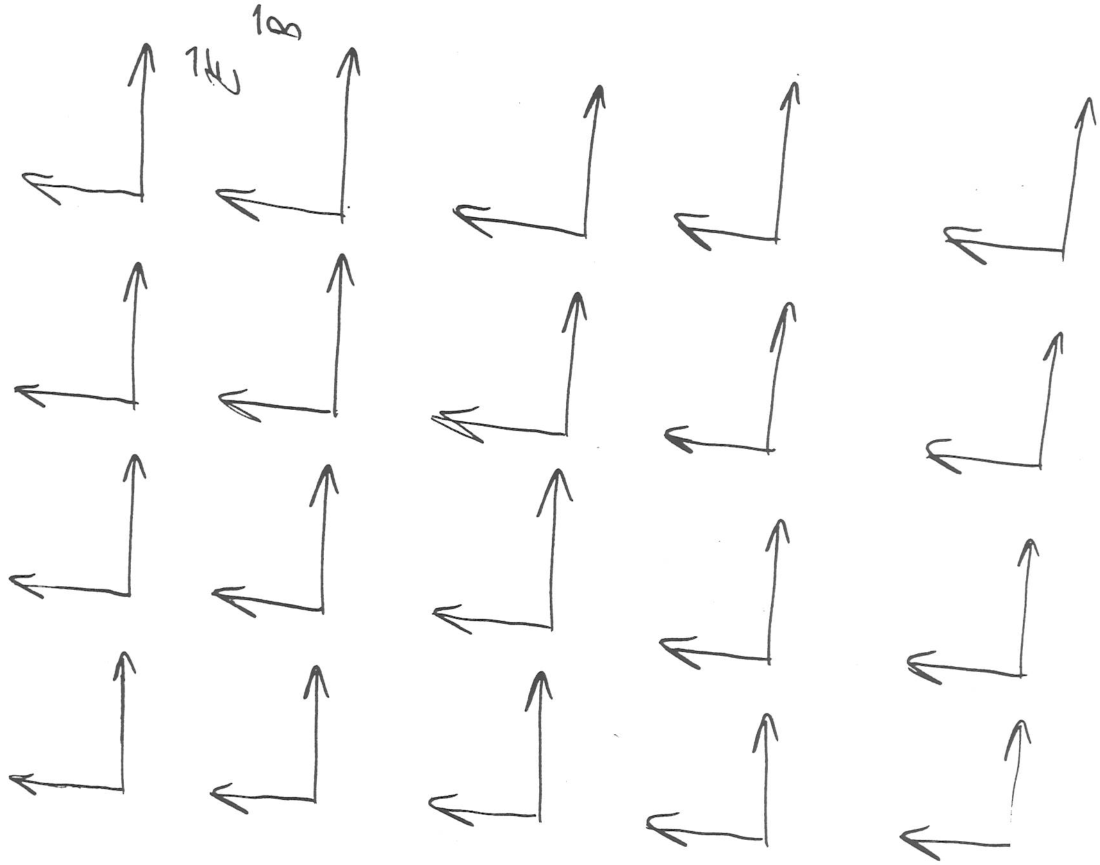
$\left(\frac{x}{c} - t\right) = 0$ when $x = 1\,m$

Notice that there is no y or z dependence in $E(\vec{r},t)$.

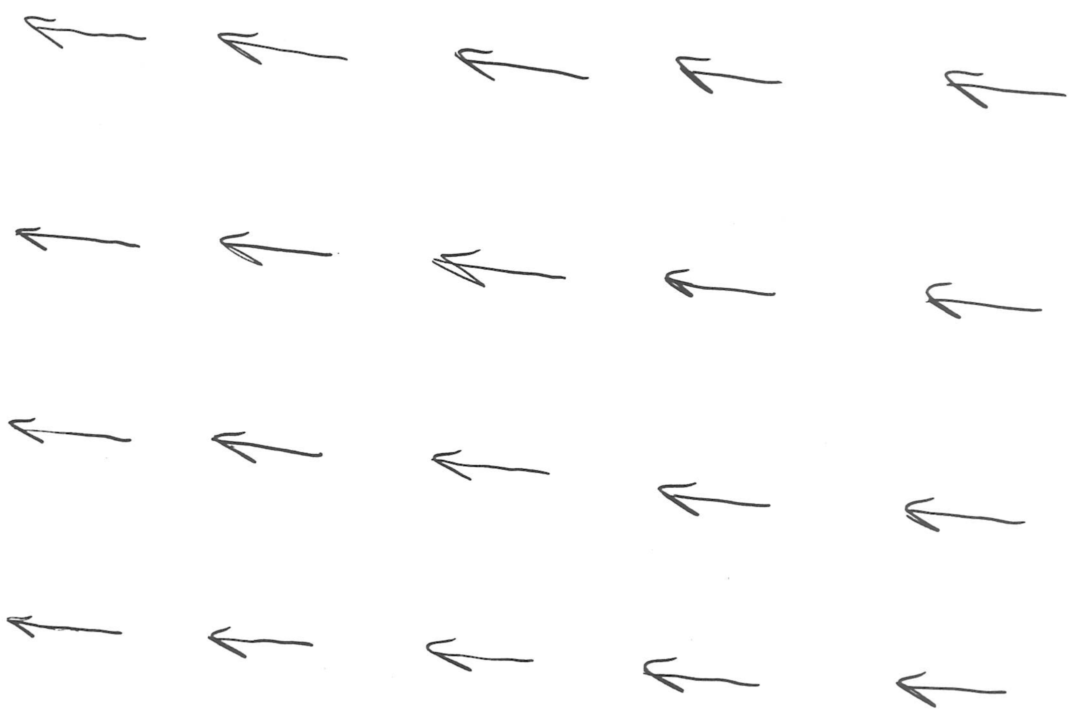
- What does this look like?
- What about the magnetic field?
- $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ are vector fields.

  They are perpendicular to each other and to the direction of propagation.

For example: $\vec{E}$ along y
$\vec{B}$ along z
motion along x



7th



In fact, the electromagnetic wave moves along the vector

$$\vec{E} \times \vec{B}$$

The direction of $\vec{E}$ is called the polarization of the wave.

$$\vec{E}(\vec{r},t) = E_{max} \sin\left[\omega\left(\frac{x}{c} - t\right)\right] \hat{j}$$

$$\vec{B}(\vec{r},t) = B_{max} \sin\left[\omega\left(\frac{x}{c} - t\right)\right] \hat{k}$$

$\frac{\omega}{c}$ is often denoted as k, the wave number.

$$\vec{E}(\vec{r},t) = E_{max} \sin(kx - \omega t) \hat{j}$$

Maxwell's Equations in vacuum put a constraint on E_{max} and B_{max}.

They are not both arbitrary. You can select one, then the other must satisfy

$$\frac{E_{max}}{B_{max}} = c \quad \text{(speed of light)}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

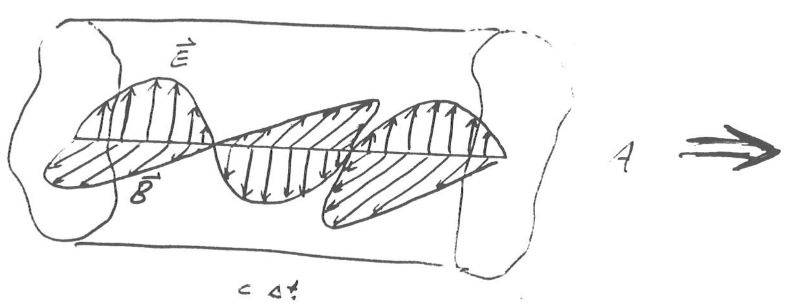
Energy in Electromagnetic Radiation

$$E(\vec{r},t) = E_{max}\sin(kx-\omega t) = E_{max}\sin\left[\omega\left(\tfrac{x}{c}-t\right)\right]$$

$$B(\vec{r},t) = B_{max}\sin(kx-\omega t) = \frac{E_{max}}{c}\sin\left[\omega\left(\tfrac{x}{c}-t\right)\right]$$

$$B_{max} = \frac{E_{max}}{c} \qquad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Consider a box of cross-sectional Area A and length $c\Delta t$. In a time Δt, all the energy in this box will fall on a screen (detector).



electric energy density $\quad u_E(\vec{r},t) = \frac{1}{2}\epsilon_0 E^2(\vec{r},t)$

magnetic energy density $\quad u_B(\vec{r},t) = \frac{1}{2\mu_0}B^2(\vec{r},t) = \frac{1}{2\mu_0}\frac{E^2(\vec{r},t)}{c^2}$

$$= \frac{1}{2}\epsilon_0 E^2(\vec{r},t) = u_E(\vec{r},t)$$

energy in box: $\quad \Delta U = (u_E + u_B)\,A\,c\Delta t$

$$= \epsilon_0 E^2(\vec{r},t)\,A\,c\,\Delta t$$

rate of energy hitting the detector per unit area:

$$\frac{\Delta U}{\Delta t}\frac{1}{A} = \epsilon_0 c\,E^2(\vec{r},t) = \frac{1}{c\mu_0}E^2(\vec{r},t) \equiv S(\vec{r},t)$$

$$= \frac{1}{\mu_0}E(\vec{r},t)B(\vec{r},t)$$

$S(\vec{r},t)$ is the magnitude of the Poynting vector.

$$\vec{S}(\vec{r},t) = \frac{1}{\mu_0}\vec{E}(\vec{r},t)\times\vec{B}(\vec{r},t)$$

direction of travel, direction of energy flow

Intensity

Intensity I is the time-averaged Poynting vector.

$$I = \frac{1}{T}\int_0^T S(\vec{r},t)\,dt = \frac{1}{c\mu_0}\,\frac{1}{T}\int_0^T E^2(\vec{r},t)\,dt$$

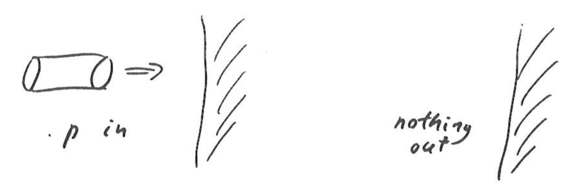
$$= \frac{1}{c\mu_0} E_{max}^2 \underbrace{\left(\frac{1}{T}\int_0^T \sin^2\left[\omega\left(\frac{x}{c}-t\right)\right] dt\right)}_{=\frac{1}{2}}$$

$$= \frac{1}{c\mu_0}\,\frac{E_{max}^2}{2} = \frac{1}{c\mu_0} E_{rms}^2$$

Light carries momentum

If the energy in the box is ΔU, then the momentum in the box is $p = \frac{\Delta U}{c}$.

Case 1: Total absorption



momentum transferred to screen : $\frac{\Delta U}{c} = \Delta p = p_f - p_i$

Case 2: total reflection

momentum transferred to screen : $\frac{2\Delta U}{c} = \Delta p$

Radiation Pressure

Average force : $F = \dfrac{\Delta p}{\Delta t}$

Average pressure : $P = \dfrac{F}{A}$ force per unit area

$$P = \frac{\Delta p}{A\,\Delta t} = \begin{cases} \dfrac{\Delta U}{cA\,\Delta t} & \text{total absorption} \\[2ex] \dfrac{2\Delta U}{cA\,\Delta t} & \text{total reflection} \end{cases}$$

$$P = \begin{cases} \dfrac{I}{c} & \text{total absorption} \\[2ex] \dfrac{2I}{c} & \text{total reflection} \end{cases}$$