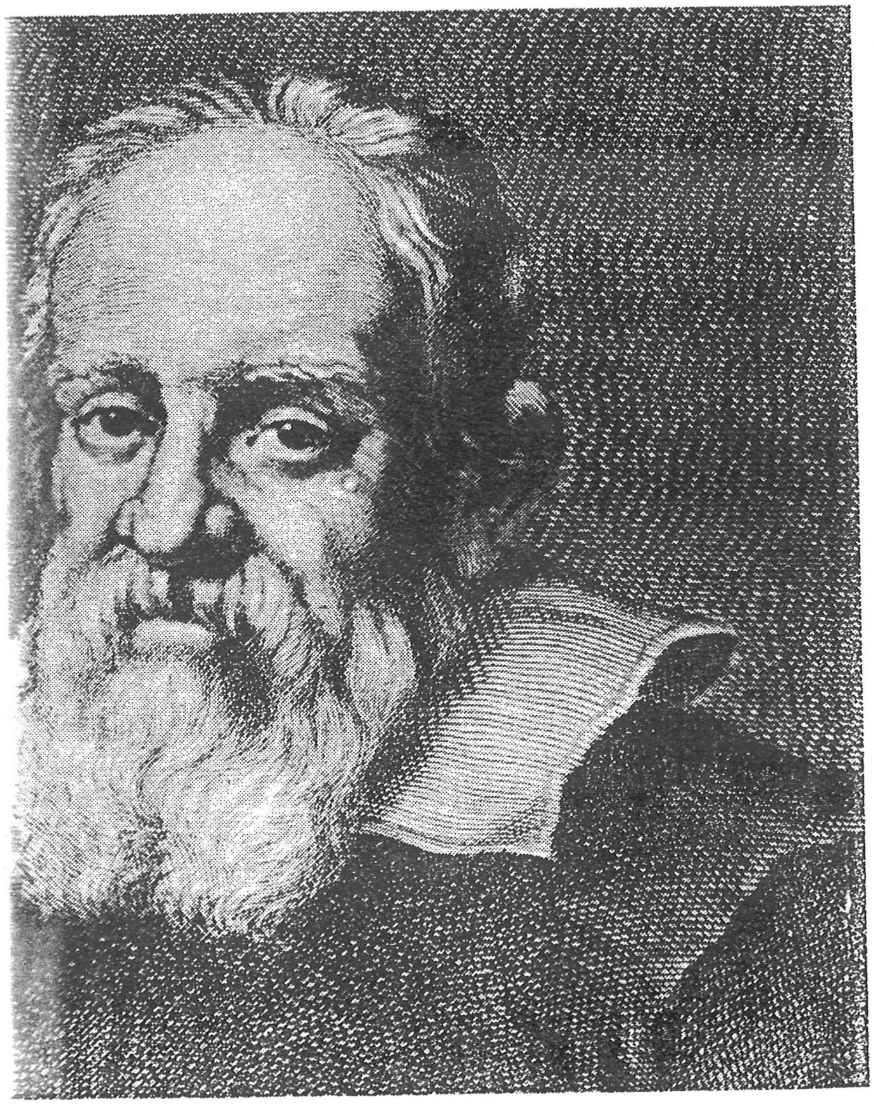


Chapter 2:
1-D Kinematics

The study of motion without regard to the cause.

- average velocity
- instantaneous velocity
- derivatives
- integrals
- average acceleration
- instantaneous acceleration
- free fall in gravity

Aristotle: (384 - 322 B.C.)

- All earthly objects tend to rest - their "natural state."
- All heavenly objects remain in circular motion forever.

Demo: Sliding block

- Heavier objects fall faster than light ones, in proportion to their weight.

twice as heavy ⇒ twice as fast
ten times as heavy ⇒ ten times as fast

Demo: Falling masses - 100g and 1000g

No experimental verification!

If it doesn't match the real world, it ain't physics.

Demo: Gedanken (thought) experiment

Galileo Galilei: (1564 - 1642)

- did not invent the telescope!

but he did change our view of the world.

- "dilated" gravity Demo: - Ramp
- used mathematics
- excellent observations
  - resolution of the sliding block puzzle - friction Demo: Book slip
  - point particles

Last time

Displacement vector: $\vec{r}_f - \vec{r}_i$

Notation

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i \qquad (\Delta ☺ = ☺_f - ☺_i)$$

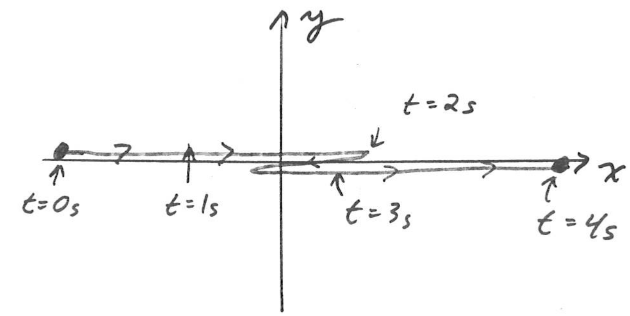
1 Dimension

- Choose an origin.
- Choose a direction for positive.
- Let $\vec{x}$ be the displacement from the origin.

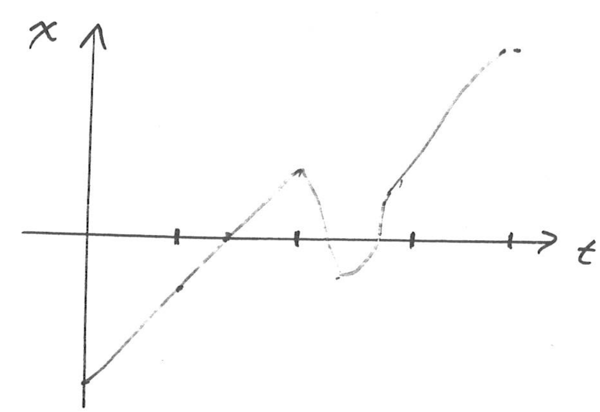
Displacement vector: $\Delta \vec{x} = \vec{x}_f - \vec{x}_i$

Path

Ex. 1-D path



Position vs. Time Graph



Average Velocity

$$\bar{v} = v_{AVG} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The <u>displacement</u> of a particle in a certain time interval divided by that time interval.

Not the total <u>distance</u> traveled ...

<u>Demo</u>: End at starting point.

<u>Demo</u>: Two paths; <u>same</u> start + end.

<u>Ex</u>. The position of a particle at time t is given by

$$x(t) = 2t^2 - 3t + 1$$

where x is in meters and t is in seconds. What is the average velocity between 2 s and 3 s?

$t_f = 3s$, $t_i = 2s$ $\Delta t = 1s$

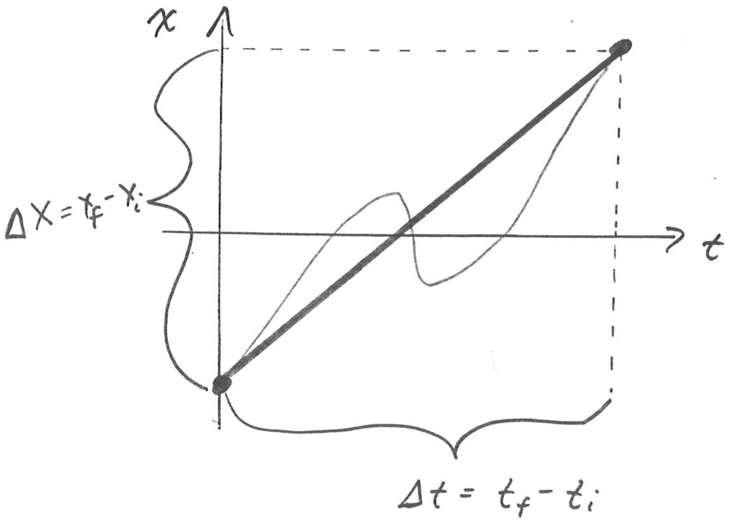
$x_f = x(3s) = 10\ m$

$x_i = x(2s) = 3\ m$

$\Delta x = x_f - x_i = 7\ m$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{7m}{1s} = \boxed{7\ m/s}$$

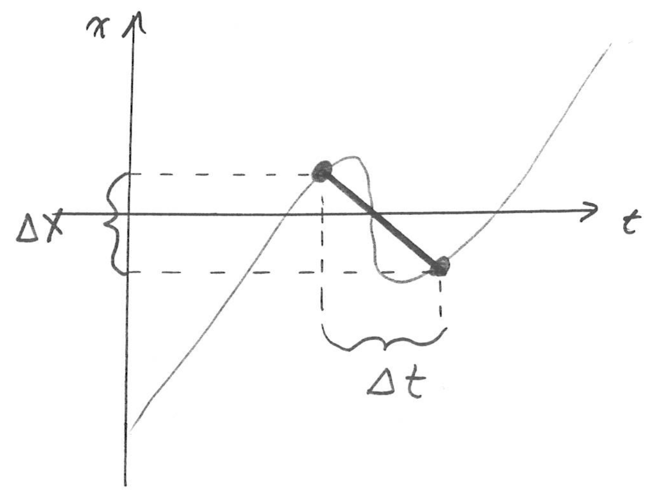
Graphically



$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\text{rise}}{\text{run}} = \text{slope}$$

= slope of the line connecting the start and end points.

For a different time interval, the average velocity will be different.



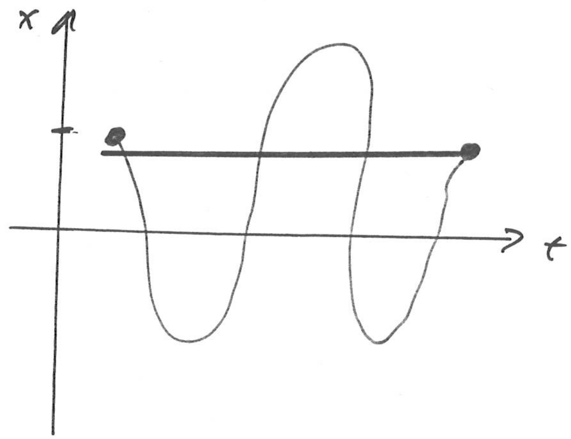
In this case the slope is negative $\Rightarrow$ $\bar{v}$ is negative.

How did this happen?

$\Delta x = x_f - x_i$ is negative.

Δt always positive.

Graph of a particle returning to its starting point.



$\Delta x = 0$

Δt is non-zero

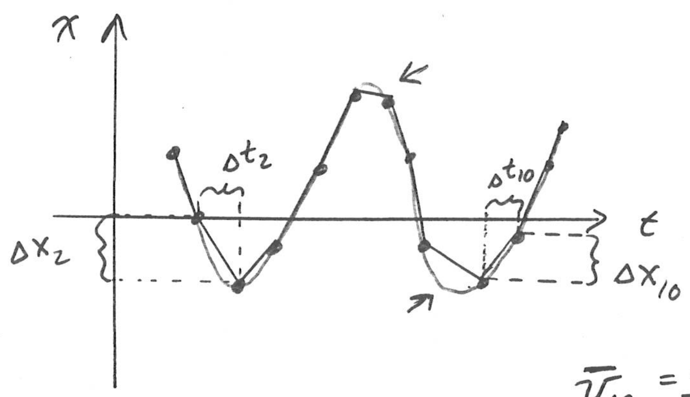
$\bar{v} = \frac{\Delta x}{\Delta t} = 0$ slope $= 0$

The concept of average velocity only makes sense over some non-zero time interval.

If the average velocity changes quite a bit over a time interval (see previous graph) then the average velocity may not accurately represent what is happening.

How can we improve our description?

Make the time interval Δt smaller.



$$\bar{v}_{10} = \frac{\Delta x_{10}}{\Delta t_{10}}$$

$$\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2}$$

How far can we take this process?

We can let $\Delta t \to 0$

Wait a minute! If $\bar{v} = \frac{\Delta x}{\Delta t}$ won't we be dividing by zero?

As $\Delta t \to 0$, the displacement over that very short time interval also shrinks to zero

$$\Delta x \to 0$$

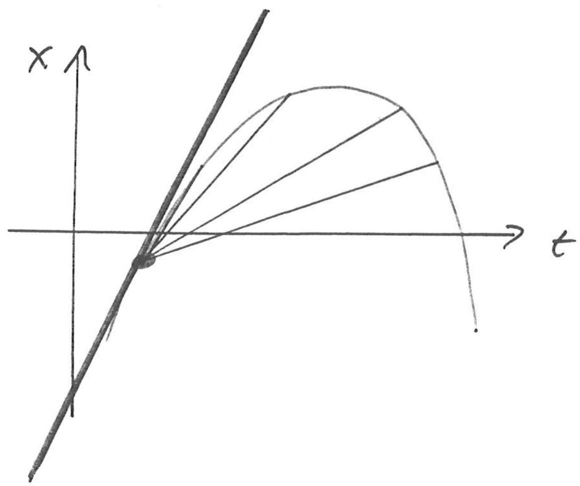
but the ratio $\frac{\Delta x}{\Delta t}$ approaches some number.

Instantaneous Velocity

$$v = v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Definition of calculus derivative.

Graphically



The instantaneous velocity at a point is the slope of the line tangent to the graph at that point.

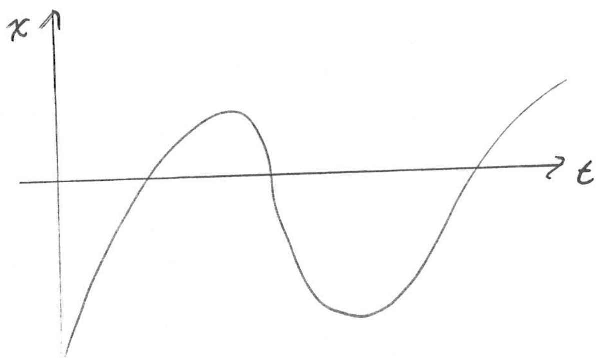
Derivatives

(Calculus in one minute)

If A is a constant, then

$$\boxed{\frac{d}{dt}(A t^n) = n A t^{n-1}}$$

What is the sign of the instantaneous velocity at various points?



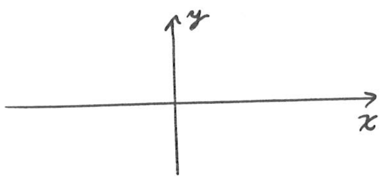
Ex $x(t) = 2t^2 - 3t + 1$

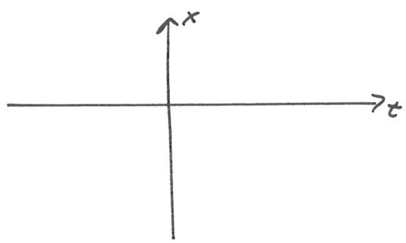
x in meters, t in seconds

What is the instantaneous velocity at 2s? At 3s?

Average Velocity and Average Speed

Ex A motorist travels West at 60 mph for 1 hr, remains at rest for $\frac{1}{2}$ hr, then travels East at 40 mph for $\frac{1}{2}$ hr.



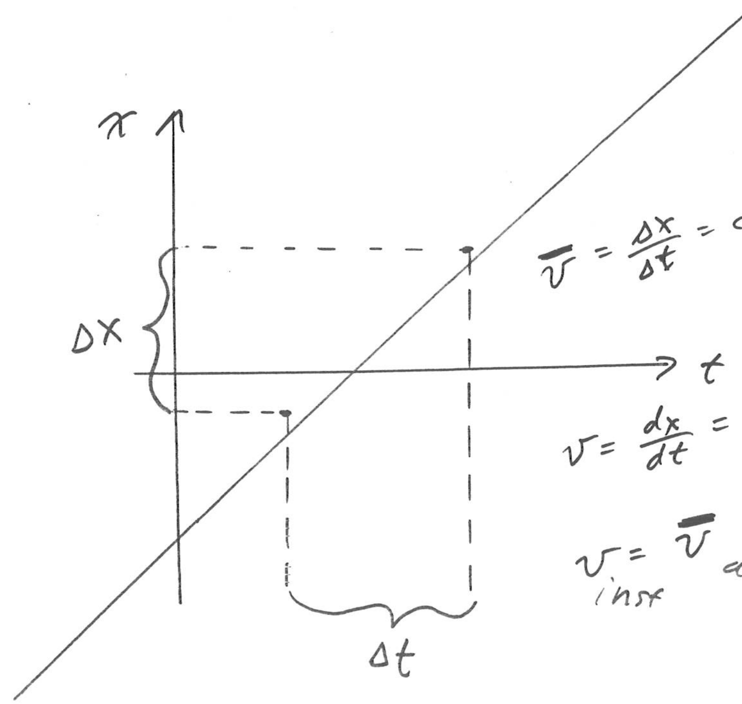


Avg. Velocity =

Avg. Speed =

A special case:

Constant positive Velocity



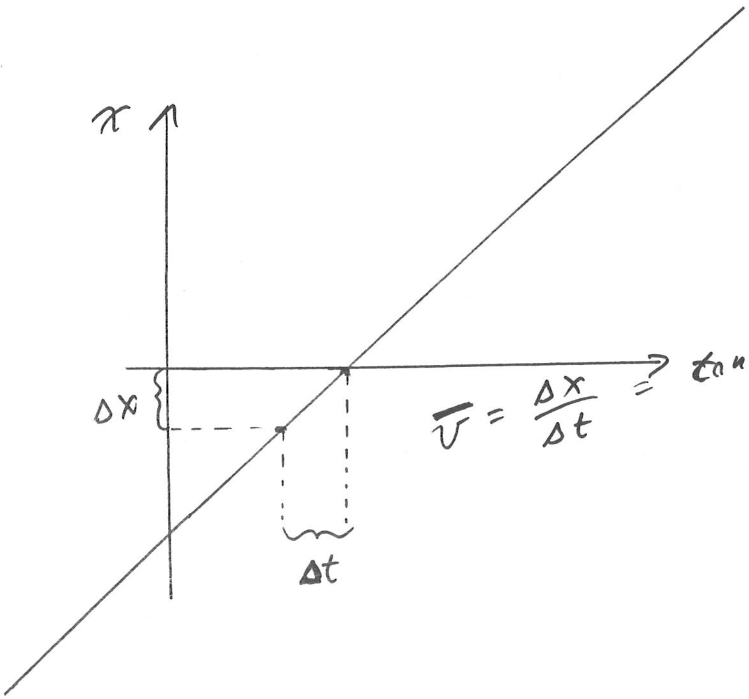
$\bar{v} = \frac{\Delta x}{\Delta t}$ = constant

$v = \frac{dx}{dt}$ = constant

$v_{inst} = \bar{v}_{avg}$

A special case:

Constant positive Velocity



$$\bar{v} = \frac{\Delta x}{\Delta t} = \text{constant}$$

Velocity vs. time graph

(for constant positive velocity)

$$v = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Start at $t_i = 0$

and call t_f just plain old t

$$v = \frac{x_f - x_i}{t}$$

$$\boxed{x_f = x_i + v\,t}$$

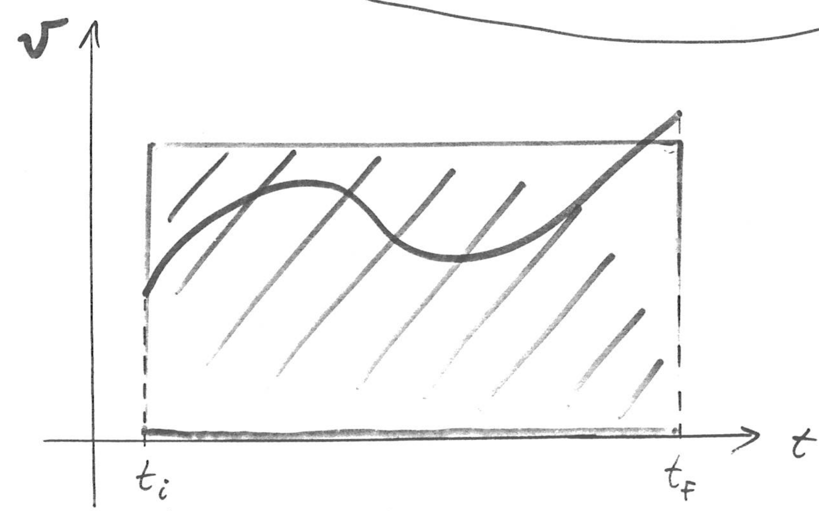
valid for $v =$ constant only !!!

Ex. I drive at 60 mph for 2 hr. in a straight line. How far do I go?

What if the velocity is not constant in time?

How do we "undo" $v = \frac{dx}{dt}$ to get x as a function of time?

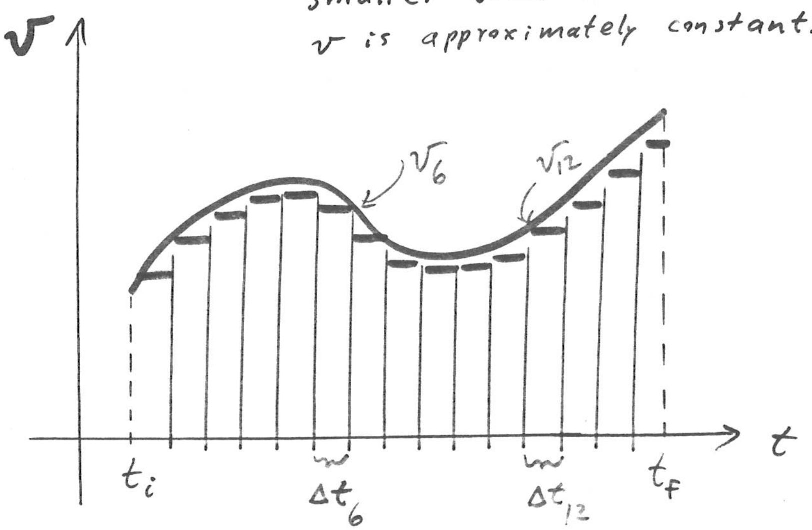
If $v =$ constant
$x_f = x_i + v\,(t_f - t_i)$



What if the velocity is not constant in time?

How do we "undo" $v = \frac{dx}{dt}$ to get x as a function of time?

Next best idea: use smaller time intervals, where v is approximately constant.



$\Delta x_6 \approx v_6 \, \Delta t_6$

$\Delta x_{12} \approx v_{12} \, \Delta t_{12}$

Or in general: $\Delta x_n = v_n \, \Delta t_n$

How does one find the total displacement $x_f - x_i$?

Add up all the skinny rectangles.

$$x_f - x_i \approx v_1 \Delta t_1 + v_2 \Delta t_2 + \dots v_n \Delta t_n$$

$$= \sum_n v_n \Delta t_n$$

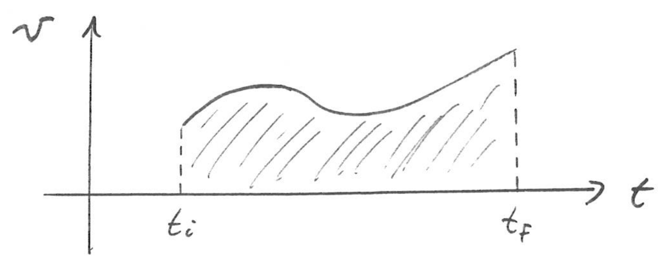
How can we get more accuracy?

Let the time intervals Δt_n shrink to zero width.

$$x_f - x_i = \sum_n v_n \Delta t_n \longrightarrow \int_{t_i}^{t_f} v_{inst}(t)\, dt$$

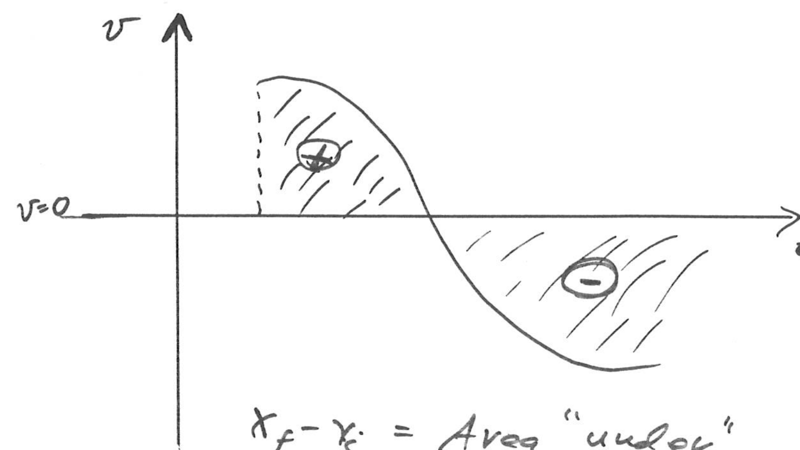
Definition of calculus <u>integral</u>.

Graphically



$\boxed{\text{$x_f - x_i$ is the area under the velocity vs. time graph.}}$

Area Under the Curve



$x_f - x_i$ = Area "under" curve
$= 0$

Integration

(more calculus in another minute)

If B is a constant, then

$$\boxed{\int (Bt^n)\,dt = \frac{Bt^{n+1}}{n+1} + \text{Const.}}$$

not $n=-1$

Definite Integrals

$$\int_{t_i}^{t_f} (Bt^n)\,dt = \left.\frac{Bt^{n+1}}{n+1}\right|_{t_i}^{t_f}$$

$$= \frac{B(t_f)^{n+1}}{n+1} - \frac{B(t_i)^{n+1}}{n+1}$$

constant cancels.

Ex. $\int_{1s}^{3s} 4t^2\,dt = \left.\frac{4}{3}t^3\right|_1^3$

$$= \frac{4}{3}(3)^3 - \frac{4}{3}(1)^3$$

$$= \frac{104}{3}$$

Derivation and Integration are inverse operations.

If C is a constant, then

$$\frac{d}{dt}\left(Ct^{n}\right) = nCt^{n-1}$$

$$\int \left(nCt^{n-1}\right) dt = Ct^{n} + \text{const.}$$

$$\frac{dx(t)}{dt} \equiv \underset{\text{cnst}}{v(t)}$$

$$\underbrace{\int v(t)\, dt}_{\text{Area under } v \text{ vs. } t} = x(t) + \text{const.}$$

Ex. The velocity of a particle varies with time according to

$$v(t) = 4t^{3} + 2$$

where v is in m/s and t is in seconds. What is the particle's displacement from $t_i = 2s$ to $t_f = 4s$?

Last Time

$$\bar{v} = v_{AVG} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$v = v_{INST} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\frac{d}{dt}(At^n) = nAt^{n-1}$$

v_{INST} is the <u>slope of the tangent</u> line on a position vs. time graph.

$$\int (Bt^n)\,dt = \left(\frac{1}{n+1}\right)Bt^{n+1} + \text{constant} \quad n \neq -1$$

x is the <u>area under the curve</u> of a velocity vs. time graph.

If $v(t)$ is a <u>constant</u>, then

$$x(t) = x_i + vt$$

movie

Average Acceleration

$$\bar{a} = a_{AVG} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

the change in a particle's velocity during a certain time interval divided by that time interval.

Instantaneous Acceleration

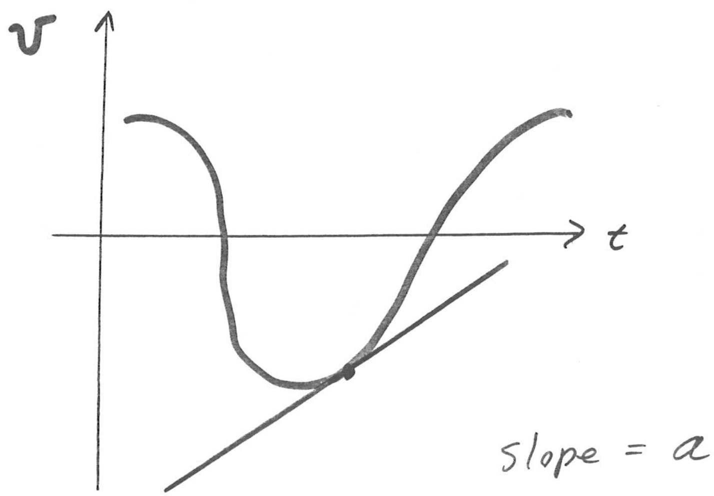
$$a = a_{INST} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

but $v = \frac{dx}{dt}$

so $a = \frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

Instantaneous

Acceleration is the slope of the tangent line on a velocity vs. time graph.



Ex $x(t) = 3t^4 + 5t^2 + 10$

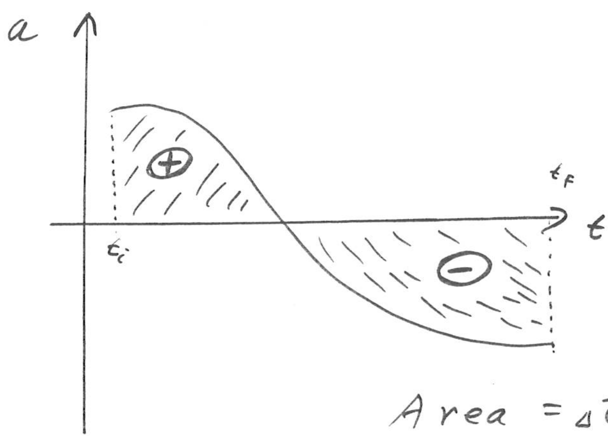
x in meters, t in seconds

What is the instantaneous acceleration at $t = 0\,s$? at $t = 1\,s$?

How do we invert $a = \frac{dv}{dt}$?

$$v(t) = \int a(t)\, dt$$

Velocity is the area under the curve on an acceleration vs. time plot.



Area $= \Delta v = v_f - v_i$

What is the average acceleration between $t = 0s$ and $t = 1s$?

$$x(t) = 3t^4 + 5t^2 + 10$$

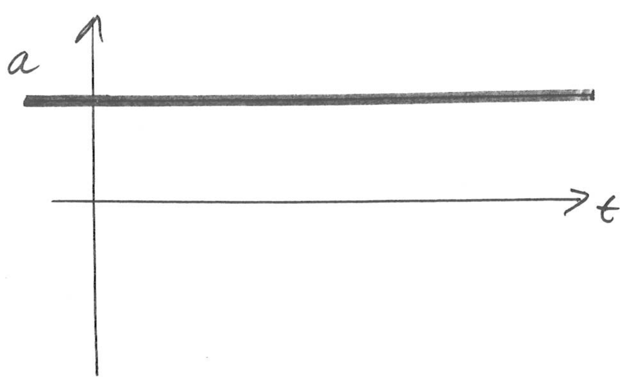
$$v(t) = 12t^3 + 10t$$

Is $\bar{a}$ just $\frac{a(0) + a(1)}{2}$?

A Special Case: Constant Acceleration

Why?

Gravity provides a nearly constant acceleration at the Earth's surface.



For <u>constant</u> acceleration

$a(t) = a$ (no time dependence)

$v_{inst}(t) = \int a \, dt = at + C_1$

arbitrary constant

$x_{inst}(t) = \int v(t) \, dt$

$= \int [at + C_1] \, dt$

$= \frac{1}{2} a t^2 + C_1 t + C_2$

a different arbitrary constant

What are C_1 and C_2 ?

What is the velocity at $t = 0\,s$?

$$v(0) = a \cdot 0 + C_1 = C_1$$

$$C_1 = v(0) \equiv v_0 \qquad \text{(constant!)}$$

What is the displacement at $t = 0\,s$?

$$x(t) = \tfrac{1}{2}at^2 + v_0 t + C_2$$

$$x(0) = \tfrac{1}{2}a \cdot 0^2 + v_0 \cdot 0 + C_2 = C_2$$

$$C_2 = x(0) \equiv x_0 \qquad \text{(constant!)}$$

Master Equations

For constant acceleration

$$\boxed{x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2}$$ no v_f

$$\boxed{v(t) = v_0 + at}$$ no x_f

The second is the time derivative of the first!

Two other equations

Solve the second for t

$$t = \frac{v - v_0}{a}$$

Substitute this value in the first.

$$x_{(t)} = x_0 + v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

Simplify

no t $$\boxed{[v(t)]^2 = v_0^2 + 2a[x(t) - x_0]}$$

or Solve the second for a

$$a = \frac{v - v_0}{t}$$

Plug into the first.

$$x = x_0 + v_0 t + \frac{1}{2}\left(\frac{v - v_0}{t}\right)t^2$$

To Get

no a $$\boxed{x(t) - x_0 = \frac{1}{2}(v_f + v_0)\,t}$$

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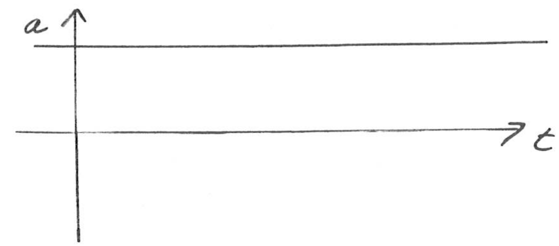
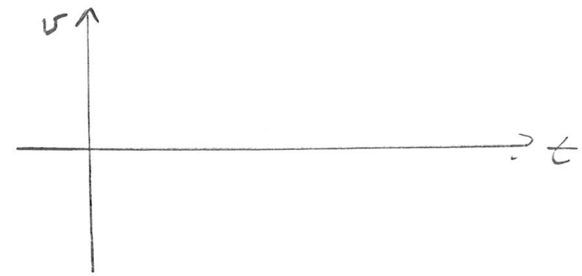
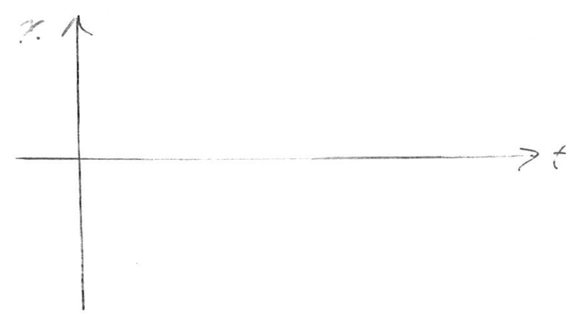
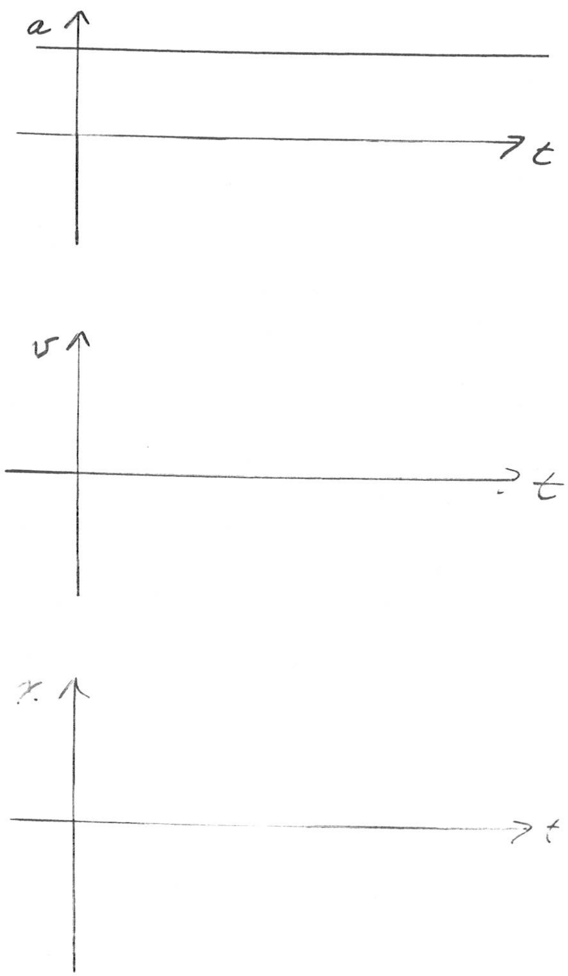
Graphically







Graphically



Gravity

Near the Earth's surface, and neglecting air resistance, <u>all</u> objects experience the same constant acceleration toward the center of the Earth.

<u>Magnitude</u> $g = 9.8\ m/s^2 = 32\ ft/s^2$

The velocity of an object in free-fall will change by $9.8\ m/s$ every second.

Be Careful!

The acceleration is a vector.
It has magnitude and direction.

An object in free-fall has an acceleration of magnitude

$$g = +9.8 \ m/s^2 \qquad \text{(}g\text{ is positive, no direction)}$$

The direction of $\vec{a}$ is downward.

$$\vec{a} = g \text{ downward}$$

The choice for positive direction is in your hands!

If you choose up to be the positive direction, then

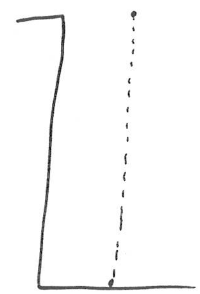
$$\vec{a} = -g = -9.8 \ m/s^2$$

But, if you choose down to be the positive direction, then

$$\vec{a} = +g = +9.8 \ m/s^2$$

Ex How long does it take a particle to fall 8 cm in Earth's gravity from rest?

Ex A ball is thrown downward with an initial speed of $10\ m/s$ from a cliff 50 m high. How long does it take to reach the ground?



1) Choose an origin of coordinates

2) Choose a direction for positive.

3) With these choices:

$y_f =$

$y_0 =$

$v_0 =$

$a =$

Demo: Dollar Drop

$$y_f = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$0 = 50 - 10t + \frac{1}{2}(-9.8)\, t^2$$

Rearrange:

$$\frac{1}{2}(9.8)\, t^2 + 10t - 50 = 0$$

Quadratic Equation:

$$At^2 + Bt + C = 0$$

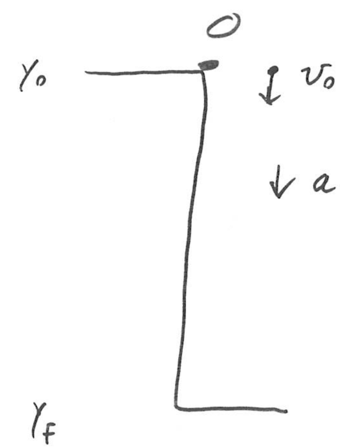
has the <u>two</u> <u>solutions</u>:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$A = 4.9$
$B = 10$
$C = -50$

$t = \underline{2.33\,s}$ <u>or</u> $-4.37\,s$

One more time with different choices.



1) Choose an origin

2) Positive direction

$\downarrow +$

3) With <u>these</u> choices:

$Y_f = +50\,m$

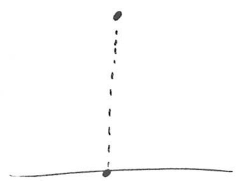
$Y_0 = 0\,m$

$v_0 = +10\,m/s$

$a = +9.8\,m/s^2$

Same solution: $t = 2.33\,s$!

Ex A ball is thrown upward with an initial speed of 10 m/s. How high does it rise?



1) Choose an origin

2) Choose positive direction

3) with these choices

$y_0 =$

$y_f =$

$v_0 =$

$a =$